System Identification Overview & Session Goals

GWADW 2015

Controls Session A: System Identification

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What is System Identification (SID)?

- SID is the process of developing or improving a mathematical representation of a physical system using experimental data
  - Black box or grey box
    (Physical insight must be used to develop the underlying model structure)
  - Time domain and frequency domain
  - Model reduction
  - Plant, or plant + controller
  - Open loop, closed loop
  - Experiment design
- SID for structural dynamics
  - Modal parameter identification *(the focus of this talk)*
  - Structural-model parameter identification *(motivation given in this talk)*
  - Control-model identification

What does SID offer?

- Does adaptive control negate the need for SID?
  - No; Adaptive techniques are essentially recursive identification algorithms applied to a specific model structure
- Does robust control design render SID obsolete?
  - No; Robust control design requires a description of the model/plant uncertainty that SID can provide.

SID Process

- Basic steps:
  - Build a physics-based model
  - Design the experiment
  - Collect the data
  - Select, filter, de-trend the data, as appropriate
  - Fit the model
  - Validate the model (compare prediction to data)
  - Iterate until converged on a validated model
- Software tools:
  - Matlab Sys ID Toolbox (Ljung)
  - Matlab Frequency-Domain Identification (Kollar)
  - Matlab System/Observer/Controller Identification Toolbox (SOCIT)
  - …

SID for structural dynamics

- Modal parameter identification
  - *the focus of this talk*
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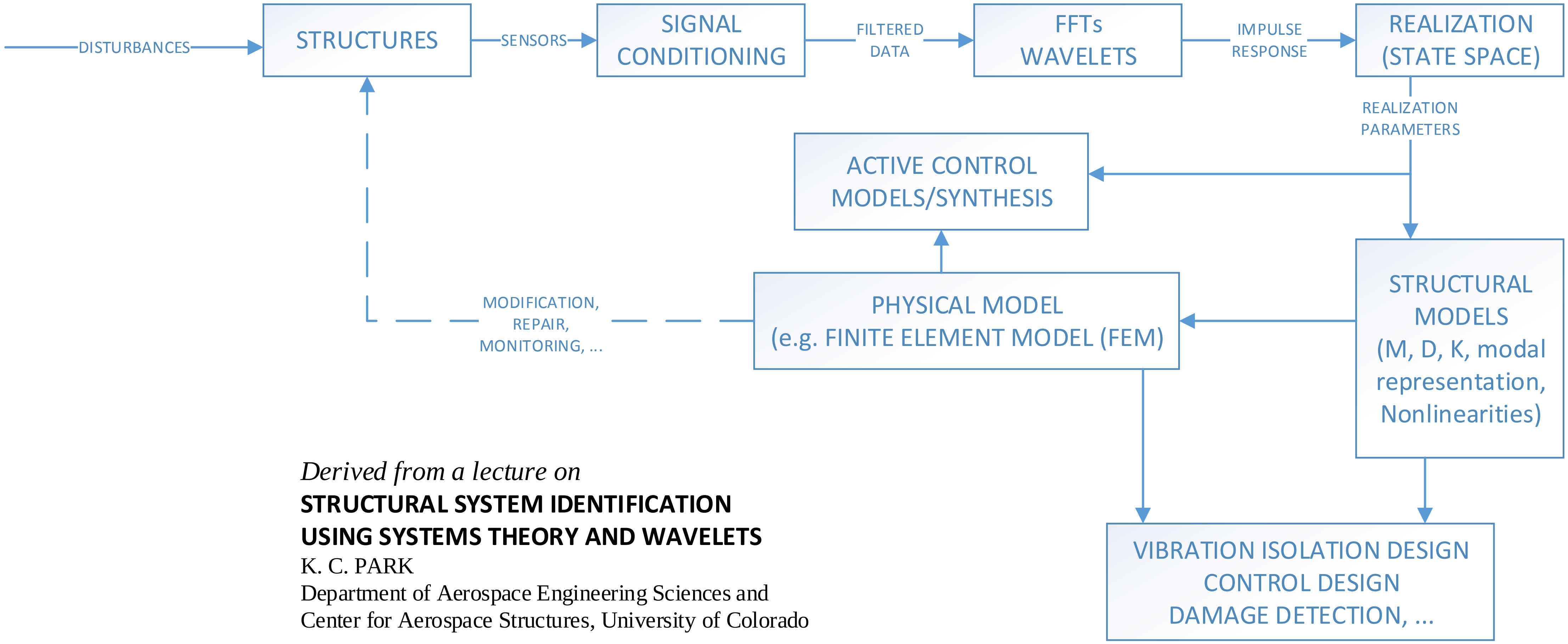
Structural Modeling & SID



Derived from a lecture on
STRUCTURAL SYSTEM IDENTIFICATION
USING SYSTEMS THEORY AND WAVELETS
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Continuous-Time, Finite Dimensional, Linear Dynamic System Model

$$M\ddot{q} + D_0\dot{q} + Kq = B_0 u$$

$$y = C_{0q} q + C_{0v}\dot{q}$$

where:

u is the input force vector (length s)

$q = \{x_1, y_1, z_1, Rx_1, Ry_1, Rz_1, x_2, y_2, z_2, Rx_2, Ry_2, Rz_2, \ldots x_n, y_n, z_n, Rx_n, Ry_n, Rz_n\}$ is the nodal displacement vector for the n nodes of the model (length is n_d = 6 n, the number of degrees of freedom)

$\{x_i, y_i, z_i, Rx_i, Ry_i, Rz_i\}$ are the 6 degrees of freedom for node i

M is the mass matrix (size n_d x n_d)

D_0 is the damping matrix (size n_d x n_d)

K is the stiffness matrix (size n_d x n_d)

B is the input matrix (size n_d x s)

y is the output vector (size r x 1)

C_{0q} is the output displacement matrix (size r x n_d)

C_{0v} is the output velocity matrix (size r x n_d)

Continuous-Time State-Space Model

The solution of the eigenproblem,

$$(K - \omega^2 M)\phi e^{j\omega t} = 0$$

results in n lowest frequency modes, with frequencies $\{\omega_1, \omega_2, \ldots \omega_n\}$, and mode shapes $\{\phi_1, \phi_2, \ldots \phi_n\}$. Defining

$\Omega = diag(\omega_1, \omega_2, \ldots \omega_n)$, the matrix of natural frequencies

$\Phi = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_n]$, the modal matrix

The modal matrix is used to diagonalize the mass and stiffness matrices, M and K:

$$M_m = \Phi^T M \Phi$$

$$K_m = \Phi^T K \Phi$$

$$Z = \left(\tfrac{1}{2}\right) M_m^{-1} D_m \Omega^{-1}$$

where the modal damping, D_m, is generally estimated and not transfo

Re-casting the governing equation in terms of a state vector,

$$x = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}$$

One obtains a state space version of the system:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

where

$$A = \begin{bmatrix} 0 & I \\ -\Omega^2 & -2Z\Omega \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ M_m^{-1}\Phi^T B_0 \end{bmatrix}$$

$$C = \begin{bmatrix} C_{0q}\Phi & C_{0v}\Phi \end{bmatrix}$$

$D = [0]$ (here I've chosen to set D to zero, as is typically the case)

Once the eigensolution results (Ω, Φ, M_m) have been read into Matlab, a modal based state space model can be created with the above equations for A, B, C and D.

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Discrete-Time, State-Space, Observer Model

- *Discrete-time:*

$$x(k+1) = A\,x(k) + B\,u(k)$$
$$y(k) = C\,x(k) + D\,u(k)$$

k = 0, 1, 2, …

where

$$A = e^{A_c\,\Delta t}$$

$$B = \int_0^{\Delta t} e^{A_c\,\tau}\,\partial\tau\,B_c$$

A_c, B_c are the continuous versions

- *Since the state vector is (generally) not accessible, a state estimator (observer) is added:*

$$x(k+1) = \bar{A}\,x(k) + \bar{B}v(k)$$
$$y(k) = C\,x(k) + D\,u(k)$$

where

$$\bar{A} = A + GC$$
$$\bar{B} = [B + GD \;\; -G]$$
$$v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}$$

System Realization Theory

- Construct a model with controllability & observability
- Seek a minimum realization (smallest state space dimensions)
- Time domain models for modal parameter identification are based on the transfer function matrix, which yields Markov parameters (i.e. pulse response)

  $x(k+1) = A\,x(k) + B\,u(k)$

  $y(k) = C\,x(k) + D\,u(k)$

  Let $u_i(0) = 1\ (i = 1, 2, \ldots, r)$ and $u_i(k) = 0\ (k = 1, 2, \ldots)$

  Pulse response matrix Y (m x r):

  $Y_0 = D;\ Y_1 = CB;\ Y_2 = CAB; \ldots;\ Y_k = CA^{k-1}B$

  where Y are the Markov parameters
- A realization is the computation of the triplet [A, B, C] (D = Y_0)

System Realization Theory

- The triplet [A, B, C] can be transformed into modal parameters by computing the eigenvectors, $\varphi = \{\varphi_1, \varphi_2, \ldots, \varphi_n\}$, with the corresponding eigenvalues, $\gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, of matrix A
- Transform to realization $[\gamma \quad \varphi^{-1}B \quad C\varphi]$

Diagonal Matrix γ contains the modal damping rates and damped natural frequencies
Re & Im parts of $\gamma_c = \ln(\gamma)\,/\Delta t$

Initial Modal Amplitudes

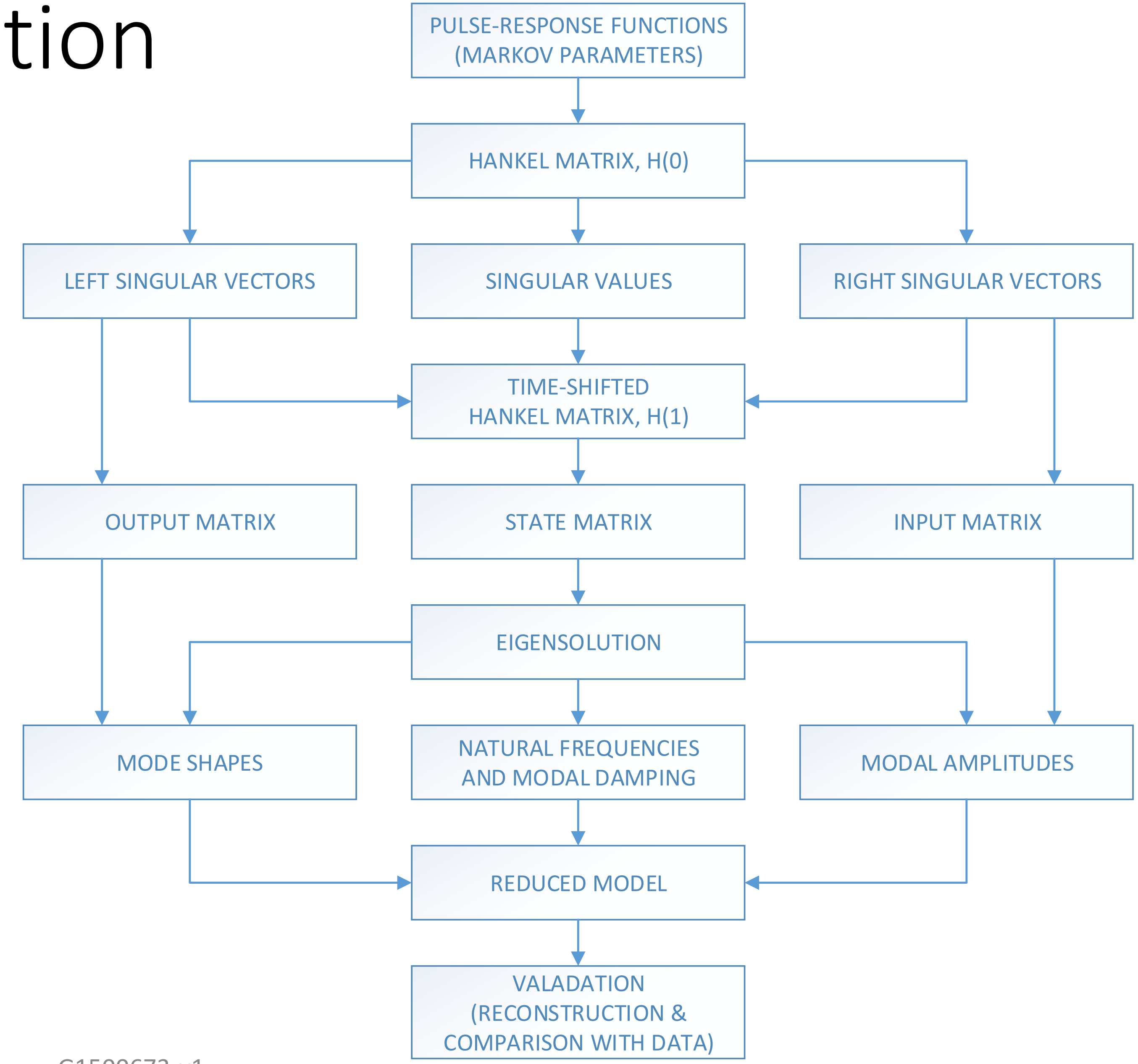
Mode Shapes at the sensor locations

Eigensystem Realization Algorithm (ERA)

- The Hankel matrix is formed from the Markov parameters

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} \cdots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \cdots \\ \cdots & & \\ Y_{k+\alpha-1} & \cdots & Y_{k+\alpha+\beta-2} \end{bmatrix}$$

- ERA is a least squares fit to the pulse response measurements
- Variant ERA/DC
  - with Data Correlations
  - Fit to the output auto-correlations and cross-correlations over a finite number of lag values
  - Juang, J. et. al., “An eigensystem realization algorithm using data correlations (ERA/DC) for modal parameter identification”, Control Theory and Advanced Technology, v4,n1,1988.



Other SID Techniques

- Ultimately we seek a recursive, real-time parameter identification of our MIMO systems (at their operating states)
- Many techniques are available; Potential candidates include:
  - Generalized Least Squares and Maximum Likelihood Estimators (e.g. the Prediction Error Method) are computationally simple
  - Observer/Kalman Filter Identification (OKID) -- time domain based, can be extended to identification of closed loop effective controller/observer combination (Observer Controller Identification, OCID)
  - State-Space Frequency Domain (SSFD) identification techniques

How and what to measure – modal testing

- Frequency Response Function (FRF) is the Fourier transform of an impulse or pulse response sequence (Markov parameters)
- Typically the FRFs are computed before application of a SID technique
- However many SID tools can analyze the free or forced response time histories directly

SID Session Goals

- Compare techniques
- Share experiences
- Exchange tools and ideas
- Define a “road map” for application to our systems and further research
  - Explore the state of the art, new tools …
  - Define suitable problems/applications
  - Define upgrades & next generation system targets (early stages of testing and commissioning)
- A new GW working group?
  - Host a wiki page?
  - Meet virtually on some cadence?