Impact of higher order modes in gravitational wave searches for binary black holes

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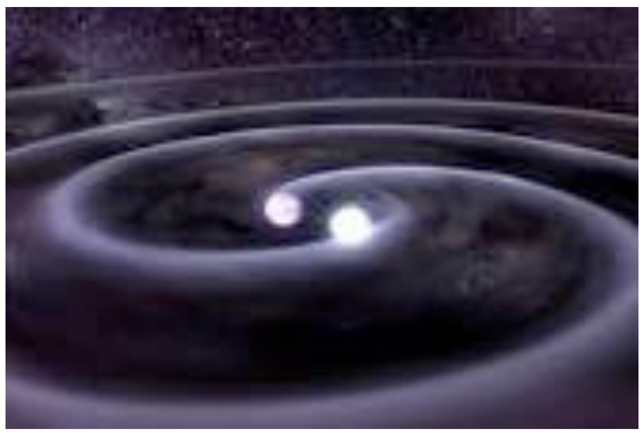
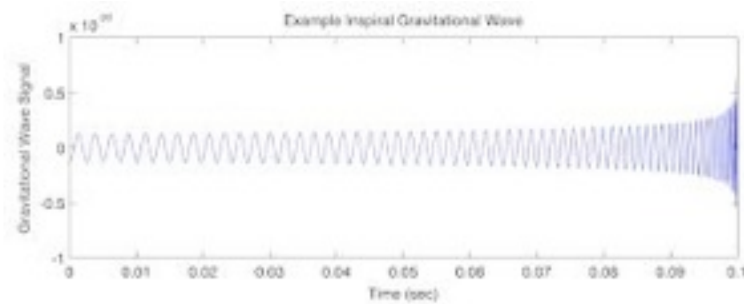
0. Outline of this talk

- 1. Introduction: higher order modes and data analysis.
- 2. Previous studies.
- 3. Analysis set up.
- 4. Effect on detection.
- 5. Parameter Bias.

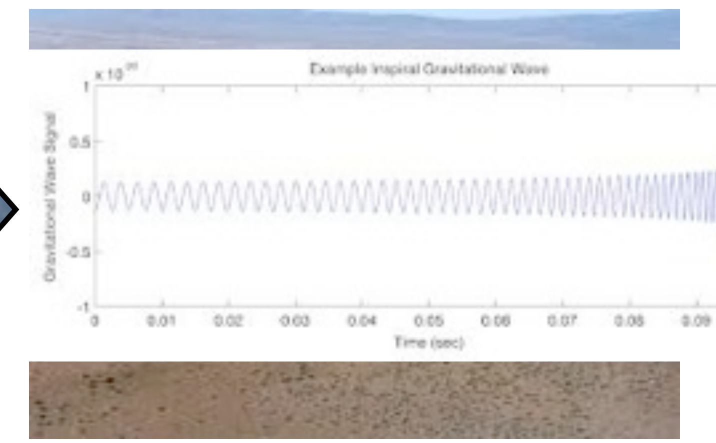
1. Gravitational Wave Search: very basics



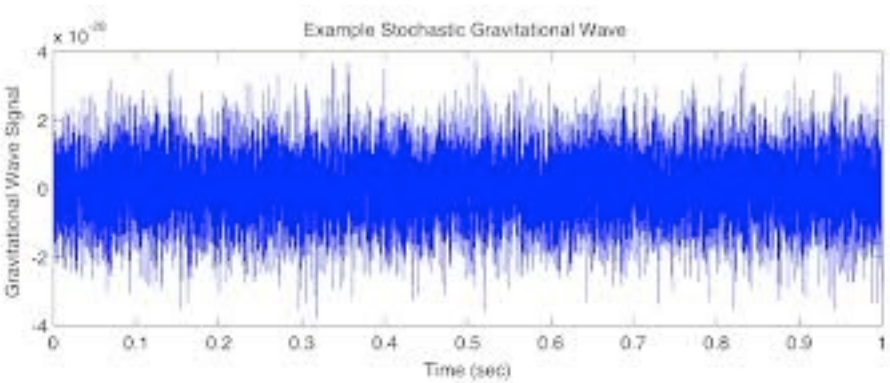
A GW is emitted



(we hope to model it well)



It reaches a detector

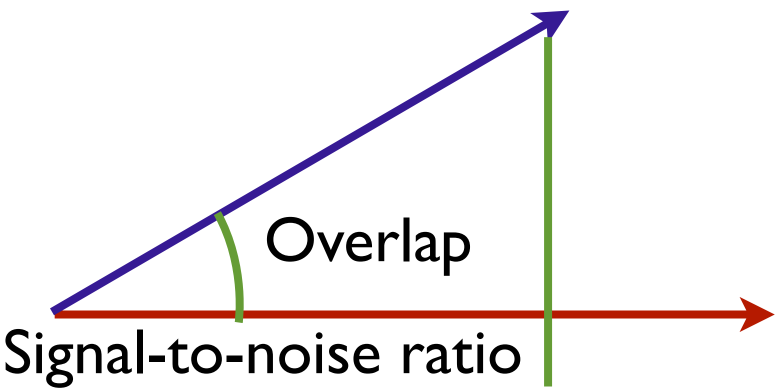


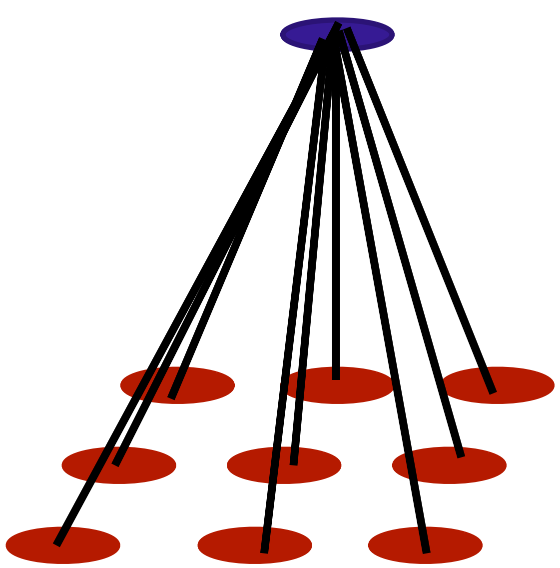
and modifies the data stream:
noise+GW

1st step: Signal-to-noise Ratio (SNR)
2nd step: Distinguish from glitches (vetoes)

Ranking statistic: newSNR...

1.2 Data analysis basics





Inner product → $\langle h|g\rangle = 4\int_{f_0}^{\infty}\frac{\tilde{h}(f)\tilde{g}^*(f)}{S_n(f)}df$

Signal-to-noise ratio → $\rho(s|h) = 2\frac{\langle s|h\rangle}{\sqrt{\langle h|h\rangle}}$

Overlap → $\mathcal{O}(h|g) = \frac{\langle h|g\rangle}{\sqrt{\langle h|h\rangle\langle g|g\rangle}}$

Fitting Factor → $FF(h|B) = max_{g\in B}\mathcal{O}(h|g)$

Horizon Distance → $h_D(h|g) \sim \rho(h|h)\times\mathcal{O}(h|g)$

1.2 Higher order modes

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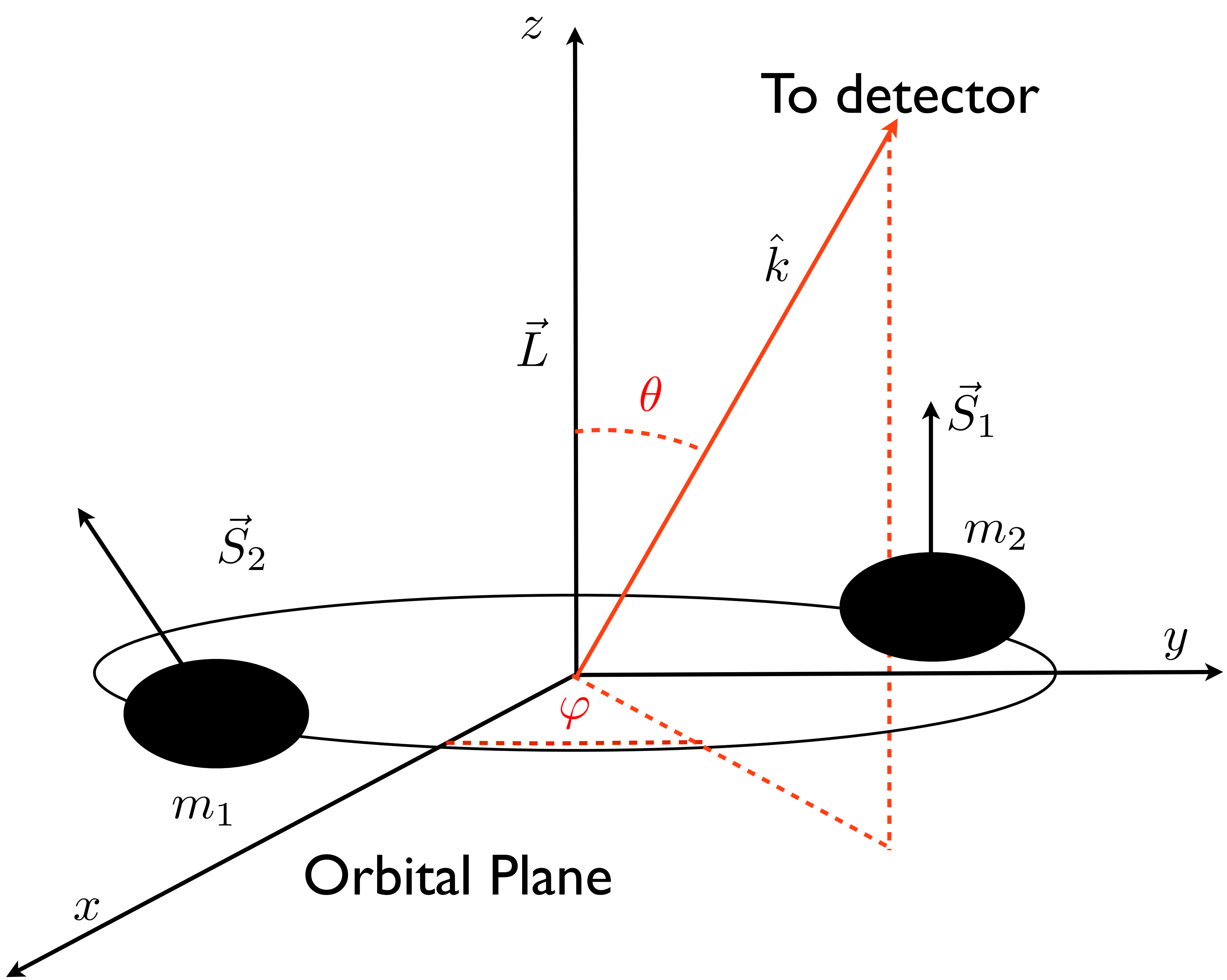
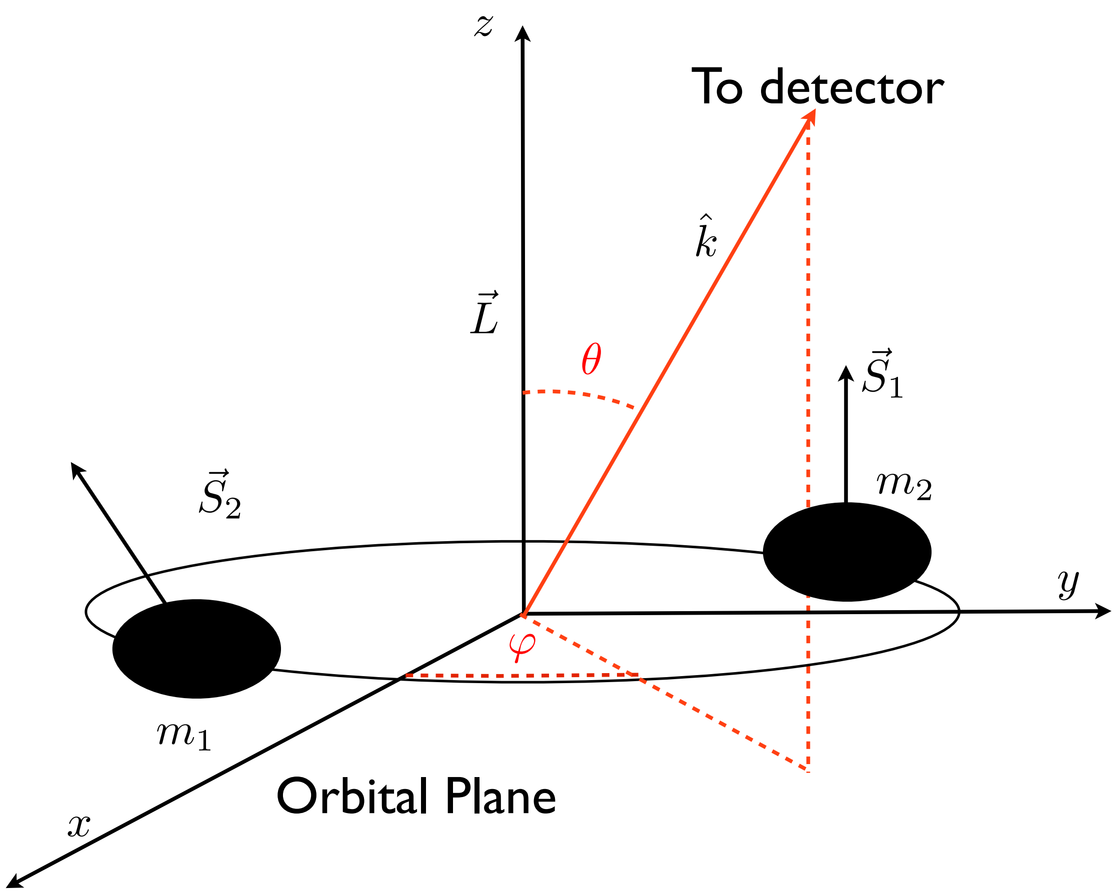
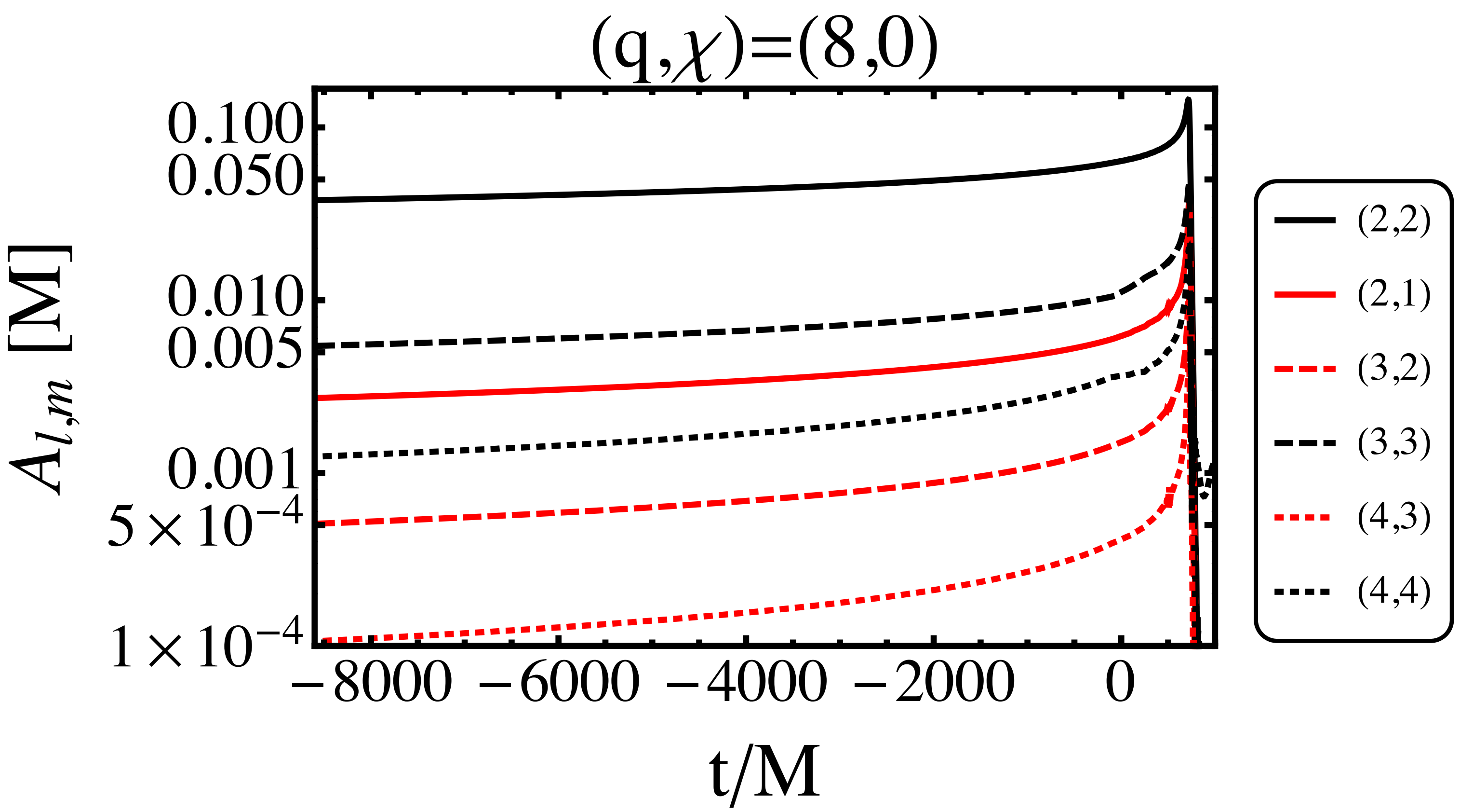


1.2 Higher order modes

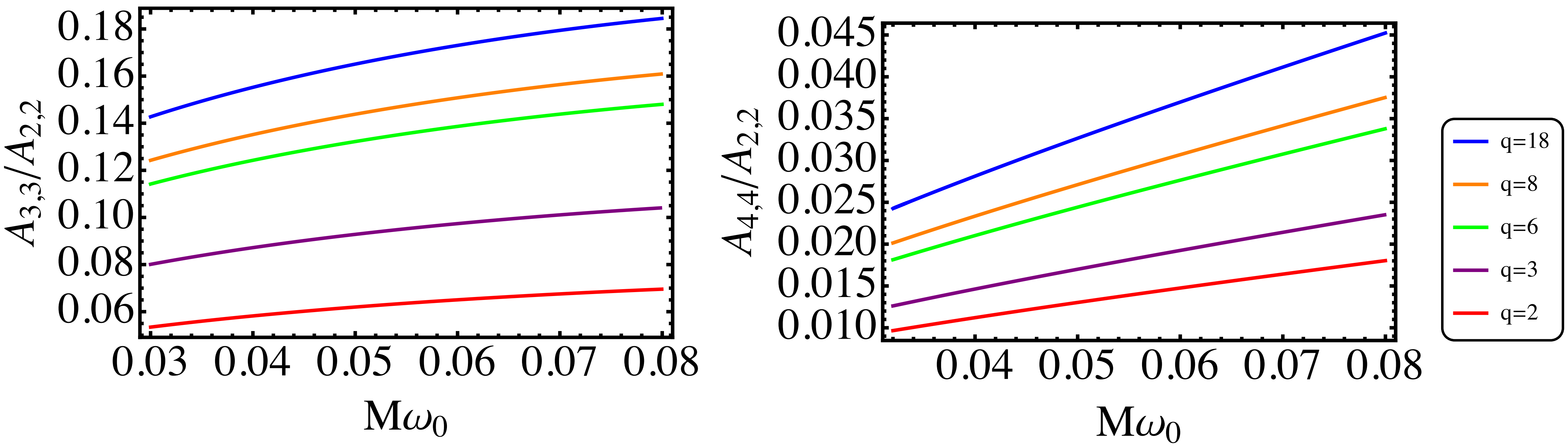




$$h(\Xi;\theta,\varphi) = \sum_{\ell,m\geq 2} Y_{\ell,m}^{-2}(\theta,\varphi)h_{\ell,m}(\Xi;t)$$

Waveforms used in current searches only include $(\ell,|m|) = (2,2)$ modes

Interesting situations



More important as q increases

Interesting situations

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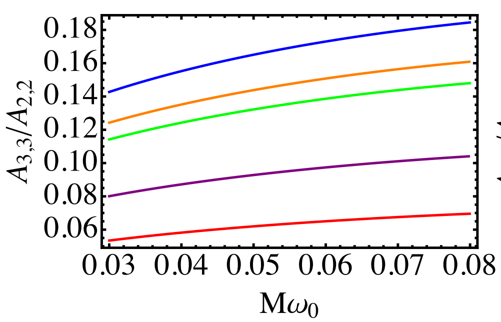
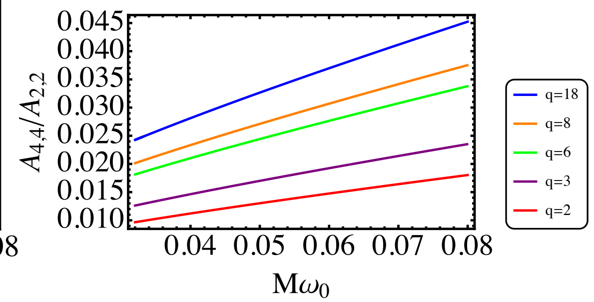
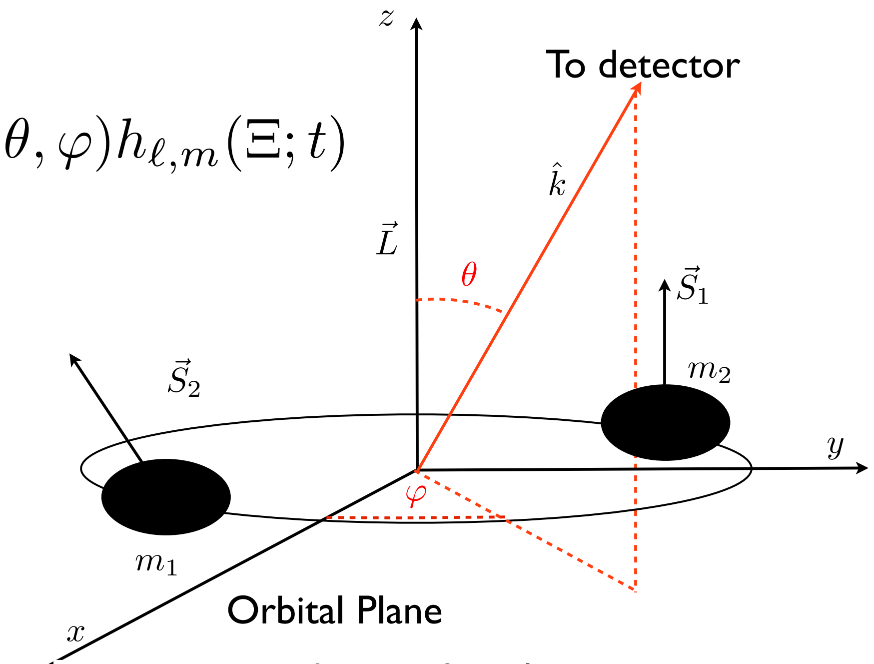
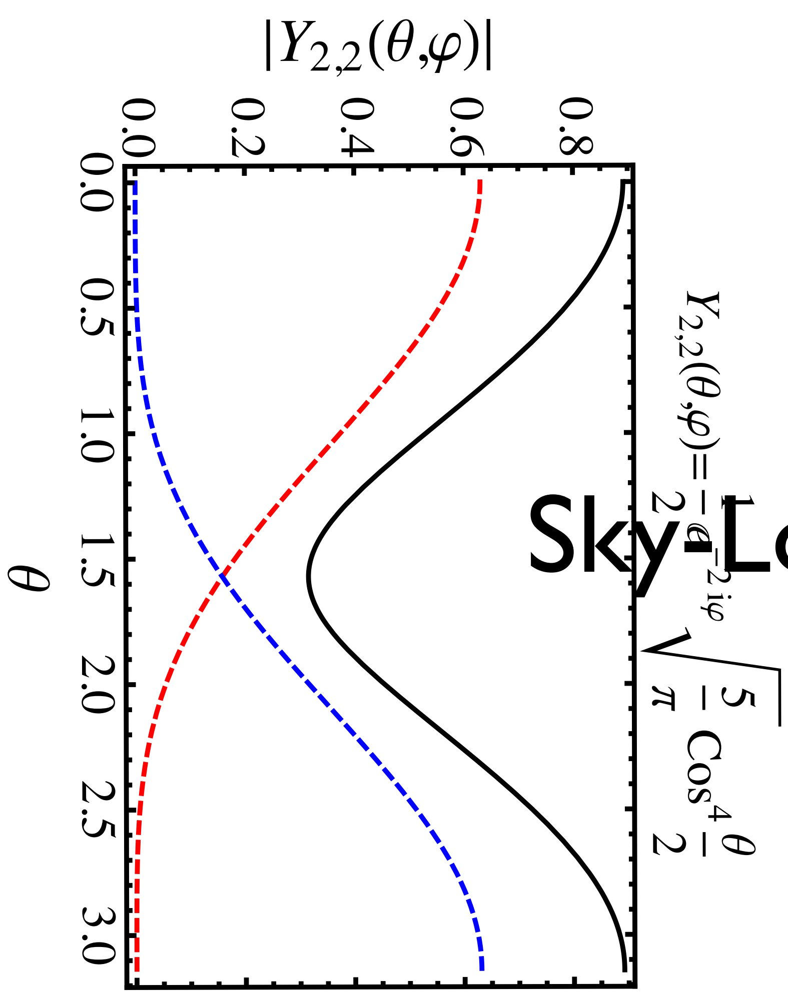
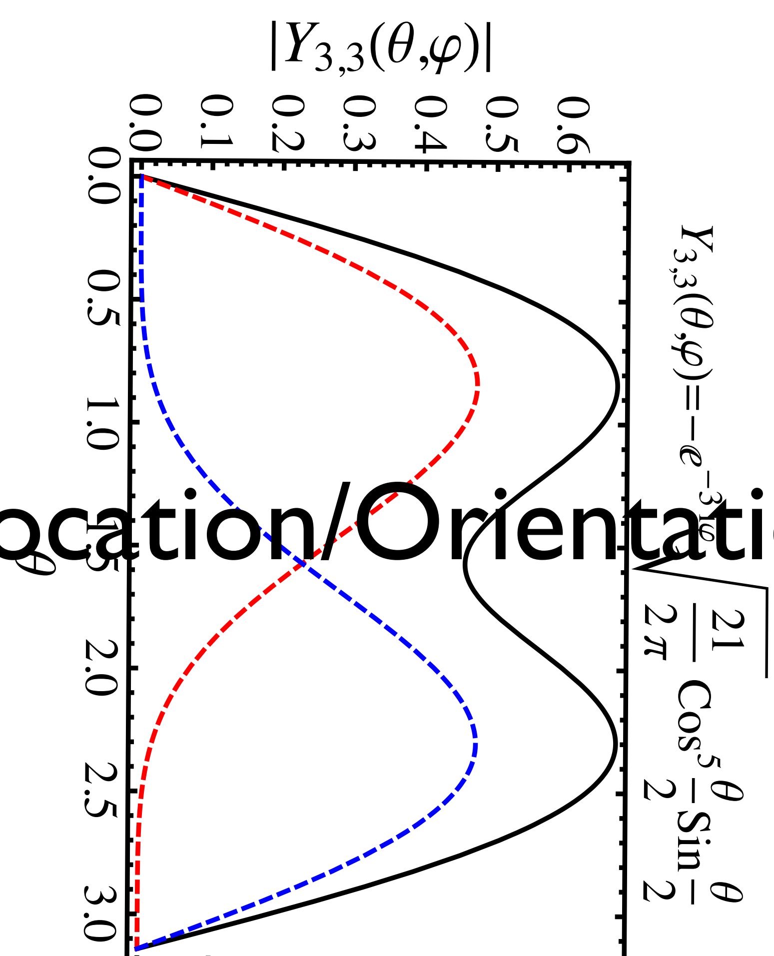


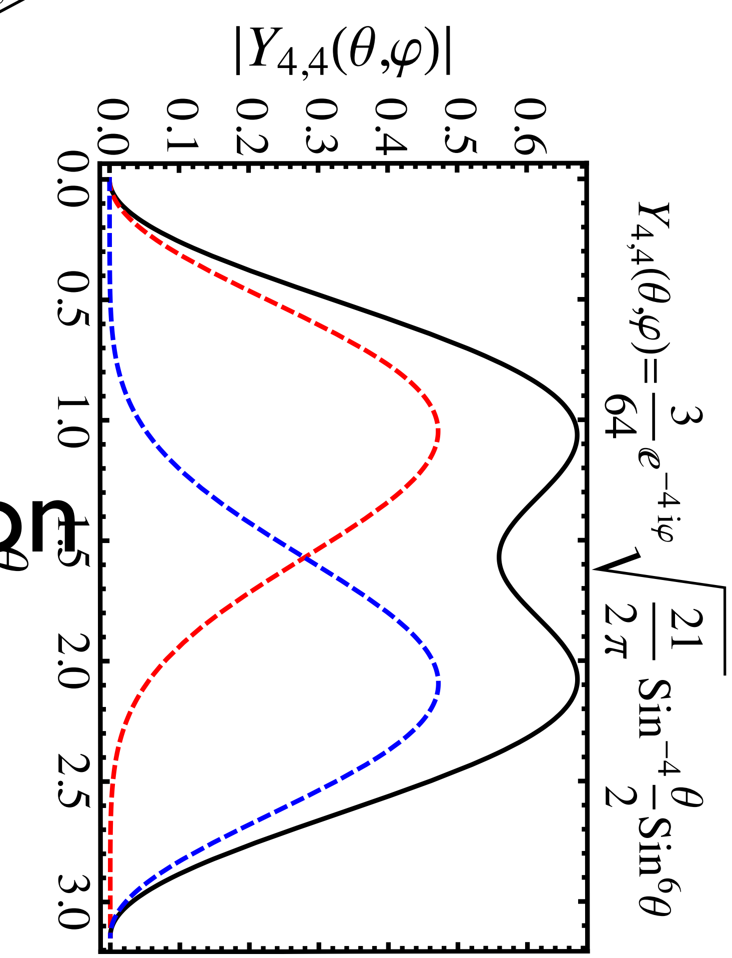




More important as q increases



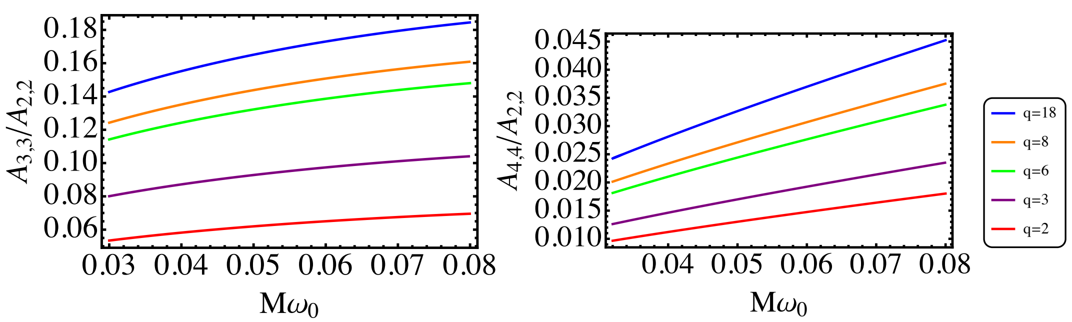


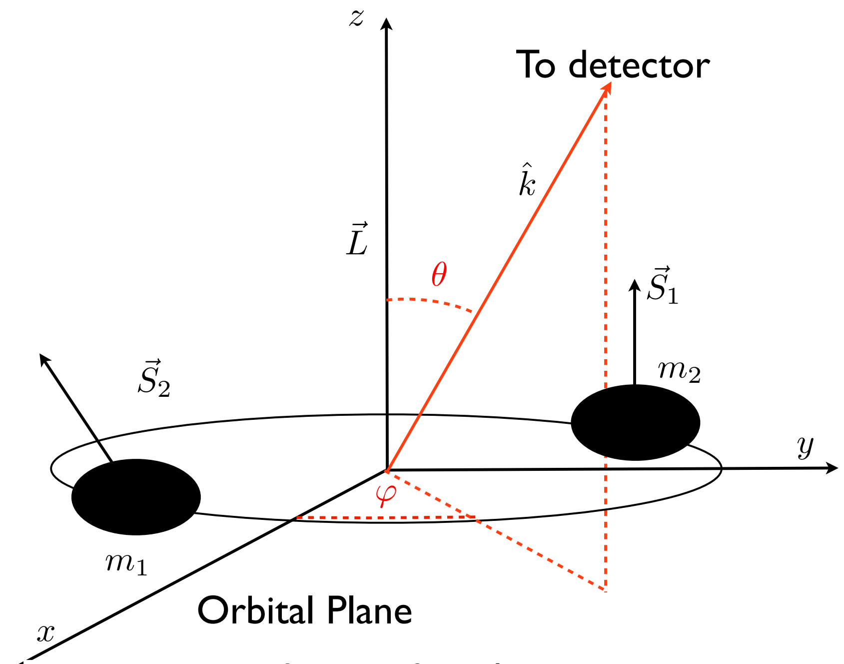


Sky-Location/Orientation

Edge-on orientations

Interesting situations





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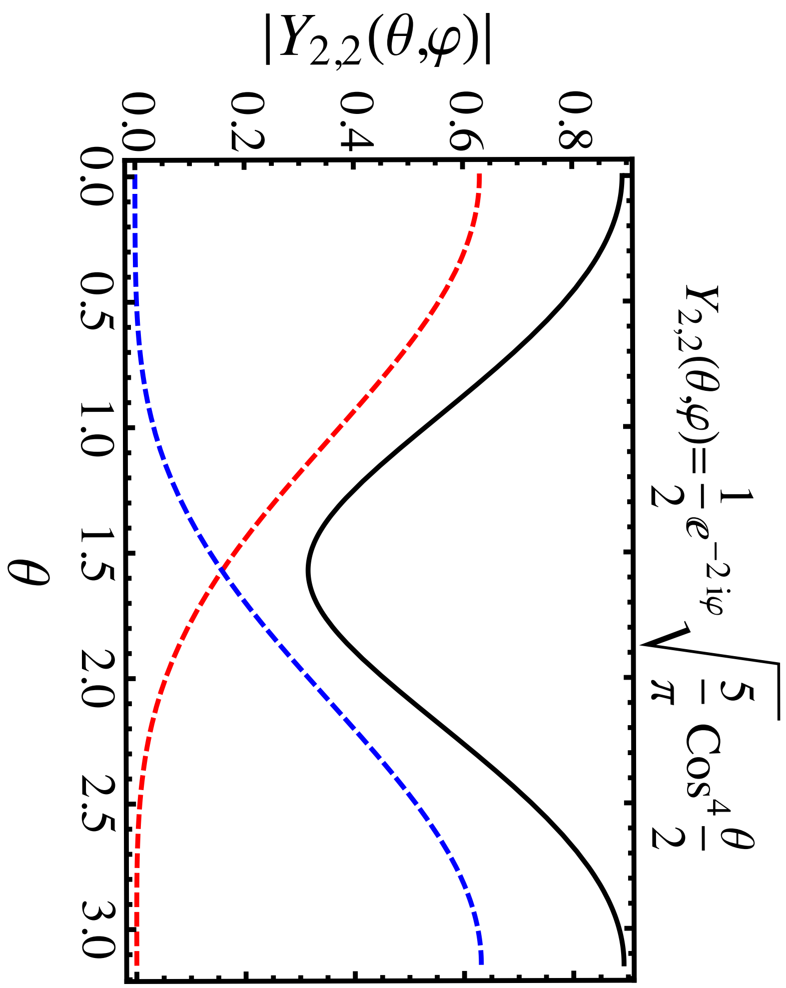
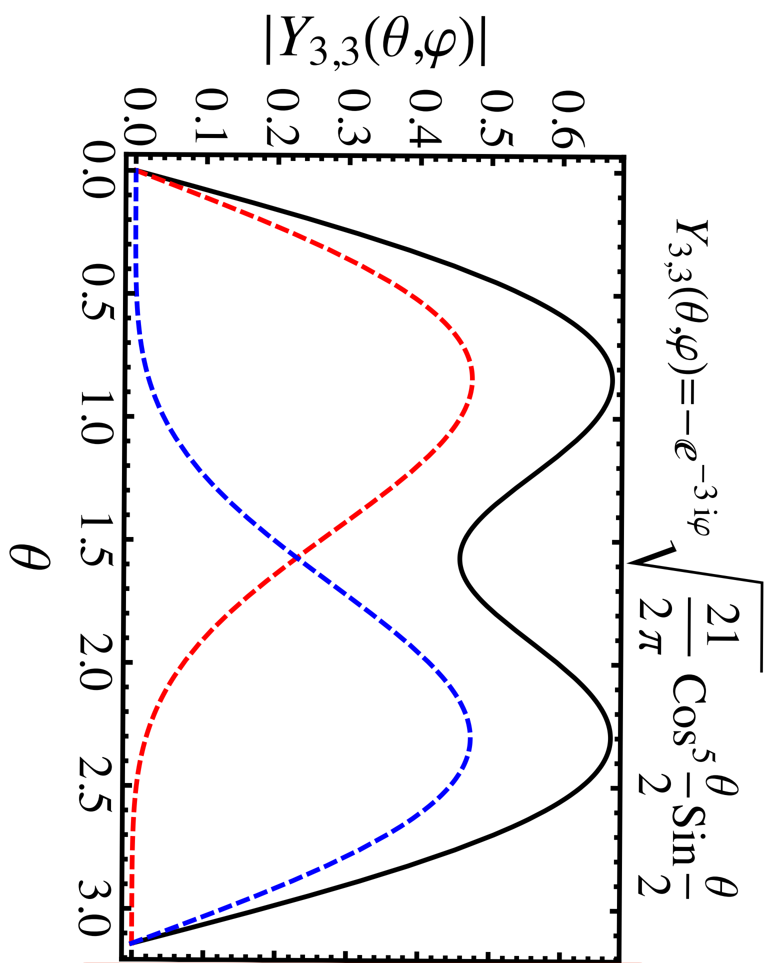
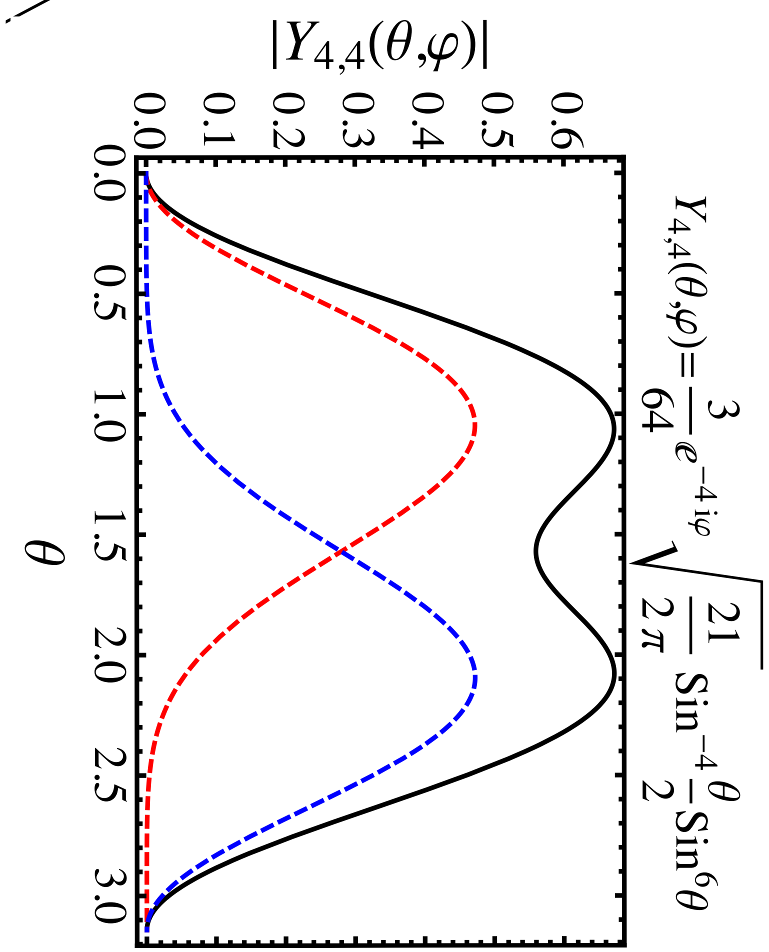
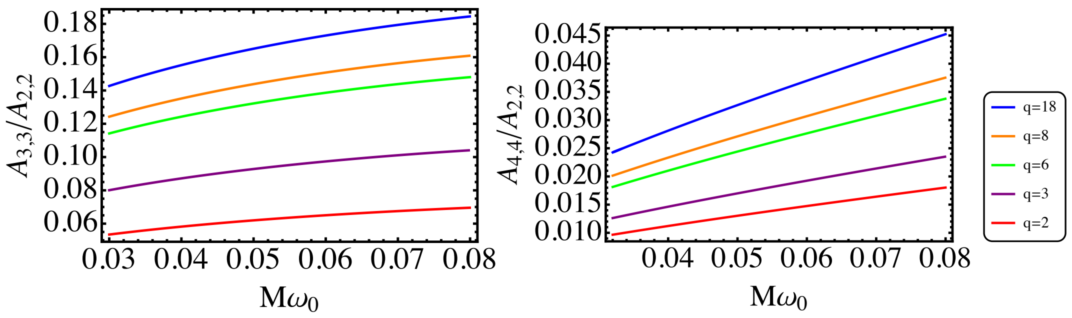




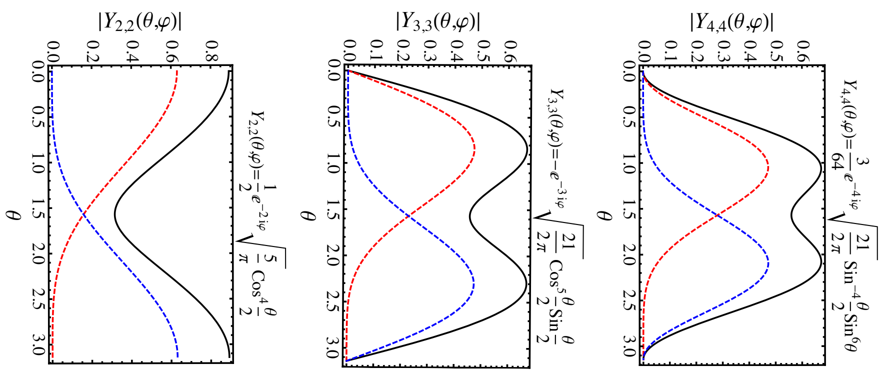


Edge-on orientations

Interesting situations



More important as q increases



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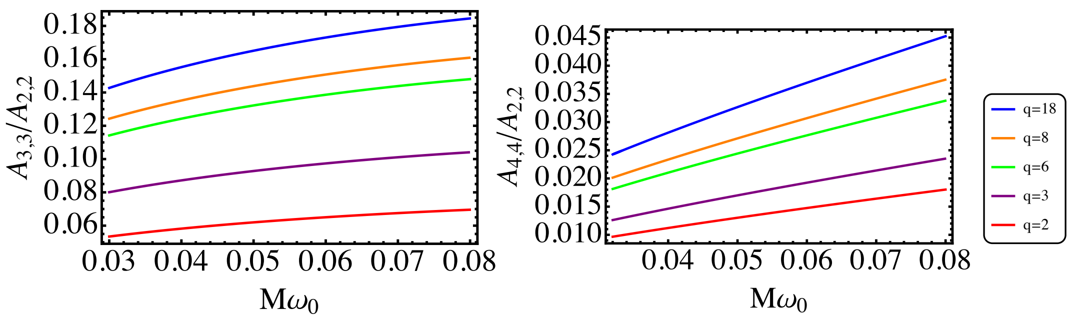

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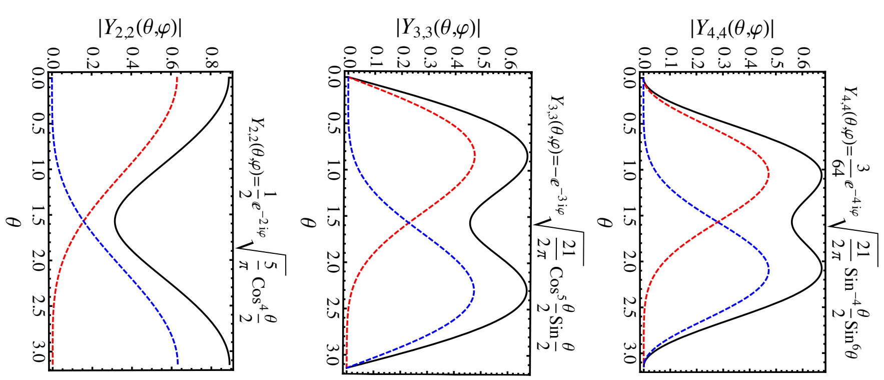
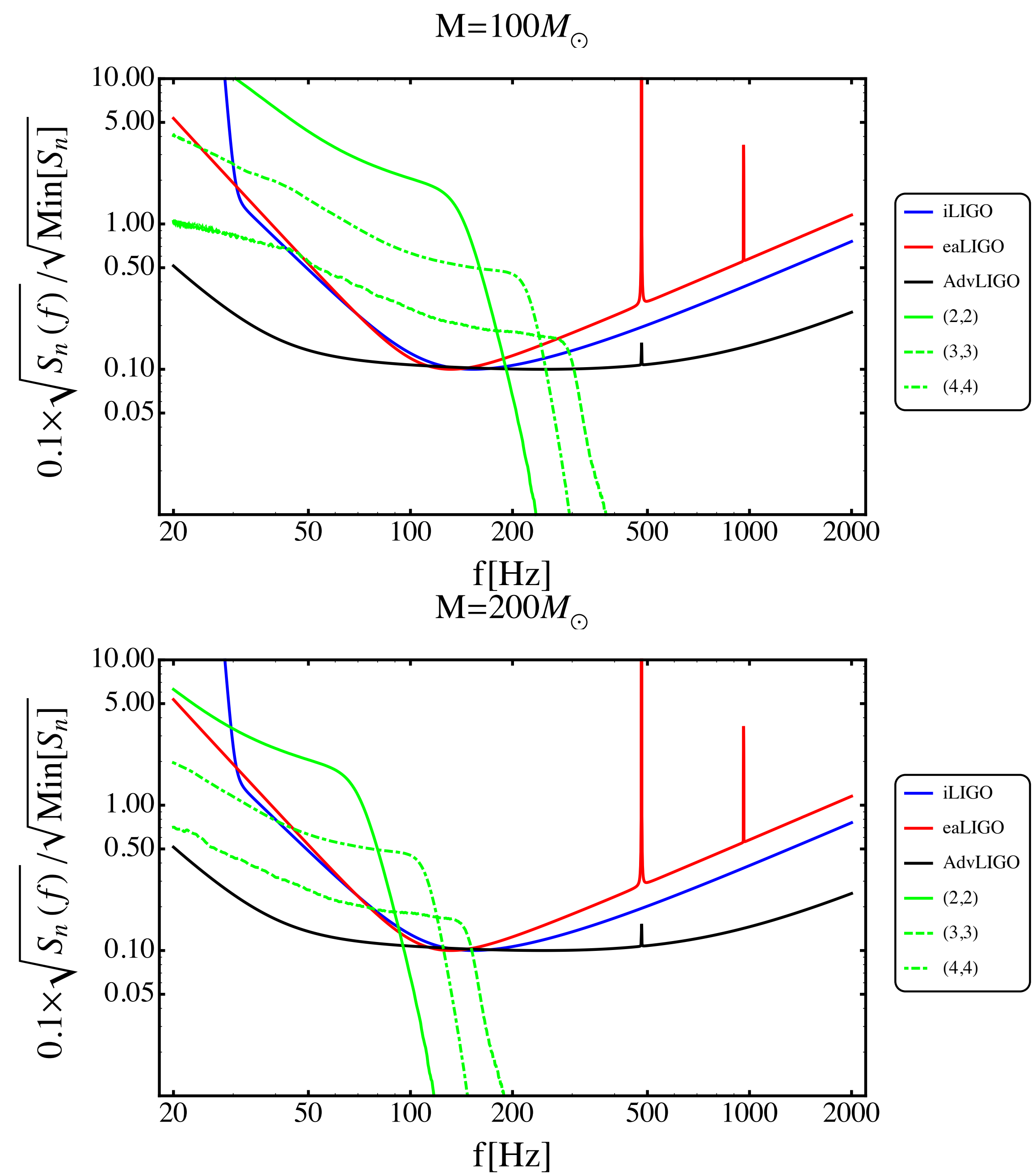


Edge-on orientations

Large Mass & lower frequency cutoff

Total Mass & Noise Curve



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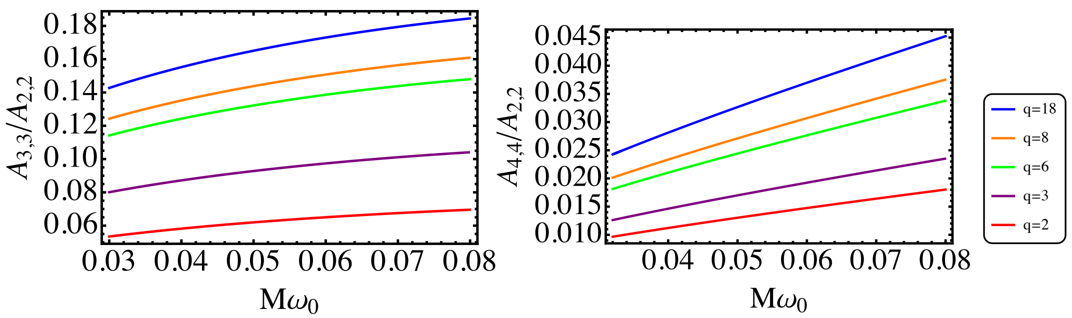

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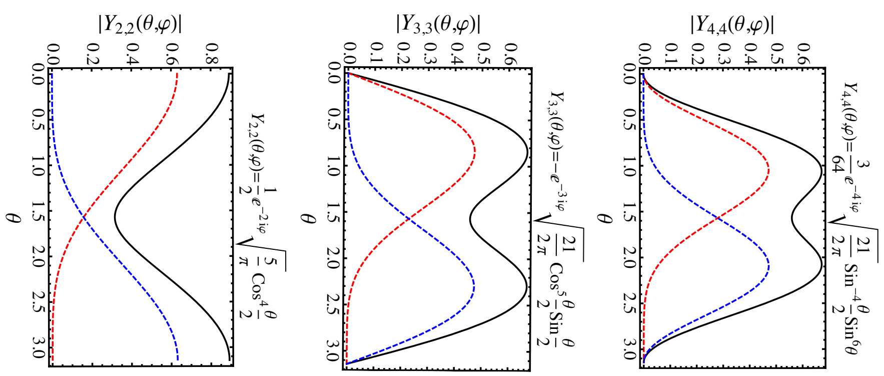
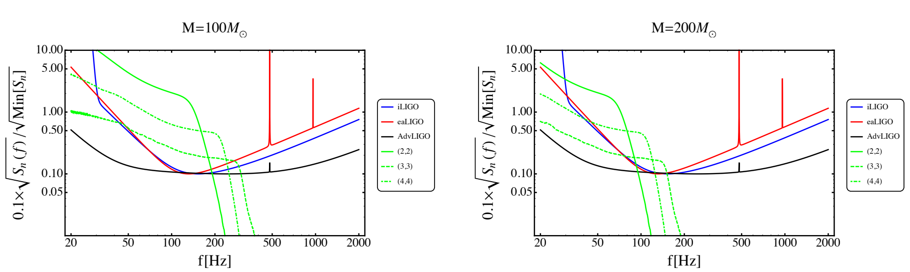
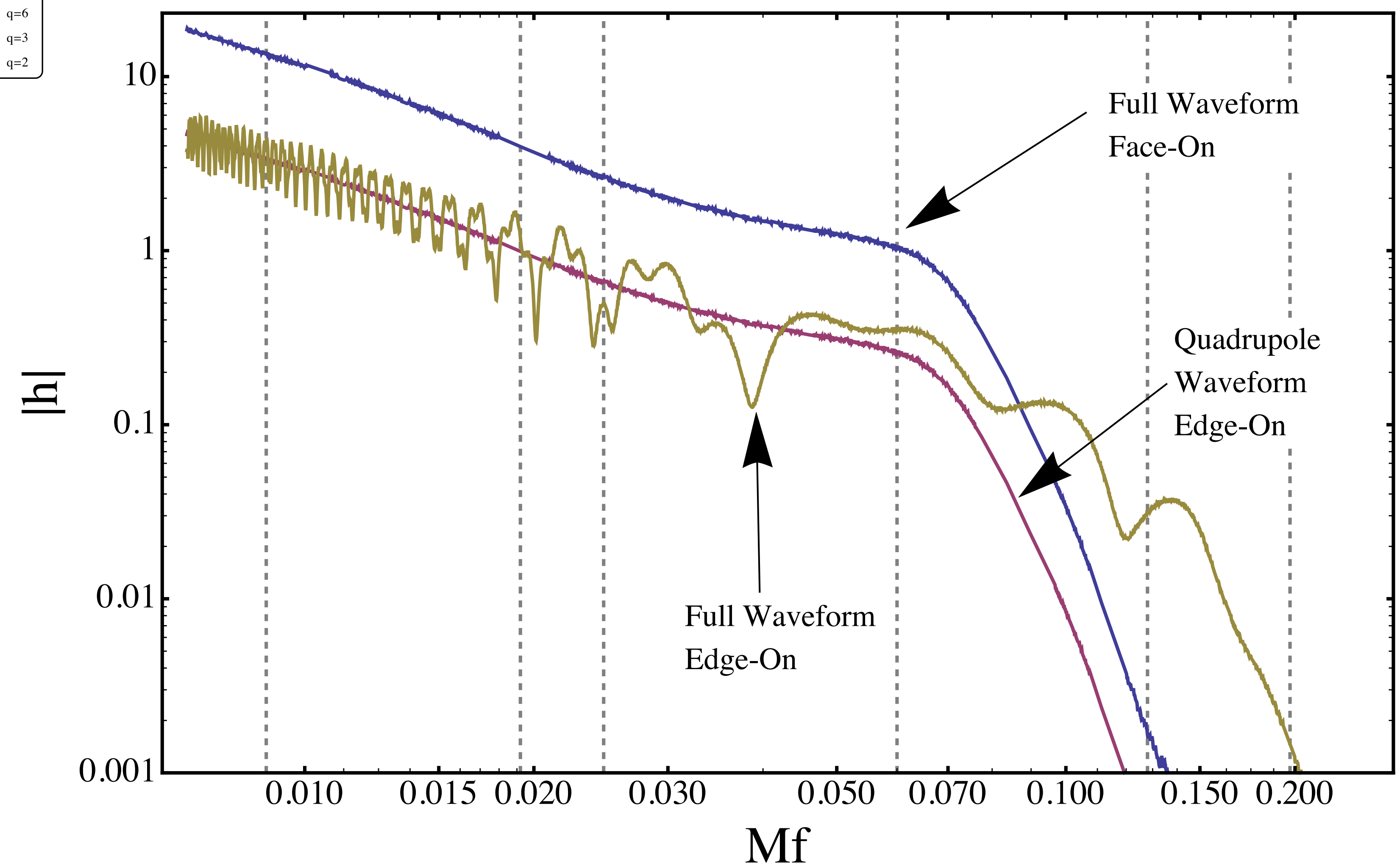


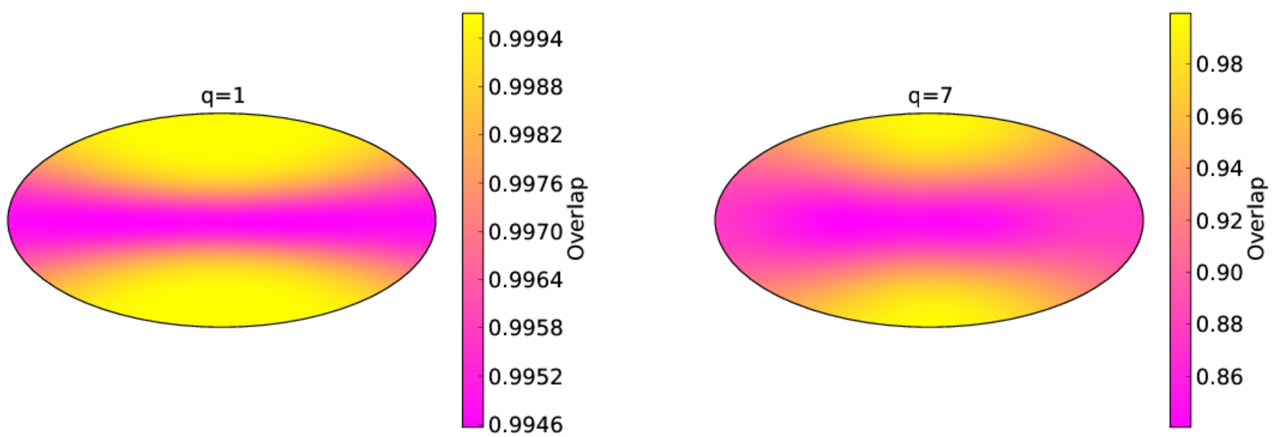
Edge-on orientations



Large Mass & high frequency cutoff

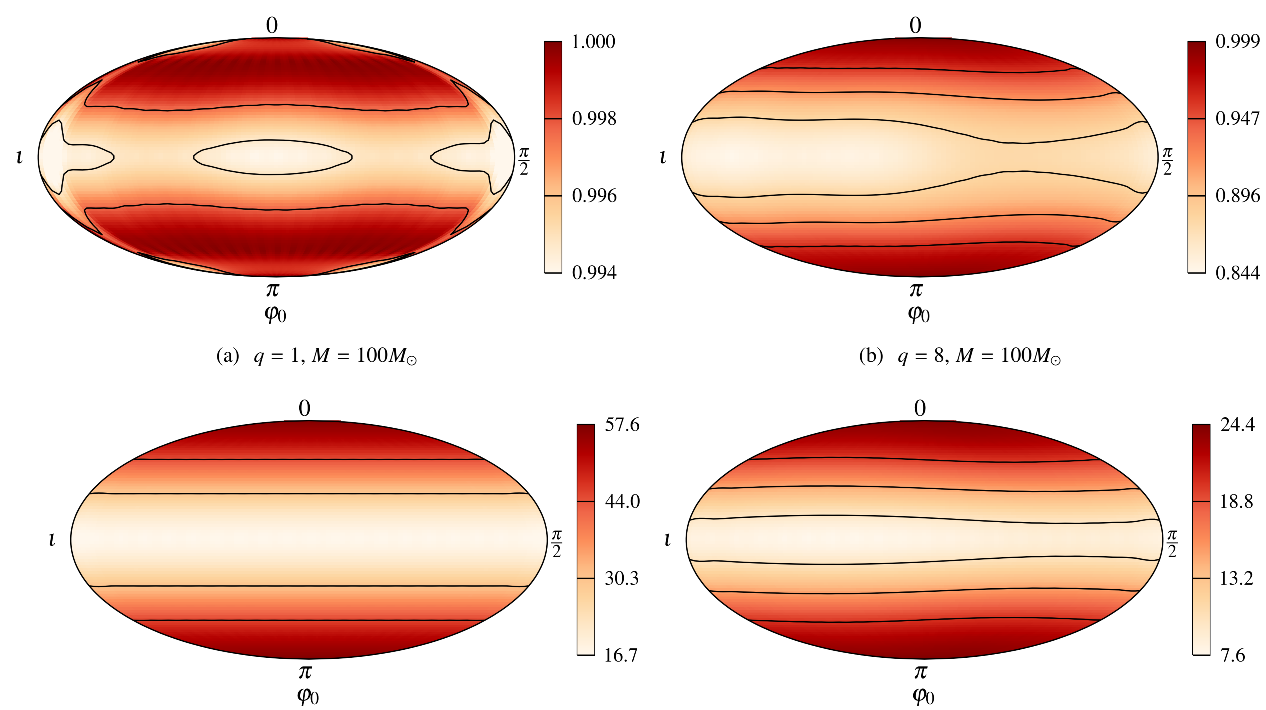


2. Previous studies



-Match lower than 0.97 for most orientations. These produce however the lowest SNR.

Pekowsky et. al.:PhysRevD.87.084008



(a) $q = 1$, $M = 100M_\odot$ (b) $q = 8$, $M = 100M_\odot$

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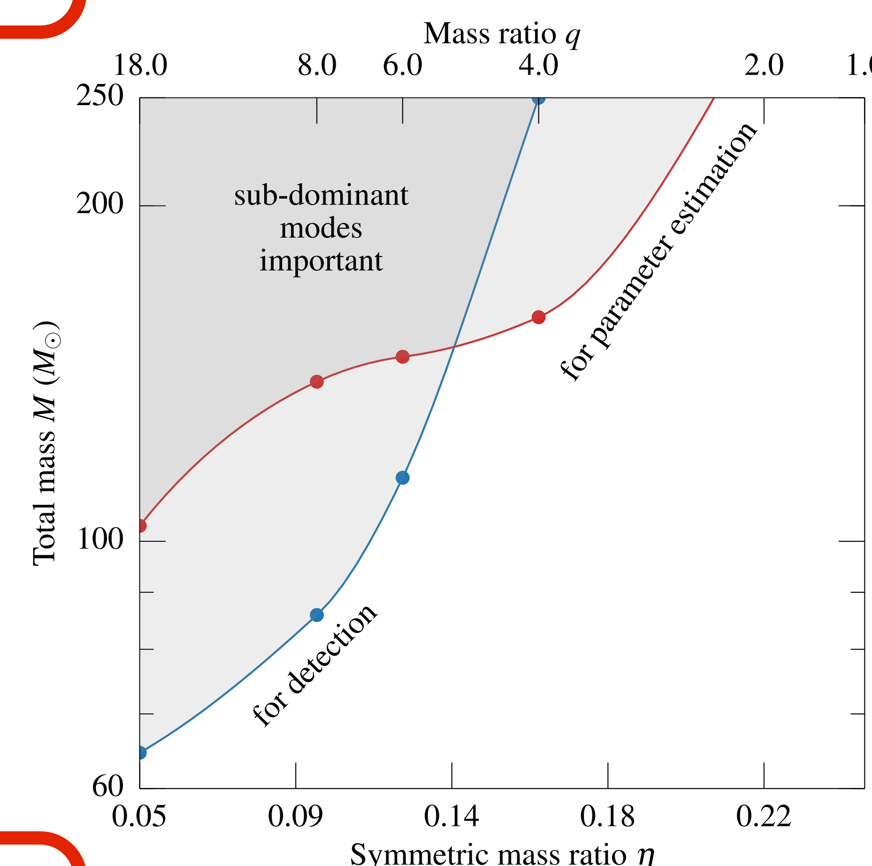
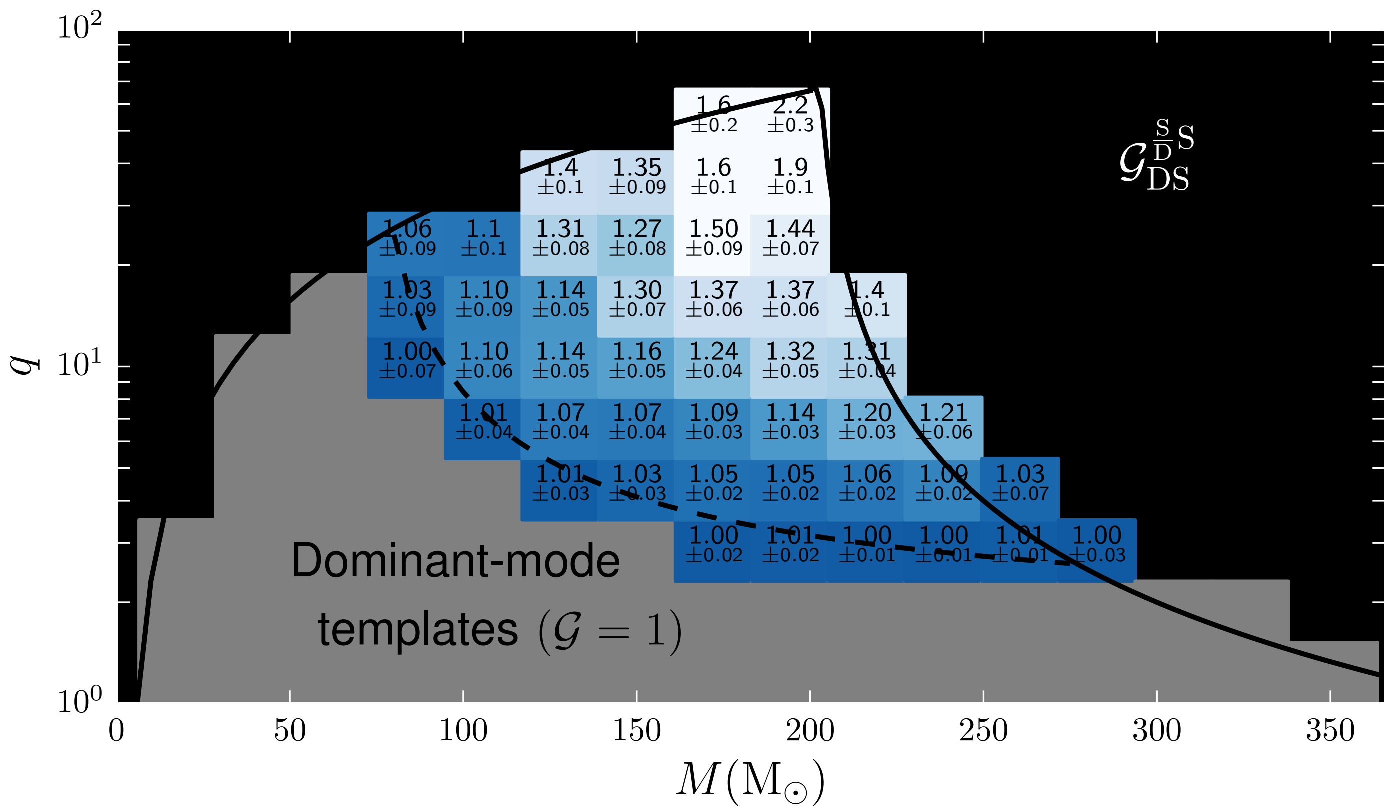


Varma et. al.:PhysRevD.90.124004

2.2 Previous studies II



-Reduction of sensitivity due to the larger number of templates.

-Worth to include higher modes for certain part of the parameter space

Capano et.al.:Phys. Rev. D 89, 102003 (2014)

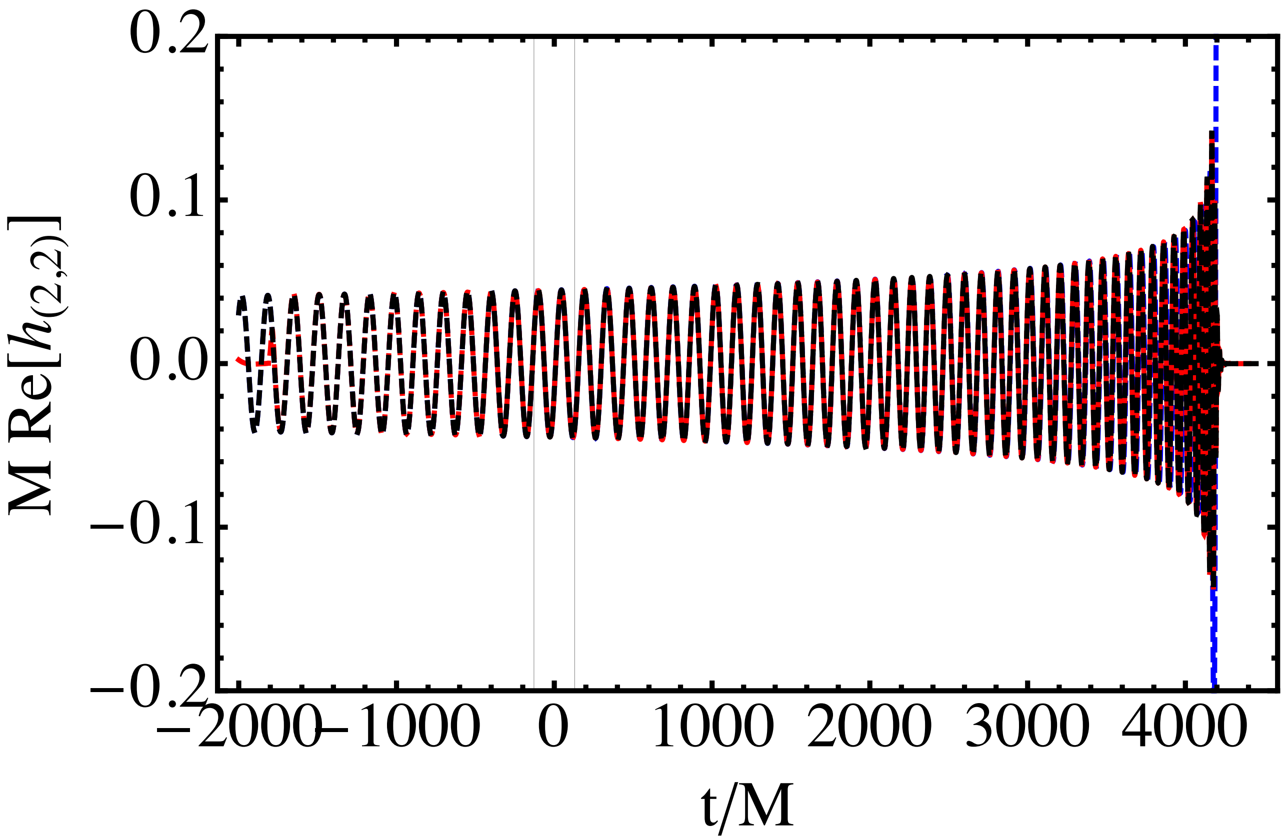
-All considered the design Advanced LIGO curve (f0=10Hz).

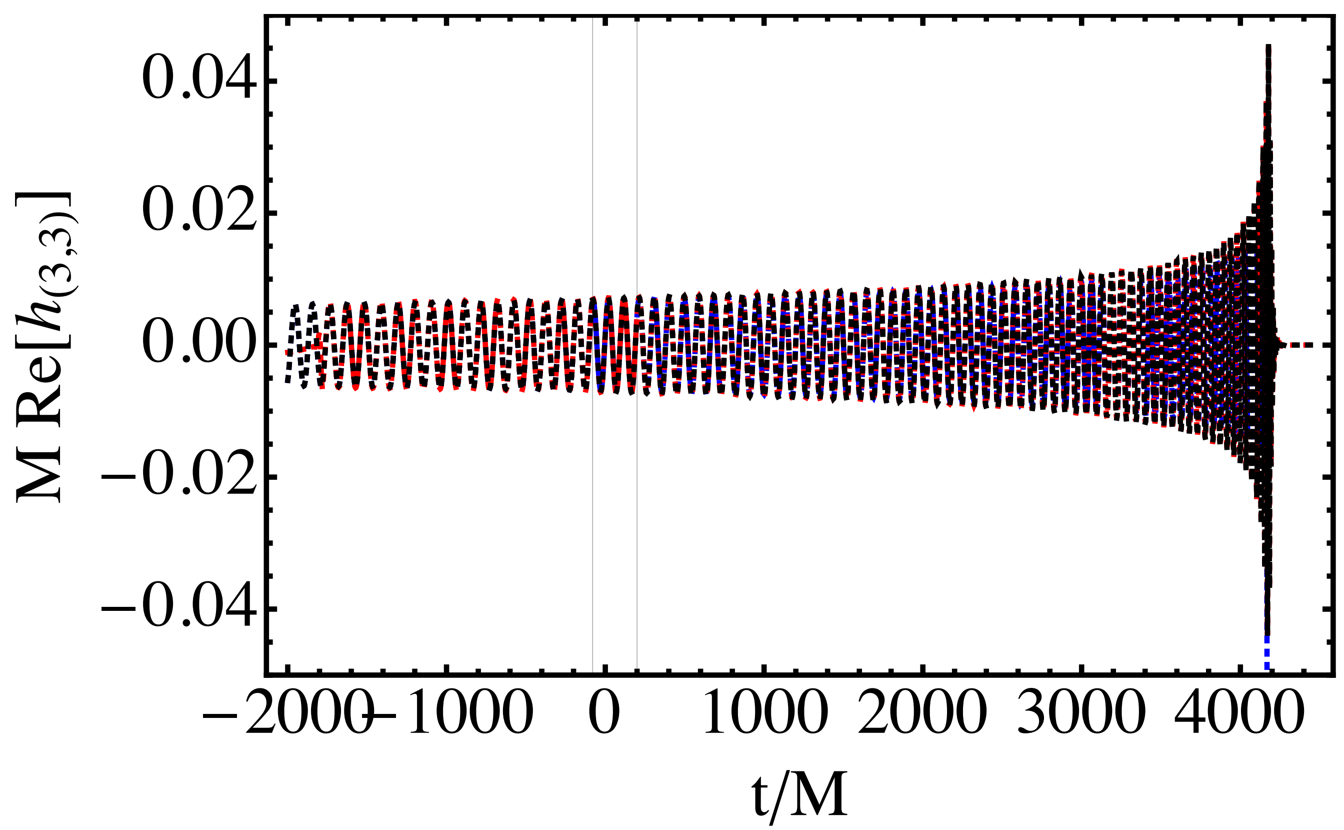
-Restricted to non-spinning targets and template banks.

-We extend to aligned-spin searches and early Advanced LIGO (f0=30Hz).

-We also consider a non-spinning search for initial LIGO (f0=30Hz).

3. Target waveforms





- Hybrid PN/NR waveforms including higher modes. [jcb et.al. **arXiv:1501.00918**]
- PN Taylor T1 to 3.5PN order phase and 3PN order amplitude. Higher modes amplitudes up to 2PN.
- Numerical Relativity from SXS catalogue [www.black-holes.org]. Data extrapolated to null infinity to order N=2.
- Templates belong to the SEOBNRv1_ROM family [Pürrer. 2014 *Class. Quantum Grav.* **31**].

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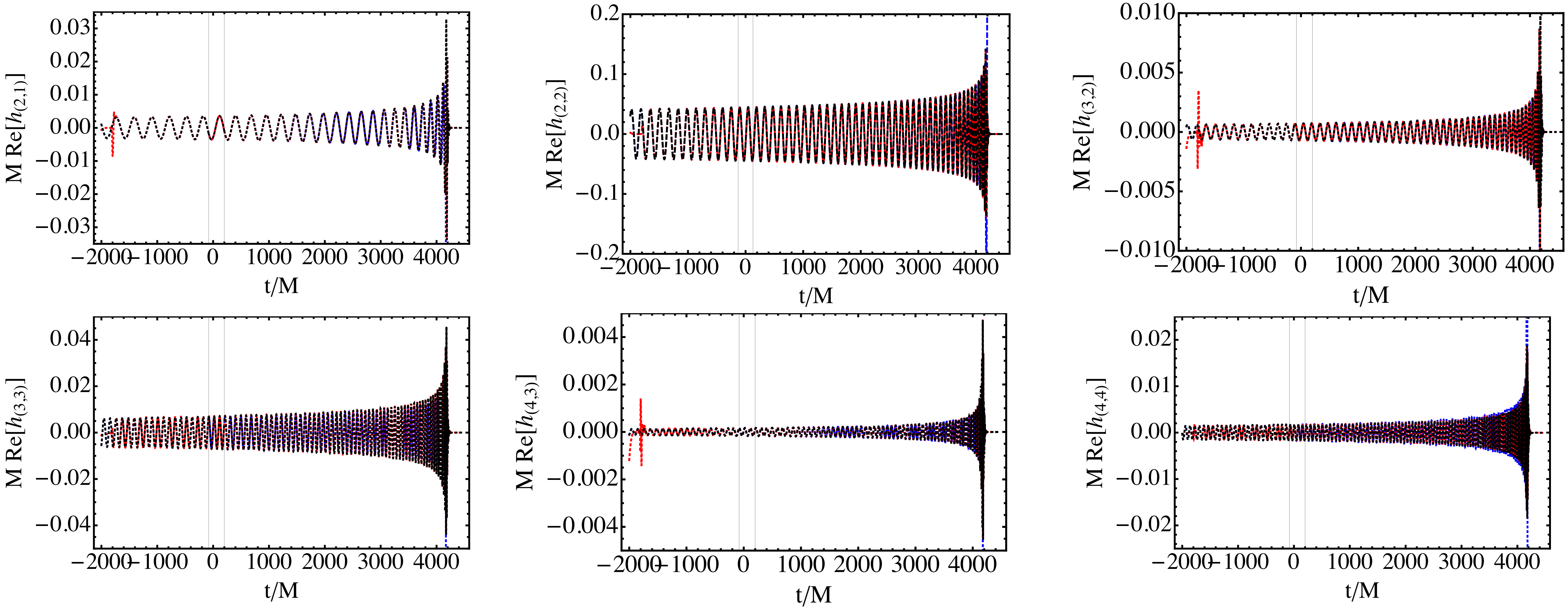

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SIM ID	q	χ	PN	$M\omega_0$
SXS:BBH:0168	3	0	T1	0.043
SXS:BBH:0167	4	0	T1	0.045
SXS:BBH:0166	6	0	T1	0.045
SXS:BBH:0063	8	0	T1	0.043
SXS:BBH:0150	1	+0.2	T1	0.035
SXS:BBH:0149	1	-0.2	T1	0.043
SXS:BBH:0046	3	+0.5	T1	0.038
SXS:BBH:0047	3	-0.5	T1	0.043

3.1 Quantities computed

- Compute fitting factor and SNR for all orientations and averaged fitting factor.

$$\Delta V[\%] = 100 \times R_i = 100 \times \left(\frac{\sum_j \mathcal{F}_{i,j}^3 \rho_{i,j}^3}{\sum_j \rho_{i,j}^3} \right) \qquad \mathcal{F}_i^{eff} = R_i^{1/3}$$

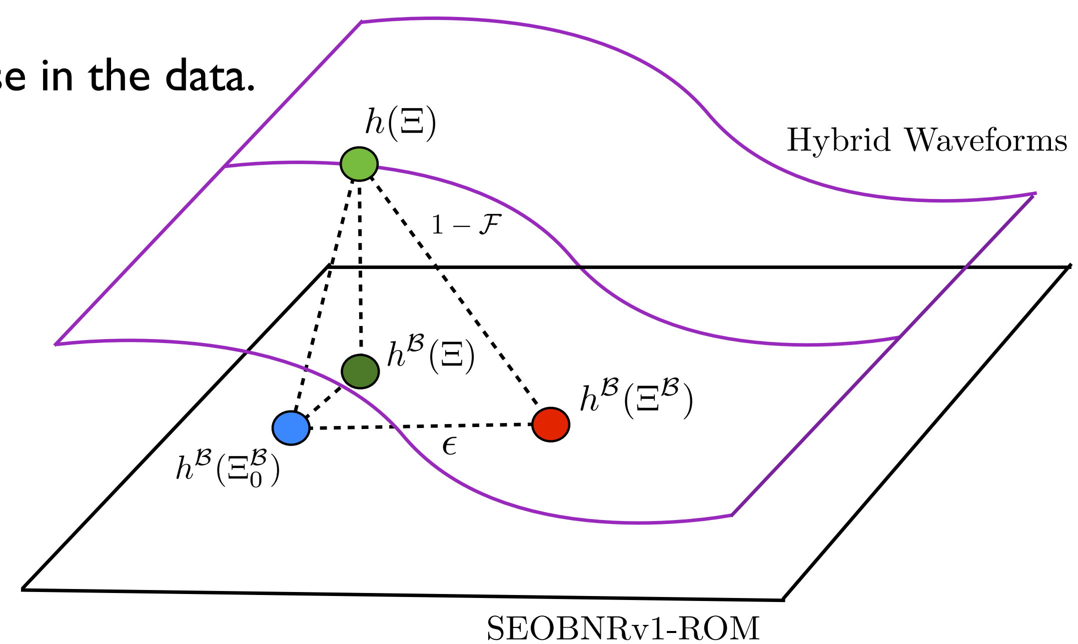
- Compute averaged parameter biases subtracting biases of the SEOBNRv1-ROM model to our quadrupolar hybrids

$$\Xi_i^{\mathcal{B}} = \left(\frac{\sum_j \Xi_{i,j}^{\mathcal{B}} \mathcal{F}_{i,j}^3 \rho_{i,j}^3}{\sum_j \mathcal{F}_{i,j}^3 \rho_{i,j}^3} \right) \qquad \Delta\Xi_i = \Xi_{i,0} - \Xi_i^{\mathcal{B}}$$

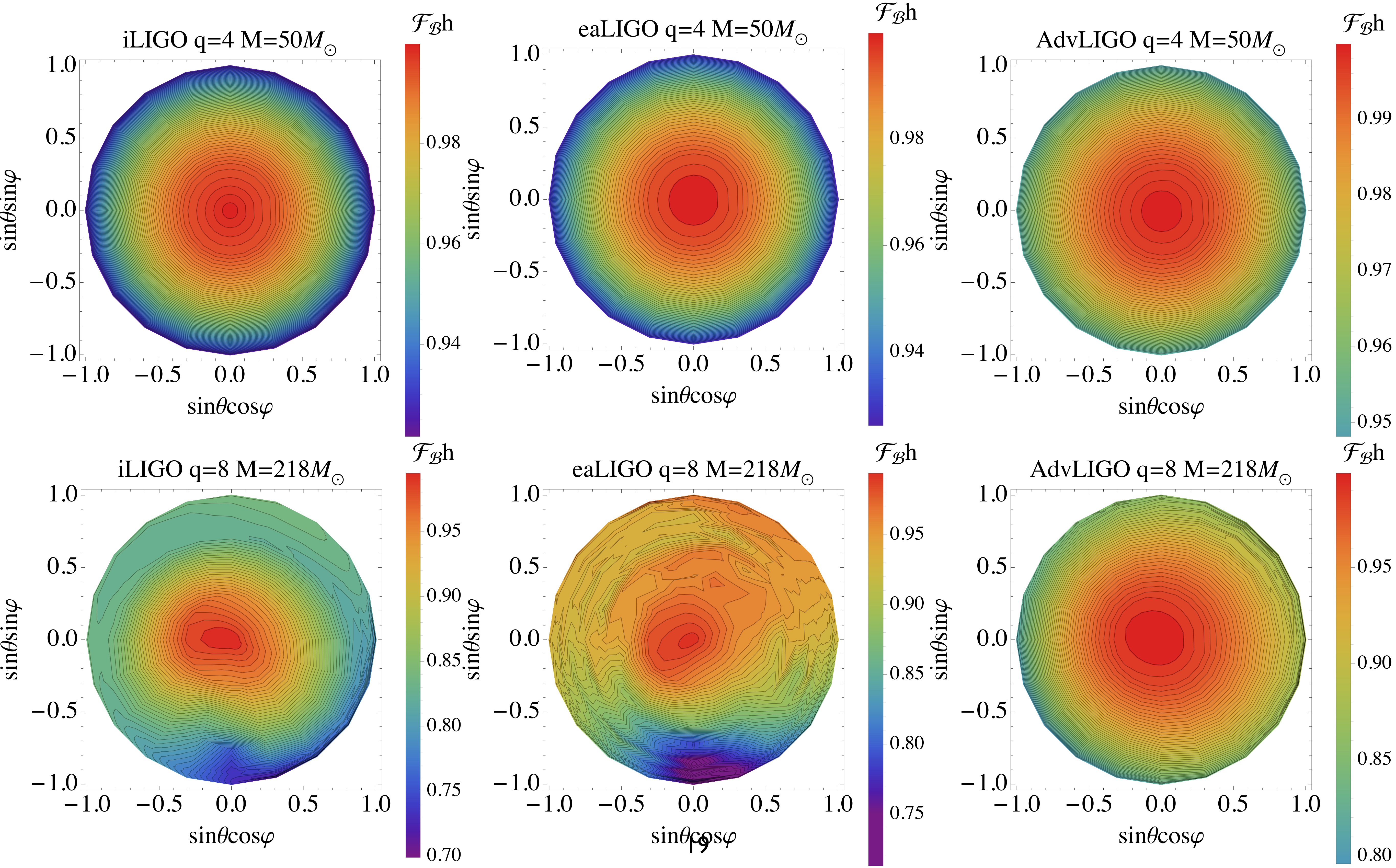
- Compare to statistical uncertainties due to presence of Gaussian noise in the data.

$$\epsilon < 1/2\rho^2$$

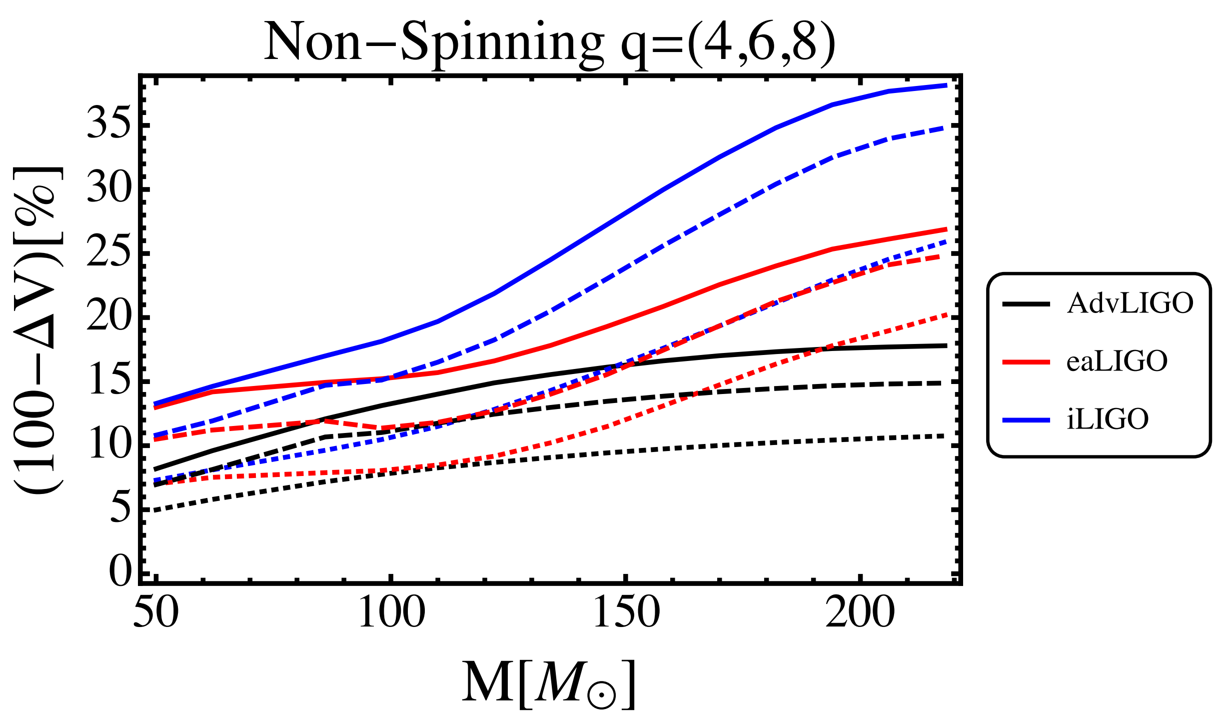
$$\rho_0 = \sqrt{1/2\epsilon}$$

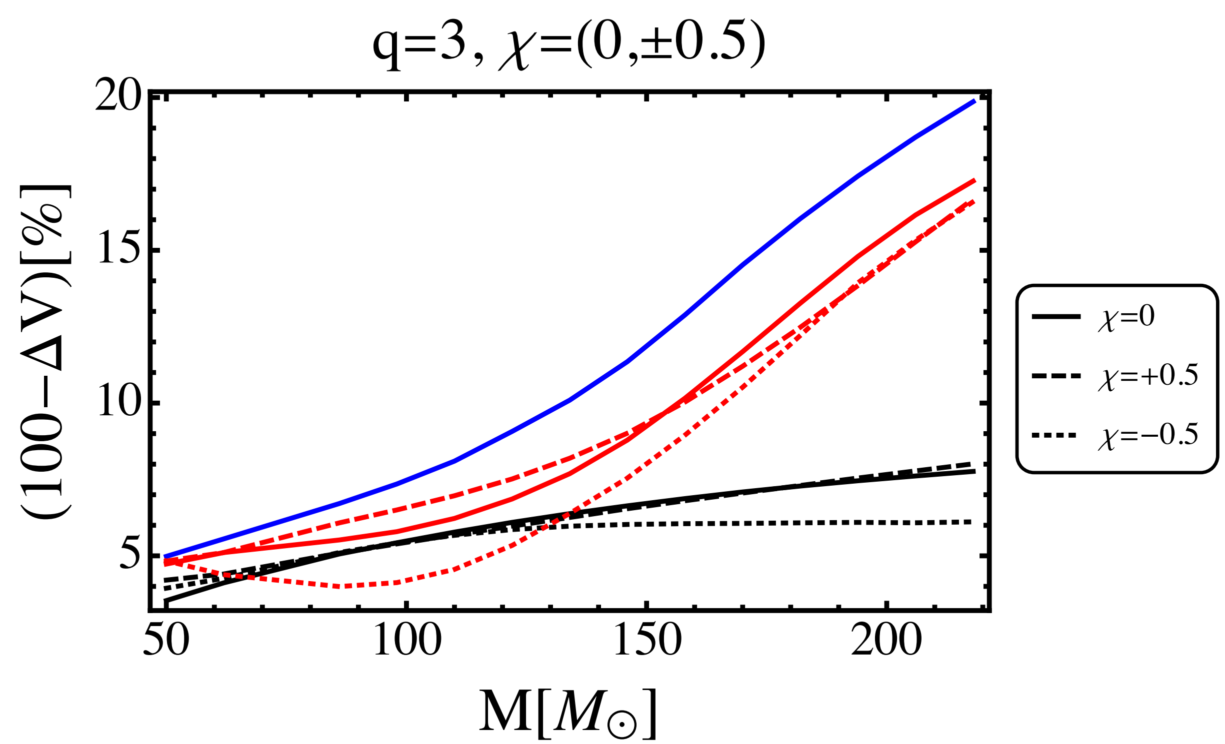


4. Effect on detection

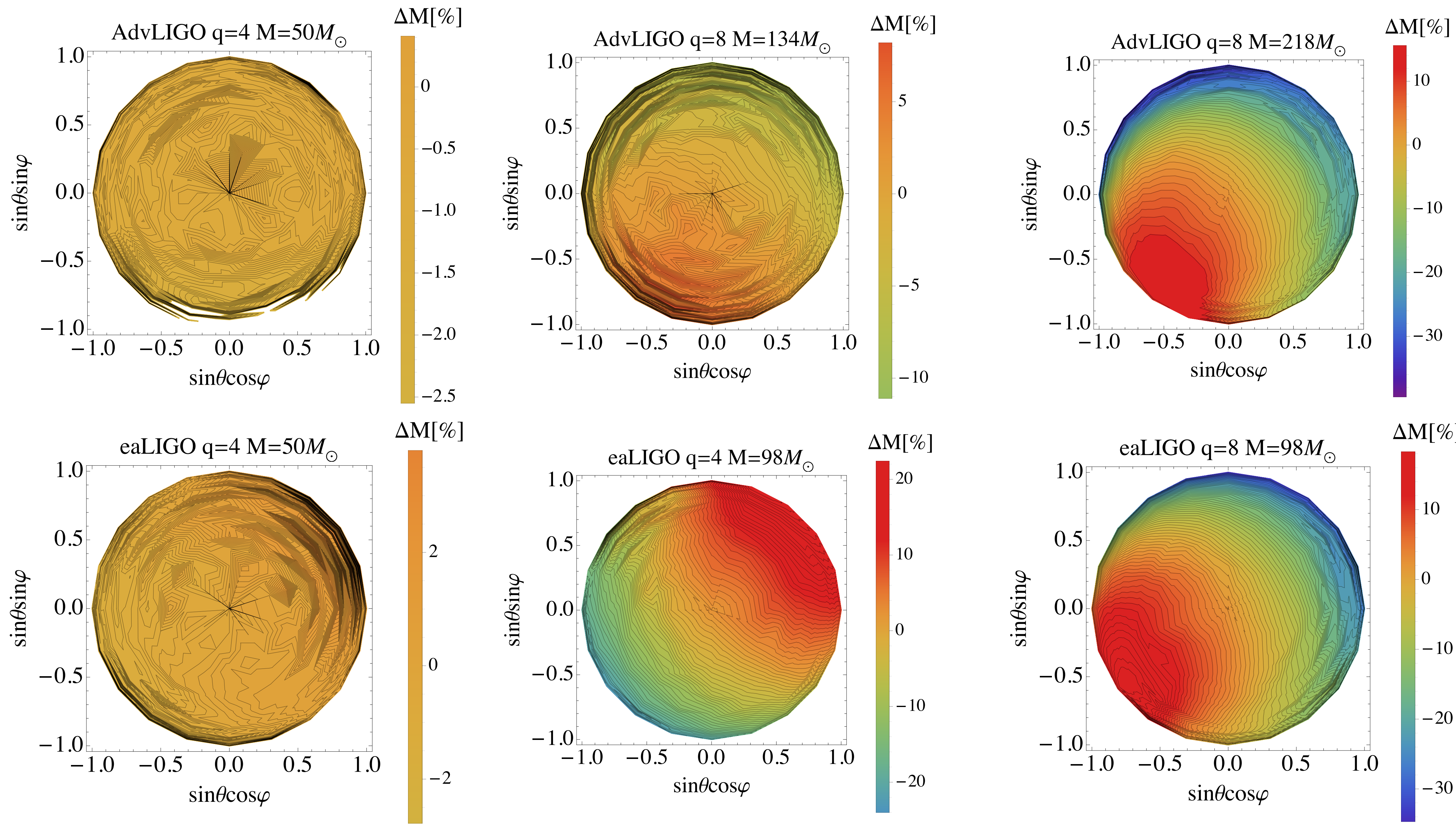


Effect on detection

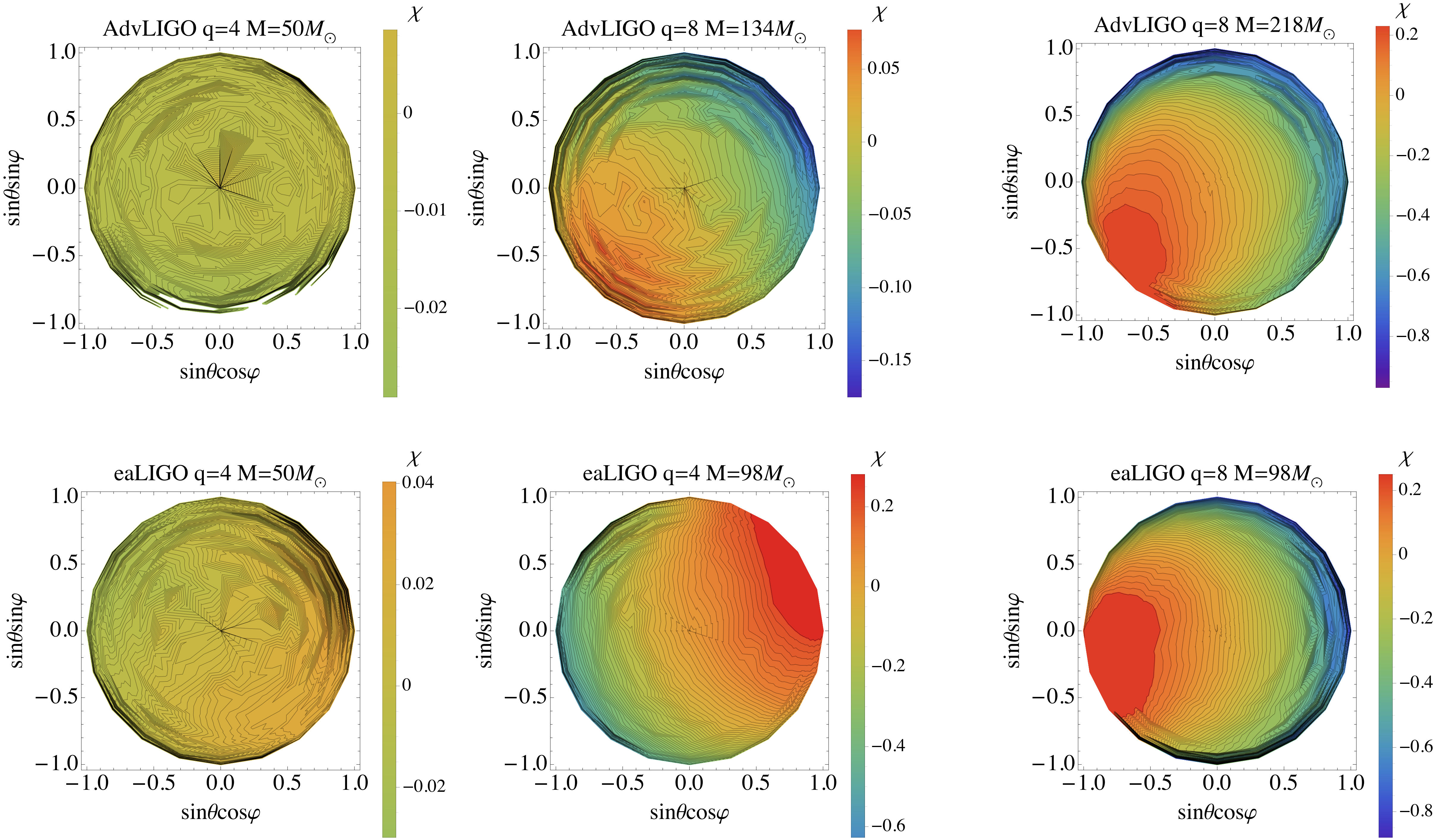




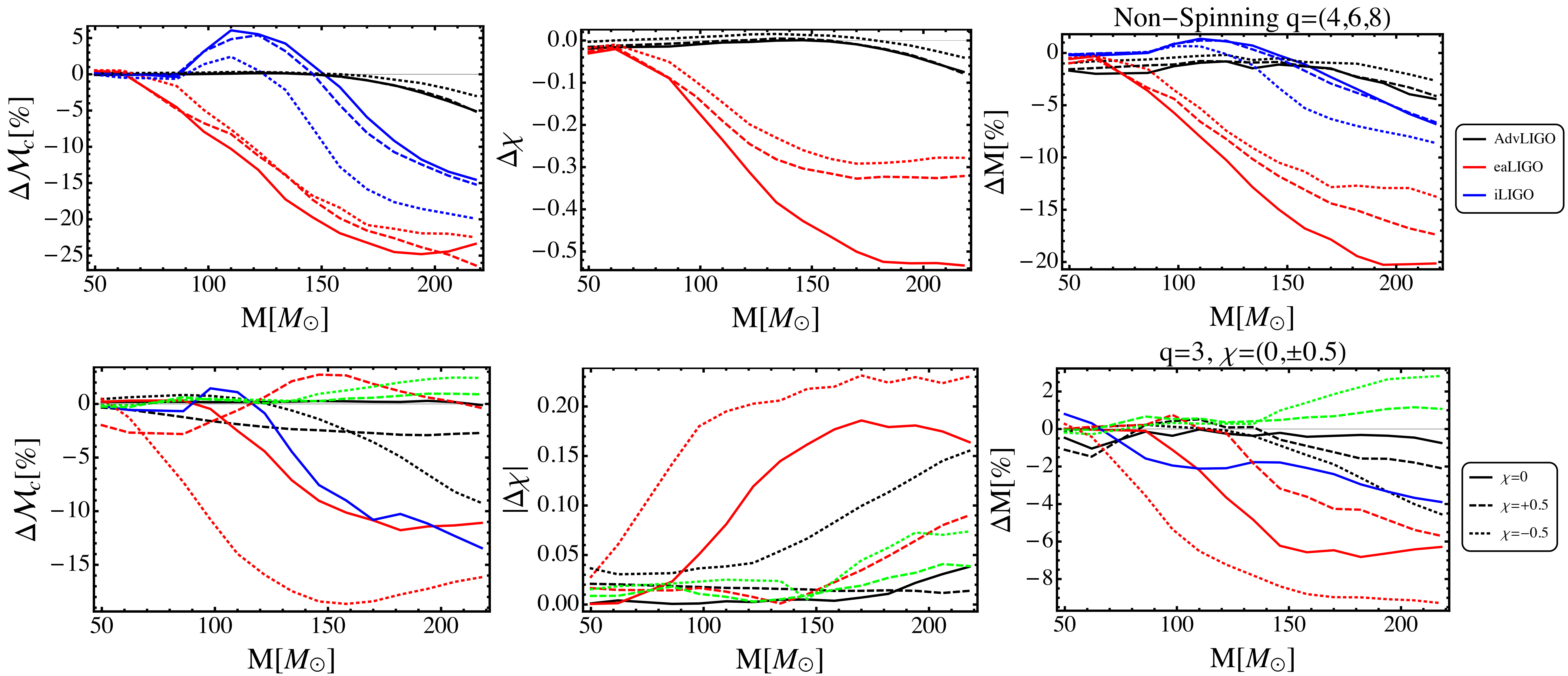
5.1 Parameter Biases (Mass)



5.2 Parameter Biases (Spin)



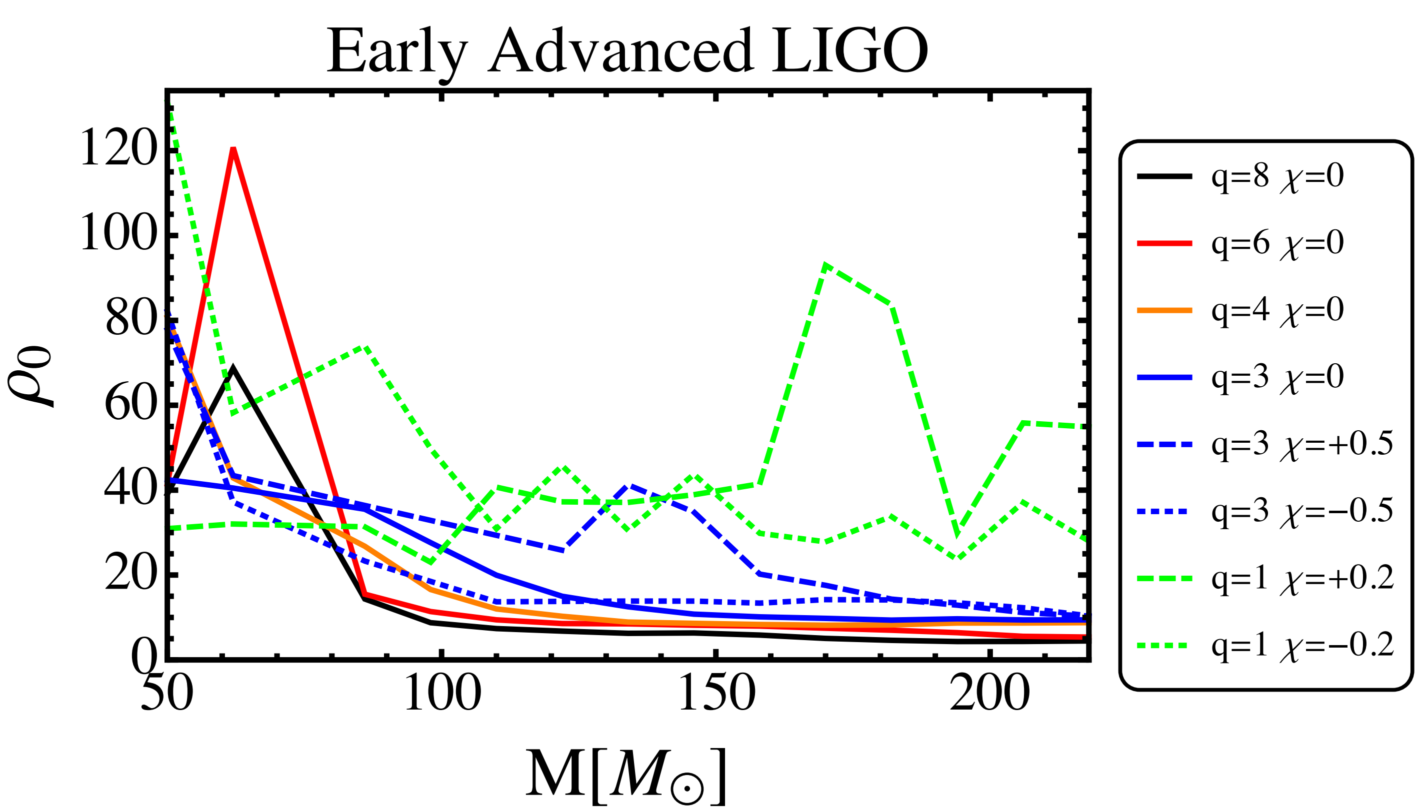
Parameter Bias II

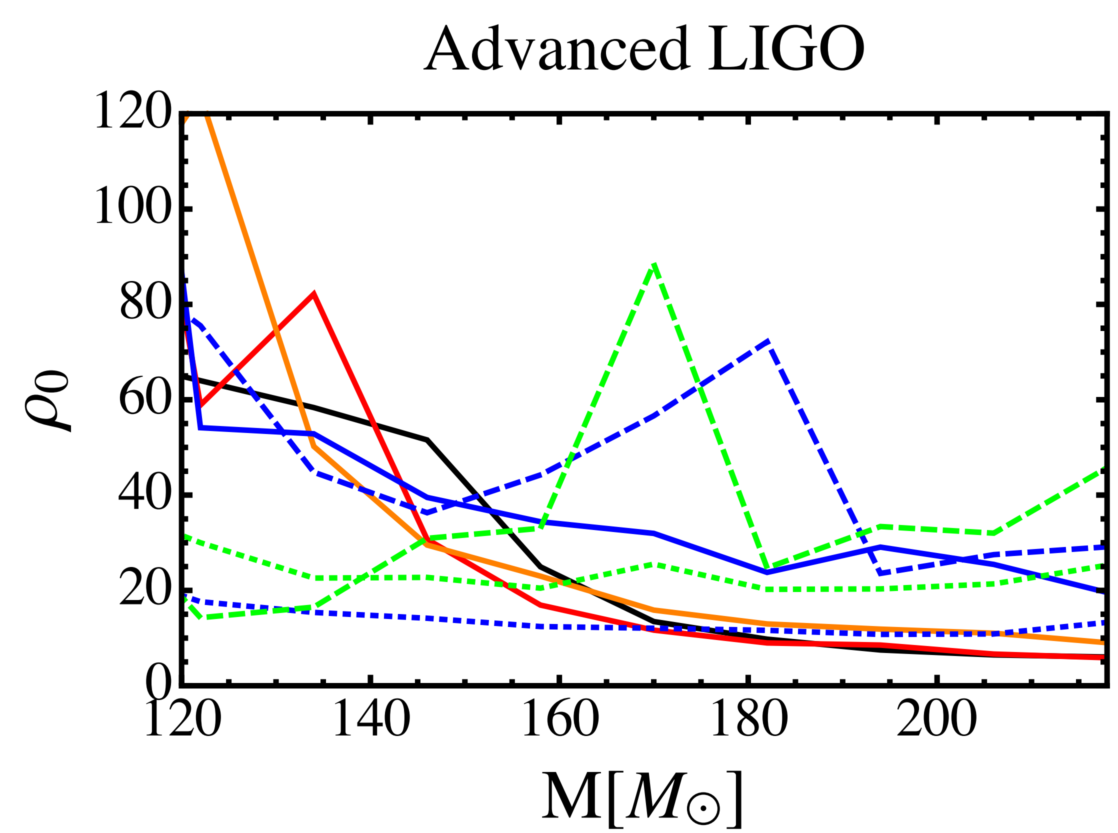


Systematic Biases vs. Statistical Uncertainty

Very sensitive to tiny errors in the estimation of the best matching template

$$\Delta\rho_0 \sim \epsilon^{3/2}\Delta\epsilon$$





$$M \geq 90 M_\odot \qquad M \geq 170 M_\odot$$

Summary

- Higher modes relevant for large mass ratio, large total mass and edge-on systems.
- Impact highly detector - dependent.

Detector	10%	Max Loss	SysDom
AdvLIGO	$(q, M) \geq (6, 100 M_\odot)$	15%	$M > 170 M_\odot$
eaLIGO	$(q, M) \geq (4, 100 M_\odot)$ $(q, M) \geq (6, 50 M_\odot)$	26%	$M > 80 M_\odot$
iLIGO	""	36%	3

- Parameter Biases towards lower spin, total mass and chirp mass.
- Large parameter biases for edge-on systems.

To Do List

- Inject our target waveforms in real noise and run a full search. (Ongoing in collaboration with AEI Hannover).
- Study of precessing systems, for which higher modes are stronger.

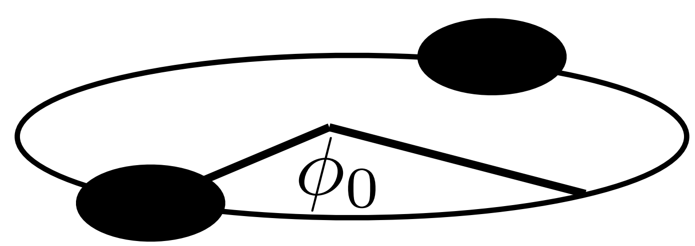
Thanks for your attention

Degrees of freedom when constructing a hybrid waveform

$$\sum_{l,m} |Y^{-2}_{l,m}(\theta,\phi)| e^{-im\phi} h_{l,m}(\Pi; t)$$

$$h_{l,-m} = (-1)^l h^*_{l,m}$$

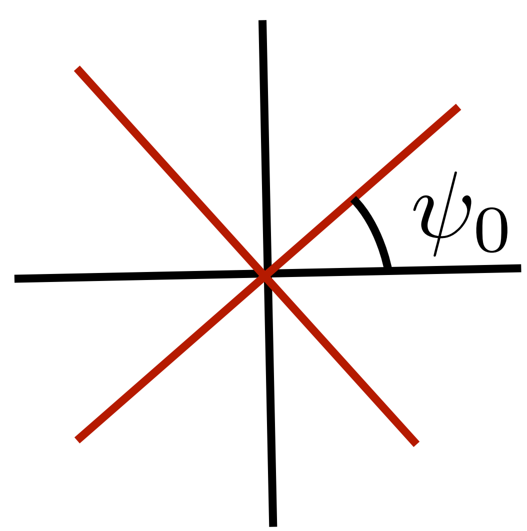
$$t \rightarrow t + \tau$$



$$\phi \rightarrow \phi + \phi_0$$

$$\psi \rightarrow \psi + \psi_0$$

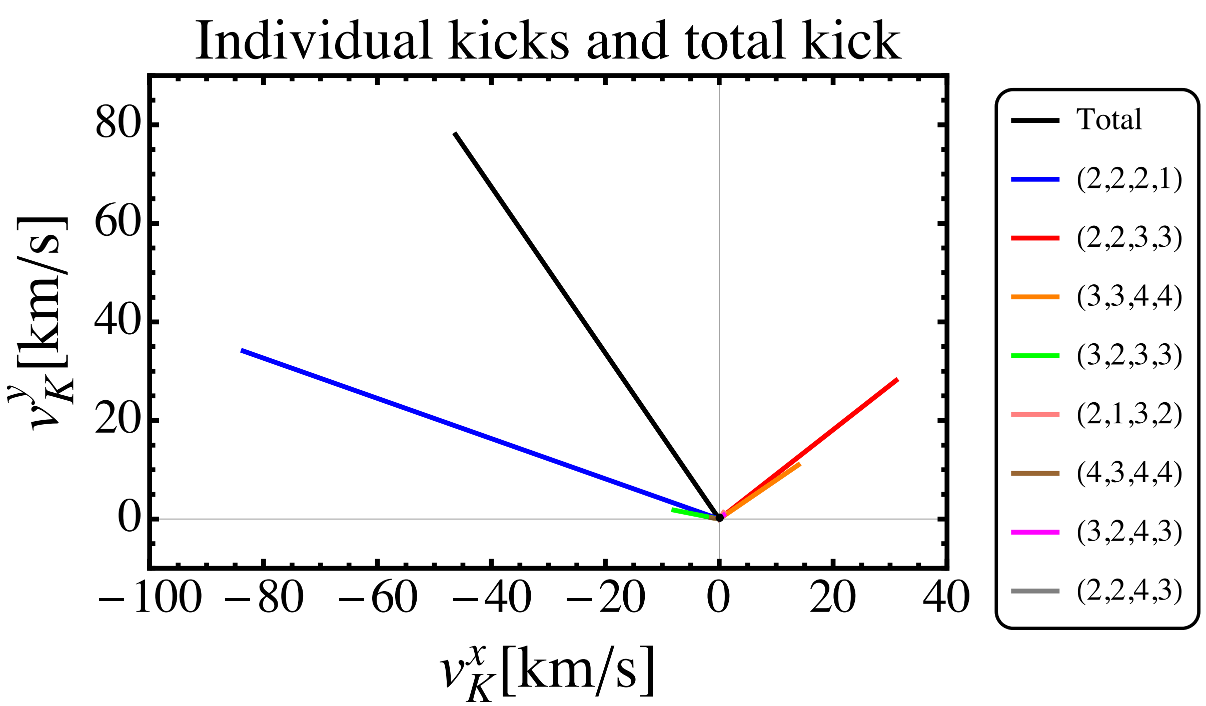
$$\Delta\psi_{l,m} = \psi_0 + m\phi_0$$

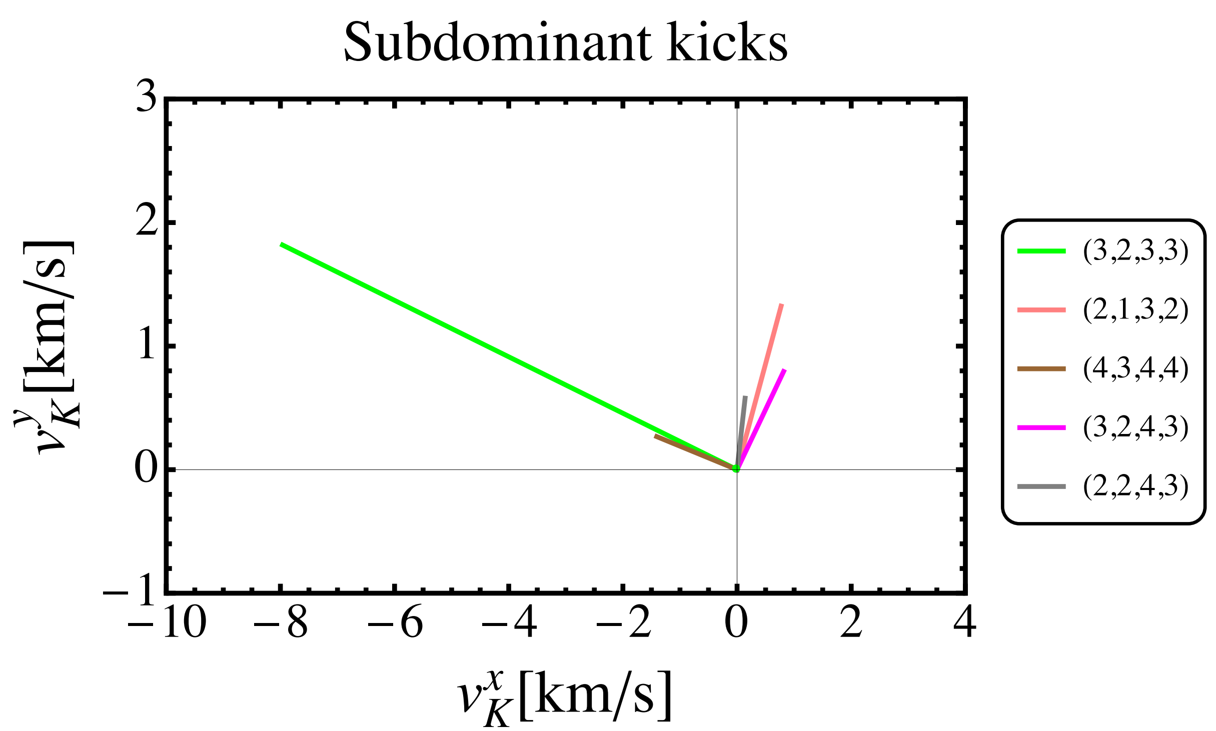


$$\psi_0 \in \{0, \pi\}$$

BH Kicks

$$\frac{dP_i}{dt} = \lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega l_i \left(\int_{-\infty}^{t}\psi_4 d\bar{t}\right)^2 d\Omega\right]$$





Black Hole Kicks