

# Model-Based Cross-Correlation Search for Gravitational Waves from Scorpius X-1

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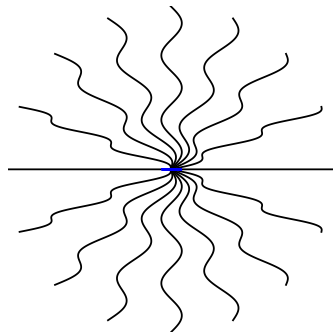
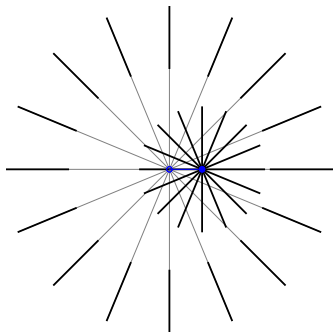
# Outline

- 1 **Searches for Gravitational Waves**
  - Crash Course in Gravitational Wave Physics
  - Gravitational-Wave Observations & Detectors
  - Gravitational Waves from Low-Mass X-Ray Binaries
- 2 **Cross-Correlation Search**
  - Fundamentals of Periodic GW Searches
  - Parameter Space Search
  - Sensitivity Estimates
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  - Detecton of Sco X-1 MDC Signals
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  - Summary and Future Outlook

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# Motivation



- In **Newtonian gravity**, force dep on distance btwn objects
- If massive object suddenly moved, grav field **at a distance** would change **instantaneously**
- In relativity, **no** signal can travel faster than light  
→ time-dep grav fields must propagate like light waves

# Gravity as Geometry

- Minkowski Spacetime:

$$\begin{aligned}
 ds^2 &= -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2 \\
 &= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu
 \end{aligned}$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Metric tensor  $\{g_{\mu\nu}(\{x^\lambda\})\}$  determined by masses  
 via Einstein's equations. (10 non-linear PDEs!)



# Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to **1st order** in  $h_{\mu\nu} \equiv$  difference btwn actual metric  $g_{\mu\nu}$  & flat metric  $\eta_{\mu\nu}$ :

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

( $h_{\mu\nu}$  “small” in weak-field regime, e.g. for GW detection)

- $h_{\mu\nu}$  analogous to electromagnetic potential  $\{A_\mu\} = \{\varphi, \vec{A}\}$
- Small coord changes induce “**gauge transformation**” on  $h_{\mu\nu}$   
Convenient choice of gauge is **transverse-traceless**:  
In this gauge:

- Vacuum Einstein eqns  $\implies$  **wave equation** for  $\{h_{ij}\}$ :

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{ij} = 0$$

- Test particles w/constant coords are **freely falling**

# Gravitational Wave Polarization States

- Far from source, GW looks like plane wave prop along  $\hat{k}$   
TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where  $h_+ \left(t - \frac{x^3}{c}\right)$  and  $h_\times \left(t - \frac{x^3}{c}\right)$  are components  
in “plus” and “cross” polarization states

- EM (spin-1 massless photon) & grav (spin-2 massless “graviton”)  
waves both have two polarization states



# Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

| Plus (+) Polarization | Cross ( $\times$ ) Polarization |
|-----------------------|---------------------------------|
|                       |                                 |





# Gravitational Wave Generation

- Generated by **moving/oscillating** mass distribution
- Lowest **multipole** is **quadrupole**
  
- Rotating neutron star w/**non-axisymmetric** perturbation gives sinusoidally-varying **quadrupole moment**.  
**Note since gravity couples so weakly, only have to worry about lowest harmonic;**  
**No complicated “pulse profile”**
- Other sources: compact binary inspiral, bursts (supernova etc), stochastic backgrounds. . .

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# Methods for Measuring Gravitational Waves

- Cosmic Microwave Background Perturbations  
( $f_{\text{gw}} \sim H_0 \sim 10^{-18} \text{ Hz}$ )
- Pulsar Timing Arrays ( $10^{-9} \text{ Hz} \lesssim f_{\text{gw}} \lesssim 10^{-7} \text{ Hz}$ )
- Laser Interferometers
  - Space-Based ( $10^{-3} \text{ Hz} \lesssim f_{\text{gw}} \lesssim 10^{-1} \text{ Hz}$ )
  - ⇒ Ground-Based ( $10^1 \text{ Hz} \lesssim f_{\text{gw}} \lesssim 10^3 \text{ Hz}$ )
- Resonant-Mass Detectors (narrowband,  $f_{\text{gw}} \sim 10^3 \text{ Hz}$ )

Note, observable GW freq cover **20** orders of magnitude, similar to EM radiation, but the frequencies are much lower ( $10^3 \text{ Hz} \lesssim f_{\text{em}} \lesssim 10^{23} \text{ Hz}$ )



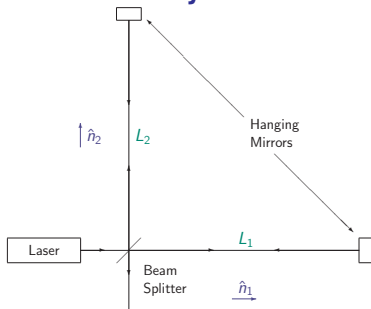
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|                       |                        |

# Measuring GWs w/Laser Interferometry

**Interferometry:** Measure GW-induced distance changes



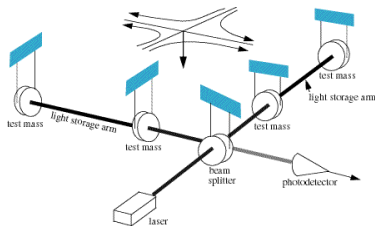
- Measure small change in

$$L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- Plausible signals:  $h \lesssim 10^{-20}$   
 → need  $L_0$  very big!
- For LIGO,  $L_0 = 4 \text{ km}$

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# Rogues' Gallery of Ground-Based Interferometers



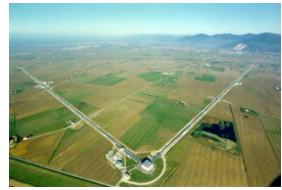
LIGO Hanford  
(Washington, USA)



GEO-600 (Germany)



LIGO Livingston  
(Louisiana, USA)



Virgo (Italy)

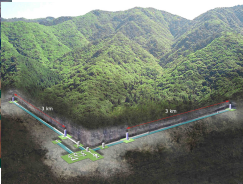
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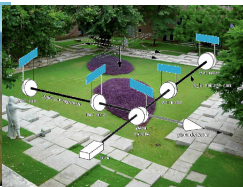
KAGRA (Japan)



LIGO Livingston  
(Louisiana, USA)



Virgo (Italy)



LIGO India



# GW Observatory Network

- “Initial Detector Era” for large ground-based interferometers ~ 2002 – 2011
- “Advanced Detector Era” starts this year
  - Germany: **GEO-600** (600m) used for technology development (laser power, squeezed light, . . . ) & “**astrowatch**” in case a **transient event** occurs when other detectors **offline**.
  - USA: **LIGO Hanford** & **LIGO Livingston** (4km)  
First observing run Fall 2015
  - Italy: **Virgo** (3km)  
Expected to start observing 2016
  - Japan: **KAGRA** (formerly LCGT)  
(3km, underground, cryogenic, under construction)
  - India: **LIGO India** (4km, planned)

Detectors distributed on the Earth useful  
for **sky localization** of transient signals

# Sensitivity of Initial & Advanced Detectors

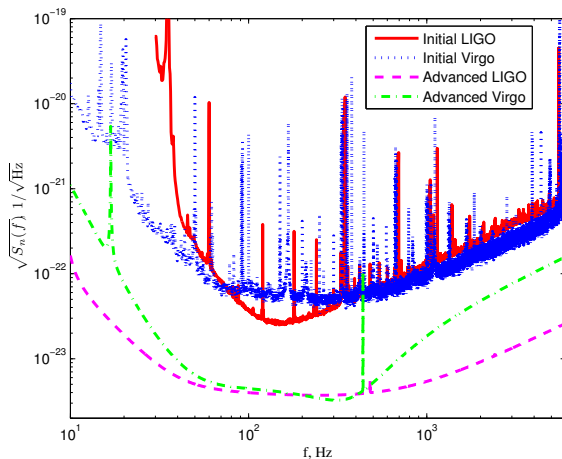


figure from [CQG 27, 173001 \(2010\)](#)

# A Few Words About Collaborations

- LIGO Scientific Collaboration : hundreds of researchers at dozens of institutions world-wide working on instrument science, data analysis, astrophysics, etc.
  - LSC scientists operate  & GEO detectors
  -  and  consortium are LSC members
-  VIRGO Collaboration operates Virgo (Italy) and includes institutions in Italy, France, Netherlands, Poland & Hungary
  - LIGO & Virgo conduct data analysis jointly
-  KAGRA: Japanese collaboration constructing detector in Kamioka mine

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# Gravitational Waves from Low-Mass X-Ray Binaries



- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides “hot spot”; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* **501**, L89 (1998)  
suggested GW spindown may balance accretion spinup;  
GW strength can be estimated from X-ray flux
- Torque balance would give  $\approx$  constant GW freq
- Signal at solar system modulated by binary orbit

# Scorpius X-1

- 2nd brightest X-Ray source in the sky, after the Sun
- **Favored model** is  $1.4M_{\odot}$  NS +  $0.42M_{\odot}$  companion  
Steeghs & Casares *ApJ* **568**, 273 (2002)  
Galloway et al *ApJ* **781**, 14 (2014)

Parameters (see Messenger et al *PRD* **92**, 023006 (2015) for refs)

| Parameter           |                  | estimate                                  | error     |
|---------------------|------------------|---|-----------|
| RA                  | $\alpha$         | $16^{\text{h}}19^{\text{m}}55^{\text{s}}$ | $0''.06$  |
| dec                 | $\delta$         | $-15^{\circ}38'25''$                      | $0''.06$  |
| distance            | $d$              | 2.8 kpc                                   | 0.3 kpc   |
| orb period          | $P_{\text{orb}}$ | 68023.70 s                                | 0.04 s    |
| time of ascension   | $t_{\text{asc}}$ | 2008-Jun-17 16:06:20 UTC                  | 100 s     |
| proj semimajor axis | $a_p$            | 1.44 lt-s                                 | 0.18 lt-s |
| eccentricity        | $e$              | 0   | 0.02      |

## Note on Inclination Angles

- Inclination  $i$  of binary orbit to line of sight
  - Projected semimajor axis  $a_p = a \sin i$  (in light seconds)
  - Amplitude of orbital Doppler modulation  $\frac{2\pi a_p}{P_{\text{orb}}}$
- Inclination  $\iota$  of neutron star spin to line of sight
  - Amplitudes of GW polarizations:  
 $A_+ = h_0 \frac{1+\cos^2 \iota}{2}$  &  $A_\times = h_0 \cos \iota$
  - Many searches sensitive to  $A_+^2 + A_\times^2$  rather than just  $h_0^2$   
 $h_0$  is amplitude of GW strain at the solar system
- Generally don't assume  $i$  and  $\iota$  are related

# GW Searches for Sco X-1

- Fully coherent  $\mathcal{F}$ -statistic search

Jaranowski, Królak & Schutz *PRD* **58**, 063001 (1998)

☞ w/6 hours of 2003 LIGO data *LSC PRD* **76**, 082001 (2007)

- Directed stochastic (“radiometer”) search

Ballmer *CQG* **23**, S179 (2006)

☞ w/2005 LIGO data *LSC PRD* **76**, 082003 (2007)

☞ w/2005-2007 LIGO data *LVC PRL* **107**, 271102 (2011)

- Sideband search Messenger & Woan *CQG* **24**, S469 (2007)

☞ w/2005-2007 LIGO data *LVC PRD* **91**, 062008 (2015)

- TwoSpect search Goetz & Riles *CQG* **24**, S469 (2007)

☞ w/2009-2010 LIGO/Virgo data *LVC PRD* **90**, 062010 (2014)

- Model-based cross-correlation search

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)



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# Coherent Maximum-Likelihood Search ( $\mathcal{F}$ -statistic)

- Divide signal parameters into
  - **amplitude params:**  $\{h_0, \iota, \psi, \phi_0\}$  (amp, orientation etc)
  - **Doppler params:**  $\lambda \equiv \{\alpha, \delta, f_0, \dot{f}_0, \dots\}$  + orb params
- Jaranowski, Królak, Schutz **PRD 58, 063001 (1998)**  
showed signal linear in  $\{\mathcal{A}^\mu\}$  (fncs of **amplitude params**)

$$h(t; \mathcal{A}, \lambda) = \mathcal{A}^\mu h_\mu(t; \lambda) \quad (\text{assume } \sum_{\mu=1}^4 = 1)$$

template waveforms  $h_\mu(t)$  depend on **Doppler params**  $\lambda$

- Optimal<sup>1</sup> detection statistic is maximized log-likelihood ratio

$$\mathcal{F} = \frac{1}{2} x_\mu(\lambda) \mathcal{M}^{\mu\nu}(\lambda) x_\nu(\lambda)$$

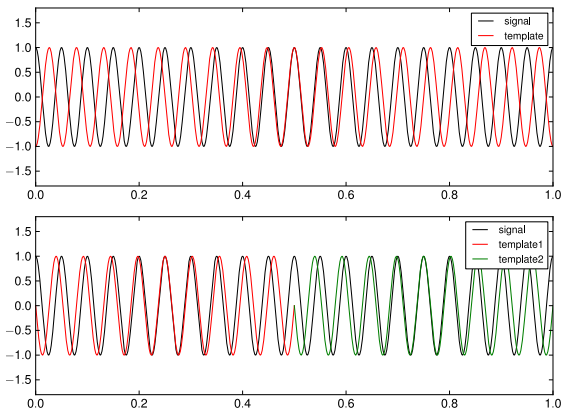
where  $\{x_\nu(\lambda)\}$  are four projections of the data

- **Problem:** long coherent searches need to try many  $\lambda$  values

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<sup>1</sup>But see Prix & Krishnan **CQG 26, 204013 (2009)**  
and JTW, Prix, Cutler & Willis **CQG 31, 065002 (2014)**

# Coherent vs Semi-Coherent Searches

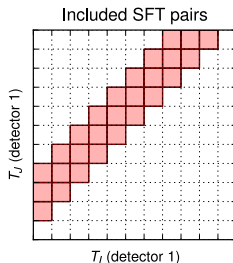
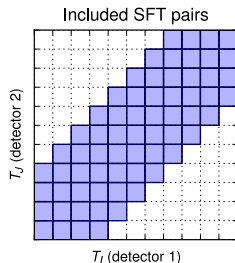


Most CW searches **semi-coherent**: deliberately limit  
**coherent integration time** & **param space resolution**  
to keep **number of templates** ( $\lambda$  points) manageable



# Motivation for Cross-Correlation Method

- $\mathcal{F}$ -stat quadratic in data; cross-terms combine all times  
→ need too many templates
- “Radiometer” method (stochastic cross-correlation search)  
only combines data from different detectors at same time;  
→ fast but much less sensitive.
- Construct quadratic cross-correlation statistic which  
combines all data segments w/  $|T_K - T_L| \leq T_{\max}$





# Basics of Cross-Correlation Method

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)  
JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)



# Basics of Cross-Correlation Method

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)

- [BTW, other targets include SN1987A supernova remnant; see Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)]

# Basics of Cross-Correlation Method

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)

- Divide data into segments of length  $T_{\text{sft}}$   
 & take “short Fourier transform” (SFT)  
 $K$  labels SFT, i.e., detector & time
- Parameters  $\lambda$  tell us which Fourier component should  
 contain signal in a given SFT;  $z_K$  is that component,  
 normalized by noise PSD  $S_n^{(K)}(f_0)$ ,  
 so  $E[z_K] = \mu_K$  and  $E[(z_K - \mu_K)(z_L - \mu_L)^*] = \delta_{KL}$
- Construct quadratic statistic  $\rho = z_K^* W_{KL} z_L$   
 with  $W_{KL} \sim \mu_K \mu_L^*$  if  $|T_K - T_L| \leq T_{\text{max}}$ , 0 otherwise.
- Normalized so that  $\text{Var } \rho = 1$  and

$$E[\rho] \propto h_0^2 \frac{\sqrt{T_{\text{obs}} T_{\text{max}}}}{\langle S_n(f_0) \rangle}$$

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# Metric for Cross-Correlation Search

JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)

- Consider dependence of  $\rho$  on parameters  $\lambda \equiv \{\lambda_i\}$
- Parameter space metric  $g_{ij} = -\frac{1}{2} \frac{E[\rho_{,ij}]|_{\lambda=\lambda^{\text{true}}}}{E[\rho^{\text{true}}]}$  from

$$\frac{E[\rho] - E[\rho^{\text{true}}]}{E[\rho^{\text{true}}]} = -g_{ij}(\Delta\lambda^i)(\Delta\lambda^j) + \mathcal{O}([\Delta\lambda]^3)$$

- Assume dominant contribution to  $E[\rho_{,ij}]$  is from variation of  $\Delta\Phi_{KL} = \Phi_K - \Phi_L$ ; get phase metric

$$g_{ij} = \frac{1}{2} \sum_{KL} \Delta\Phi_{KL,i} \Delta\Phi_{KL,j} |W_{KL}|^2 \equiv \frac{1}{2} \langle \Delta\Phi_{KL,i} \Delta\Phi_{KL,j} \rangle_{KL}$$

- If you ignore that weighting factor you get back usual metric

$$\langle \Phi_{I,i} \Phi_{I,j} \rangle_I - \langle \Phi_{I,i} \rangle_I \langle \Phi_{J,j} \rangle_J$$

# Computing Cost Scaling

JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)

- Resolution is  $\Delta f_0 \propto T_{\max}^{-1}$ ,  $\Delta a_p \propto \frac{P_{\text{orb}}}{f_0 T_{\max}}$ ,  $\Delta t_{\text{asc}} \propto \frac{P_{\text{orb}}^2}{a_p f_0 T_{\max}}$   
(if  $T_{\max} \ll P_{\text{orb}}$ )
- Number of templates  $\propto f_0^2 T_{\max}^3$   
(for Sco X-1,  $P_{\text{orb}}$  well constrained observationally)
- Number of pairs of data segments  $\propto \frac{T_{\text{obs}} T_{\max}}{T_{\text{sft}}^2}$
- Note: need  $T_{\text{sft}} \propto (f_0)^{-1/2}$  to avoid signal loss  
from Doppler phase acceleration
- Rough cost scaling  $\propto f_0^2 T_{\max}^4$
- Work in progress on algorithmic improvements e.g. resampling  
à la Patel, Siemens, Dupuis & Betzwieser *PRD* **81**, 084032 (2010)

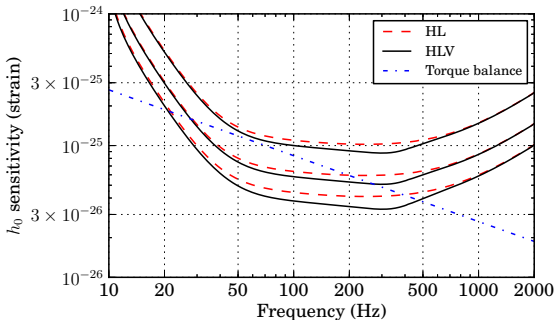
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# Sensitivity Estimates

- Sensitivity of search  $h_0 \propto (S_n)^{1/2} (T_{\text{obs}} T_{\text{max}})^{-1/4}$
- Expected signal strength from torque balance  $h_0 \propto f_0^{-1/2}$
- Compare for 1 yr advanced detector data w/  $T_{\text{max}} = 6, 60, 600 \text{ min}$   
(Single-template false alarm prob  $5 \times 10^{-10}$ )



JTW, Sundaesan, Zhang & Peiris *PRD* **91**, 102005 (2015)

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# Mock Data Challenge

Messenger et al *PRD* **92**, 023006 (2015)

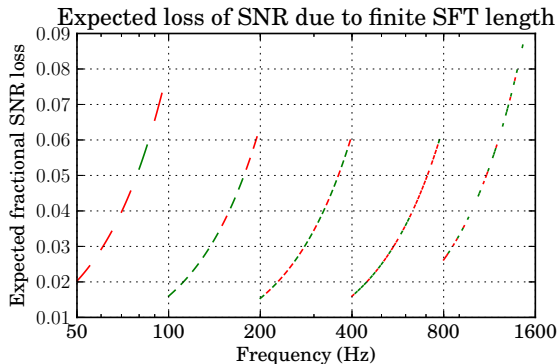
- “Apples-to-apples” comparison of search methods
- One year simulated white gaussian LIGO (2 sites) & Virgo noise, with gaps,  $(S_n)^{1/2} = 4 \times 10^{-24} \text{ Hz}^{-1/2}$  (advanced design)
- 100 simulated signals  
(50 “open” w/published parameters, 50 “closed”) injected into specified 5 Hz bands from 50-1450 Hz
- Log-normal distribution of  $6 \times 10^{-26} \lesssim h_0 \lesssim 2 \times 10^{-24}$   
Mostly above torque-balance level; chosen for detectability
- Participants: Radiometer\*, Sideband\*, TwoSpect\*, Polynomial, CrossCorr\*
  - \* has been used in LSC/LVC observational paper
  - \* “late entrant” in self-blinded mode

# Cross-Correlation Configuration

Zhang, JTW & Krishnan *in preparation*

- SFT length varied with frequency (Doppler acceleration)

|           |            |            |            |             |
|-----------|------------|------------|------------|-------------|
| 50–100 Hz | 100–200 Hz | 200–400 Hz | 400–800 Hz | 800–1450 Hz |
| 900 s     | 600 s      | 420 s      | 300 s      | 240 s       |



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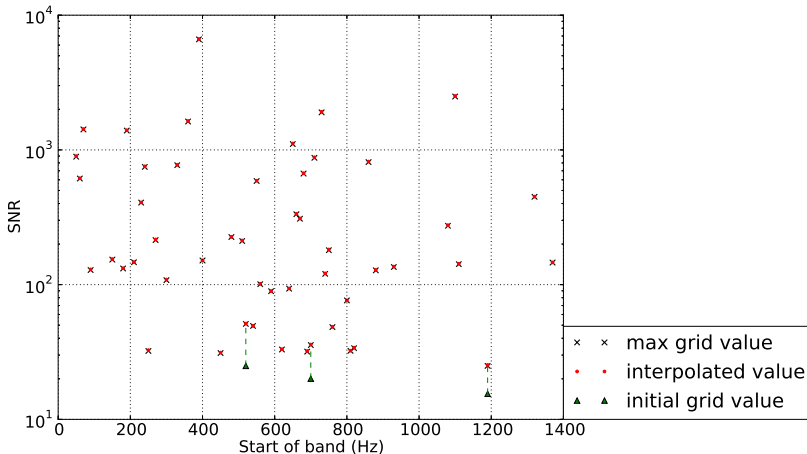
- Search  $\pm 3\sigma$  in  $a_p$  and  $t_{asc}$
- Adjust  $T_{max}$  to keep cost per 5 Hz band roughly constant
- Consider budget per band of  $N_{pair} N_{tmplt} \leq 5 \times 10^{14}$   
 [ $\mathcal{O}(500)$  CPU-days on AEI atlas cluster]
- Choose two different  $T_{max}$  values for each freq octave  
 (deeper search within  $1.5\sigma$  of most likely orb params)

| 50–100 Hz | 100–200 Hz | 200–400 Hz | 400–800 Hz | 800–1450 Hz |
|-----------|------------|------------|------------|-------------|
| 5400 s    | 2400 s     | 2100 s     | 1140 s     | 780 s       |
| 3600 s    | 1200 s     | 840 s      | 840 s      | 540 s       |





# Detection of Closed Signals: 50 for 50!



Zhang, JTW & Krishnan *in preparation*



# Detection: Open and Closed

- All closed signals were detected with  $\rho > 15.4$ 
  - Detectable without zoom followup  
(Did it on 3 quietest anyway to increase SNR)
  - Could in principle use followup to improve param est;  
Limited by “spin wandering” (imperfect torque balance)
- Open signals more interesting:
  - 48 of 50 detected with  $\rho > 11$
  - One signal (in 960 Hz band, with  $h_0 = 4.96 \times 10^{-26}$ ) had  $\rho = 6.7$ ;  
zoom followup brought this to  $\rho = 14.7$
  - One signal (in 490 Hz band, with  $h_0 = 3.85 \times 10^{-26}$ ) **not** detected  
Loudest  $\rho$  (not at signal point) was 6.07; zoom brought to 5.02  
( $\rho$  at true signal params was 3.5)

Zhang, JTW & Krishnan **in preparation**

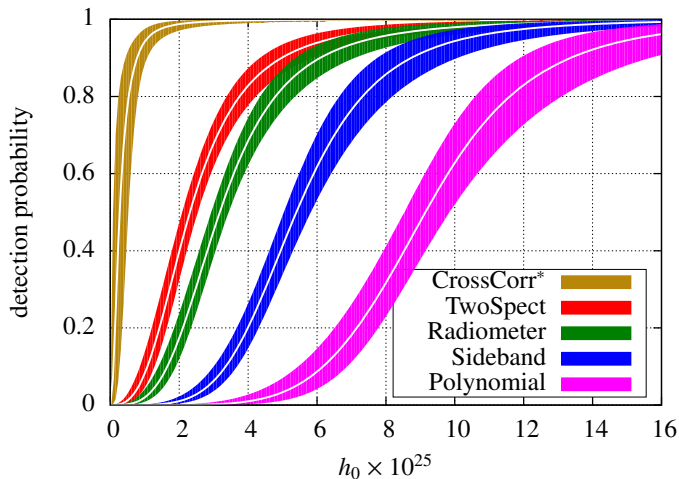
# Comparison of Detection Efficiencies

Out of 50 closed signals:

- CrossCorr: found 50 with  $h_0 \gtrsim 6.8 \times 10^{-26}$
- TwoSpect: found 34 with  $h_0 \gtrsim 1.3 \times 10^{-25}$
- Radiometer: found 28 with  $h_0 \gtrsim 2.2 \times 10^{-25}$
- Sideband: found 16 with  $h_0 \gtrsim 3.6 \times 10^{-25}$
- Polynomial: found 7 with  $h_0 \gtrsim 7.7 \times 10^{-25}$

Messenger et al *PRD* **92**, 023006 (2015)

# Comparison of Detection Efficiencies

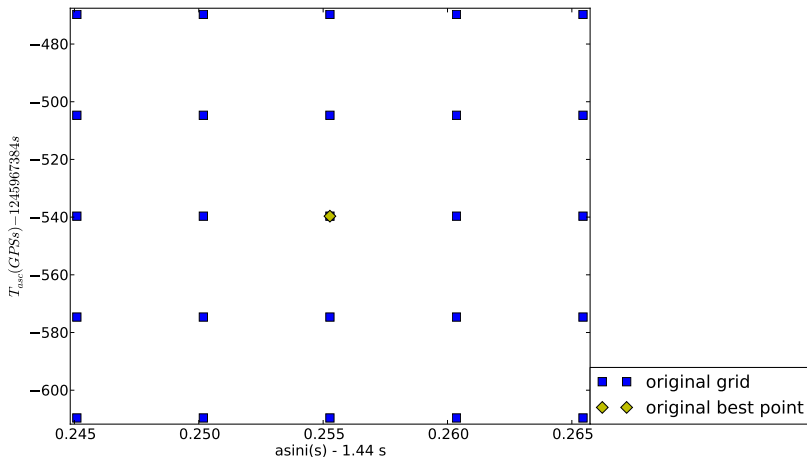


Messenger et al *PRD* **92**, 023006 (2015)

# Outline

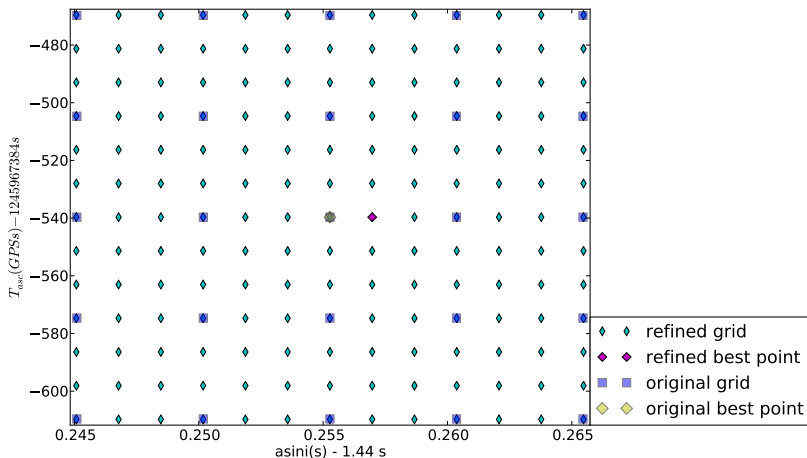
- 1 Searches for Gravitational Waves
  - Crash Course in Gravitational Wave Physics
  - Gravitational-Wave Observations & Detectors
  - Gravitational Waves from Low-Mass X-Ray Binaries
- 2 Cross-Correlation Search
  - Fundamentals of Periodic GW Searches
  - Parameter Space Search
  - Sensitivity Estimates
- 3 **Scorpius X-1 Mock Data Challenge**
  - Detecton of Sco X-1 MDC Signals
  - **Parameter Estimation**
  - Summary and Future Outlook

# Original Search Grid (excerpt)



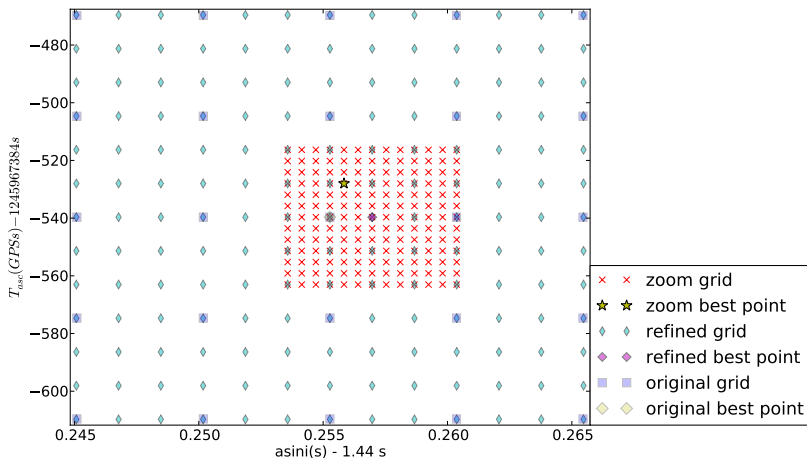
Zhang, JTW & Krishnan in preparation

# Refined Grid (same $T_{\max}$ )



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# Zoomed Grid (increase $T_{\max}$ if desired for followup)



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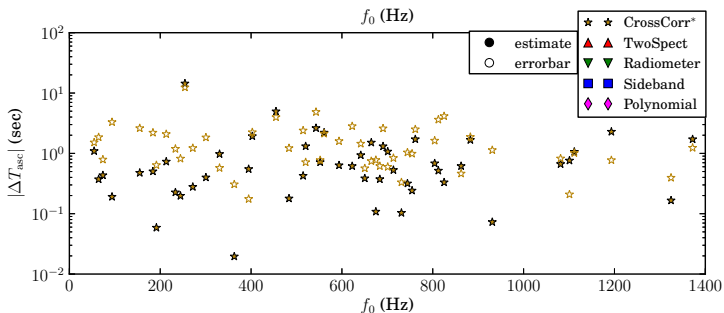


# CrossCorr Parameter Estimation

- In 1 year Gaussian noise  $\rho \gtrsim 8$  means false alarm  $< 10^{-13}$   
☞ with trials factor  $\sim 10^8$ , still  $\lesssim 10^{-5}$
- For such “detections”, can estimate parameters:
  - $h_0$  from  $\rho$  value and noise level  
dominant error is unknown inclination of NS spin  
 (“could” estimate, but we didn’t for MDC)
  - $f_0, a_p, t_{asc}$  from values around best-fitting template
- In  $3 \times 3 \times 3$  box around “best” point, fit parabola to  $\rho(\lambda)$   
☞ allows resolution below grid spacing
- Sources of error:
  - Systematic: unknown inclination means max CC stat not at true param values, even without noise
  - Statistical: noise fluctuations offset max CC stat
  - Interpolation: determination of sub-grid



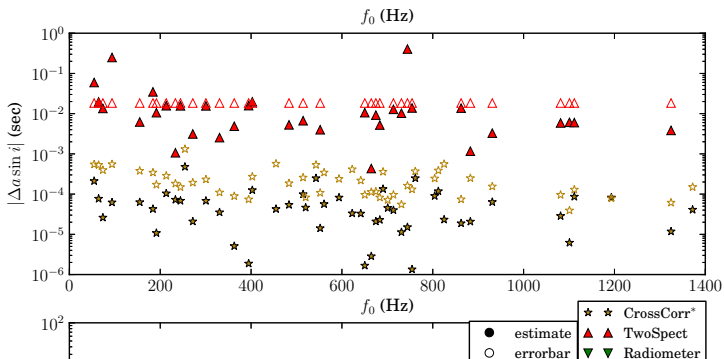
# Parameter Estimation: Time of Ascension



- No one else estimated  $t_{asc}$
- Errors consistent w/errorbars  
(after high-SNR empirical correction based on open data)

Messenger et al [PRD 92, 023006 \(2015\)](#)

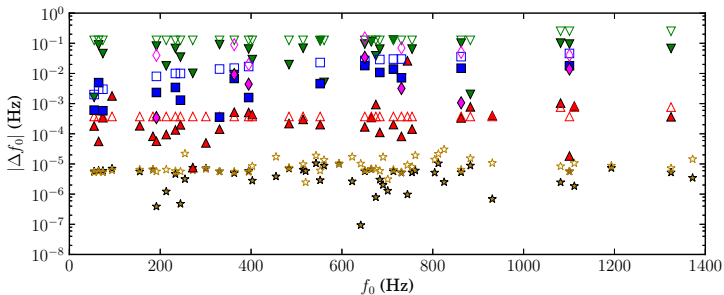
# Parameter Estimation: Projected Semimajor Axis



- Removed systematic bias at low frequencies (high-SNR empirical correction based on open data)
- Even w/conservative errorbars, most precise  $a_p$  measurement

Messenger et al [PRD 92, 023006 \(2015\)](#)

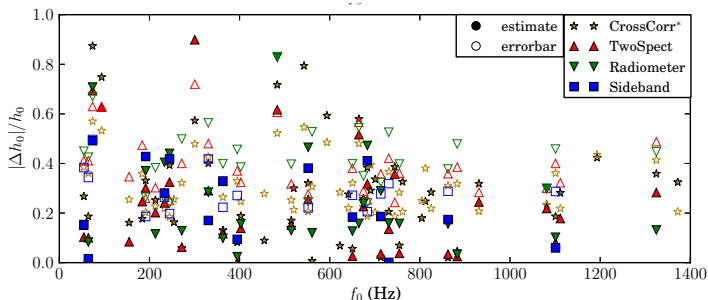
# Parameter Estimation: Frequency



- Systematic error dominant
- Interpolation allows very precise  $f_0$  estimate

Messenger et al *PRD* **92**, 023006 (2015)

# Parameter Estimation: Amplitude



- Dominant error from unknown spin inclination (for everyone)

Messenger et al *PRD* **92**, 023006 (2015)

# Outline

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# Summary and Future Outlook

- Advanced **Gravitational Wave** detectors about to begin observing in 10-4000 Hz band
- Promising target is the low-mass X-ray binary **Scorpius X-1**
- **Cross-correlation method** adapted for **CW signals** allows balance of sensitivity and computing cost via semicoherent analysis
- **Advanced detector era** sensitivity should reach **torque balance prediction**
- Validated by recent Sco X-1 Mock Data Challenge
- Note: proposed stacked  $\mathcal{F}$ -stat search (Leaci and Prix **PRD 91, 102003 (2015)**) has similar sensitivity scaling; aims to achieve longer coherence time