# Magnetic noise from lead glass recoil mass

Garilynn and Peter Fritschel have asked me to look into the displacement noise of the test mass by magnetic field fluctuations in the chamber causing motions of the the new design ring recoil mass. The magnetic susceptibility of the lead glass of the ring is larger than that of fused silica and is a possible worry.

The end result of the estimate in this note is that even with large uncertainties in the magnetic susceptibility of the glass, the noise is negligible by many orders of magnitude.

The mechanism for the noise begins with the magnetic field of the Earth, $B_{earth}$, inducing a magnetic moment in the glass. The magnetic moment per volume in the glass is $M = \frac{\chi_v B_{earth}}{\mu_0}$ , $\chi_v$ is the magnetic susceptibility per volume of the glass. The magnetic moment interacts with the gradient of the fluctuation magnetic field B(f) in the chamber. For the sake of the calculation, the typical scale length of the fluctuating field is $d_g$ . The displacement of the recoil mass becomes

$$x_{recoil}(f) = \frac{V_{ring}\chi_v B_{earth} B(f)}{m_{ring}\mu_0 \omega^2 d_g}$$

The recoil mass displacement fluctuations cause the force between the test mass and recoil mass to vary if there is already a net electrostatic force from the ESD. The largest such force would come from a bias voltage. The electrostatic force on the test mass is $< F_{esd} > = \frac{6.6x10^{-16} V_{bias}^2}{g^{2.46}}$ where g is the gap between the test mass and the recoil mass. The change of the ESD force with displacement of the recoil mass is

$$F_{tm}(f) = \frac{2.46 < F_{esd} >}{g} x_{recoil}(f)$$

The noise displacement of the test mass becomes

$$x_{tm}(f) = \frac{2.46 < F_{esd} > B_{earth} B(f) \chi_v}{g\, m_{tm}\, \mu_0\, \rho_{ring}\, d_g\, \omega^4}$$

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|---|---|---|---|
| $<F_{esd}>$ | 1.2e5 N | $\rho_{ring}$ | 3.86e3 kg/m$^3$ |
| $V_{bias}$ | 200 V | B(f) | 3.0e-10 T/sqrt(HZ)@ 4Hz assumed flat |
| $d_g$ | 0.06 m | $B_{earth}$ | 2.0e-5 T |
| g | 5.0e-3 m | $\chi_v$ | 1e-3 -> 1e-4 MKS units |
| $m_{tm}$ | 40 kg | | |

The end results is $x_{tm}(f) < 10^{-21}/ f^4$ meter/sqrt(Hz), completely negligible.