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MODELLING CALIBRATION ERRORS IN CBC WAVEFORMS

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ABSTRACT

We present a useful parameterisation and a flexible model for the effects of calibration errors in gravitational wave detectiors on measured gravitational waveforms.

1. CALIBRATION ERRORS

When a gravitational wave with a frequency-domain waveform $\tilde{h}(f)$ enters our detector, we assume it records a data stream (again in the frequency domain) that is an additive combination of a waveform and noise:

$$\tilde{d}(f) = \tilde{h}_{\rm obs}(f) + \tilde{n}(f). \tag{1}$$

Because the detector is not perfectly calibrated, however, there are frequency-dependent amplitude and phase errors in $\tilde{h}_{\rm obs}$ with respect to $\tilde{h}$:

$$\tilde{h}_{\rm obs}(f) = \tilde{h}(f)\left(1 + \delta A(f)\right)\exp\left(i\delta\phi(f)\right). \tag{2}$$

We expect these calibration errors to be small, and smoothly varying in frequency. Because we expect to have to implement calibration adjustments to our waveforms in an MCMC, it is better to use the slightly-modified model

$$\tilde{h}_{\rm obs}(f) = \tilde{h}(f)\left(1 + \delta A(f)\right)\frac{2 + i\delta\psi(f)}{2 - i\delta\psi(f)}. \tag{3}$$

The ratio involving $\delta\psi$ is chosen so that it always has complex amplitude 1 (i.e. it is a pure phase shift), and it agrees with the complex exponential to third order in $\delta\phi$ or $\delta\psi$:

$$\frac{2 + i\delta\psi(f)}{2 - i\delta\psi(f)} = \exp(i\delta\psi) + \mathcal{O}\left(\delta\psi^3\right), \tag{4}$$

but it nonetheless involves only algebraic operations, not transcendental ones and is therefore more efficient in its implementation. A plot of the relationship between $\delta\psi$ and $\delta\phi$ appears in Figure 1.

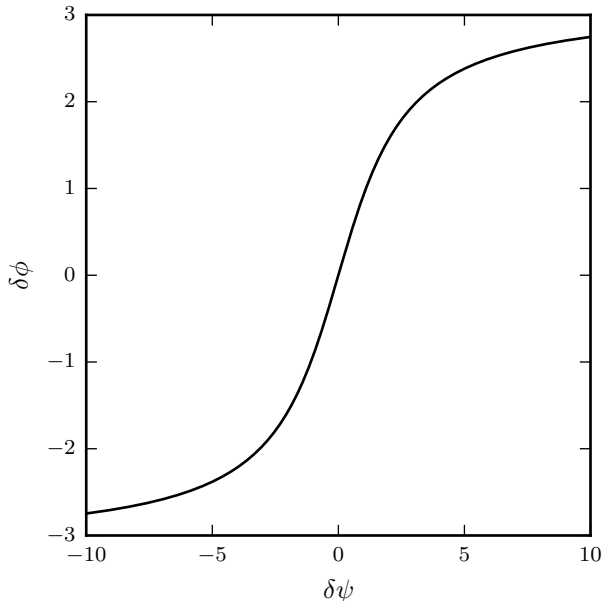

Fig. 1.— The relationship between $\delta\psi$ (Eq. (3)) and the phase shift $\delta\phi$. In the range $-\infty < \delta\psi < \infty$, $-\pi < \delta\phi < \pi$.

2. SPLINE MODEL

Given a parameterised model for the calibration errors, we could fit them out. A good model for the smooth calibration errors could be splines, for example

$$\delta A(f) = p_s\left(f; \{f_i, \delta A_i\}\right) \tag{5}$$

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and

$$\delta\psi(f) = p_s\left(f; \{f_i, \delta\phi_i\}\right), \tag{6}$$

where p_s is a cubic spline polynomial, the f_i are the nodes of the polynomial in frequency, and δA_i and $\delta\psi_i$ are the values of the spline at those nodal points. The parameters of this model are then the δA_i and the $\delta\psi_i$ (and a more general model could also include the nodal points, f_i, as parameters). Each detector will have independent calibration parameters in a multi-detector analysis. Following Vitale *et al.* (2012), we will choose nodal points equally spaced in $\log f$; this choice constrains the correlation length of the calibration errors in frequency space.

Because the calibration errors are expected to be small, it seems reasonable to place a Gaussian prior on the calibration error parameters

$$p\left(\delta A_i\right) = N(0, \sigma_A) \tag{7}$$

and

$$p\left(\delta\psi_i\right) = N(0, \sigma_\psi), \tag{8}$$

where σ_A and σ_ψ characterise our expected uncertainty about the magnitude of the calibration error at these frequencies. These parameters can then be fit and the corresponding calibration errors marginalised over in a run of one of the `LALInference` samplers.

The facility described in this note has been added by the authors to `LALInference`, and investigations are ongoing.

3. *AR*(1) MODEL

If we assume, as in Vitale *et al.* (2012), that the correlation length of the calibration errors at a given frequency is proportional to that frequency, then the calibration errors have a constant correlation length in $\log f$. It may be reasonable to assume that the errors are stationary in $\log f$ and have an autocovariance function of the form

$$\left\langle \delta A\left(f_i\right) \delta A\left(f_j\right) \right\rangle = \sigma_A^2 \exp\left(-\frac{\left|\log f_i - \log f_j\right|}{\tau_A}\right), \tag{9}$$

where σ_A^2 is the variance of the amplitude error generating process, and τ_A the correlation length. As shown in Farr (2014), such an autocorrelation can be generated by an unevenly sampled[1] $AR(1)$ (i.e. single-step, linear, recursive) process:

$$\delta A\left(f_{i+1}\right) = \alpha_i dA\left(f_i\right) + \beta_i, \tag{10}$$

where

$$\alpha_i = \exp\left(-\frac{\left|\log f_{i+1} - \log f_i\right|}{\tau_A}\right) \tag{11}$$

and

$$\beta_i \sim \sigma_A \sqrt{1 - \alpha_i^2} \times N(0, 1). \tag{12}$$

To start the recursion, we have the initial value

$$\delta A(f_0) \sim N\left(0, \sigma_A\right). \tag{13}$$

A sample track of δA generated by such a process appears in Figure 2.

In this model, the σ_A, τ_A, σ_ψ, and τ_ψ are hyperparameters. and the values of the calibration errors at each frequency are latent variables generated by the above process controlled by the hyperparameters. The dimensionality of the calibration model is very large (two calibration parameters per frequency bin), but perhaps the constraint of the AR(1) model with only two hyperparameters is strong enough to enable good fitting.

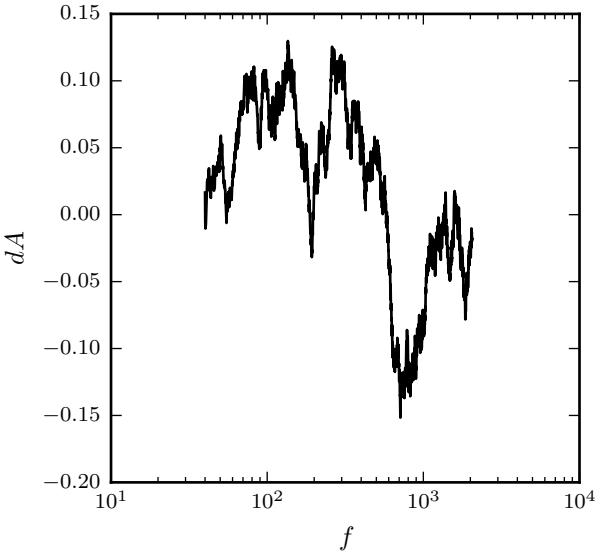

FIG. 2.— A sample of the latent calibration parameters drawn from the $AR(1)$ process of § 3 with parameters $\sigma_A = 0.05$ and $\tau_A = 1$.

This model is not yet implemented in `LALInference`.

REFERENCES

S. Vitale, W. Del Pozzo, T. G. F. Li, C. Van Den Broeck, I. Mandel, B. Aylott, and J. Veitch, Phys. Rev. D **85**, 064034 (2012), arXiv:1111.3044 [gr-qc].

W. M. Farr, "Efficient time-domain noise modeling," (2014), unpublished

[1] Unevenly sampled because the frequencies are not uniformly spaced in $\log f$.