LIGO-G1401391-v1

Higher order laser modes in gravitational wave detectors

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Higher order laser modes

- **Longitudinal sensing and control**
  - **Plane wave calculation was sufficient**

- **Alignment, mode matching, mode selection**
  - **higher order modes need to be taken into account**

Eigenmodes of the lasers

- **Solution of Maxwell's equation for propagating electromagnetic wave under the paraxial approximation**

- **=> Laser beams change their intensity distributions and wavefront shapes as they are propagated**

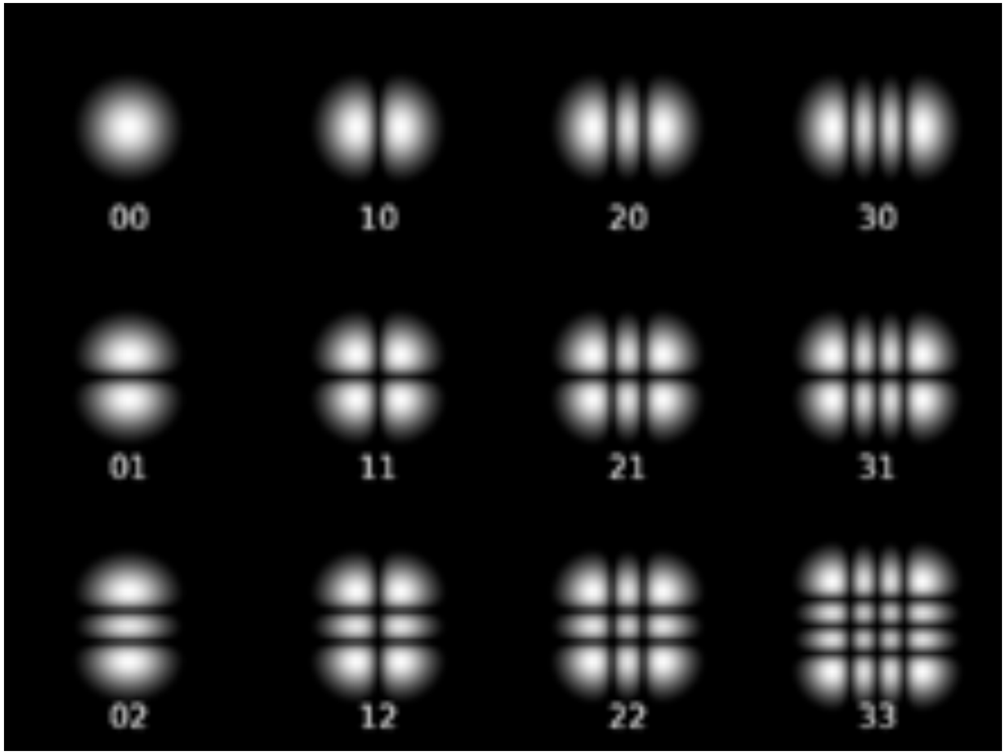
=> Any laser beam can be decomposed and expressed as a <u>unique</u> linear combination of eigenmodes

In this sense, a (given) set of eigenmodes are ortho-normal basis

Hermite Gaussian modes

- **HG modes (TEM mods) : one example of the eigenmodes**

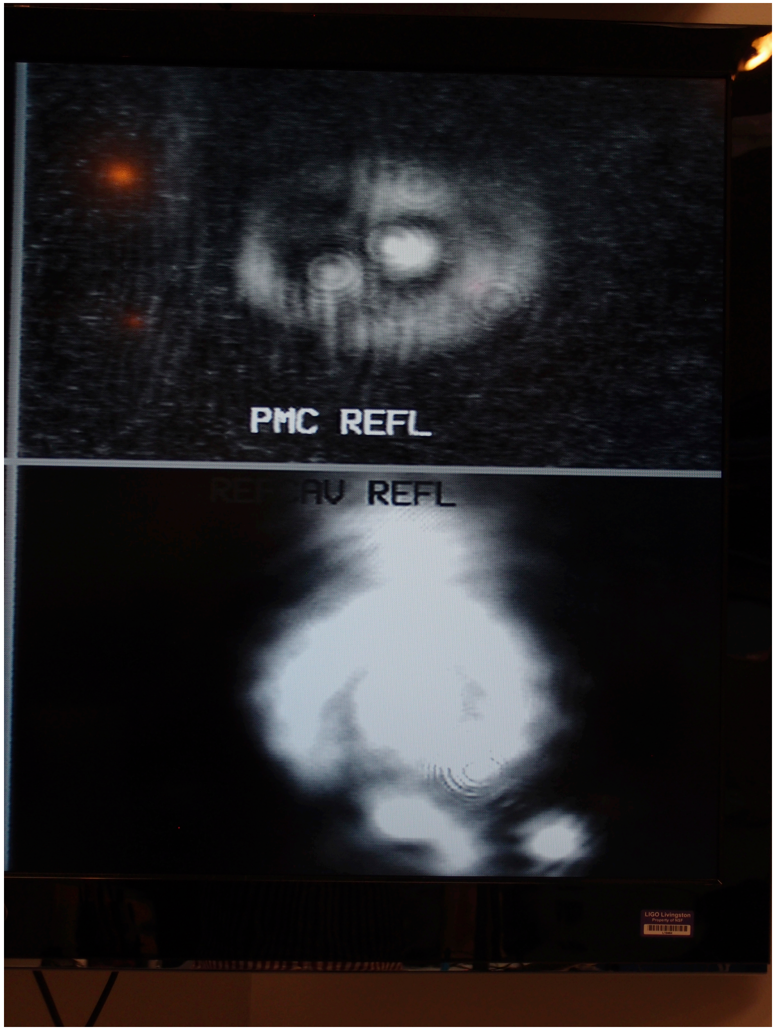
$$E_{mn}(x,y,z) = E_0 \frac{w_0}{w} H_m\left(\frac{\sqrt{2}x}{w}\right) H_n\left(\frac{\sqrt{2}y}{w}\right) \exp\left[-(x^2+y^2)\left(\frac{1}{w^2}+\frac{jk}{2R}\right) - jkz - j(m+n+1)\zeta(z)\right]$$



A. E. Siegman, *Lasers*, University Science Books, Mill Valley, CA (1986)
H. Kogelnik and T. Li, Appl. Opt. 5 (1966) 1550-1567
Wikipedia http://en.wikipedia.org/wiki/Transverse_mode

Any beam can be decomposed…

- **Astronaut**



- **World cup football**

Hermite Gaussian modes (TEM modes)

Trivia

- **There are infinite sets of HG modes**
  - **A TEMoo mode for an HG modes can be decomposed into infinite modes for other HG modes**

- **The complex coefficients of the mode decomposition is invariant along the propagation axis**
  - **Where ever the decomposition is calculated, the coefficients are unique.**

- **No matter how a beam is decomposed, the laser frequency stays unchanged!**

  (sounds trivial but frequently misunderstood)

Useful to note

- **Beam size at z**

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2},$$

- **Wavefront curvature at z**

$$R(z) = z + \frac{z_R^2}{z},$$

- **Gouy phase**

$$\eta(z) = \tan^{-1} \left(\frac{z}{z_{\mathrm{R}}} \right)$$

- **Rayleigh range**

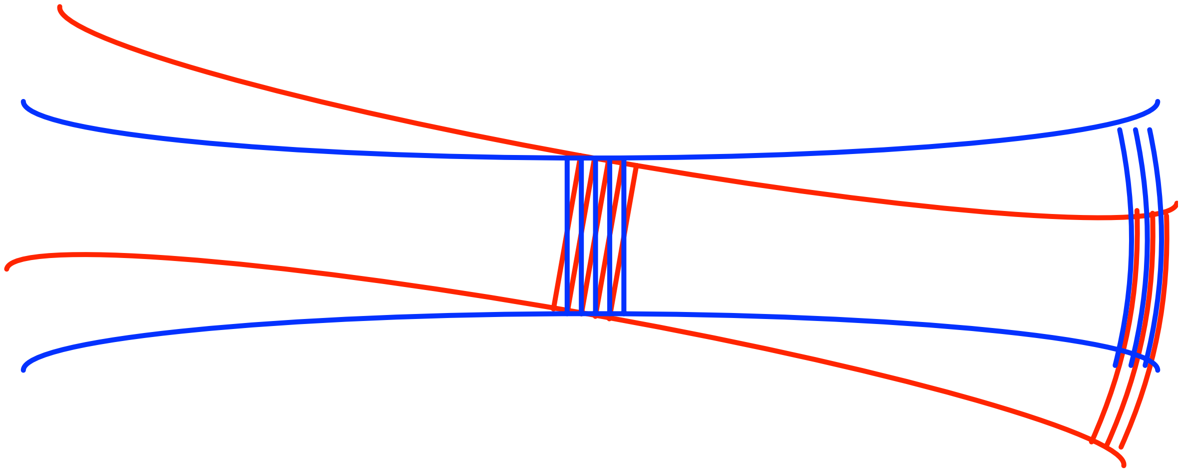
$$j \frac{\pi w_0^2}{\lambda} = j z_R.$$

cf. Huygens' principle

Gouy phase shift

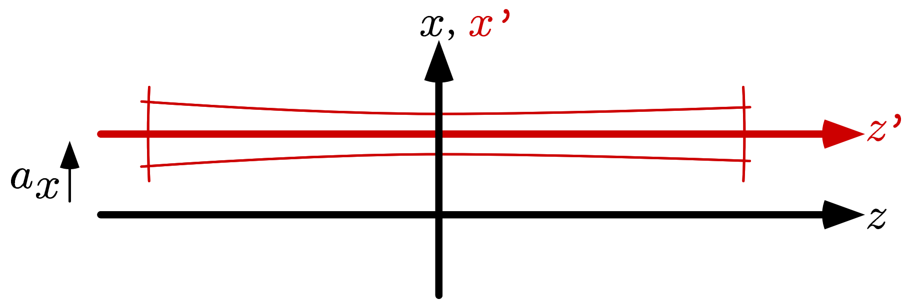
$$E_{mn}(x,y,z) = E_0 \frac{w_0}{w} H_m\left(\frac{\sqrt{2}x}{w}\right) H_n\left(\frac{\sqrt{2}y}{w}\right) \exp\left[-(x^2+y^2)\left(\frac{1}{w^2}+\frac{jk}{2R}\right) - jkz - j(m+n+1)\zeta(z)\right]$$

- **Gouy phase shift:**
  Relative Phase shift between the transverse modes
  - **Different optical phase of the modes for the same distance**
    => Different resonant freq in a cavity (will see later)
  - **"Near field" and "Far field"**



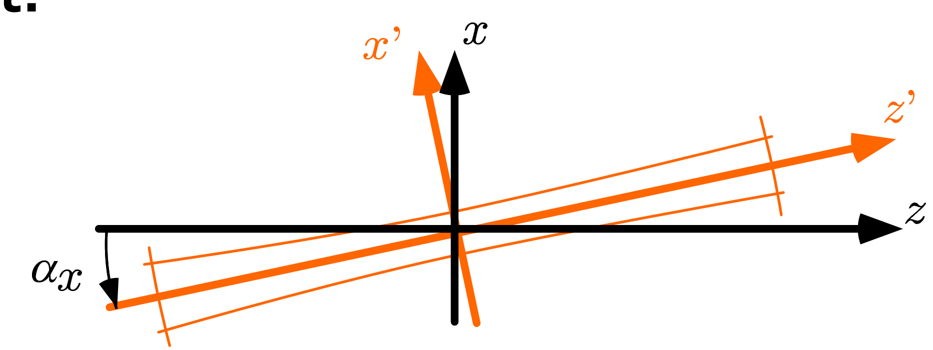
Decomposition of misaligned modes

- **Lateral shift:**



$$P_x(a_x) * U_{00+}(x,y,z) \simeq \left[1 - \frac{1}{2}\left(\frac{a_x}{w_0}\right)^2\right] U_{00+} + \frac{a_x}{w_0} U_{10+}$$

- **Rotational shift:**



$$R_x(\alpha_x) * U_{00+}(x,y,z) \simeq \left[1 - \frac{1}{2}\left(\frac{\alpha_x}{\alpha_0}\right)^2\right] U_{00+} - i\frac{\alpha_x}{\alpha_0} U_{10+}$$

K. Kawabe Ph.D thesis: http://t-munu.phys.s.u-tokyo.ac.jp/theses/kawabe_d.pdf

"Cavity" eigenmodes

- **TEM modes with matched wavefront RoC**

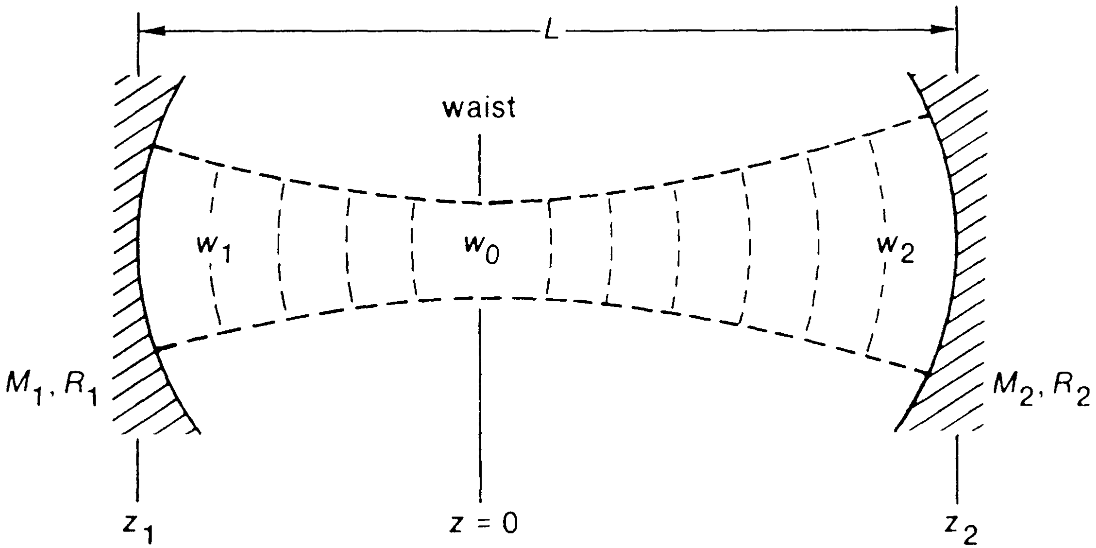
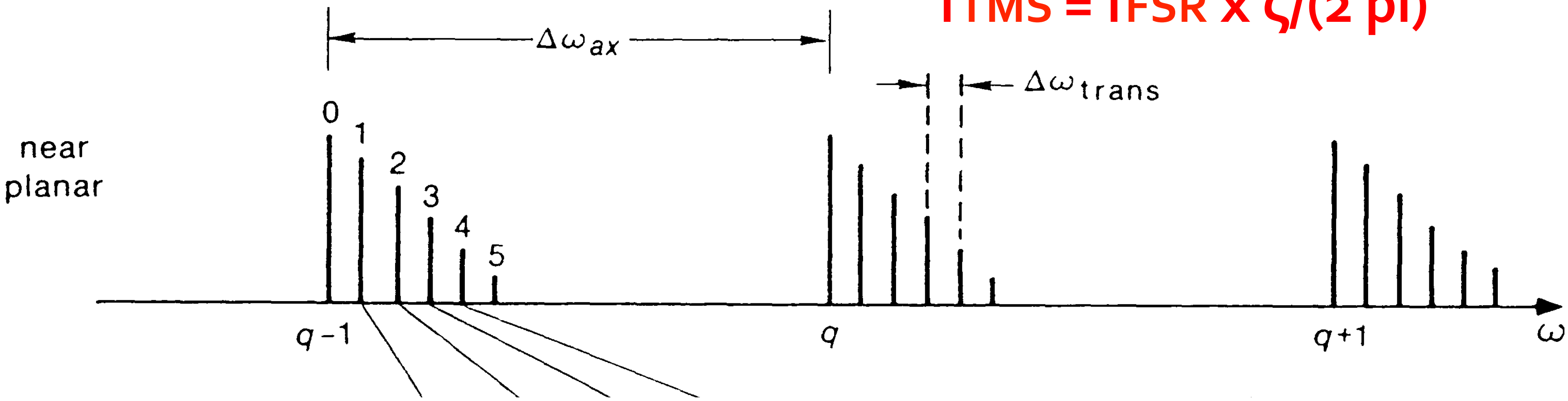


FIGURE 19.2
Notation and analytical model for analyzing a simple stable two-mirror cavity.

A. E. Siegman, *Lasers*, University Science Books, Mill Valley, CA (1986)

"Cavity" eigenmodes

- **Due to different Gouy phase shifts between TEM modes, their resonant frequencies are different**



ζ: cavity round trip Gouy phase shift

A. E. Siegman, *Lasers*, University Science Books, Mill Valley, CA (1986)

Optical resonator stability

- **g-factors**

$$g_1 \equiv 1 - L/R_1$$
$$g_2 \equiv 1 - L/R_2$$

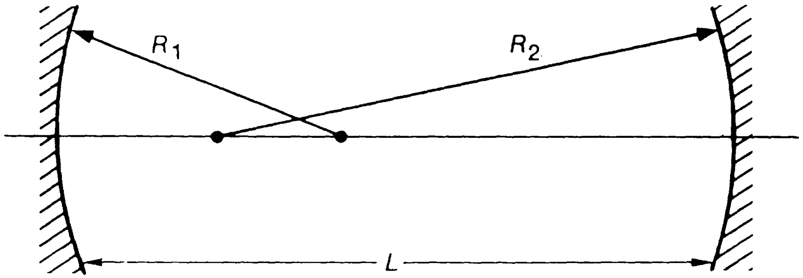


FIGURE 19.3
The resonator g parameters.

- **Stability criteria**

$$0 \leq g_1 g_2 \leq 1.$$

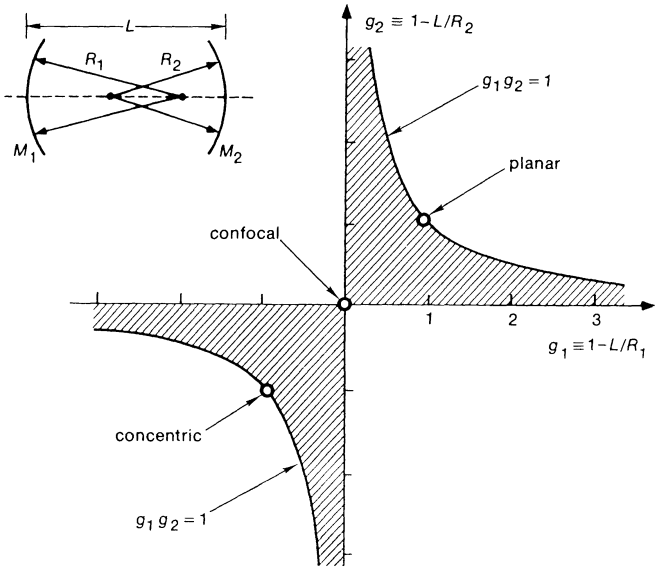


FIGURE 19.4
The stability diagram for a two-mirror optical resonator.

A. E. Siegman, *Lasers*, University Science Books, Mill Valley, CA (1986)

Optical resonator stability

- **General case**

derived that **the accumulated round-trip Gouy phase shift can be computed only from the round-trip ABCD matrix of the cavity** as:

$$\zeta = \mathrm{sgn}B \cdot \cos^{-1}\left(\frac{A+D}{2}\right) \quad , \tag{12}$$

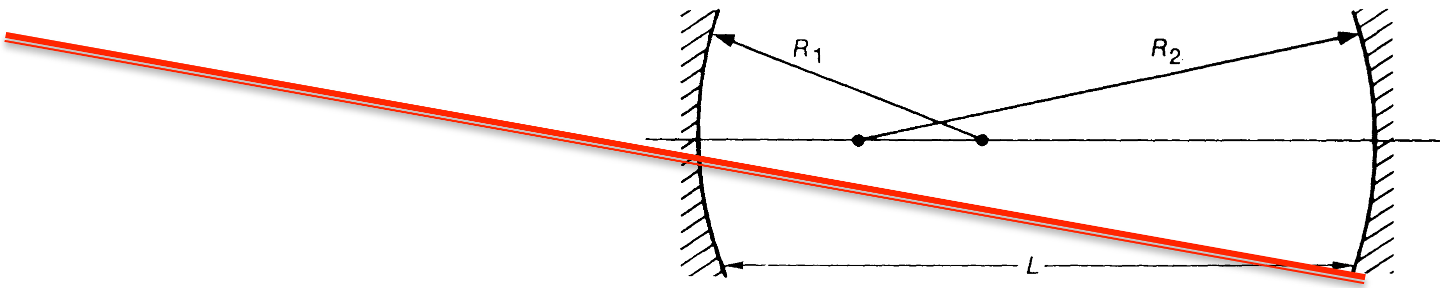
- **The cavity is stable when this quantity ζ exists**

$$-1 \leq \frac{A+D}{2} \leq 1.$$

T1300189 "On the accumulated round-trip Gouy phase shift for a general optical cavity" Koji Arai https://dcc.ligo.org/LIGO-T1300189

Cavity alignment

- **To match the input beam axis and the cavity axis**



- **Corresponds to the suppression of TEM01/10 mode in the beam with regard to the cavity mode**

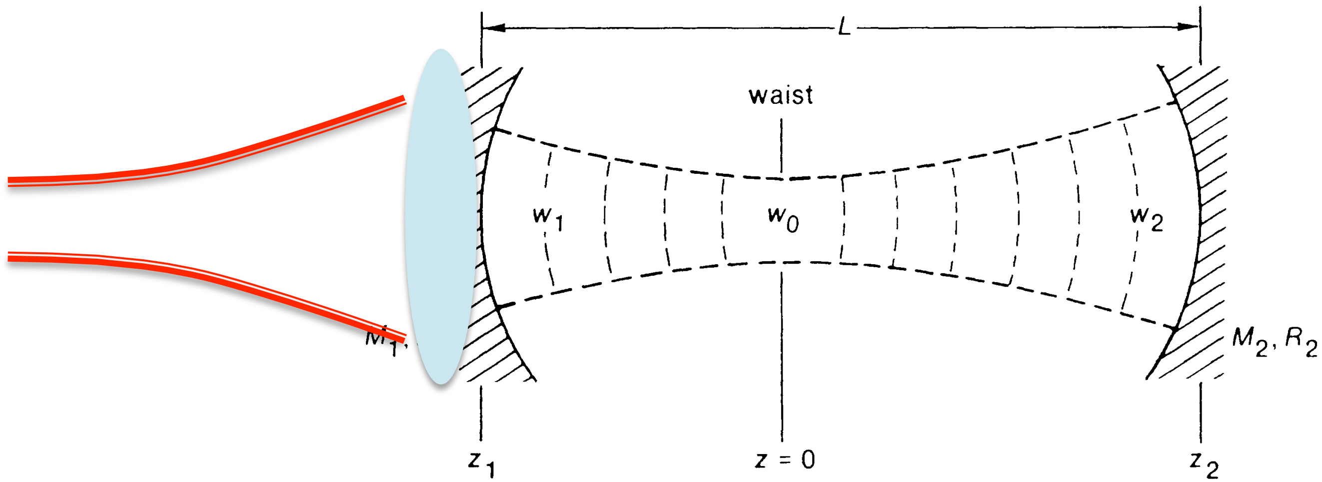
- **4 d.o.f.: (Horizontal, Vertical) x (translation, rotation)**

Note: it is most intuitive to define the trans/rot at the waist

- **To move the mirrors or to move the beam?**

Cavity mode matching

- **To match the waist size and position of the input beam to these of the cavity**



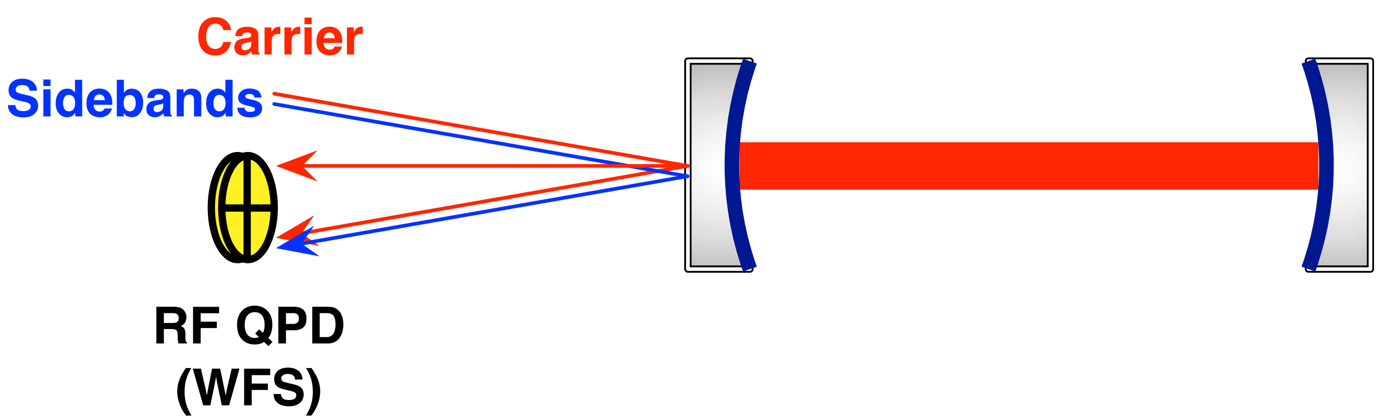
- **Corresponds to the suppression of TEM02/11/20 mode in the beam with regard to the cavity mode**

Angular global control

- Wave Front Sensing
  - Misalignment between the incident beam and the cavity axis
  - The carrier is resonant in the cavity
  - The reflection port has
    - Prompt reflection of the modulation sidebands
    - Prompt reflection of the carrier
    - Leakage field from the cavity internal mode

(Prompt reflection of the modulation sidebands and Prompt reflection of the carrier: no signal)

(All three: spatially distributed amplitude modulation)

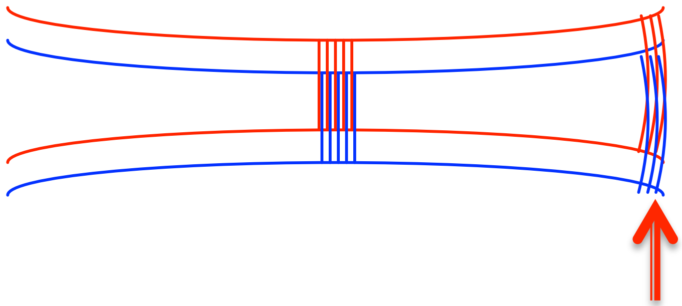


E Morrison et al Appl Optics **33** 5041-5049 (1994)

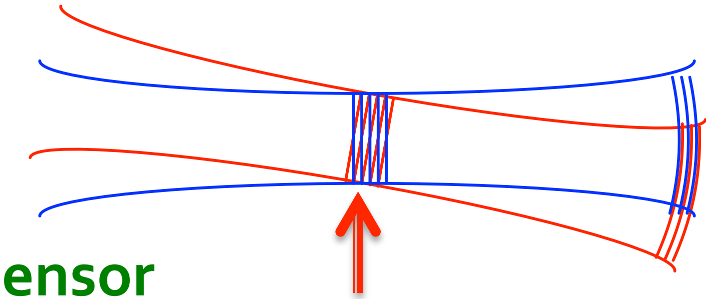
Angular global control

- Wave Front Sensing
  - WFS becomes sensitive
    when there is an angle between
    the wave fronts of the CA and SB
  - Can detect rotation and translation
    of the beam separately,
    depending on the "location" of the sensor
  - Use lens systems to adjust the "location" of the sensors.
    i.e. Gouy phase telescope

Sensitive at the far field



Sensitive at the near field

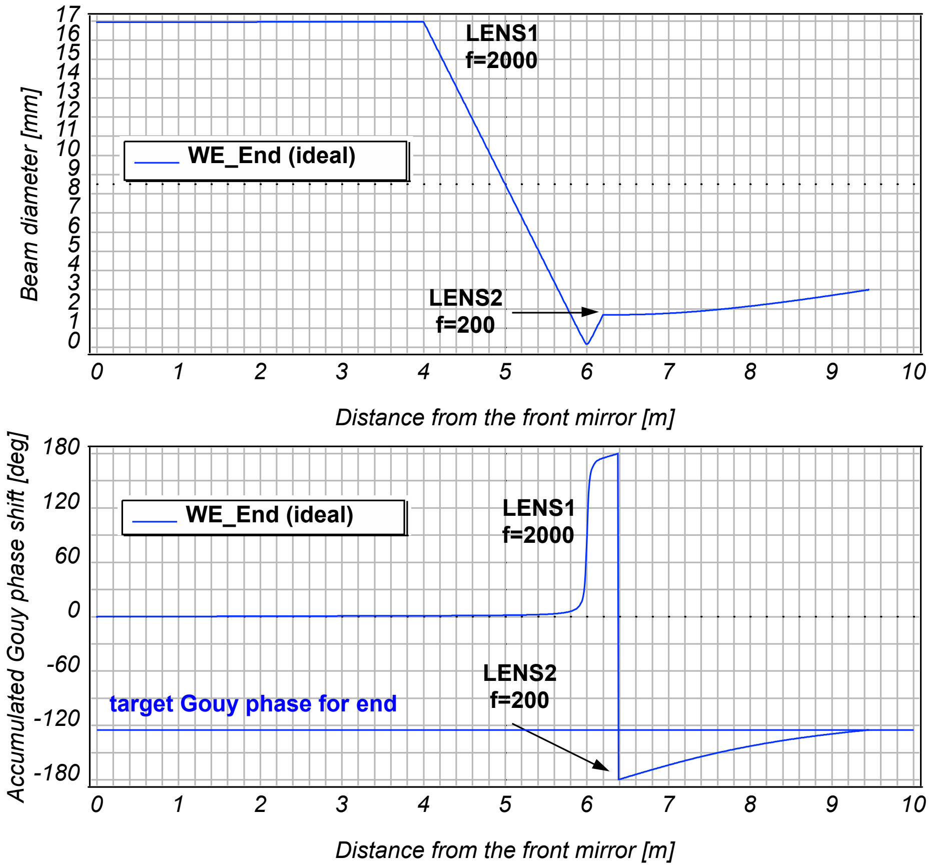


Angular global control

- **Frequent mistake:**

  What we want to adjust is the accumulated Gouy phase shift!

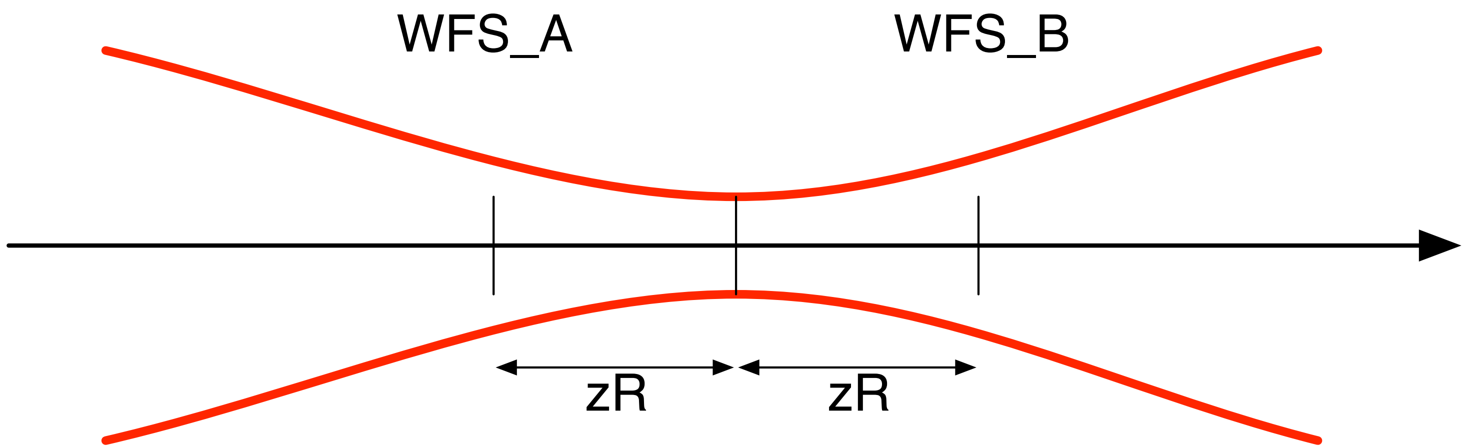
  Not the one for the final mode!



Angular global control

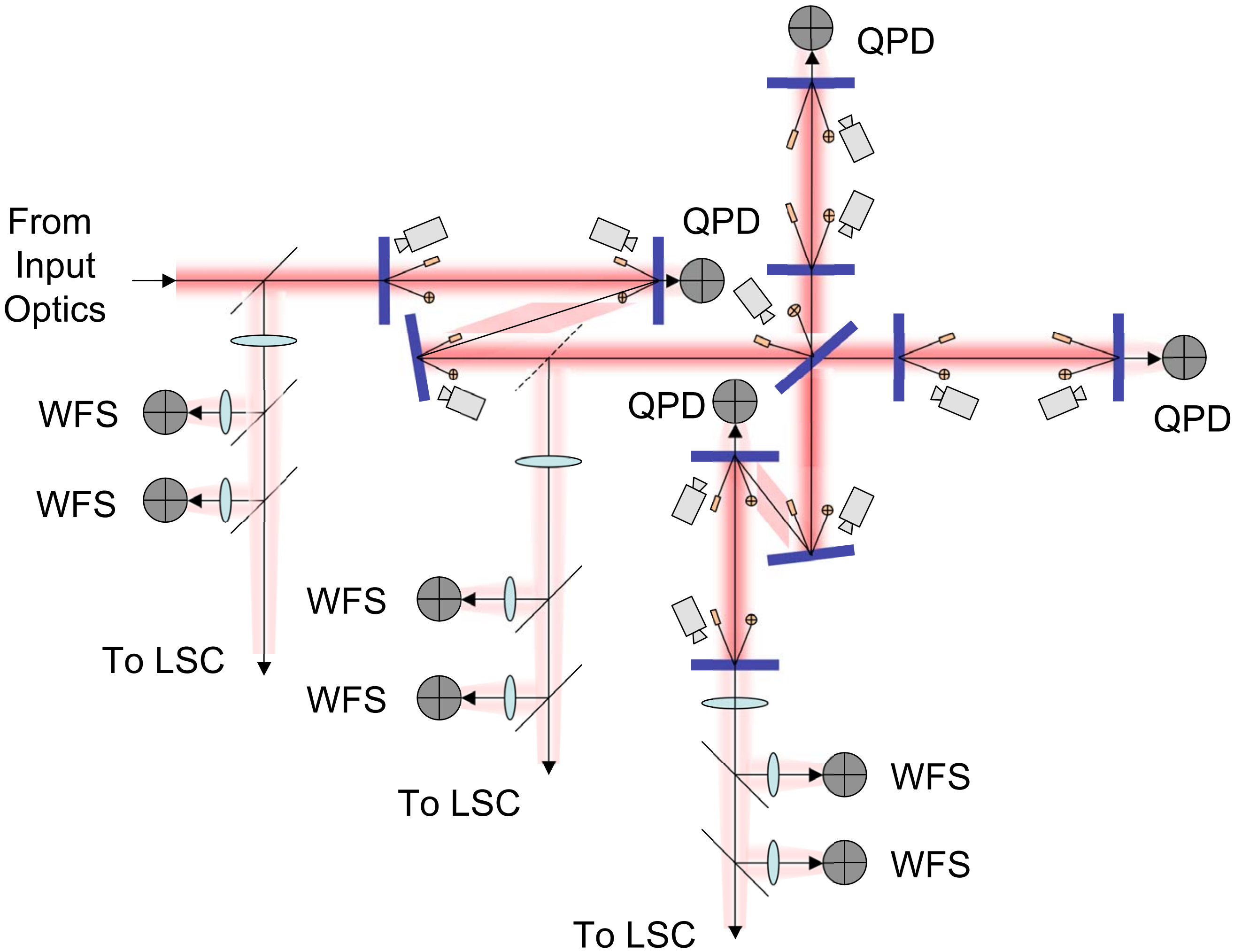
- **aLIGO implementation:**

Separate Gouy phase of a set of two WFSs with 90 deg



aLIGO angular control

- **Combine WFS, DC QPD, digital CCD cameras**



Other topics

- **PRC/SRC Degeneracy**

- **Sigg-Sidles instability & alignment modes**
  - G0900594
- **Impact on the noise**
  - G0900278 / P0900258

- **Parametric Instability**
  - HOM in the arm cavity
    ->Rad Press.
    ->Mirror acoustic mode
    ->Scattering of TEM00->HOM