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**Cavity performance with large beam size**

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## 1 Introduction

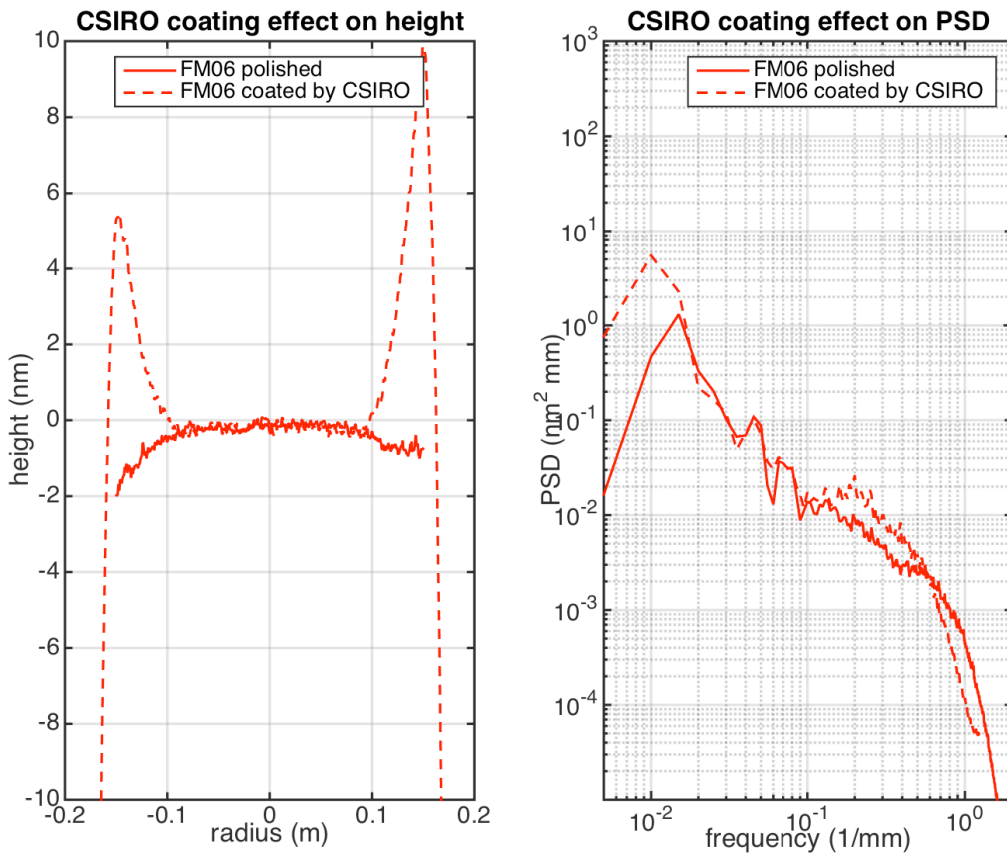
Round trip loss and higher order mode as a function of beam size based on aLIGO polished surface data and the best available coating.

## 2 Cavity

Symmetric cavity with 4km arm, 34cm mirror size if not otherwise mentioned.

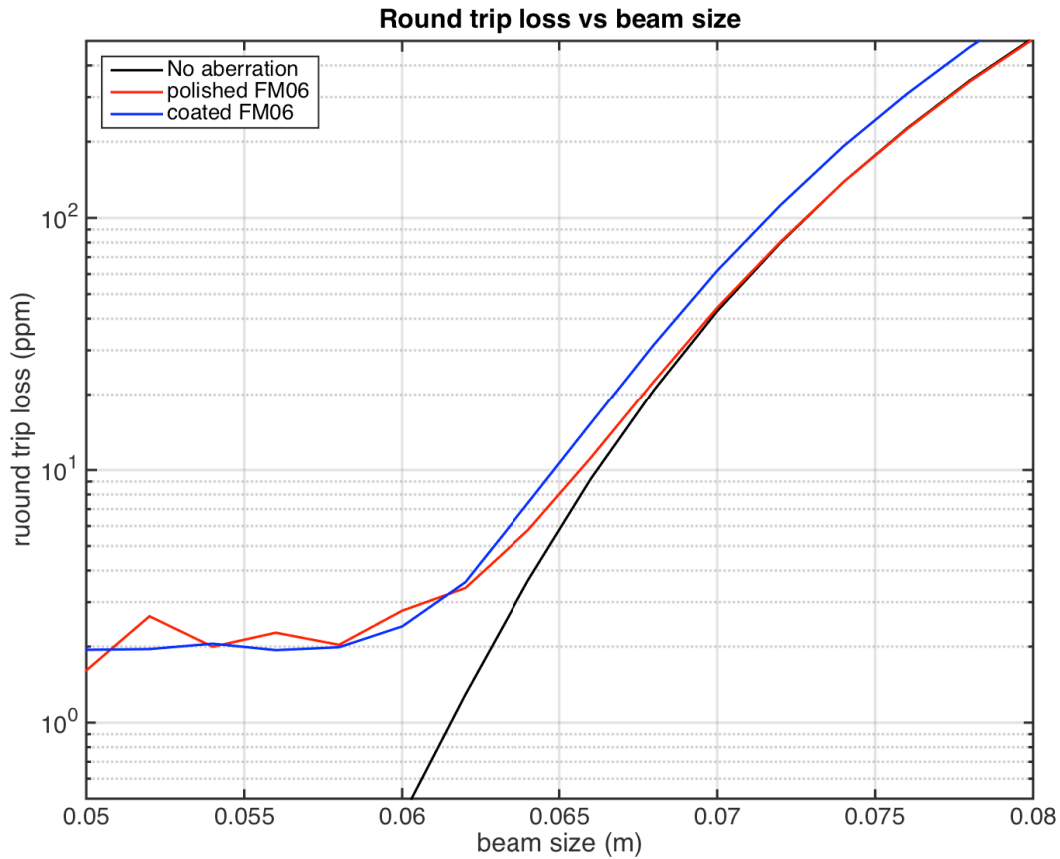
## 3 Best coating

CSIRO coated a folding mirror 06, which showed the best coating profile so far from the uniformity point of view.



**Figure 1 CSIRO coating on FM06**

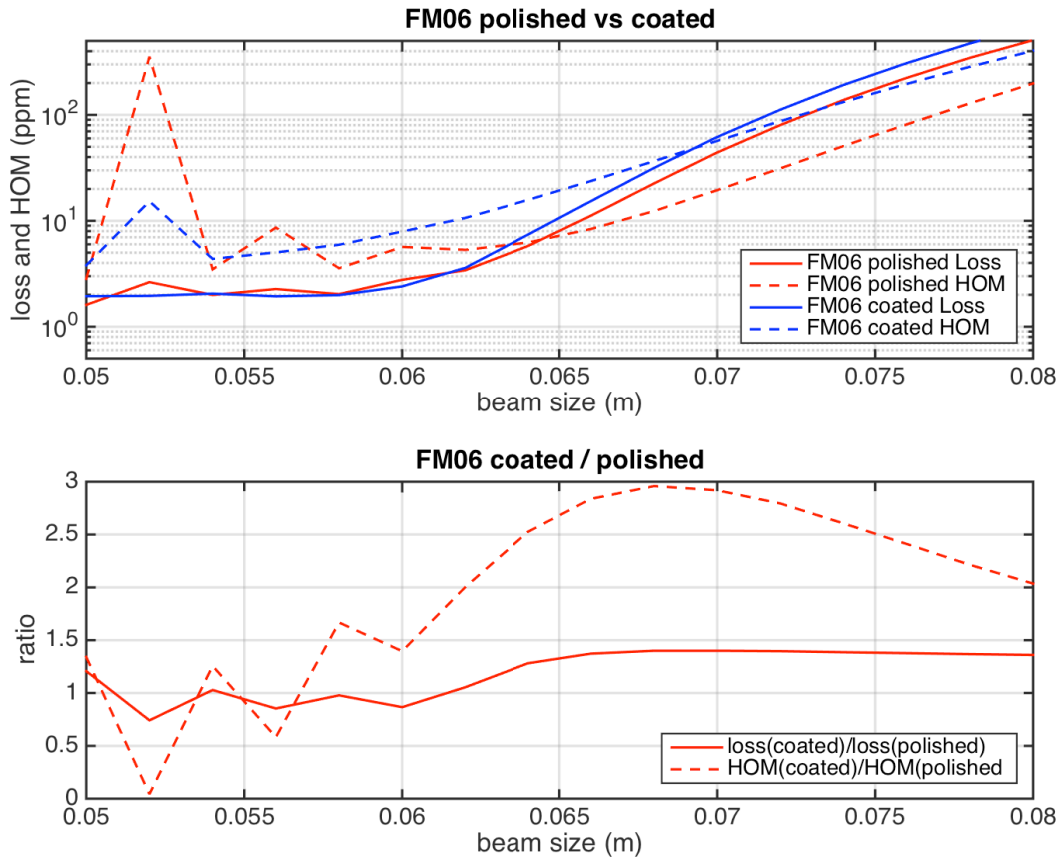
The coating uniformity is very good in 20cm aperture, but the thickness varies a lot outside of this uniform region. A similar behavior is observed in the coatings by LMA.



**Figure 2 Round trip loss : No aberration vs polished vs coated**

This compares the loss in a cavity without aberration, with polishing and with coating. The comparison of black and red lines show that the polished surface aberration is important for a small beam size, i.e., the physical boundary is not important, but is no more an important factor for larger beam size, whose loss is dominantly determined by the physical boundary.

The comparison of red and blue lines shows that the coating aberration at the boundary affects the loss even for a larger beam size case.



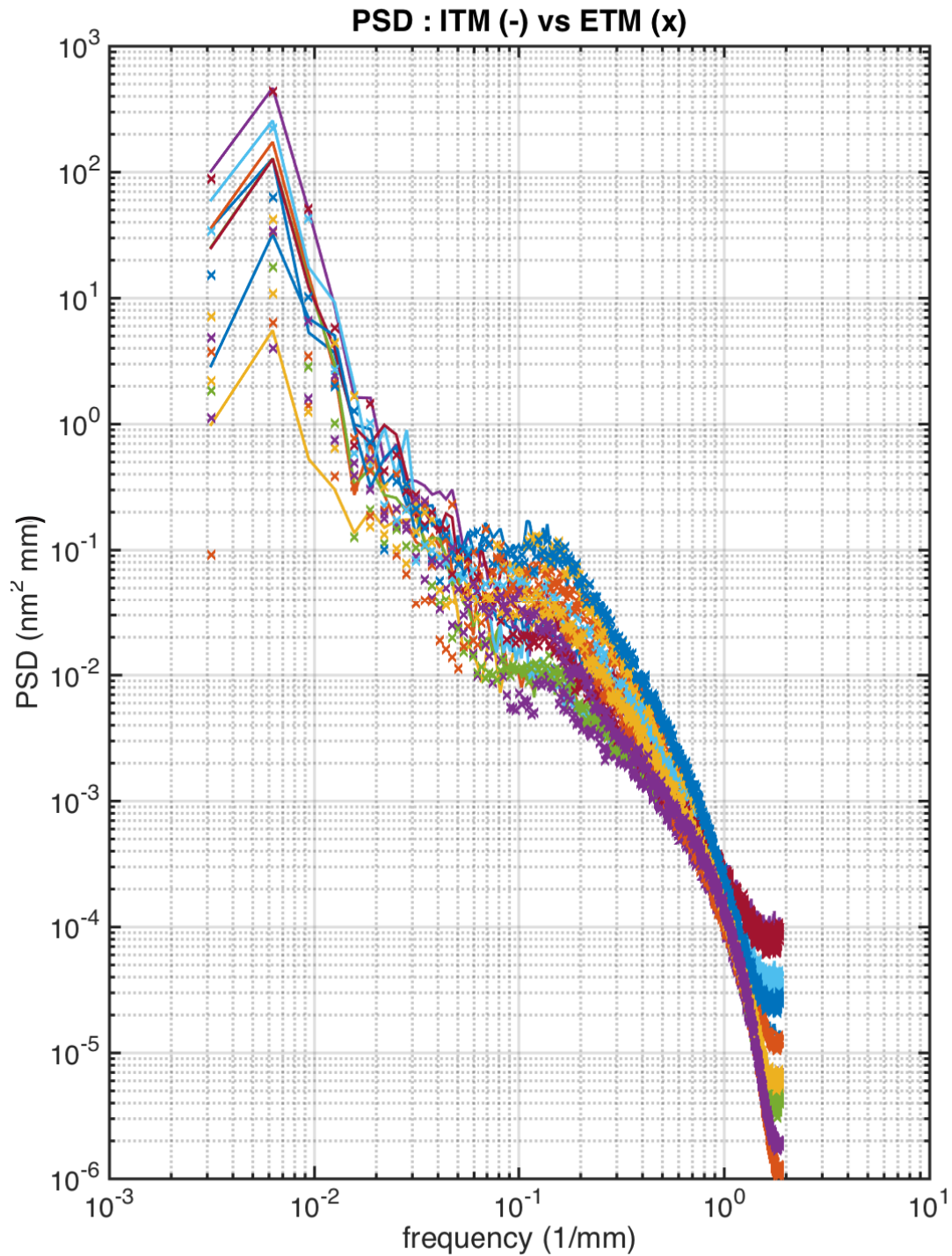
**Figure 3 Loss and HOM in a cavity with FM06 mirrors**

The FM06 mirror phasemaps without and with coating are used for ITM and ETM can calculated the round trip loss and HOM fraction. The round trip loss with coating degrades by 30-40% relative to the loss without coating for a cavity with beam size larger than 6.5cm.

HOM fraction is  $\sim 100$ ppm, and this will be able to be negligible compared to the deformation by the ITM transmission maps.

The structure for small beam size (5cm  $\sim$  6cm) are due resonances of higher order modes.

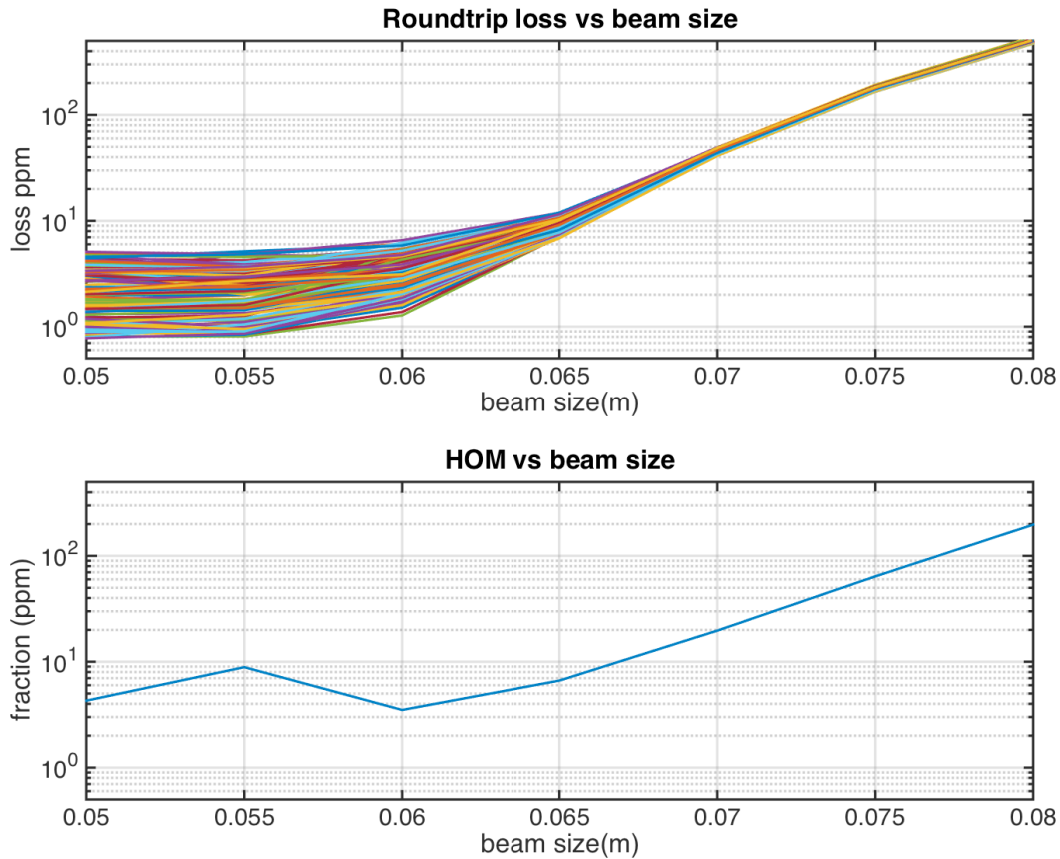
## 4 aLIGO polished mirror qualities



**Figure 4 PSDs of aLIGO mirrors (polished)**

aLIGO mirrors polished by Zygo. 8 ITMs and 10 ETMs after subtracting tilt and power. Both are of same quality after removing curvatures. The following analysis used cavities formed by paring two of these maps.

## 5 Loss in a cavity vs beam size



**Figure 5 Round trip loss and HOM with aLIGO polished mirror maps**

By using multiple sets of pairs of polished mirror maps mentioned in Sec.4, round trip losses were calculated for FP cavities with length of 4km and mirror aperture of 34cm. Each line corresponds to loss vs beam size with one set of mirror map pair. The HOM plot has multiple lines but all are almost identical.

The round trip loss is  $\sim 10$ ppm with a beam size of 6.5cm and is  $\sim 50$ ppm with 7cm beam. This is the result using polished mirror maps, and the coating can increase the loss by several 10%.

## 6 Extrapolation to larger area

Based on the polishing and coating data available for aLIGO mirrors, mirror surface maps with larger apertures are created as follows and those maps are used in the following analysis.

The polished surface is extended to outer area,  $r > r_0$ , by using the following formula.

$$extendedMap(r, \theta) = originalMap(2r_0 - r, \theta) \times \left(1 + \frac{r - r_0}{r_0} \alpha\right)$$

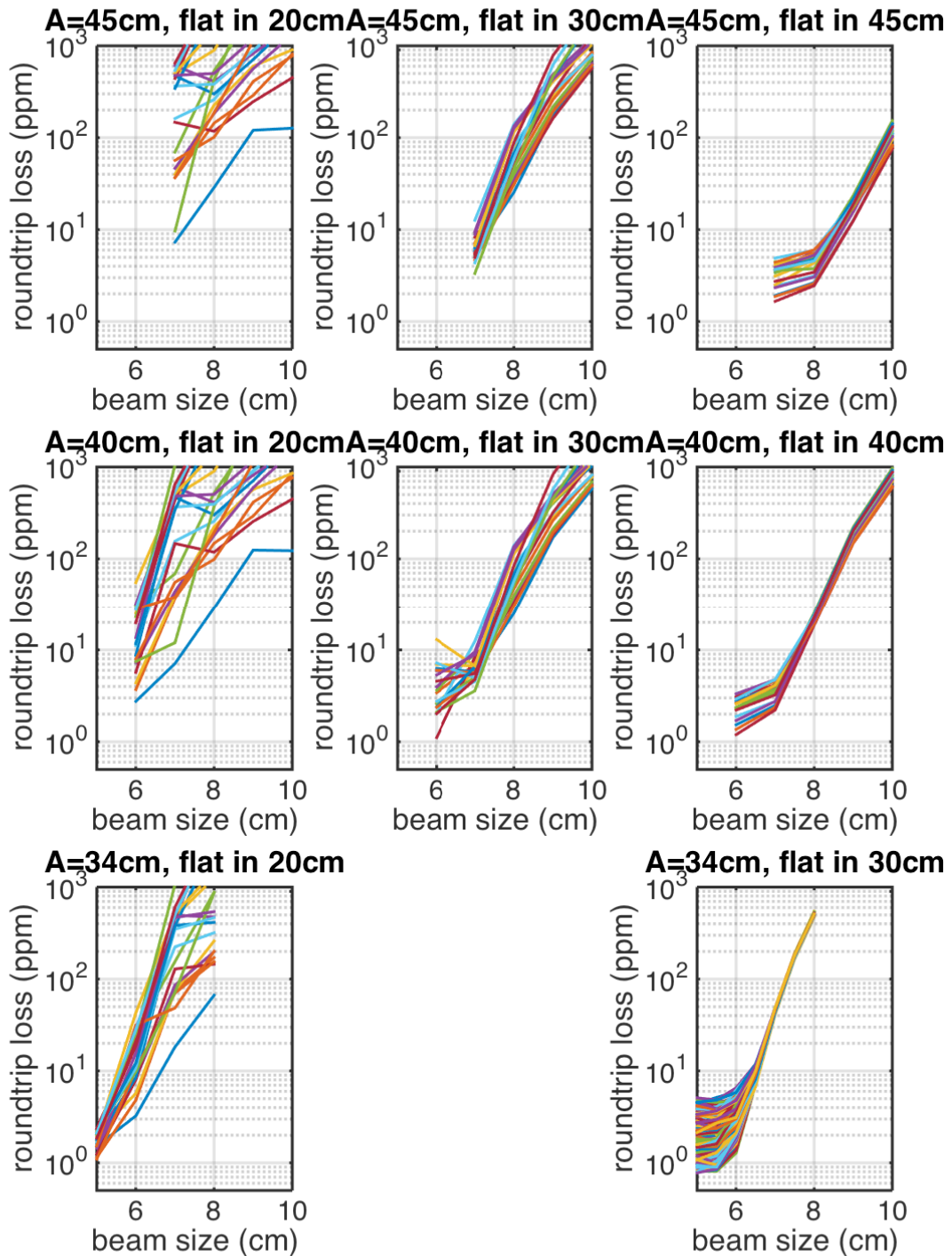
In this formula,  $r_0$  is the outmost radius of the original map data available, and  $\alpha$  is used to worsen the aberration outside of  $r_0$ . In the following analysis,  $\alpha = 1$  is used.

The extrapolation of the coated surface is more tricky and ambiguous. A map is fit by Zernike terms within a radius of  $r_0$ , and the original map is separated to low frequency part and high frequency part. The low frequency part is a sum of Zernike terms with  $n+m \leq 8$ , and the high frequency part is the rest. The high frequency part is extended using the formula above to the larger area. The low frequency part is extended using the sum of lower order Zernike terms just by using larger  $r$  value. Because of the nature of Zernike functions, this low frequency part becomes large for  $r > r_0$ . The sum of the low frequency part and the high frequency part, which were extended using separate methods, are added together to form a new extended map.

The new map has the following features. First, within the radius  $r_0$ , it is close to the original map. The coating in the central region is very good and it is good enough approximation to use the polished surface map. Second, short wavelength variations are almost the same in the entire mirror surface, somewhat worse at the outer edge. Third, the map varies very rapidly outside of the region where the fit was applied, i.e.,  $r > r_0$ . The rapid blowup just outside of  $r_0$  is not the exact reproduction of the coating uniformity blowup shown in Fig.1, but the shape and the magnitude is close when  $r_0$  is chosen properly.

This method of the extrapolation of the coating profile outside of the central region is not a real simulation of the coating structure. But this will serve to show the effect of the non-uniformity of the coating in semi-quantitatively.

## 7 Loss estimation with larger mirror aperture



**Figure 6 Loss vs Mirror size**

Top row is the case with mirror size of 45cm, second 40cm and third 34cm. First column is the case that the coated surface is flat only in 20cm aperture, which is the case of FM06. Middle

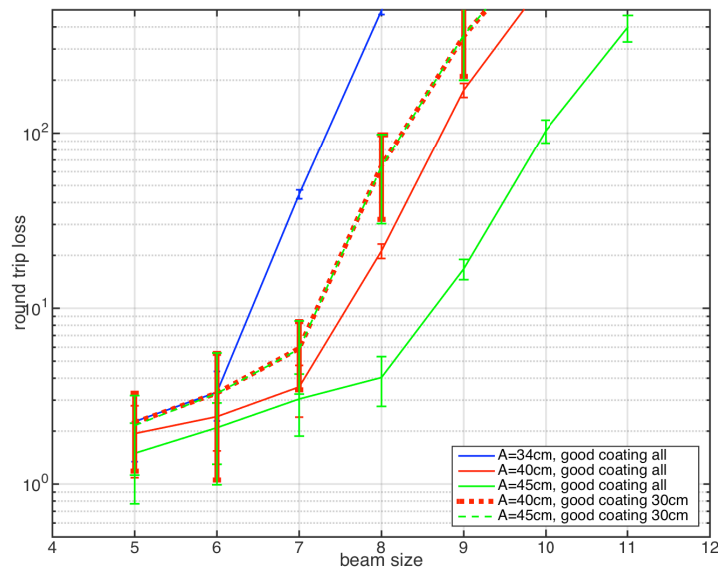


column is the case that the coating is flat in the central 30cm aperture and the right column is the case that the coating will not degrade the polishing roughness.

Multiple lines in each plot correspond to results using maps in Fig.4. The coating effect is more important than the polishing quality.

Right most column (no deformation by coating) shows that the round trip loss for beam size of 8cm configuration is less than 10ppm, ~20ppm and several 100ppm for the mirror size of 45cm, 40cm and 34cm respectively. Or, if the requirement of the round trip loss due to the figure is 30ppm, the beam size limit is 9.5cm for 45cm mirror, 8cm for 40cm and 7cm for 34cm mirror.

If the coating changes the thickness at 30cm aperture, 40cm or 45cm mirror size does not matter.



**Figure 7 Round trip loss vs Mirror size – summary**

Figure 7 shows the summary of the plots in Fig.6. Each line and associated error bars in Fig.7 are the average and mean of lines in each plot in Fig.6. E.g., the green solid line, “A=45cm, good coating all”, corresponds to the top-right plot in Fig.6.

Three solid lines, blue, red and green, show that the loss becomes smaller as the mirror size becomes larger if the mirror surface aberration is determined by the polishing quality. The size of the error bar of each line indicates that variation of polishing, shown in Fig.4, does not change the major characteristics of the dependence of the loss on the mirror and beam size.

Red and blue dotted lines over lap each other. This means that, if the surface aberration is compromised by the coating outside of 30cm aperture, the reduction of the loss by using larger mirror size is minimal. This statement is strongly dependent on the assumption how the coating thickness variation would be at the outer region, but it is indicative that it is very important that the coating uniformity at the mirror boundary is important to utilize the larger mirror size to reduce the round trip loss.

## 8 Estimation of the higher order modes by the ITM transmission maps