Testing The Strong Field Dynamics of General Relativity Using Compact Binary Systems - Progress Report #1

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Einstein's Theory of General Relativity has been well tested in the *weak* field regime over the past century. However, such tests have not been carried out in the inherently non-linear and dynamical *strong* field regime. Recent advancements in ground based gravitational wave detectors, (e,g. Advanced LIGO, VIRGO), will allow us probe the *strong* field regime of general relativity by investigating gravitational waves produced by astrophysical systems with strong gravitational fields such as compact binary coalescences. While current search techniques utilize standard GR waveforms to determine when a candidate has been detected, alternative theories of gravity predict signals that differ significantly from GR in the *strong* field. We investigate the ability of aLIGO to detect non GR gravitational wave signals by studying the detailed effects of modifications to standard GR waveforms.

I. INTRODUCTION

Gravitational waves are propagating oscillations in the gravitational field caused by the acceleration of massive bodies [1]. They are akin to light and radio waves, which are emitted by the acceleration of charged particles. However, gravitational waves are different in that they require a minimum of a pair of geodesics for one to probe the effect of the gravitational field of the radiation caused by the emitter. This stems in part from the fact that GR describes the effects of both stationary gravitational fields and gravitational radiation by the tidal forces they impart on free test masses. Additionally, the tidal forces produced by the canonical Newtonian potential of a self-gravitating source is described by the corresponding Poisson equation for gravity, $\nabla^2\phi = 4\pi G\rho$. For a stationary field the tidal force falls off as inverse cubed, $\nabla^2\phi \propto r^{-3}$ at a distance r from the massive body. However, the tidal force due to the gravitational wave amplitude results in only inversely proportional decrease,$\nabla^2 h \propto r^{-1}\lambda^{-2}$, where h is the amplitude of the gravitational wave at a particular wavelength, λ. This results means that for close distances to the object ($r \lesssim h$), the tidal force due to the stationary field will dominate. At large distances ($r \gg h$), the tidal forces caused by gravitational radiation will have a more significant effect on the tidal forces experienced by the test mass. As a result, efforts to detect gravitational waves are inherently biased towards high energy events which would be extremely luminous in gravitational radiation [2]. Astrophysical objects can be characterized by the "compactness" parameter, $\epsilon \sim Gm/Rc^2$, where G is the gravitational constant, m is the characteristic mass of the object, R characteristic radius, and c the speed of light in a vacuum. Compact binary coalescences (CBC) are ideal sources for the detection of gravitational waves [3–5] while also allowing the ability to test the non-linear strong field regime of GR, a region which has not been tested before.

Previous efforts have tested a myriad of aspects of the Einstein Equivalence Principle (EEP) including: the Weak Equivalence Principle (WEP), local Lorentz invariance, and local position invariance. Current work is in progress to push these constraints even further [6]. For example, the WEP, which states the trajectory of a freely falling test body is independent of its internal structure and composition, has been tested in a variety of methods. Tests of the WEP include measuring the fractional difference in acceleration between two bodies. This difference is referred to as the "Eötvös ratio" and is defined by $\eta \sim 2|a_1 - a_2|/|a_1 + a_2|$, where a_1 and a_2 refer to the acceleration of the respective bodies considered for the test. One specific example performed at the University of Washington was able to reach a value of $\eta \sim 2 \times 10^{-13}$ [7–9]. Future efforts to constrain this parameter are ongoing [10].

Although GR has been well tested in the weak field regime over the past century, an equally robust set of experiments have yet to test GR and alternative theories of gravitation in the strong-field. Up until now, tests have been limited to the technological capabilities of observational tools or the inherently weak, $\epsilon \lesssim 10^{-5}$, gravitational field of our Solar System. To test the non-linear and highly dynamical strong field regime of GR, one can probe GW signals emitted by nearby astrophysical systems which include: core-collapse supernovae or coalescing binary systems containing black holes (BH) or

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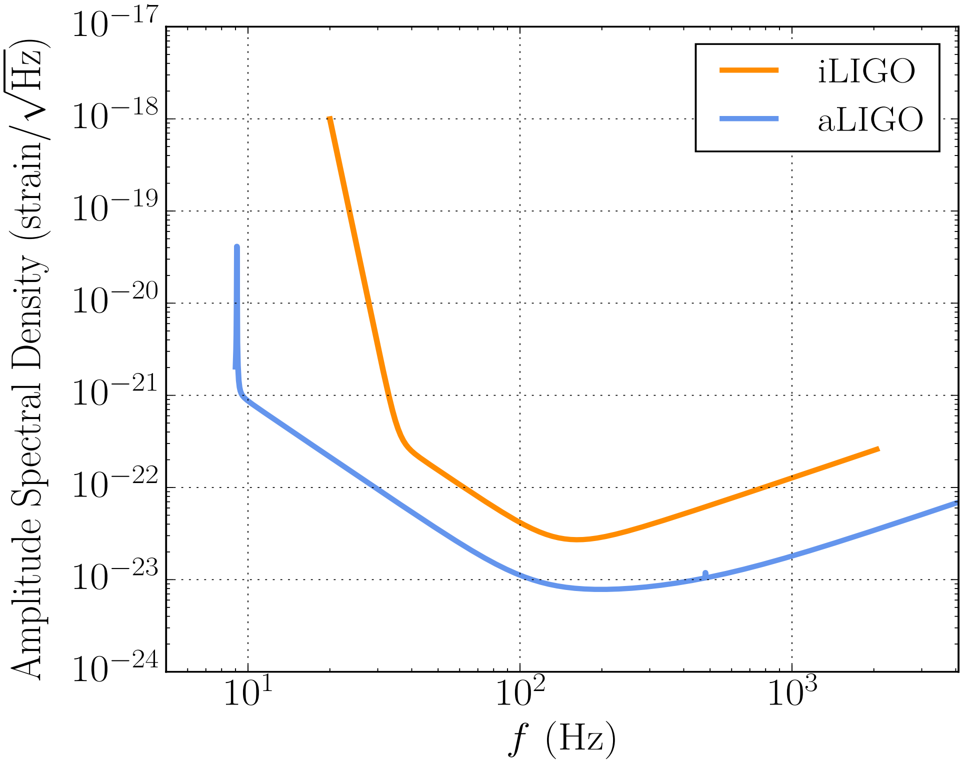

FIG. 1. Diagram showing the noise amplitude spectrum for aLIGO.

neutron stars (NS). Such signals have not been detectable in the past due to a combination of insufficient detector sensitivity and the statistically small number of events predicted to occur within the current detector horizon. However, next generation gravitational wave detectors will have the capability of detecting and measuring gravitational waveforms between frequencies of $f \sim 10$ Hz to $f \sim 500$ Hz, with maximum sensitivity to strain at $f \sim 100$ Hz of $h \sim 10^{-22}$ (see Fig1) for aLIGO [11], along with an increased horizon distance of up to $\sim$ 100Mpc. This increase in detector sensitivity also increases the number of possible events to 10-100 events per year. Detection of GW signals from these highly dynamical astrophysical sources will allow for the very first empirical test of GR in the strong field.

Current techniques for determining if an observed signal originated from an astrophysical source as opposed to a non gaussian glitch or instrumental error include: checking for coincident triggers within a small timeframe, statistically minimizing the known noise from the detector, and then comparing the observed signal with a template bank of waveforms. The techniques used to model the gravitational waveforms of different astrophysical phenomena come with one caveat, they accept GR as a null result. However, alternative theories of gravity lead to solutions of GR in the weak-field but could diverge strongly in events beyond that; such as the merging of two compact objects orbiting at velocities ratios of $0.1 \lesssim \beta \lesssim 0.6$, where $\beta = v/c$. In the event that GR is not the complete theory of gravitation, a detection from such a highly relativistic source that emits a GW that deviates significantly from GR could bypass detection for a template bank utilizing only standard GR waveforms and even introduce unexpected degeneracies with inferred intrinsic parameters of the system. Therefore, it is paramount that the methods by which incident signals are analyzed thoroughly account for physically motivated deviations from GR that have been inferred by alternative theories of gravity.

In this paper we investigate the effect of non-GR deviations in simulated gravitational waveforms used to analyze detection signals by the next generation GW detector, aLIGO. For this investigation, we consider binary systems composed of compact objects such as neutron stars and black holes. We perform numerical calculations to model these gravitational waveforms from a variety of binary systems constructed using the standard GR approach. We then introduce an arbitrary function to the standard GR gravitational waveform which produces a significant deviation from the standard waveform. Then, we perform a quantitative assessment of the properties of these modified waveforms and their implications on next generation detectors. In Sect. III we discuss our methods, in Sect. IV we present our numerical calculations, in Sect. V we discuss our results and their implications to next generation GW detectors, and in Sect. VI we discuss our conclusions.

II. OBJECTIVES

This investigation aims to asses the ability to test and constrain deviations from GR by analyzing gravitational waves from detected binary black hole mergers in the presence of expected statistical and systematic errors. We will address statistical errors governed by the number of binary signals observed, their associated signal to noise ratio (SNR) distribution, and the detector noise spectrum. Systematic errors considered include: the accuracy of the waveform models used, effects from black hole spin, detector calibration, and glitches in detector noise that deviates from a Gaussian distribution.

III. METHODS

We begin our investigation by devising a means by which we can implement deviations to standard waveforms as determined by GR. Such an implementation may include variations of fitting coefficients used in the modeling of waveforms that make the assumption that GR correctly model the physical phenomena in question. After variations to the standard waveforms have been introduced, we will test how efficient `gstlal`, the low-latency GW detection pipeline used by the LIGO Scientific Calibration (LSC), is at detecting waveforms which deviate from those of standard GR. `gstlal` uses a large test bank against which it performs statistics that allow it to determine whether a known waveform has been found within the incoming data. However, should GR be incorrect for astrophysical phenomena in the *strong* field regime, then a detection may be missed by detection pipelines. We will perform injections to the incoming stream of data to `gstlal`, first with GR waveforms, followed by waveforms with the introduced deviations.

We can see this by a simple approximation wherein we treat the system as a "centrifuge", or a system in which two stars of similar masses are orbiting in a circular radius, R. If we also only consider one polarization in this approach, the strain amplitude emitted by the GW can be written as

$$h_+ = \frac{4G^2\mu M}{c^4 D a}\cos(\Phi(\mathrm{t})), \tag{1}$$

with

$$\Phi = \int 2\pi\Omega dt, \tag{2}$$

where M is the total mass of the system, Ω is the frequency of the emitted GW, D is the distance from the system, μ is the reduced mass, and a is the orbital seperation of two objects in a tight circular orbit. From this, the approximate gravitational wave luminosity is found to be

$$L = \frac{1}{4\pi D^2}\frac{32G^4\mu^2M^3}{5c^5a^5}. \tag{3}$$

As a result, nearby binary systems consisting of compact objects such as neutron stars or black holes are the most luminous systems and are one of the most likely sources of detection [12, 13]. The energy carried away by these waves will cause the orbit of the binary to shrink and the frequency of the waves observed to increase with time. Binary systems in which the objects are approaching merger are a powerful source of gravitational radiation.

Examples of the waveforms considered for this investigation include the restricted TalyorF2 waveforms which were used by [16] to study the inpiral of compact binaries consisting of two neutron stars (BNS). In the LIGO Algorithms library [17], TaylorF2 is implemented as

$$h(f) = \frac{1}{D}\frac{\mathcal{A}(\theta,\phi,l,\psi,\mathcal{M},\eta)}{\sqrt{\dot{F}(\mathcal{M},\eta;f)}}f^{2/3}e^{i\Psi(t_c,\phi_c,\mathcal{M},\eta;f)}, \tag{4}$$

where D is the luminosity distance to the source, (θ,ϕ) specify the sky position, (l,ψ) give the orientation of the inspiral place with respect to the line of sight, $\mathcal{M}$ is the chirp mass, and η is the symmetric mass ratio. In terms of component masses (m_1, m_2), one finds $\eta = m_1m_2/(m_1+m_2)^2$ and $\mathcal{M} = (m_1+m_2)\eta^{3/5}$.

Bayesian inference is a method which will allow us to characterize different hypotheses $\mathcal{H}_i$ and $\mathcal{H}_j$ which may correspond to the null hypothesis of GR or a particular deviation. Using Bayesian inference we can then compute the posterior probability of a particular hypothesis $\mathcal{H}_i$ by applying Bayes' theorem. This yields a posterior probability of the hypothesis $\mathcal{H}_i$ given the data

$$P(\mathcal{H}_i|d,\mathrm{I}) = \frac{P(\mathcal{H}_\mathrm{i}|\mathrm{I})P(d|\mathcal{H}_\mathrm{i},\mathrm{I})}{P(d|\mathrm{I})}, \tag{5}$$

where $P(\mathcal{H}_i|\mathrm{I})$ is the prior probability of the hypothesis and $P(d|\mathcal{H}_i,\mathrm{I})$ is the marginal likelihood (evidence) for $\mathcal{H}_i$. For this investigation, we will use the `LALInference` software which is also a part of the LALSuite software stack. This program will allow efficient implementation of Bayesian analysis on the data collected from our waveform study.

A. Progress

Week 1 included gaining access to the computing clusters available for LSC members. Additionally, an exercise to solve the inspiral of two 1.4 $\mathrm{M}_\odot$ neutron stars was completed. This exercises accomplished many objectives including: helping gain familiarity with Python data structures, initiating a gain in physical intuition for the systems being studied, and providing an order of magnitude estimation for compact binary systems. To gain familiarity with LIGO computing environment, we preformed a variety of exercises using `lalsim`, a software program that generates GW waveforms for various astrophysical events. We used different waveforms and varied physical parameters such as: mass of the companions, $m_{1,2}$, distance to source, D, and the dimensionless reduced spin quantity, χ. This analysis furthered out tuition for compact binary systems and also gave a quantitative estimation for the astrophysical sources which could be detectable by the expected early aLIGO sensitivity.

Week 2 was spent completing our analysis of different physical input quantities and their affect of the waveform in both the time and frequency domain. We found that the similarities between the time domain and the frequency domain can be described by the overall strain amplitude. For a system in which the strain amplitude is large, we see that a large strain amplitude in the time domain corresponds to a larger ASD that operates well within that of the expected aLIGO curve. Conversely, we see for smaller strain amplitudes such as those found in systems for $D \geq 100$ (Mpc), show a much lower ASD curve that operates well below the capabilities of aLIGO. We also worked on developing a means by which we can implement our non GR parameters. Specific steps towards this included creating an additional instance of LALSuite on the cluster so that the source code could be modified to include non GR values.

We have had to confront the following challenges: determining the most efficient method to implement non GR behavior, how to utilize previous efforts to help aid in the efficiency of our study, identifying particular alternative theories that we expect to play the largest role. Our goals for this coming month include: determining and/or developing an efficient method for implementing non-GR effects using `LALSimulation`, performing test injections for both GR and non-GR, devising a comprehensive study extending the study of these non-GR parameters, the use of `LALInference` to perform Bayesian statistics on our data, and lastly, drawing what implications this study can have on the effectiveness of `gstlal` in detecting non-GR waves and providing an efficient means to integrate non-GR waveform variations into detection pipelines.

Updated steps by which our goals will be met are detailed below:

Objectives	
Week	Focused Efforts
3-4	Non-GR parameter implementation *Create method to implement non GR params.* *Perform first GR & non-GR injection* **Progress Report 1 Due, July 7th**
5-6	Optimize method for larger scale study *Determine full grid, perform large parameter sweep* *Bayesian analysis with LALInference*
7-8	Post processing, Bayesian Analysis *Test certain theories using data* *analyze results, implications for aLIGO* **Progress Report 2 Due, August 3rd**
9-10	Explain results, implications, scalability *Generalize method for future use* *Draw conclusion, prepare final presentation*

IV. RESULTS

V. DISCUSSION

VI. CONCLUSIONS

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