of curves reveals several points of similarity. First, all curves for metals appear to have a "cyclic" stress sensitivity limit" σ_L (indicated by solid squares in Fig. 36.15) below which the curve representing damping vs. stress is a straight line on a log-log plot and displays no stress-history effect. However, above σ_L, two changes occur: (1) the slope of the curve changes with increasing stress, in some cases quite abruptly, and damping exponent n may attain very high values (e.g., 30 for mild steel), and (2) in practically all metals the damping is not only a function of stress amplitude but also of the number of prior stress cycles. Thus, a different curve is required for each stress history. Stress history in this region may cause large changes in damping; e.g., a factor of 30 for mild steel, at its fatigue strength. The limit stress σ_L usually falls somewhat below the fatigue strength of the material.

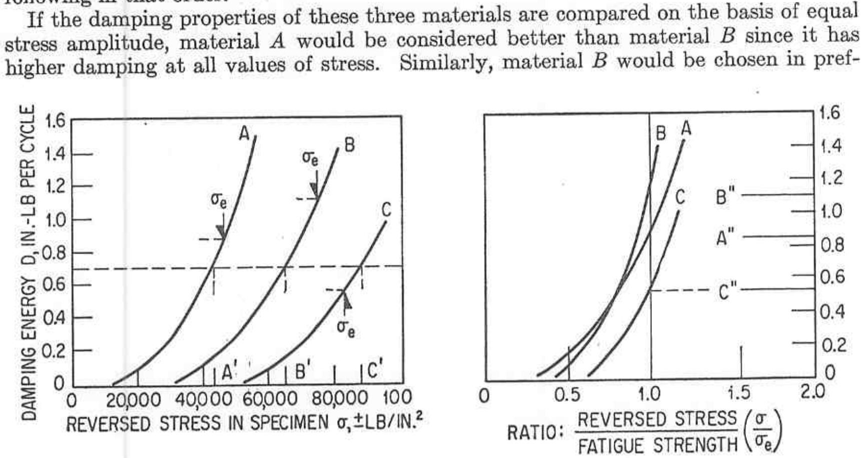
To facilitate comparisons between the reference damping units, loss factor η and D under uniform stress ($\alpha/\beta = 1$), the loss factor also is plotted in Fig. 36.15. Since the relationship between D and η depends on the value of E, a family of lines for the range of $E = 5 \times 10^6$ to 30×10^6 lb/in.^{2} is indicated for $\eta = 1$. The lines for the other values of η correspond to a value of $E = 15 \times 10^6$ lb/in.^{2}

COMPARISON OF VARIOUS MATERIAL DAMPING MECHANISMS AND REPRESENTATIVE DATA FOR ENGINEERING MATERIALS

The general qualitative characteristics of the various types of damping are summarized in Table 36.2 by comparing the effects of different testing variables. The data tabulated indicate that, in general, anelastic mechanisms do not contribute significantly to total damping at intermediate and high stresses; in these regions magnetoelastic and plastic strain mechanisms probably are the most important from an engineering viewpoint.

In comparing the damping properties of several materials for engineering design and particularly for resonant fatigue considerations, both the magnitude of damping energy at a given stress and the proximity of that stress to the fatigue strength should be considered. Two methods for comparing the damping properties of materials are shown in Fig. 36.16.[19] These are hypothetical curves representative of comparisons commonly observed in real materials. In Fig. 36.16A, the damping properties of three materials A, B, and C are shown as a function of stress (linear scales). The arrows marked σ_e indicate the fatigue strengths for the materials, C having the highest value and B and A following in that order.

If the damping properties of these three materials are compared on the basis of equal stress amplitude, material A would be considered better than material B since it has higher damping at all values of stress. Similarly, material B would be chosen in preference to material C. However, in many engineering applications the relative fatigue strengths of the three materials also must be considered. When operated near resonance, a part increases in stress until the energy input to the part is absorbed by means of damping. Thus, in choosing materials for high resonant fatigue strength, comparisons should be made not only on the basis of damping at a given stress but also on the basis of stress for equal energy absorption and the proximity of this stress to the fatigue strength. For example, assume that the amount of energy which the material is required to absorb is 0.7 in.-lb/cycle. The horizontal line drawn at this value of damping energy in Fig. 36.16A indicates that the three materials would be stressed to values of A', B', and C' (abscissa). In comparing these stresses with the corresponding fatigue strength, material C is stressed significantly above its fatigue strength.



(A) (B)

FIG. 36.16. Hypothetical curves for three materials to illustrate methods of comparing damping properties.

The relationship between damping and fatigue behavior for the three materials A, B, and C is replotted in Fig. 36.16B. In this graph, the abscissa is the ratio of the induced stress to the fatigue strength of the material. At stresses corresponding to the fatigue strength, materials A, B, and C have damping energy values given by A'', B'', and C'' in Fig. 36.16B. On comparing the relative damping properties of the three materials in this manner, material C is the least effective energy absorber at its fatigue strength. Material A has the highest damping at low stress ratios, but material B exhibits superior damping properties for values of stress ratio in excess of 0.75.

Damping vs. stress ratio data have been determined for a variety of common structural materials at various temperatures.[2] Some of these data are listed in Table 36.5 (all tests at 20 cpm). For a large variety of structural materials (not particularly selected for large magnetoelastic or plastic strain damping), the data are found to lie within a fairly well-established band shown in Fig. 36.17. The approximate geometric-mean curve

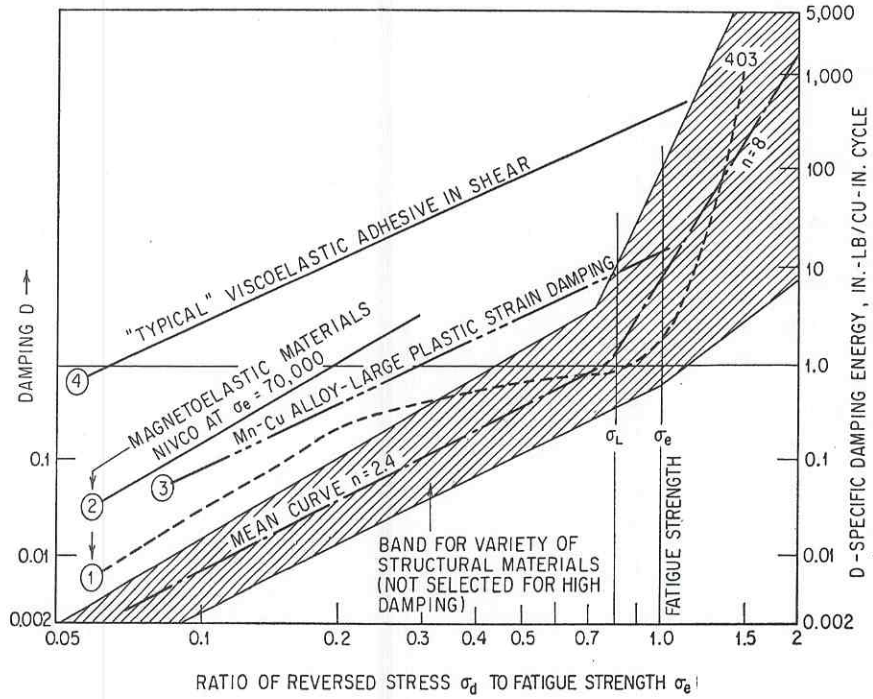


FIG. 36.17. Range of damping properties for a variety of structural materials. The shaded band defines the damping for most structural materials.

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Table 36.5. Damping, Elasticity, Fatigue, and Static Properties of Various Structural Materials

Type of material	Material (and test temp. if above room temp.)	Static properties: Mod. of elas. E, lb/in.^{2}	Static properties: Yield stress (0.2% offset), lb/in.^{2}	Static properties: Tensile strength, lb/in.^{2}	Fatigue behavior: Fatigue strength σ_e, lb/in.^{2}	Fatigue behavior: Cyclic stress limit σ_L, lb/in.^{2}	Fatigue behavior: Ratio σ_L/σ_e	Damping properties: Up to σ_L: J	Damping properties: Up to σ_L: n	D at σ_L	D at 60% σ_e	D at σ_e after $10^{1.3}$ cycles	D at σ_e after 10^6 cycles	D at 120% σ_e: $10^{1.3}$ cycles	D at 120% σ_e: Max. No. of cycles
Superalloys	Type 403	30.5×10^6	111×10^3	129×10^3	65×10^3	66×10^3	1.02	* 0.25×10^{-12}	2.9 *	1.40	0.85	1.4	1.4	5	10
	N-155	30.0	60.5	119	53	33	0.62	2.0	2.5	0.4	0.4	45	25	180	220
	N-155 (1350°F)	22.3	...	...	40	26	0.65	0.13	3.0	2.3	1.8	140	14	700	30
	N-155 (1500°F)	21.3	...	...	29	25	0.86	0.19	3.0	3	1.0	10	11	50	60
	Stellite 31 AC	32.9	65.1	92.6	38	24	0.63	0.14	2.9	0.7	0.6	8	0.9	50	20
	Stellite 31 AC (1200°F)	28.2	...	...	38	35	0.92	774	2.1	2.7	1.1	12	3	200	13
	Stellite 31 AC (1350°F)	26.4	...	...	32	33	1.03	4,040	2.0	4.4	1.6	4.2	4.2	23	11
	Stellite 31 AC (1500°F)	25.4	...	...	54	54	1.00	566	2.0	1.65	0.60	1.65	1.65	6	11
	Type 403 (700°F)	27.0	...	...	47	46	0.98	19.6	2.45	5.2	1.65	5.4	5.8	15	20
	Type 403 (900°F)	25.3	...	...	72	72	1.00	9.52 †	2.4 †	1.6	0.58	1.6	1.6	8	25
	Lapelloy	31.2	112	129	39.5	45	1.14	325	2.19	5.04	1.2	3.8	3.8	6	7
	Lapelloy (900°F)	25.6	...	...											
Titanium	RC130B	16.9	139	152	86	95	1.10	279	2.0	2.0	0.65	1.7	1.7	2.7	3.5
	RC130B (600°F)	14.5	...	...	62	50	0.81	2,000	1.9	1.7	0.90	1.9	5	4.4	25
	RC55 CW	14.6	81	86	45	15	0.33	102	2.1	0.06	1.15	6	20	13	100
	RC55 CW (600°F)	11.5	...	...	22	20	0.91	248	2.2	0.72	0.30	3.3	1.8	9.8	16
	RC55 A	13.8	57.0	75.7	41	24	0.59	278	2.0	0.16	0.17	10	150	50	750
	RC55 A (600°F)	10.6	...	...	20.5	10.5	0.51	635	2.0	0.07	0.5	38	10	65	40
Other ferrous materials	Sandvik (O & T)	29.2	177	204	92	100	1.09	8.9	2.3	2.8	0.8	2.3	2.3	4.5	30
	Sandvik (N)	29.2	116	186	76	55	0.72	0.61	2.6	1.3	0.8	12	70	100	220
	SAE 1020 steel	29.4	46.6	71.3	35.5	30	0.85	500	2.0	0.45	0.23	0.7	20	5	100
	Gray iron	19.4	...	20.3	9.5	6.5	0.69	494	2.4	0.7	0.5	2	1.2	3.3	2.3
Other nonferrous metals	24S-T4 aluminum	10.6	48.6	72.8	27	24	0.88	780	2.0	0.45	0.2	1	0.6	0.9	2.2
	J-1 magnesium	6.5	33.5	46	17	8	0.47	1,560	2.0	0.1	0.13	1.1	0.5	3.5	1
	Magnesium-silicon alloy	...	...	...	3.6	1.5	0.42	3,720	2.5	0.35	0.7	17	1.0		
	Manganese-copper alloy	...	60	90	19.0	18	0.95	12	2.82	12	3.3	13	13	25	21
Nonmetals	Glass laminate	3.4	...	40	10.5	18	1.6	4.2	2.9	...	0.5	2	2	3.3	3.3

* Up to σ = 12,000 lb/in.^{2}; above this value, n decreases. At σ = 20,000 lb/in.^{2}, n = 1.5; at σ = 30,000 lb/in.^{2}, n = 1.05; for values of σ > 40,000 lb/in.^{2}, n = 0.63.
† Up to σ = 14,000 lb/in.^{2}; at σ = 20,000 lb/in.^{2}, n = 1.7; at σ = 30,000 lb/in.^{2}, n = 1.5; at σ = 40,000 lb/in.^{2}, n = 1.3; at σ = 50,000 lb/in.^{2}, n = 1.3; at σ = 60,000 lb/in.^{2}, n = 1.45.

is shown. The equations for this two-segment curve are

$$D = 1.7\left(\frac{\sigma}{\sigma_e}\right)^{2.4} \qquad \left[\frac{\sigma}{\sigma_e} < 0.8\right]$$
$$D = 6.0\left(\frac{\sigma}{\sigma_e}\right)^{8} \qquad \left[\frac{\sigma}{\sigma_e} > 0.8\right] \tag{36.42}$$

However, the following single equation defines the mean damping with sufficient accuracy for engineering purposes:

$$D = \left(\frac{\sigma}{\sigma_e}\right)^{2.3} + 6\left(\frac{\sigma}{\sigma_e}\right)^{8} \tag{36.43}$$

The approximate bandwidth about this geometric mean curve for the various structural materials included in the band is as follows: from ⅓ to 3 times the mean value at a stress ratio of 0.2 or less; from ⅕ to 5 times at a ratio of 0.6; from ⅒ to 10 times at a ratio of 1.0; and 1/50 to 50 times the mean value at a ratio of 1.2.

Also shown in Fig. 36.17 for comparison purposes are data for four materials having especially high damping. Materials 1 and 2 are the magnetoelastic alloys Nivco 10 and 403. Nivco 10 retains its high damping up to the stresses shown (data not available at higher stresses). However, the 403 alloy reaches its magnetoelastic peak at a stress ratio of approximately 0.2 and increases less rapidly beyond this point; when plastic strain damping becomes dominant (at stress ratio of approximately 0.8), damping increases very rapidly. By contrast, material 3, a manganese-copper alloy with large plastic strain damping, retains its high damping up to and beyond its fatigue strength.[20] Material 4 is a "typical" viscoelastic adhesive (G_2 = 138 lb/in.^{2}), assuming that the permissible cyclic shear strain is unity (experiments show that a shear strain of unity does not cause deterioration in this adhesive even after millions of cycles).[21] The magnetoelastic material has a damping thirty times as large as the average structural material in the stress range shown in Fig. 36.17, and the viscoelastic damping is over ten times as large as the magnetoelastic damping.

The range of D observed for common structural materials stressed at their fatigue limit is 0.5 to 100 in.-lb/in.^{3}/cycle, with a mean value of 7.0. For materials stressed at a rate of 3,600 cpm under uniform stress distribution (tension-compression), 1 in.^{3} of a typical specimen will safely absorb and dissipate 0.064 hp (48 watts). Some high-damping materials (those having D = 100 at the fatigue limit) can absorb almost 1.0 hp (746 watts)/in.^{3} in the safe stress range, assuming no significant frequency or stress-history effects.

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Figure 37.1 shows what can be accomplished at resonance by application of a commercial damping material to a circular 18-gage steel panel mounted in rigid clamping rings of 12-in. internal diameter. The panel was driven through resonance by an electromagnetic exciter actuated by an audio oscillator; then, the sound pressure level was measured 3 in. away. A 20-db reduction of sound level at resonance is demonstrated, with practically no reduction of sound level at frequencies away from resonance. The decrease of resonant frequency after application of treatment is due mostly to an increase in the composite mass of the treated panel without compensating increase of stiffness.

The curves of Fig. 37.1 represent the frequency response of a distributed system in its fundamental flexural mode of vibration. The resemblance to the forced-vibration response of a lumped spring-mass system lends plausibility to the idealization of the normal modes of distributed systems by equivalent masses and springs. Since such idealization can be rigorously justified (see below), the mathematical treatment of viscous damping acting on lumped-parameter systems becomes applicable to isolated mode response in distributed systems, and the various concepts and measurement parameters that are useful in specifying damping in the single degree-of-freedom lumped-parameter system find their usefulness greatly extended. For reference purposes, the relations between the common quantities are summarized in Table 37.1. In terms of the response of the simple spring-mass-dashpot systems, the following definitions are applicable:

c/c_c = fraction of critical damping for viscous or equivalent-viscous damping, where c is the damping constant of the dashpot acting on the mass m in parallel to the spring stiffness k and c_c is the critical damping constant as given by Eq. (2.12)

η = loss factor of spring with complex stiffness given by $k(1 + j\eta)$

κ = decay constant of system in free vibration given in terms of viscous damping by $\kappa = c/2m$

Δ = logarithmic decrement of free vibration of system defined in terms of successive peak displacements during decay

$\mathbf{D}$ = decay rate of free vibrations expressed in decibels per second during exponential decay of amplitude

Δf = bandwidth of steady-state resonance of system as related at the half-power points

Q = quality factor of steady-state resonance of system as related to the resonant amplification factor, A_R

ψ = specific damping capacity of system in terms of ratio between energy dissipated per cycle and elastic energy-storage capacity of the system

The first two quantities, c/c_c and η, are dimensionless parameters for specifying the energy-dissipation capacity of a spring-mass system in general relation to dynamic response. The next three quantities, κ, Δ, and $\mathbf{D}$, constitute properties of the motion of the system that are observable during the decay of free vibration at the damped natural period of the system. The last three quantities, Δf, Q, and ψ, constitute properties of the steady-state response of the system that are observable from measurements made during forced vibration in the neighborhood of resonance. The interrelations displayed in Table 37.1 will be shown to be useful in interpreting and discussing damping measurements involving practical distributed systems, despite their definition in terms of single degree-of-freedom system response.

In practice, the existence of resonances in a machine can be detected aurally by making a slow continuous variation of the speed of a machine from a frequency considerably below the normal operating range to a frequency considerably above. If, as speed is increased, definite increases in loudness of the noise from the machine are noted at certain critical speeds (assuming the elimination of standing-wave effects by averaging through several observation points), this is probably due either to excitation of different resonances or to successive excitation of a single resonance by different exciting components. When resonances are so prominent that they can be detected by loudness changes in this way, the large amplitude of the resonating part will permit its easy location.

More frequently, however, resonances will manifest themselves during speed change by a change in the quality of the sound emitted as the resonant component appears from

Table 37.1. In

	c/c_c
c/c_c	c/c_c
η	$\eta/2$ $\eta \ll 1$
κ	κ/ω_R
Δ	$\Delta/2\pi$ $\Delta \ll 2\pi$
$\mathbf{D}$	$\mathbf{D}/a\omega_R$
Δf	$\pi\,\Delta f/\omega_R$ $\Delta f \ll \omega_R/\pi$
Q	$1/2Q$ $2Q \gg 1$
ψ	$\psi/4\pi$ $\psi \ll 4\pi$

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Table 37.1. Interrelations among Common Damping Quantities Idealized for Lumped-parameter Systems of One Degree-of-freedom

$a = 20 \log e = 8.68 \qquad \omega_R = 2\pi f_R \qquad {\omega_N}^2 = {\omega_R}^2 - \kappa^2$

	c/c_c	η	κ	Δ	D	Δf	Q	ψ
c/c_c	c/c_c	$2c/c_c$ $c \ll c_c$	$\omega_R c/c_c$	$2\pi c/c_c$ $c \ll c_c$	$a\omega_R c/c_c$	$\omega_R c/\pi c_c$	$c_c/2c$	$4\pi c/c_c$ $c \ll c_c$
η	$\eta/2$ $\eta \ll 1$	η	$\omega_R\eta/2$ $\eta \ll 1$	$\pi\eta$ $\eta \ll 1$	$a\omega_R\eta/2$ $\eta \ll 1$	$\omega_R\eta/2\pi$ $\eta \ll 1$	$1/\eta$ $\eta \ll 1$	$2\pi\eta$ $\eta \ll 1$
κ	κ/ω_R	$2\kappa/\omega_R$ $\kappa \ll \omega_R$	κ	$2\pi\kappa/\omega_N$	$a\kappa$	κ/π	$\omega_R/2\kappa$ $\kappa \ll \omega_R$	$4\pi\kappa/\omega_R$ $\kappa \ll \omega_R$
Δ	$\Delta/2\pi$ $\Delta \ll 2\pi$	Δ/π $\Delta \ll 2\pi$	$\omega_N\Delta/2\pi$	Δ	$a\omega_N\Delta/2\pi$	$\omega_N\Delta/2\pi^2$	$\pi/(\Delta - \Delta^2)$ $\Delta \ll 1$	$2(\Delta - \Delta^2)$ $\Delta \ll 1$
D	$\mathrm{D}/a\omega_R$	$2\mathrm{D}/a\omega_R$ $\mathrm{D} \ll a\omega_R$	D/a	$2\pi\mathrm{D}/a\omega_N$	D	$\mathrm{D}/a\pi$	$a\omega_R/2\mathrm{D}$ $\mathrm{D} \ll a\omega_R$	$4\pi\mathrm{D}/a\omega_R$ $\mathrm{D} \ll a\omega_R$
Δf	$\pi\,\Delta f/\omega_R$ $\Delta f \ll \omega_R/\pi$	$2\pi\,\Delta f/\omega_R$ $\Delta f \ll \omega_R/\pi$	$\pi\,\Delta f$	$2\pi^2\,\Delta f/\omega_N$	$a\pi\,\Delta f$	Δf	$\omega_R/2\pi\,\Delta f$ $\Delta f \ll \omega_R/\pi$	$4\pi^2\,\Delta f/\omega_R$ $\Delta f \ll \omega_R/\pi$
Q	$1/2Q$ $2Q \gg 1$	$1/Q$ $2Q \gg 1$	$\omega_R/2Q$ $2Q \gg 1$	π/Q $2Q \gg 1$	$a\omega_R/2Q$ $2Q \gg 1$	$\omega_R/2\pi Q$ $2Q \gg 1$	Q	$2\pi/Q$
ψ	$\psi/4\pi$ $\psi \ll 4\pi$	$\psi/2\pi$ $\psi \ll 4\pi$	$\omega_R\psi/4\pi$ $\psi \ll 4\pi$	$\psi/2$ $\psi \ll 4\pi$	$a\omega_R\psi/4\pi$ $\psi \ll 4\pi$	$\omega_R\psi/4\pi^2$ $\psi \ll 4\pi$	$2\pi/\psi$	ψ

and disappears into the background noise. In discrete-frequency noise and vibration spectra measured at different speeds, the existence of resonance is made known by undue prominence of successive noise components as the speed passes through values that excite the fixed frequencies of resonance. The possibility must not be overlooked that some resonance may occur at each speed by correspondence of some of the many excitation components with any of the large number of natural frequencies possible. This makes the continuously changing character of the noise the only clue to resonance as speed is changed, because total loudness may be comparatively unchanged until the several resonances have been mitigated. The absence of discrete peaks in narrow-band vibration and noise spectra is no assurance of the absence of resonant conditions (see below). Resonances can be excited and may require damping when the system is driven either by broad-band mechanical excitation or by impinging airborne sound.

DIRECT IMPULSE EXCITATION AND SHOCK EXCITATION

Whenever a blow is delivered to a solid body of any kind, its natural frequencies are excited. The inherent damping or vibration-damping treatment in this case determines both the length of time each mode of vibration continues to persist and, to a lesser extent, the amplitude of the initial excitation. As decay proceeds, the higher modes of vibration generally disappear more rapidly and the fundamental mode of vibration persists, especially in simple systems like unconstrained bars and plates.

In sheet-metal structures and housings, the impulse sound produced by the motion of an undamped panel is characterized by what is called "tinniness." In the consumers' mind, such noise has come to be associated with cheap, flimsy construction. For this reason, vibration-damping treatments are used in many commercial products, such as metal furniture and fixtures.

The rating of automobile body panels and similar pressed steel constructions, by means of the quality of sound resulting from a blow with the knuckles, is particularly subject to abusive comparisons when the quality of products comes under question. The automotive damping treatment is a multipurpose application required to minimize

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material composite or a structural joint. In Fig. 2.5 the properties of the total specimen S are designated by the subscript s. Thus, D_s is the energy dissipated by the entire member S (made of a material having a unit damping value of D) and U_s is the total elastic energy stored in the specimen for which U is the unit strain energy.

The unit damping properties of the material may generally be determined directly from the properties of the entire specimen. The relationships between material properties and specimen properties are given in Chapter VI for a specimen or member made of macroscopically uniform materials. Using these relations the unit damping energy D of the material can be determined from the total damping energy D_s and the unit strain energy U from the total strain energy U_s.

The principal units for defining material, specimen, composite structures, and dynamic systems are tabulated in Fig. 2.6. Nomenclature for the properties of materials and members is tabulated in rows I A, B, and C. The properties of composites, structures, etc. (see Section 2.3), are identified under major row II in Fig. 2.6. Nomenclature for the response of typical systems is decribed in rows III D, E, and F. This response may be temporal decay (row D), spatial attenuation (row E), or near resonant response (row F).

Nomenclature for Linear Members and Materials. Under sinusoidal loading the hysteretic loops for linear materials and members are elliptical in shape and increase in area with the square of stress or strain amplitude. Of all the units used to define the properties of linear materials the complex notation of linear viscoelasticity is in the most widely used, has a sound mathematical basis, and has been agreed upon as a standard nomenclature (57 Le; also see COMPLEX in Appendix C). The complex modulus, compliance, and other notations which rheologists now consider quite conventional for linear viscoelastic materials not only provide a basic, minimum and mathematically consistent system, but one which is very useful in the practical sense.

In order to clarify further the difference between specimen and material properties, we consider first the properties of a *linear* specimen such as shown in Fig. 2.5. The spring constant of such a linear specimen can be defined in complex notation as follows:

$$k_s^* = k_s' + ik_s'' \qquad \text{(Eq. d)}$$

where k_s^* = complex spring constant of the specimen or member; k_s' = elastic spring constant or storage constant of spring; k_s'' = loss constant of spring.

Some of the features of complex notation for linear materials are discussed in detail in Section 3.4 for Voigt viscoelasticity and some of the differences in notation required for non-Voigtian units are reviewed in Section 3.5.

Generally, the unit complex stiffness properties or moduli of a material can be determined from the overall complex stiffness k_s^* of a specimen of the material if the type of loading and stress distribution in the specimen are known. Under simple loading conditions the elementary equations of the theory of elasticity are appropriate. Under uniform normal load:

$$\sigma = P/A_c = \varepsilon E = eE/l = ek_s/A_c. \qquad \text{(Eq. e)}$$

Thus $$E^* = (l/A_c)\,k_s^*,\ E' = (l/A_c)\,k_s',\ \text{and}\ E'' = (l/A_c)\,k_s''. \qquad \text{(Eq. f)}$$

Under torsional and flexural loads similar equations apply:

$$\tau = M_z c/I_z = \gamma G = c\theta G/l$$
$$\sigma = M_x c/I_x. \qquad \text{(Eqs. g)}$$

Using such elementary equations, or more refined approaches as necessary, the properties k_s^* of the elasto-dissipative specimen can be reduced to the unit moduli of materials, such as E for Young's Modulus (normal stress), and G for the shear modulus. These are defined below in complex notation.

$$E^* = E' + iE'' \qquad \text{(Eq. h)}$$
$$G^* = G' + iG'' \qquad \text{(Eq. i)}$$

where: E^* and G^* are the complex moduli of the material; E' and G' are the elastic or storage moduli; E'' and G'' are the loss moduli. Bulk modulus K^* and longitudinal modulus M^* can be defined in a similar manner.

Another unit frequently used to define the damping properties of linear (and nonlinear) materials is relative damping energy which is the ratio of damping energy to strain energy given below:

$$\text{loss coefficient of a material} = \eta = D/2\pi U$$
$$\text{loss coefficient of a member} = \eta_s = D_s/2\pi U_s. \qquad \text{(Eqs. j)}$$

For linear materials the loss coefficient is independent of stress amplitude as shown by the following equation:

$$\eta = \frac{J\sigma_a^2}{2\pi\sigma_a^2/2E} = \frac{JE}{\pi}. \qquad \text{(Eq. k)}$$

Since η may exceed unity for viscoelastic materials (see row 7 of Fig. 2.4), a distinction must be made between k_s^* and k_s' and between E^* and E'. For example, if $\eta > 1$, then $|E^*| > 1.4E'$ (see equations given in Section 3.4). Complex notation provides an excellent nomenclature for linear materials, such as polymers, that have large damping.

Linear damping phenomena in metals at low stress (anelasticity) can also be defined in terms of complex notation similar to that used in linear viscoelasticity. However, in contrast with polymeric damping, anelastic damping is usually very small (η is almost always less than 0.1 and generally less than 0.01). Thus, a distinction between E^* and E' is usually unnecessary in anelastic materials (if $\eta < 0.1$, then E^* is within 1% of E'). The units η and E (where $E \simeq |E^*| \simeq E'$) are generally used to specify the anelastic and elastic properties of a metal.

Anelasticity studies have been made mostly by solid-state metallurgists concerned with internal friction as a macrostructural research tool. Relative energy units have been used almost exclusively for this purpose, the unit $Q^{-1} = \eta$ (or $Q = 1/\eta$) being the most popular. For anelastic and other linear phenomena no distinction needs to be made between Q and Q_s (since $Q \simeq Q_s \simeq 1/\eta \simeq 1/\eta_s$ as reviewed in Fig. 2.6 and Section 6.6). Since η has been adopted as the standard energy ratio unit in this monograph, it will be used hereafter for presenting data on anelastic damping.

Nomenclature for Nonlinear Members and Materials. The rheological equations and nomenclature for nonlinear materials have not been brought to a mathematical level comparable to that attained for linear materials. Many of the procedures and assumptions (equation of state, stable structure, linear viscosity, etc., see Section 4.2) that have proved successful for linear viscoelastic materials are generally unrealistic for structural materials, particularly at stress levels of interest in structural mechanics. Simple elliptical loops are generally not observed and complex notation is generally inappropriate.

The two types of units generally used for defining the damping properties of nonlinear materials are damping energy dissipation per cycle of stress (see row A of Fig. 2.6) and ratios of damping energy to strain energy (see row C). Such energy units are very useful for certain types of engineering computations (if specified in unique, meaningful terms) but are less satisfactory than the more compact notation of linear viscoelasticity theory. Furthermore, the damping energy units specify only the area within a stress–strain hysteresis loop and not its shape (structural metals at high stress, for example, generally display hysteresis loops with sharp ends rather than ellipses characteristic of linear viscoelasticity). Several of the curves shown in Fig. 2.2 have approximately the same values for D_s (same area) although they are significantly different in shape. In the analysis of the dynamic response of nonlinear systems, for example, both the area within the hysteretic loop and its shape are important. In this context energy units do not completely define hysteretic effects. However, in spite of the limitations, damping energies D or D_s appear to be the most satisfactory nomenclature now available for nonlinear materials for most types of response problems.

Metals and other structural materials sometimes display quadratic damping (damping exponent $n = 2$) particularly at low and intermediate stress. Thus, the relative energy units are independent of stress. However, if the hysteretic loops are pointed, rather than elliptical (see Fig. 2.2), this type of damping cannot be categorized as truly linear. Nevertheless, for analytic

purposes proportional damping of this type can be approximated by complex notation. Aeroelastic and other types of structural vibration problems have been approached in this manner (51 S, 56 My, 60 Bis, also see 5.5). In many cases the damping is sufficiently small so that $E^* \simeq E'$, and the damping properties can be specified in terms of the relative energy unit.

Most structural materials at intermediate and high stress display nonquadratic damping, and in some cases a very high degree of nonlinearity exists ($n = 15$ and more). This means that the relative damping units are often critically dependent on amplitude of stress or strain. Furthermore, conventional strain energy definitions are not satisfactory for high-damping, nonlinear materials (see Section 4.7). On the other hand, if such damping properties were to be approximated by complex notation, E'' and in some cases E' would be a function of amplitude. In view of these difficulties relative damping units and complex notation appear to be less satisfactory for highly nonlinear materials than unit damping energy D.

Summary of Units for Members and Materials. Of the many units and nomenclature used for designating the damping properties of materials those listed in Fig. 2.6, rows A, B, C, and summarized below are considered standard in this monograph.

(A) *Absolute Damping Energy Units* (applicable to both linear and nonlinear materials)

Unit value for a material (damping energy per unit volume): D in units of in-lb/in^3-cycle.

Damping energy dissipated in total specimen or member: D_s in units of in-lb/cycle.

(B) *Complex Moduli* (generally applicable to linear materials only)

Unit property of materials, such as:

$$E^* = E' + iE'' \quad \text{(units of lb/in}^2\text{)} \qquad \text{(see Eq. h)}$$

$$G^* = G' + iG'' \qquad \text{(see Eq. i)}$$

Property of total specimen or member:

$$k_s^* = k_s' + ik_s'' \qquad \text{(see Eq. d)}$$

(C) *Relative Energy Units* (applicable to both linear and nonlinear materials)

These are dimensionless ratios of damping energy to strain energy.

Unit properties of material:

$$\text{loss coefficient } \eta = D/2\pi U \qquad \text{(see Eqs. j)}$$

$$\text{quality factor } Q = 2\pi U/D = 1/\eta \qquad \text{(Eq. l)}$$

where U = unit strain energy of the material at maximum strain.

A. Slocum, Precision Machine Design, Prentice Hall, c 1992.

the thermal diffusivity of the material. In addition, the manner in which a part is made (e.g., a solid section from granite compared to a hollow section from steel), will affect the temperature gradient across the part and hence the thermal deformations of the part. For example, when subject to a given heat flux across its height, similar shaped beams designed for use in a coordinate measuring machine experienced the following bending deformations (normalized to granite): $\delta_{\text{Granite}} = 1.00$, $\delta_{96\%\ \text{Alumina (cored)}} = 0.60$, $\delta_{\text{Solid aluminum}} < 0.10$, $\delta_{\text{Hollow aluminum}} = 0.25$, $\delta_{\text{Hollow steel}} = 1.80$.[52] As discussed in Section 2.3.5, often the best strategy for control of thermal deformations is to isolate heat sources and actively cool them, insulate the structure, maximize thermal conductivity of structural elements, and actively attempt to control the temperature of the machine and the environment.

Manufacturability

A design engineer must integrate part configuration with material choice and manufacturing methods, particularly when the part will be produced in high volumes. Ideally, a design engineer would be familiar with all materials and manufacturing processes, so the situation would never arise where a part could not be manufactured, or a better configuration bypassed because the design engineer thought the part could not be manufactured when in reality it could. Fortunately, material manufacturers are usually well aware of different manufacturing methods and are usually happy to help with the selection of materials and manufacturing methods.

7.3.3 Material Damping[53]

The effect of material damping can be readily observed by placing your ear against a desk and then hitting it (i.e., the desk) and listening to the sound as it decays. In a machine tool, vibrations can be induced by cutting action or by some other excitation mechanism (e.g., a rotating component that is slightly out of balance) that causes the toolpoint to move as it passes by; hence it is very important to build a structure that has high damping to minimize this effect. Vibrations in a structure are dampened by energy losses in the material and in the interfaces between components.[54]

Although it has been extensively studied, the mechanism of damping in a material is difficult to quantify and one must generally rely on empirical results.[55] In fact, damping is highly dependent on alloy composition, frequency, stress level and type, and temperature. Structural damping levels are often quite low, and frequently the dominant source of damping is the joints in an assembly. In fact, one must be extremely wary of damping data that is presented in the literature, because often it is presented without a discussion of the design of the test setup.

There are several damping quantifiers that are used to describe energy dissipation in a structure. The quantifiers include:

- η Loss factor of material
- η_s Loss factor of material (geometry and load dependent)
- A_r Resonance amplification factor
- ϕ Phase angle ϕ between stress and strain (hysteresis factor)
- δ_{Ld} Logarithmic decrement[56]
- ΔU The energy dissipated during one cycle
- ζ The damping factor associated with second order systems

52 K. H. Breyer and H. G. Pressel, "Paving the Way to Thermally Stable Coordinate Measuring Machines," Progress in Precision Engineering, P. Seyfried, et al. (Eds.), Springer-Verlag, New York, 1991, pp. 56-76.

53 *"The danger of more or less perpetual vibration of significant magnitude is one of the bugbears of designers of accurate instruments, and research leading to some practical data on this subject for various types of members is urgently required."* T. N. Whitehead.

54 Mechanical dampers (e.g, shear and tuned mass dampers) are discussed in Section 7.4.1. Joint damping is discussed in Section 7.5 (e.g., see Figure 7.5.8).

55 A discussion of the many different microstructural mechanisms that generate damping in materials is beyond the scope of this book. For a detailed discussion see B. J. Lazan, Damping of Materials and Members in Structural Mechanics, Pergamon Press, London, 1968.

56 Most texts on vibration refer to the log decrement as δ; however, to avoid confusion with discussions on displacement termed δ, the log decrement will be referred here to as δ_{Ld}.

The various damping terms are related in the following manner:

$$\eta = \frac{1}{A_r} = \frac{\delta}{\pi} = \phi = \frac{\Delta U}{2\pi U} \tag{7.3.5}$$

Condition	Damping energy integral α	Strain energy integral β
Tension/compression	1	1
Rectangular beam (uniform bending)	$\frac{2}{n+2}$	0.5
Cylindrical beam (uniform bending)	$\frac{1}{n+1}$	0.33

Figure 7.3.2 Stress distribution and damping functions. Note that b/a = 1 for all cases if n = 2. (After Lazan.)

The loss factor η_s can be determined experimentally by subjecting a specimen to various frequencies and stresses while measuring the amplification. This allows for the damping to be determined as a function of frequency and stress. The loss factors are equated by the following relation:

$$\eta = \eta_s \frac{\beta}{\alpha} \tag{7.3.6}$$

where α and β are functions of load and geometry as shown in Figure 7.3.2. The factor n is a measure of the stress in the material. When n = 2.0, the material is subject to a low stress.

The phase angle is the ratio of the apparent modulus of elasticity E_2 at low frequencies and the apparent modulus of elasticity E_1 at high frequencies:

$$\phi = \frac{E_2}{E_1} \tag{7.3.7}$$

The logarithmic decrement δ_{Ld} is a measure of the relative amplitude between N successive oscillations of a freely vibrating system (one struck by an impulse):

$$\delta_{Ld} = \frac{-1}{N} \log_e \left(\frac{a_N}{a_1}\right) \tag{7.3.8}$$

The logarithmic decrement can also be related to the damping factor ζ, velocity damping factor b, mass m, and natural frequency ω_n of a second order system model:

$$\zeta = \frac{\delta_{Ld}}{\sqrt{4\pi^2 + \delta_{Ld}^2}} \tag{7.3.9}$$

$$b = 2m\zeta\omega_n \tag{7.3.10}$$

Note that the amplification at resonance of a second order system is given by

$$A_r = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} \quad (\zeta \leq 0.707) \tag{7.3.11}$$

Damping values for various materials are given in Table 7.3.2. The amount of damping one obtains from a material is very low compared to the amount of damping that one can obtain with the addition of a damping mechanism. Damping mechanisms can range from simple sand piles to more complex shear dampers or tuned mass dampers as discussed in Section 7.4.1.

Material	Load	T_1 (°K)	T_2 (°K)	σ_1 (ksi)	σ_2 (ksi)	f_1 (Hz)	f_2 (Hz)	ζ_1	ζ_2	A_{r1}	A_{r2}
Alumina								5.00E-06	1.50E-05	100000	33300
Aluminum (6063-T6)	bending			1	6			2.50E-04	2.50E-03	2000	200
Aluminum (pure annealed)	axial	50	300					3.50E-06	1.00E-05	143000	50000
Beryllium (18.6%Be)	unspec.			2	50			7.50E-03	4.10E-01	66.7	1.3
Copper (brass)	bending					50	600	1.50E-03	3.00E-03	333	167
Copper (pure annealed)	bending					20	550	3.50E-03	1.00E-03	143	500
Glass	bending					10	100	1.00E-03	3.00E-03	500	167
Granite (Quincy)	bending					140	1600	2.50E-03	5.00E-03	200	100
Iron (cast, annealed)	bending					100	2000	6.00E-04	1.50E-03	833	333
Iron (mild steel)	bending			2.5	5.5			4.50E-04	7.00E-04	1110	714
Lead	bending					20	160	4.00E-03	7.00E-03	125	71.4
Polymer concrete	bending							3.50E0-03		143	
Portland cement concrete	bending							1.20E-02		41.7	
Quartz (ground, piezo)	unspec.					65k		5.00E-06		100000	
Sand (loose on an Al beam)											
beam alone	bending					1000	4000	1.00E-03		500	
50% wt. layer of sand	bending					1000	4000	4.00E-02	9.95E-02	12.5	5.1
100% wt. layer of sand	bending					1000	4000	9.95E-02	4.10E-01	5.1	1.3
Silica (fused, annealed)	axial	73	1073					5.00E-07	5.00E-05	1000000	10000
Silicon nitride (n)	unspec.							1.25E-05		40000	
Soil (misc.)	unspec.					6	30	4.99E-02		10.0	

Table 7.3.2 Damping factors for a various materials for $\beta/\alpha = 1$.[57]

7.3.4 Environmental Properties

In addition to being able to withstand mechanical loads, a part must be configured and a material chosen to ensure that performance in adverse environments will be satisfactory. A precision machine may operate in a clean temperature-controlled room, but thermal performance and corrosion resistance must still be considered.

Thermal Properties[58]

Table 7.3.1 listed common precision engineering materials and their thermal properties. The thermal conductivity is a measure of how well heat is conducted through the material. Materials with low thermal conductivities tend to develop hot spots or large temperature gradients. As illustrated in Section 2.3.5, gradients cause bending moments, which lead to Abbe errors. So even if a material has a very low coefficient of thermal expansion (e.g., Invar or Zerodur), if it is subjected to large thermal gradients or local heat sources, a part made from it may deform more than if the part where made from a material which diffuses heat well (e.g., aluminum). In practice, however, precision instruments rarely encounter such large gradients, so a minimal coefficient of thermal expansion is often the dominant property that affects material choice. The specific heat of a material is a measure of how much thermal energy can be stored in the material. Along with the thermal conductivity, this allows the design engineer to determine how long it may take for the part to reach thermal and hence dimensional equilibrium.[59] Radiation coupling is governed by the surface geometry, temperature difference, and thermal emissivity. The latter is a strong function of the surface finish and chemistry. The decision on which material to use based on thermal growth considerations may require careful finite element modeling or a simple experiment to simulate the operating environment and machine configuration.

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57 Most of the information in this table is from the earlier reference by Lazan. The value for polymer concrete was provided by Jack Kane of Gandalf Inc., 206 San Jose Drive, Dunedin, Florida 34698.

58 Although this section on thermal properties may seem short, one should remember the emphasis placed on thermal errors in Chapter 2. Ideally, every machine designer should have taken a good course in heat transfer.

59 See the conceptual design case study in Section 8.8.2.