Three stage actuation "kappas" (v0.02)
(lines are canceled with Pcal)

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First attempt

κ_U, κ_P and κ_T are calculated in sequence.

$$d_{\rm err}(f) = \frac{C(f)}{1+G(f)}\big(A_{\rm U}(f)x_{\rm U} + \Delta L_{\rm pcal}(f)\big) \tag{1}$$

$$\to d_{\rm err}(f) = \frac{C(f)}{1+\underbrace{\big(A_{\rm U}(f)+A_{\rm P}(f)+A_{\rm T}(f)\big)D(f)C(f)}_{G(f)}}\big(A_{\rm U}(f)x_{\rm U} + \Delta L_{\rm pcal}(f)\big) \tag{2}$$

$$\to \Big(1+\big(A_{\rm U}(f)+A_{\rm P}(f)+A_{\rm T}(f)\big)D(f)C(f)\Big)\frac{d_{\rm err}(f)}{C(f)} = A_{\rm U}(f)x_{\rm U} + \Delta L_{\rm pcal}(f) \tag{3}$$

$$\to A_{\rm U}(f)D(f)d_{\rm err}(f) + \big[C^{-1}(f) + \big(A_{\rm P}(f)+A_{\rm T}(f)\big)D(f)\big]d_{\rm err}(f) = A_{\rm U}(f)x_{\rm U} + \Delta L_{\rm pcal}(f) \tag{4}$$

$$\to A_{\rm U}(f)\big(x_{\rm U} - D(f)d_{\rm err}(f)\big) = \big[C^{-1}(f) + \big(A_{\rm P}(f)+A_{\rm T}(f)\big)D(f)\big]d_{\rm err}(f) - \Delta L_{\rm pcal}(f) \tag{5}$$

$$A_{\rm U}(f) = \kappa_{\rm U}A_{\rm U,0}(f) \tag{6}$$

$$\kappa_{\rm U} = \frac{1}{A_{\rm U,0}(f)}\frac{\big[C^{-1}(f) + \big(A_{\rm P}(f)+A_{\rm T}(f)\big)D(f)\big]d_{\rm err}(f) - \Delta L_{\rm pcal}(f)}{x_{\rm U} - d_{\rm err}(f)D(f)} \tag{7}$$

Similarly $\kappa_{\rm P}$ and $\kappa_{\rm T}$ are calculated as:

$$\kappa_{\rm P} = \frac{1}{A_{\rm P,0}(f)}\frac{\big[C^{-1}(f) + \big(\kappa_{\rm U}A_{\rm U,0}(f)+A_{\rm T}(f)\big)D(f)\big]d_{\rm err}(f) - \Delta L_{\rm pcal}(f)}{x_{\rm P} - d_{\rm err}(f)D(f)} \tag{8}$$

$$\kappa_{\rm T} = \frac{1}{A_{\rm T,0}(f)}\frac{\big[C^{-1}(f) + \big(\kappa_{\rm U}A_{\rm U,0}(f)+\kappa_{\rm P}A_{\rm P,0}(f)\big)D(f)\big]d_{\rm err}(f) - \Delta L_{\rm pcal}(f)}{x_{\rm T} - d_{\rm err}(f)D(f)} \tag{9}$$

where $A_{\rm U,0}$, $A_{\rm P,0}$ and $A_{\rm T,0}$ are the reference time actuation functions.

Second attempt

κ_{U}, κ_{P} and κ_{T} are calculated simultaneously. Starting the same way as in the first attempt.

$$d_{\mathrm{err}}(f_{\mathrm{U}}) = \frac{C(f_{\mathrm{U}})}{1 + \underbrace{C(f_{\mathrm{U}})D(f_{\mathrm{U}})\big[A_{\mathrm{U}}(f_{\mathrm{U}}) + A_{\mathrm{P}}(f_{\mathrm{U}}) + A_{\mathrm{T}}(f_{\mathrm{U}})\big]}_{G(f_{\mathrm{U}})}}\big(A_{\mathrm{U}}(f_{\mathrm{U}})x_{\mathrm{U}} + \Delta L_{\mathrm{pcal}}(f_{\mathrm{U}})\big) \tag{10}$$

$$\rightarrow \Big(1 + C(f_{\mathrm{U}})D(f_{\mathrm{U}})\big[A_{\mathrm{U}}(f_{\mathrm{U}}) + A_{\mathrm{P}}(f_{\mathrm{U}}) + A_{\mathrm{T}}(f_{\mathrm{U}})\big]\Big)\frac{d_{\mathrm{err}}(f_{\mathrm{U}})}{C(f_{\mathrm{U}})} = A_{\mathrm{U}}(f_{\mathrm{U}})x_{\mathrm{U}} + \Delta L_{\mathrm{pcal}}(f_{\mathrm{U}}) \tag{11}$$

$$\rightarrow \frac{d_{\mathrm{err}}(f_{\mathrm{U}})}{C(f_{\mathrm{U}})} + \big[A_{\mathrm{U}}(f_{\mathrm{U}}) + A_{\mathrm{P}}(f_{\mathrm{U}}) + A_{\mathrm{T}}(f_{\mathrm{U}})\big]D(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}}) = A_{\mathrm{U}}(f_{\mathrm{U}})x_{\mathrm{U}} + \Delta L_{\mathrm{pcal}}(f_{\mathrm{U}}) \tag{12}$$

$$\rightarrow \big(D(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}}) - x_{\mathrm{U}}\big)A_{\mathrm{U}}(f_{\mathrm{U}}) + D(f_{\mathrm{U}})A_{\mathrm{P}}(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}}) + D(f_{\mathrm{U}})A_{\mathrm{T}}(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}}) = \Delta L_{\mathrm{pcal}}(f_{\mathrm{U}}) - \frac{d_{\mathrm{err}}(f_{\mathrm{U}})}{C(f_{\mathrm{U}})} \tag{13}$$

Now introduce time-dependences: $A_{\mathrm{U}}(f) = \kappa_{\mathrm{U}}A_{\mathrm{U},0}(f)$ and write equations for all 3 lines:

$$\begin{aligned}
\underbrace{\big(D(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}}) - x_{\mathrm{U}}\big)A_{\mathrm{U},0}(f_{\mathrm{U}})}_{a_{11}}\kappa_{\mathrm{U}} + \underbrace{D(f_{\mathrm{U}})A_{\mathrm{P},0}(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}})}_{a_{12}}\kappa_{\mathrm{P}} + \underbrace{D(f_{\mathrm{U}})A_{\mathrm{T},0}(f_{\mathrm{U}})d_{\mathrm{err}}(f_{\mathrm{U}})}_{a_{13}}\kappa_{\mathrm{T}} &= \underbrace{\Delta L_{\mathrm{pcal}}(f_{\mathrm{U}}) - \frac{d_{\mathrm{err}}(f_{\mathrm{U}})}{C(f_{\mathrm{U}})}}_{b_1} \\
\underbrace{D(f_{\mathrm{P}})A_{\mathrm{U},0}(f_{\mathrm{P}})d_{\mathrm{err}}(f_{\mathrm{P}})}_{a_{21}}\kappa_{\mathrm{U}} + \underbrace{\big(D(f_{\mathrm{P}})d_{\mathrm{err}}(f_{\mathrm{P}}) - x_{\mathrm{P}}\big)A_{\mathrm{P},0}(f_{\mathrm{P}})}_{a_{22}}\kappa_{\mathrm{P}} + \underbrace{D(f_{\mathrm{P}})A_{\mathrm{T},0}(f_{\mathrm{P}})d_{\mathrm{err}}(f_{\mathrm{P}})}_{a_{13}}\kappa_{\mathrm{T}} &= \underbrace{\Delta L_{\mathrm{pcal}}(f_{\mathrm{P}}) - \frac{d_{\mathrm{err}}(f_{\mathrm{P}})}{C(f_{\mathrm{P}})}}_{b_2} \\
\underbrace{D(f_{\mathrm{T}})A_{\mathrm{U},0}(f_{\mathrm{T}})d_{\mathrm{err}}(f_{\mathrm{T}})}_{a_{31}}\kappa_{\mathrm{U}} + \underbrace{D(f_{\mathrm{T}})A_{\mathrm{P},0}(f_{\mathrm{T}})d_{\mathrm{err}}(f_{\mathrm{T}})}_{a_{32}}\kappa_{\mathrm{P}} + \underbrace{\big(D(f_{\mathrm{T}})d_{\mathrm{err}}(f_{\mathrm{T}}) - x_{\mathrm{T}}\big)A_{\mathrm{T},0}(f_{\mathrm{T}})}_{a_{33}}\kappa_{\mathrm{T}} &= \underbrace{\Delta L_{\mathrm{pcal}}(f_{\mathrm{T}}) - \frac{d_{\mathrm{err}}(f_{\mathrm{T}})}{C(f_{\mathrm{T}})}}_{b_3}.
\end{aligned} \tag{14}$$

Which we can write it in matrix form and solve it for κ's:

$$\mathbf{A}\boldsymbol{\kappa} = \boldsymbol{b} \tag{15}$$

$$\boldsymbol{\kappa} = \mathbf{A}^{-1}\boldsymbol{b} \tag{16}$$

where $\mathbf{A}$ and $\boldsymbol{b}$ calculated from 3 calibration lines and the reference time sensing and actuation models.

During nominal operation d_{err} at all 3 frequencies averages to 0, and $x_i > 0$ at $f = f_i$ for $i \in \{\mathrm{U}, \mathrm{P}, \mathrm{T}\}$. thus $\mathbf{A}$ is mostly diagonal ($\mathbf{A}^{-1}$ exists).