LIGO-G1401386-v1

Interferometer Length Sensing in Gravitational Wave Detectors

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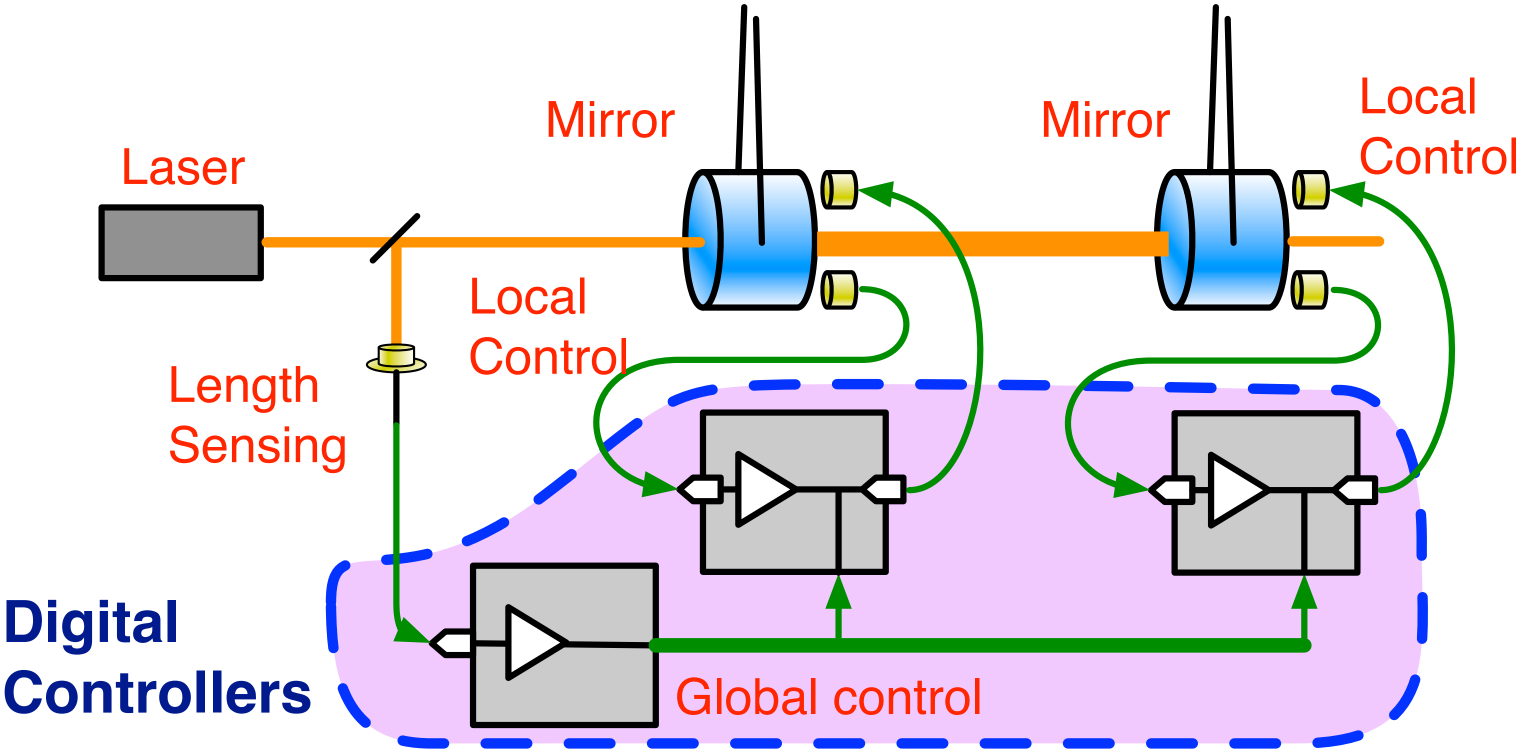
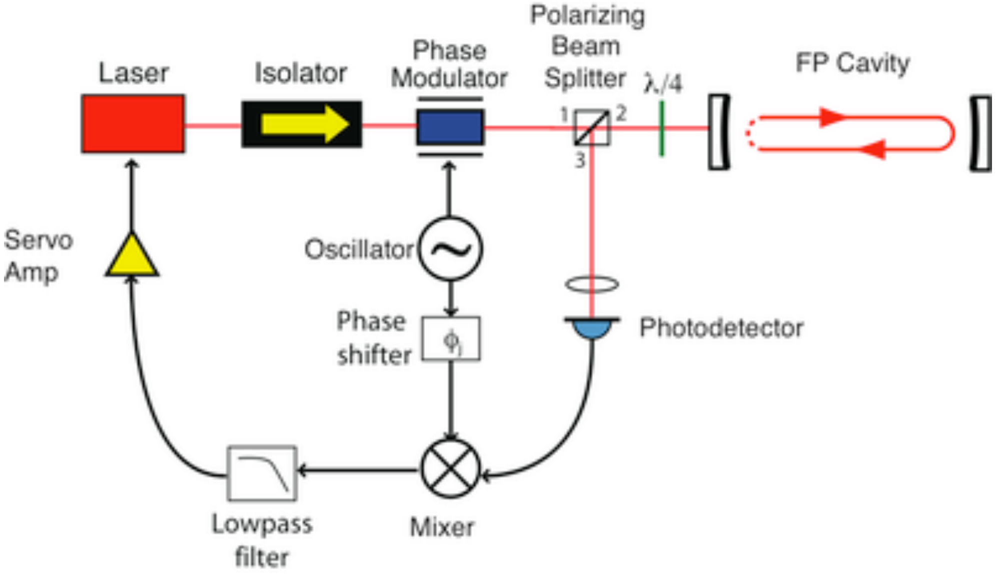
Global control

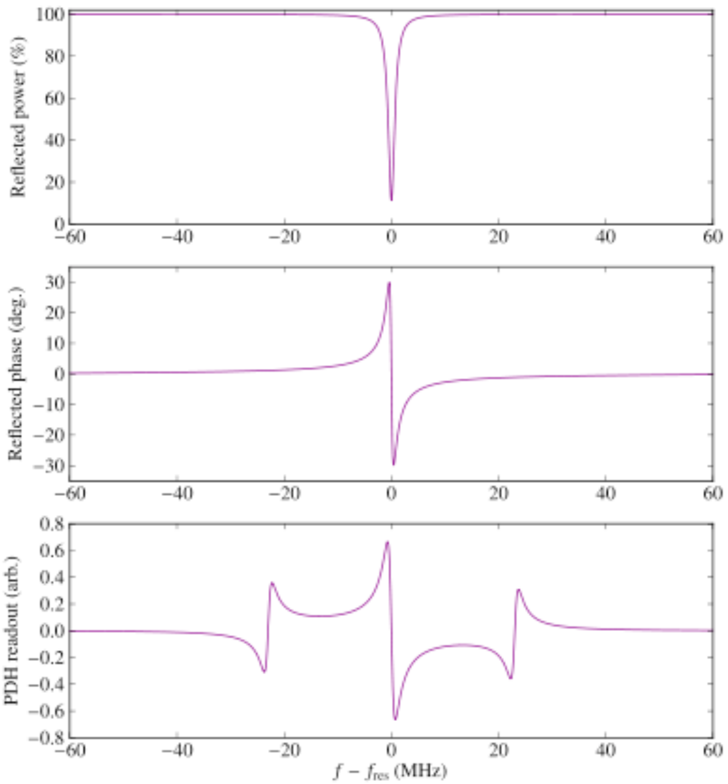
- Interferometer control using the main laser beam
  - On the top of the local control, optical path lengths and the mirror alignment need to be kept at the most sensitive state of the interferometer



Cavity length control

- Pound-Drever-Hall (PDH) technique
  - We want to keep the cavity at the TOP of the resonance
  - Phase of the cavity reflection is linear to the cavity detuning
  - Use modulation / demodulation





Cavity Ref Power

Cavity Ref Phase

PDH Error signal

Detuning

Drever, R. W. P., et al Appl Phys B **31** 97-105 (1983)

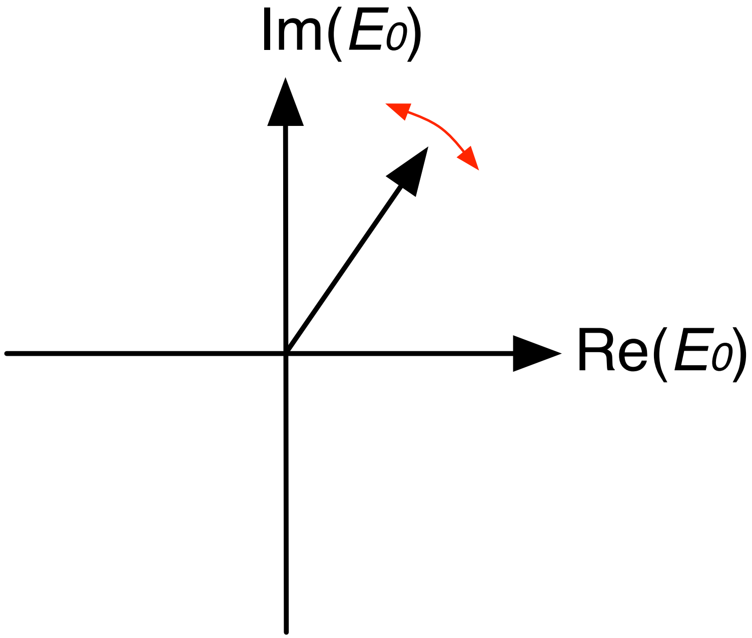
http://en.wikipedia.org/wiki/Pound%E2%80%93Drever%E2%80%93Hall_technique

http://www.sjsu.edu/faculty/beyersdorf/Archive/Phys208F07/Sideband%20generation%20in%20LIGO.pdf

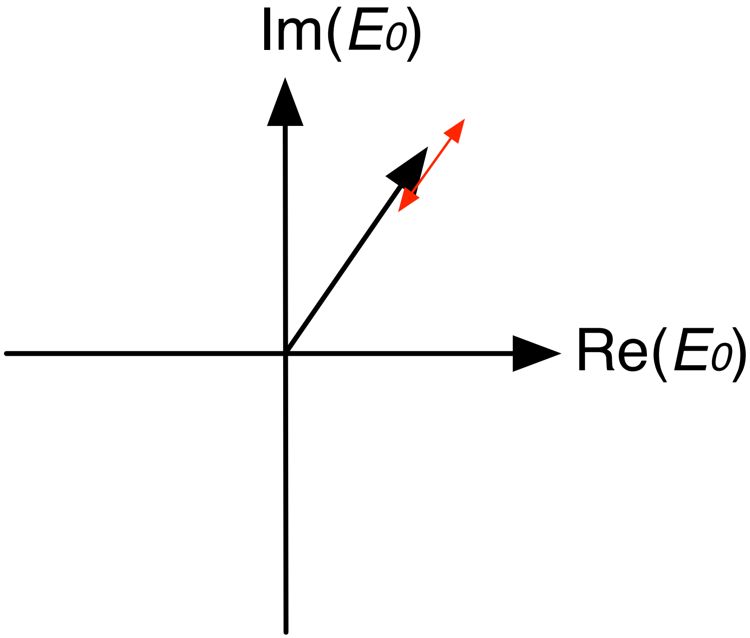
Modulation

- **Phasor diagram**

$$E = E_0(t) \exp(-\Omega t)$$



Phase modulation



Amplitude modulation

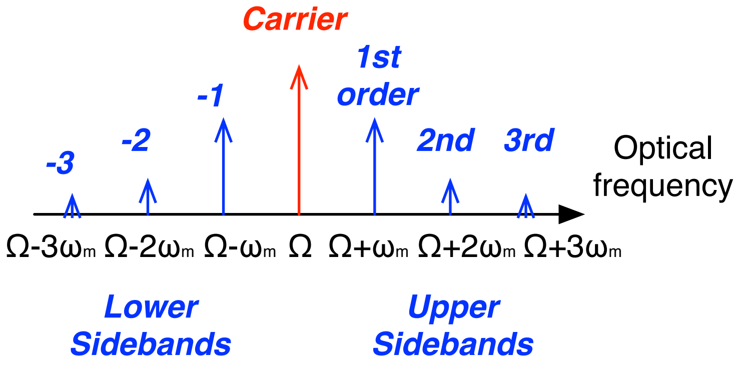
Modulation

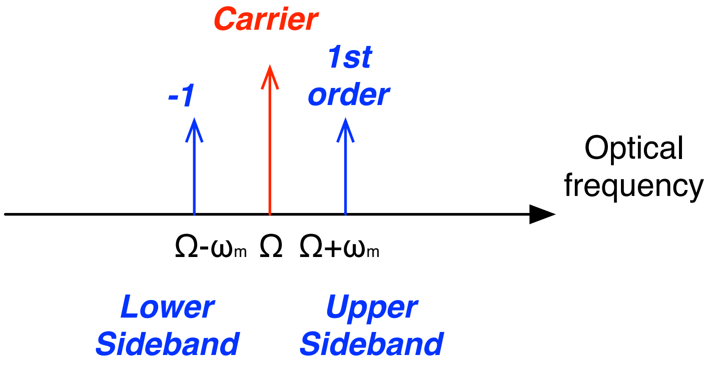
- **Modulation sidebands**
  - **Phase modulation**

$$
\begin{aligned}
E &= E_{\mathrm{in}} e^{i(-\Omega t + m\cos\omega_{\mathrm{m}} t)} \\
&= i^{|n|} J_{|n|}(m) e^{in\omega_m t} E_{\mathrm{in}} e^{-i\Omega t} \\
&= E_{\mathrm{in}} e^{-i\Omega t} \left[J_0(m) \right. \\
&\quad + iJ_1(m) e^{i\omega_m t} + iJ_1(m) e^{-i\omega_m t} \\
&\quad - J_2(m) e^{i2\omega_m t} - J_2(m) e^{-i2\omega_m t} \\
&\quad \left. + \cdots \right]
\end{aligned}
$$

Amplitude modulation

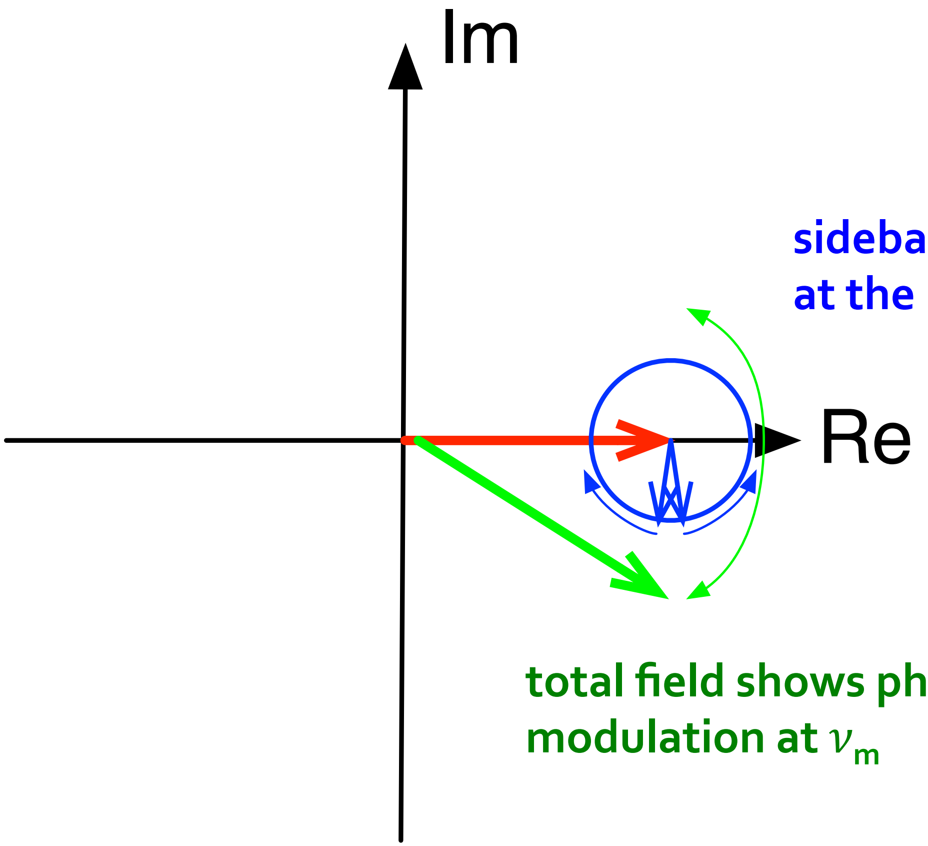
$$
\begin{aligned}
E &= E_{\mathrm{in}} (1 + m\cos\omega_{\mathrm{m}} t) e^{-i\Omega t} \\
&= E_{\mathrm{in}} e^{-i\Omega t} \left[1 + \frac{m}{2} e^{i\omega_m t} + \frac{m}{2} e^{-i\omega_m t} \right]
\end{aligned}
$$





Modulation

- Phasor diagram vs Sideband picture



sidebands rotate at the rate of ν_m

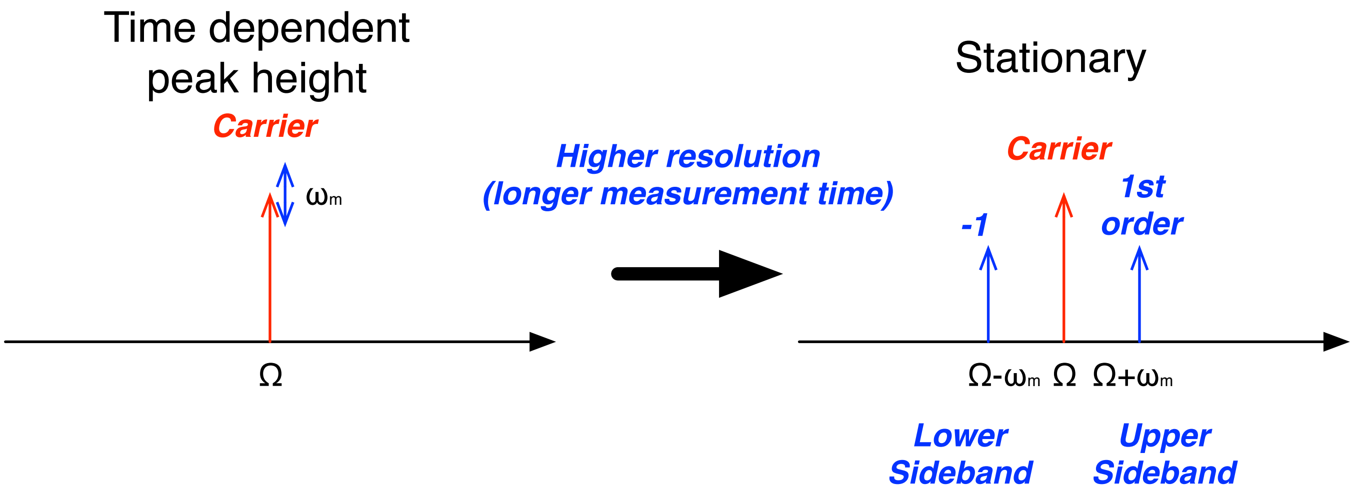
total field shows phase modulation at ν_m

- 2nd order? 3rd order?

Modulation

- Time dependent electric field vs Sideband picture

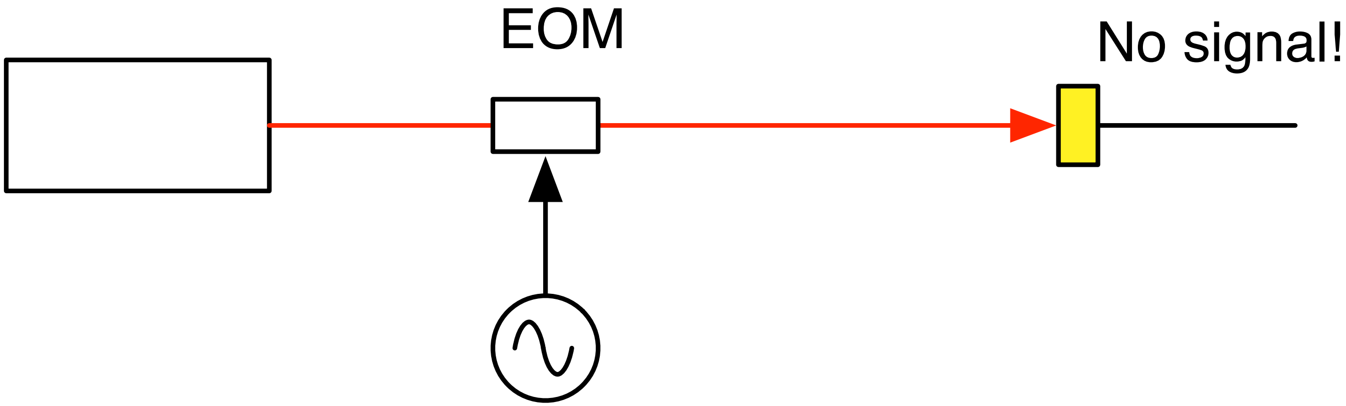
Spectrum analyzer



They are equivalent!

Modulation

- **Note: Photodetectors don't feel phase modulation!**



$$
\begin{aligned}
EE^* &= E_{\rm in}^2 e^{i(-\Omega t + m\cos\omega_{\rm m} t)} e^{-i(-\Omega t + m\cos\omega_{\rm m} t)} \\
&= E_{\rm in}^2 \quad (\text{constant})
\end{aligned}
$$

Modulation

- Two quadratures of PM

$$E_{\rm in}e^{i(-\Omega t+m\cos\omega_{\rm m}t)} = E_{\rm in}e^{-i\Omega t}\left[J_0(m)+iJ_1(m)e^{i\omega_m t}+iJ_1(m)e^{-i\omega_m t}\right]$$

$$E_{\rm in}e^{i(-\Omega t+m\sin\omega_{\rm m}t)} = E_{\rm in}e^{-i\Omega t}\left[J_0(m)+J_1(m)e^{i\omega_m t}-J_1(m)e^{-i\omega_m t}\right]$$

- Two quadratures of AM

$$E_{\rm in}(-1+m\cos\omega_{\rm m}t)e^{i\Omega t} = E_{\rm in}e^{-i\Omega t}\left[1+\frac{m}{2}e^{i\omega_m t}+\frac{m}{2}e^{-i\omega_m t}\right]$$

$$E_{\rm in}(-1+m\sin\omega_{\rm m}t)e^{i\Omega t} = E_{\rm in}e^{-i\Omega t}\left[1+i\frac{m}{2}e^{i\omega_m t}-i\frac{m}{2}e^{-i\omega_m t}\right]$$

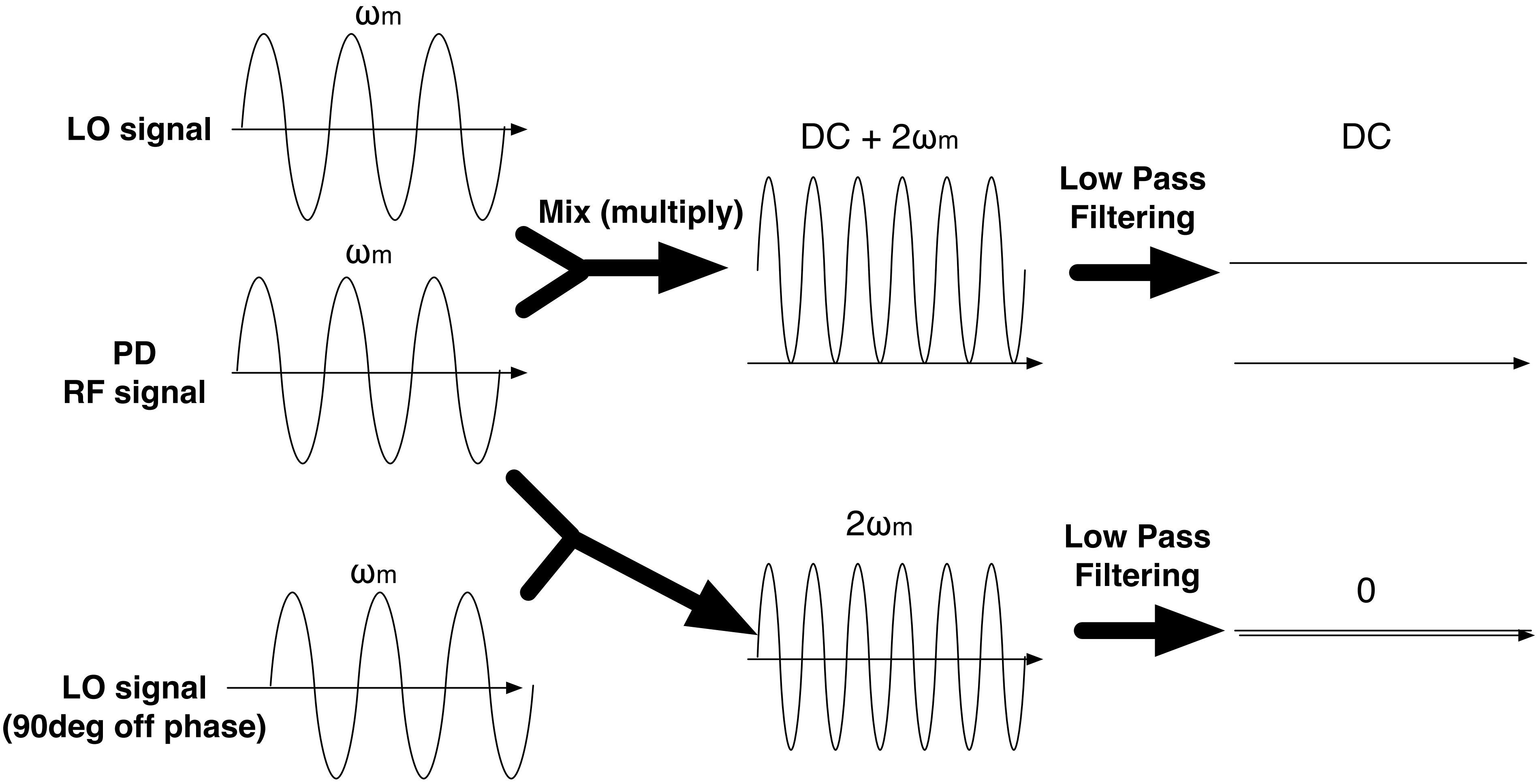
- Relationship between PM and FM (freq modulaition)

$$\phi(t) = \int 2\pi f(t)dt \quad\Longrightarrow\quad f(t)=\dot{\phi}(t)/2\pi$$

$$\phi(t) = m\sin(\omega t) \quad\Longrightarrow\quad f(t)=\frac{m\omega}{2\pi}\cos(\omega t)$$

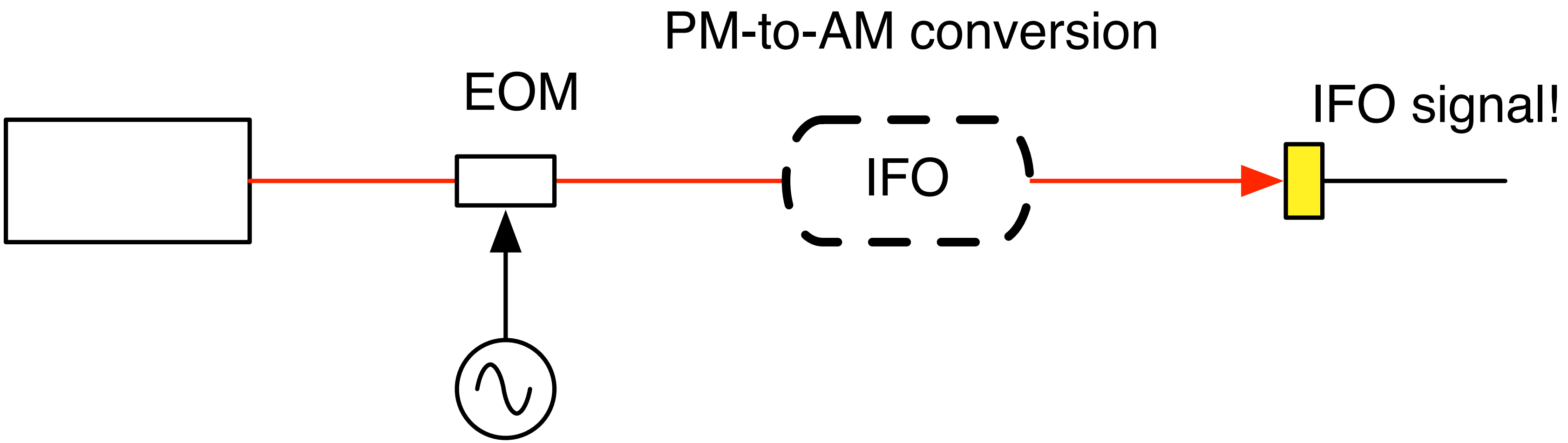
Demodulation

- RF x LO & LPF
- Two demodulation quadratures (I&Q)

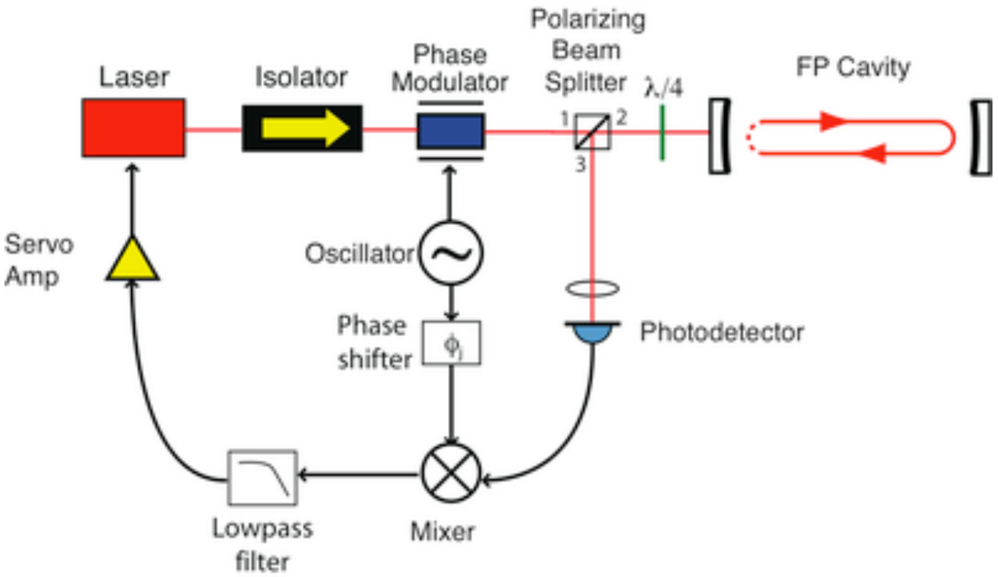


RF detection ~ Interferemeter LSC

- Use the interferometer as PM-AM converter



Pound Drever Hall technique



- **Sideband picture**
  - Carrier experiences “fast phase change” depending on the length of the interferometer

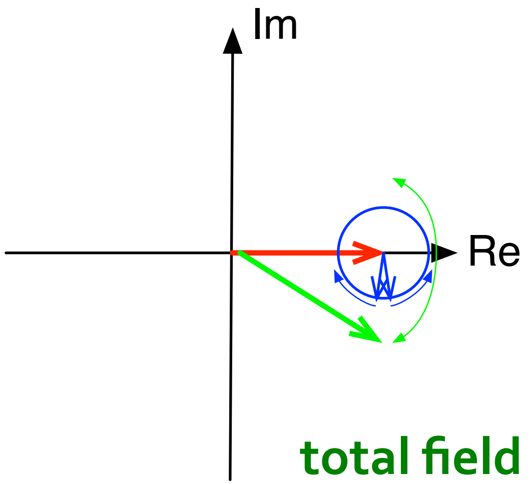
$$E_{\mathrm{in}}e^{i(-\Omega t+m\cos\omega_{\mathrm{m}}t)}\times r_{\mathrm{cav}}$$

$$=E_{\mathrm{in}}e^{-i\Omega t}\left[r_{\mathrm{cav}}(\Omega)J_0(m)+iJ_1(m)e^{i\omega_m t}+iJ_1(m)e^{-i\omega_m t}\right]$$

Pound Drever Hall technique

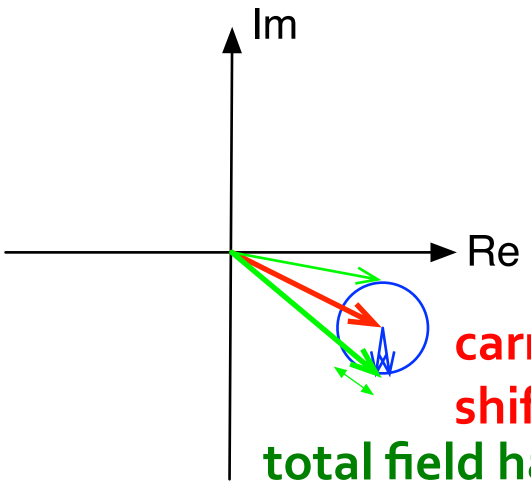
- Phasor picture
  - Reflected field on Resonance

Detuned



total field only has
phase modulation at ν_m

No power modulation



carrier phase shift

total field has
amplitude modulation at ν_m

Power modulation at ν_m

=> Photocurrent demodulated at ν_m
shows the linear signal to the detuning!

Michelson length control

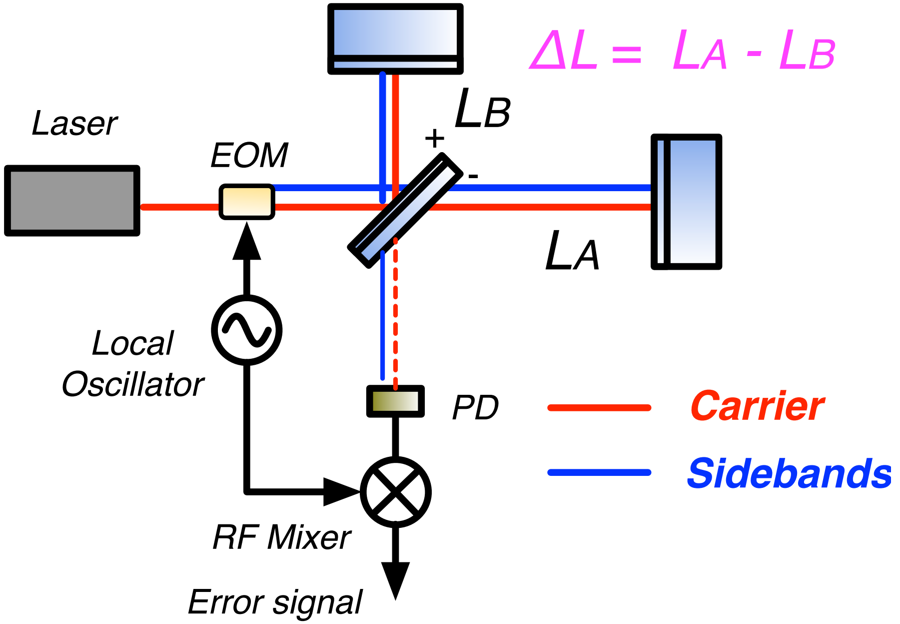
- Michelson is operated at the dark fringe

  for the shot noise and the power recycling

  - At the dark fringe, DC signals can't be a good error signal
  - Schnupp asymmetry:

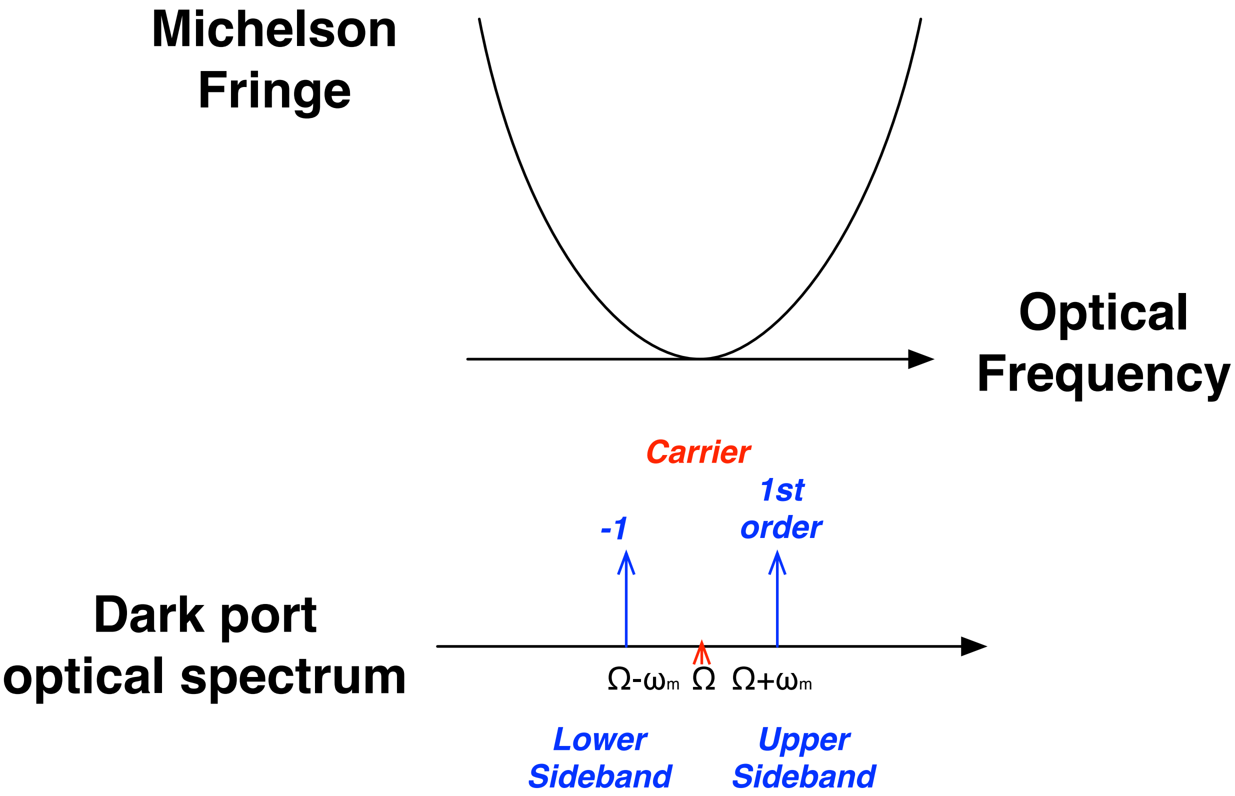
    Introduce small arm length asymmetry

    => RF sidebands leaks to the dark port



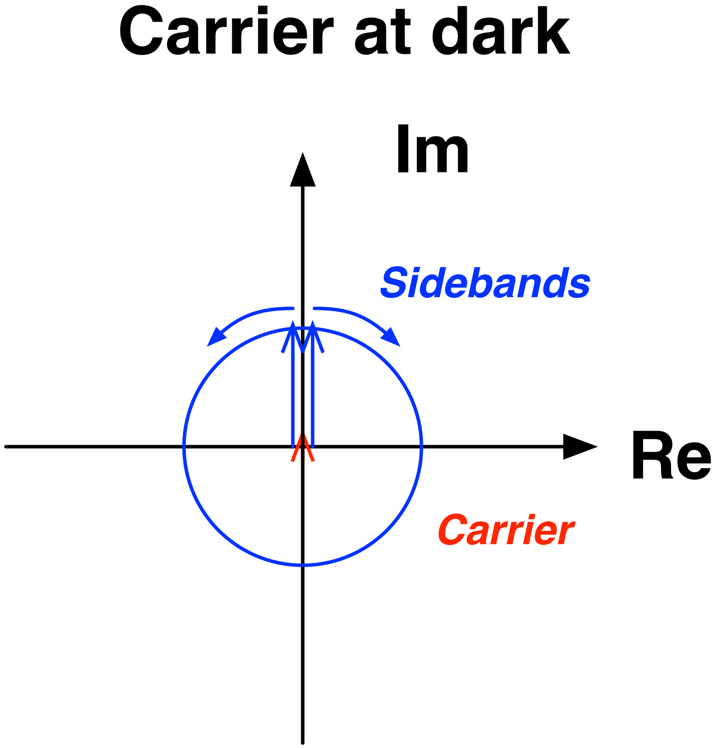
Michelson length control

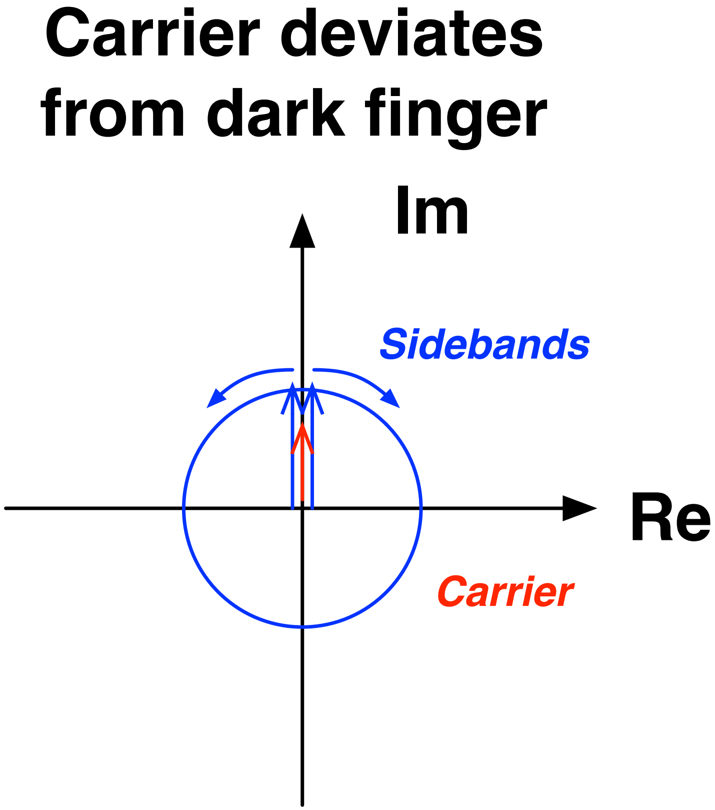
- Schnupp asymmetry ~ sideband picture
- Because of the asymmetry, the dark port is no longer dark for the sidebands even if the carrier is at dark.



Michelson length control

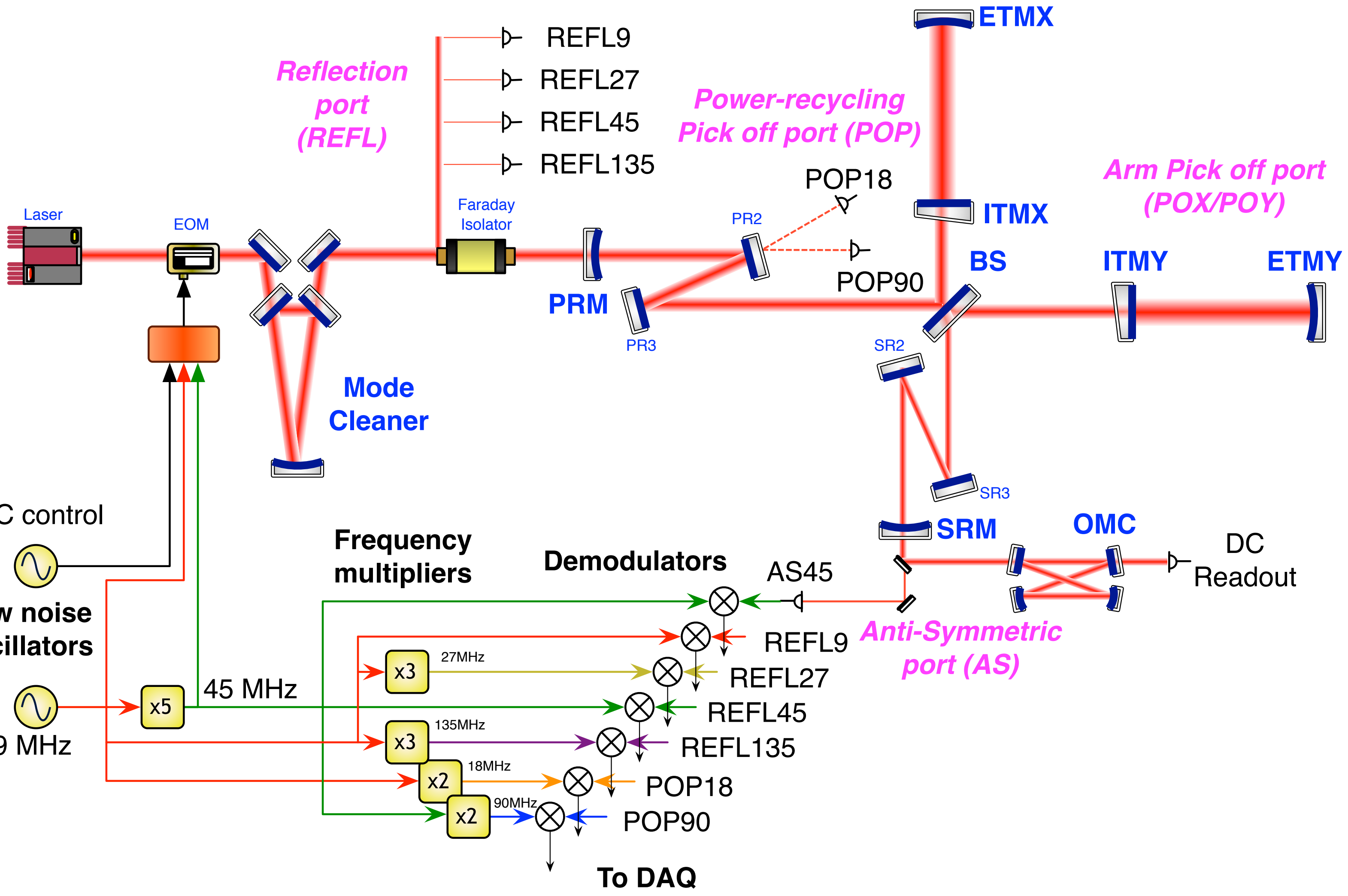
- Schnupp asymmetry ~ phasor picture





aLIGO length control

- Actual aLIGO Length control scheme



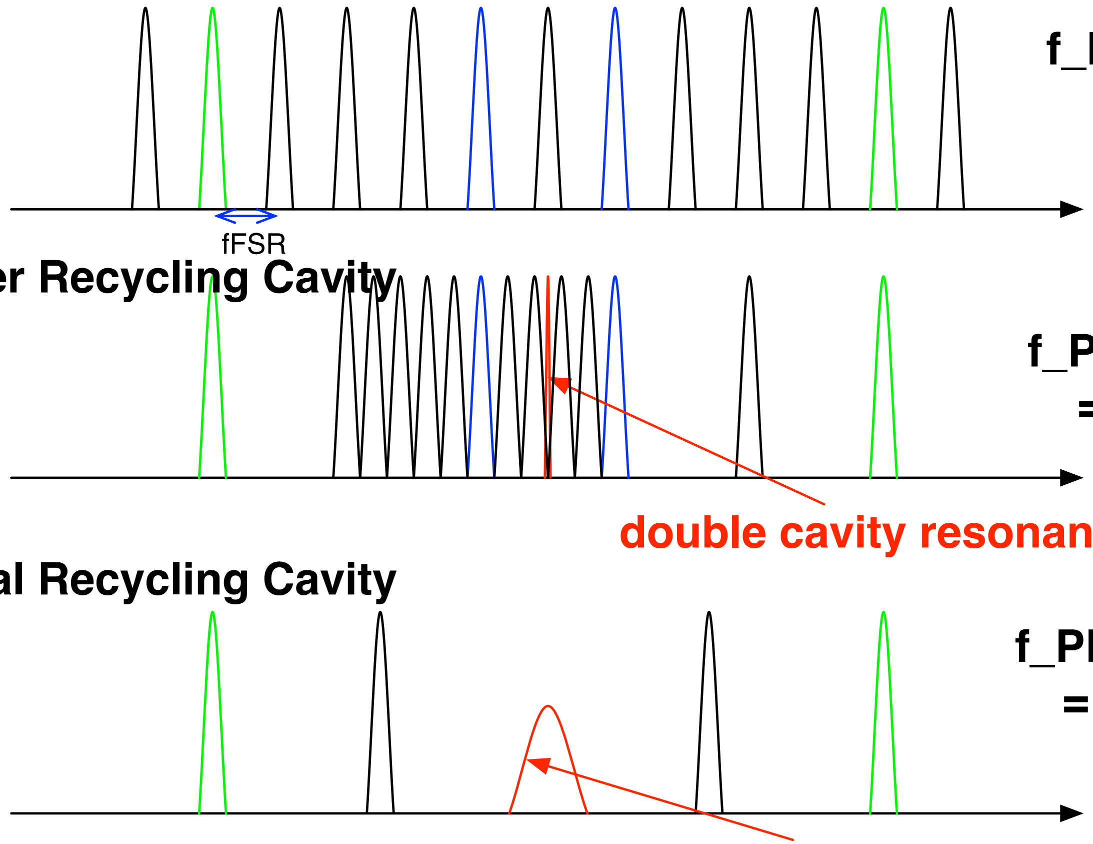
Resonant conditions

- For signal extraction purpose we need to resonate two sidebands (f1 and f2).
  - f1 +1/-1 SBs should be resonant in the PRC
  - f2 +1/-1 SBs should be resonant in both the PRC and SRC

- The carrier, f1 SBs, and f2 SBs needs to go through the resonances of the input mode cleaner.

(f1 = n fFSR_MC, f2 = m fFSR_MC)

Resonant conditions



Mode cleaner resonances

f_MC = 9.1MHz

Power Recycling Cavity

f_PRC = 2.6 MHz
= f_MC/(7/2)

double cavity resonance

Signal Recycling Cavity

f_PRC = 2.68 MHz
= (f_MC*5)/17

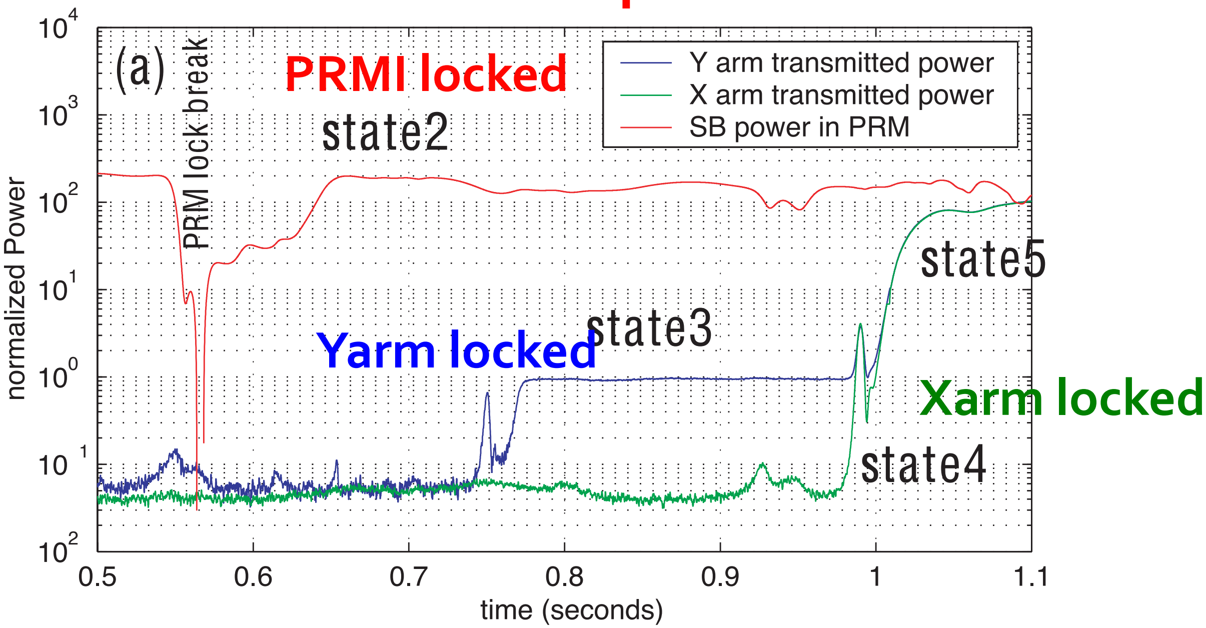
lowered finesse of the arm cavity resonance

(See also T070247)

Lock Acquisition: Real and Simulated

- Transition from non-operational state to the final linear state
  - Nonlinear process ~ Lock Acquisition

iLIGO Lock acquisition

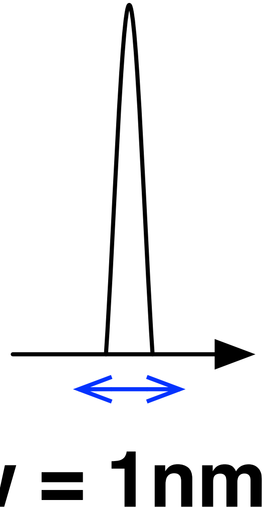


Arm powers are normalized by the power when one arm is locked.
SB power is normalized by the input SB power.

- Before the lock, we can’t make a diagnosis of the control loops Without diagnosis, the lock is difficult. (Chicken & egg problem)

Lock Acquisition: Real and Simulated

- Mirror velocity vs Required actuator force
  - Mirror momentum: p = m v
  - Interaction time: dt = w / v
  - Required force: F dt = m v
    => F = m v^2 / w



- For single arm case:
  - m: 40kg, v: 1um/s, w: 1nm => 0.04N
- For common mode (double cavity) case:
  - m: 40kg, v: 1um/s, w: 0.03nm => 1.3N
- For DC control of a 1m pendulum for 1um displacement
  - F = m g / l x = 0.4 mN
  - 3000 times smaller!
- We want longer interaction time! (quieter mirror)

Lock acquisition strategy

- Probabilistic lock & Deterministic lock
- Arm Length Stabilization (aka green locking)
- Transmission locking
- 3f locking
- PDH linearization

Lock acquisition strategy

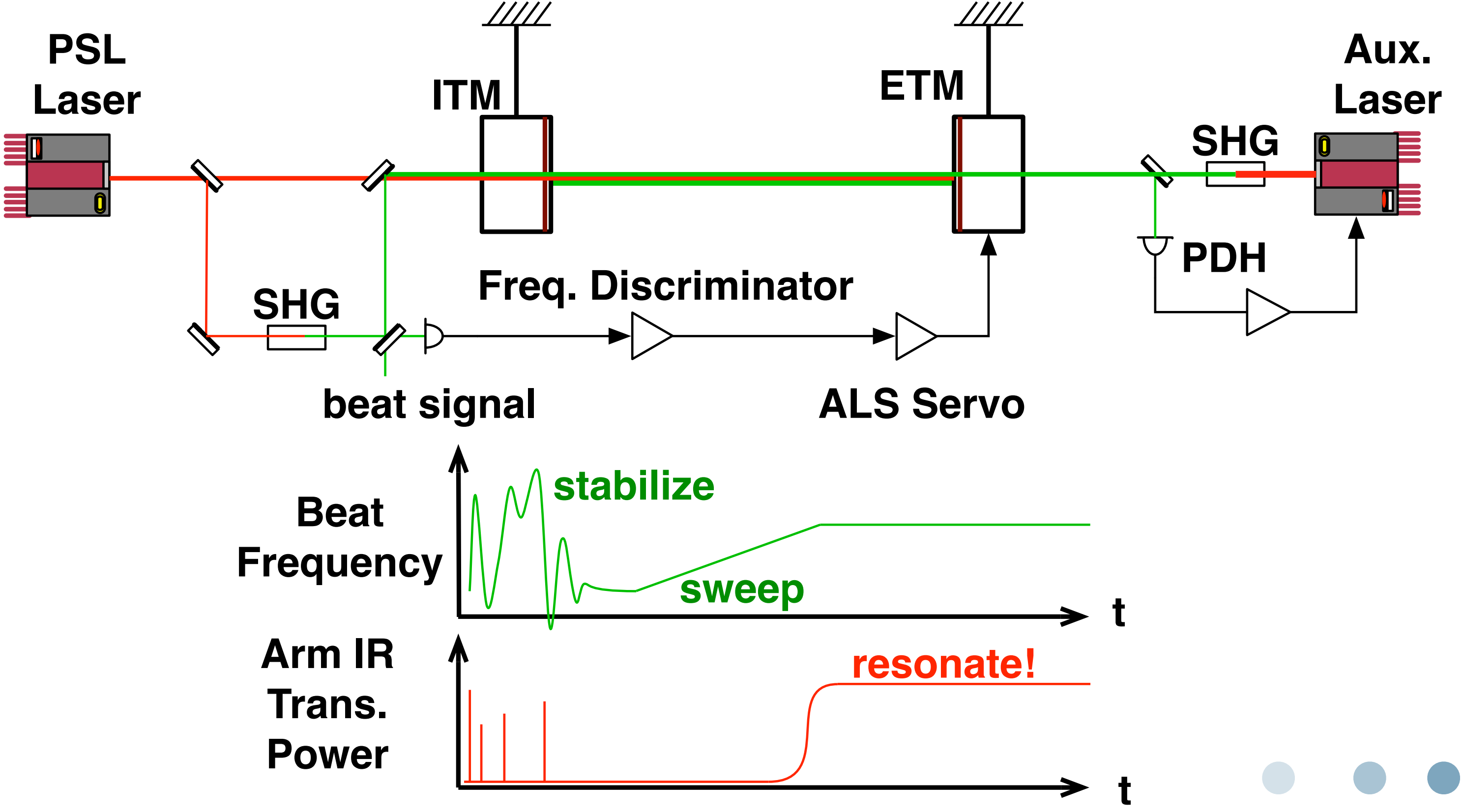
Green Locking

K. Izumi er al, "Multicolor cavity metrology", J. Opt. Soc. Am. A **29** (2012) 2092-2103

- Stabilizes the arm lengths using green beams

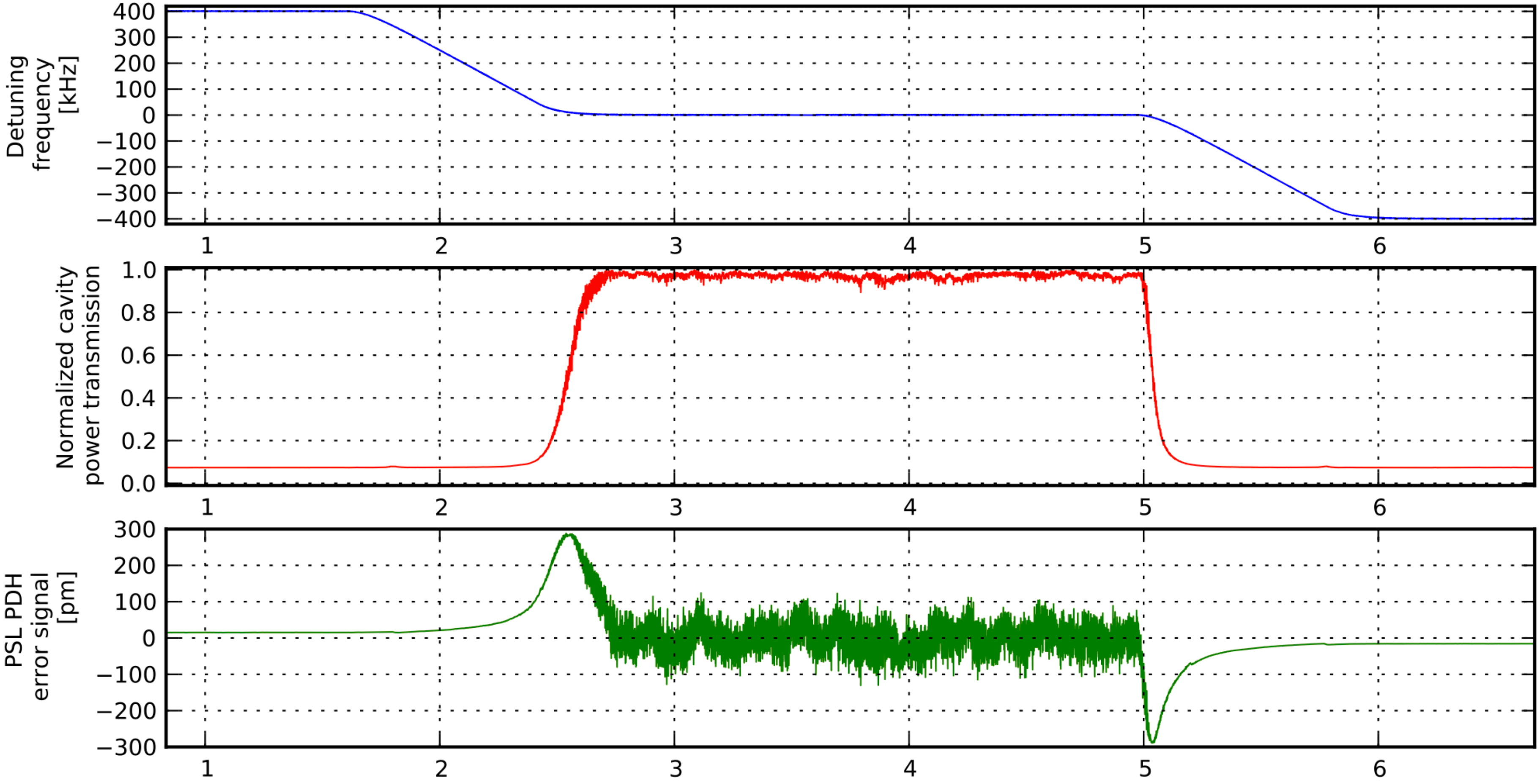
- Stabilizes arm fluctuation: from ~1um (~10MHz) to ~100pm (~1kHz)

- For deterministic lock acquisition



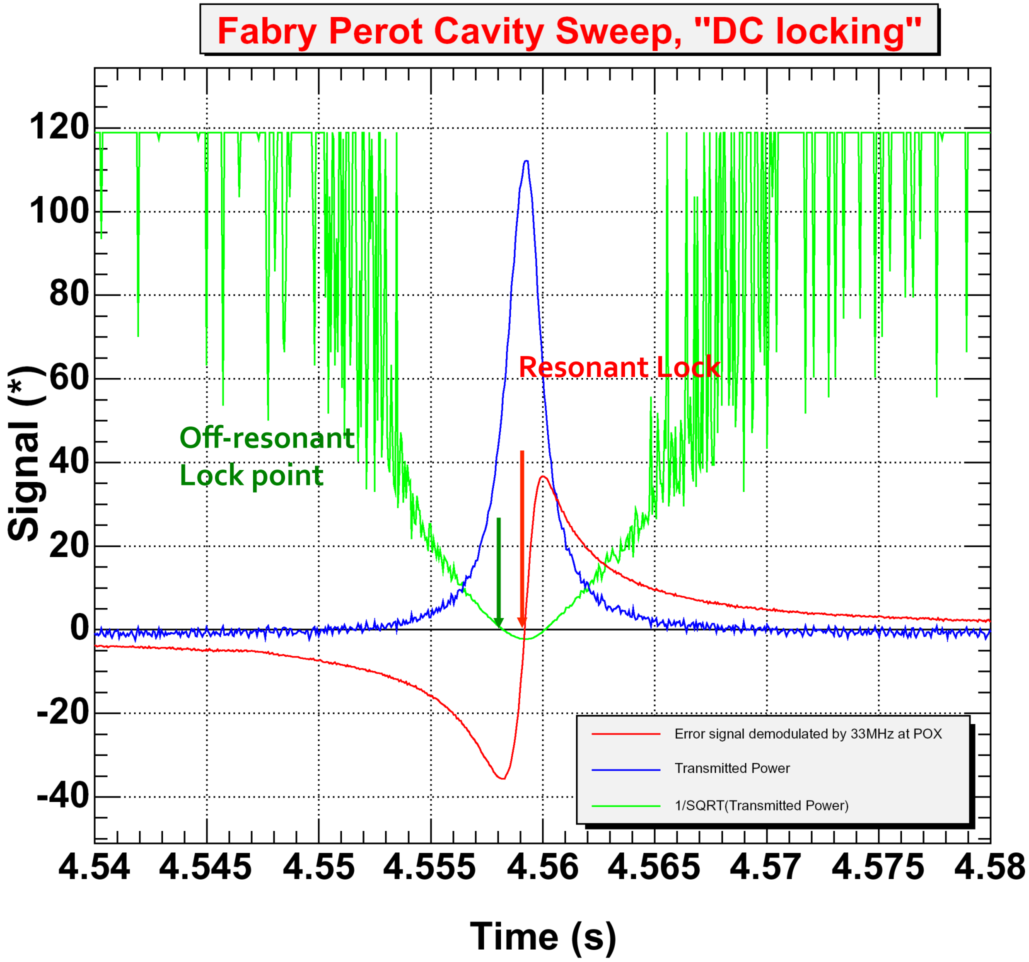
Lock acquisition strategy

- **Green Locking for one arm (40m prototype)**



Lock acquisition strategy

- Transmission locking



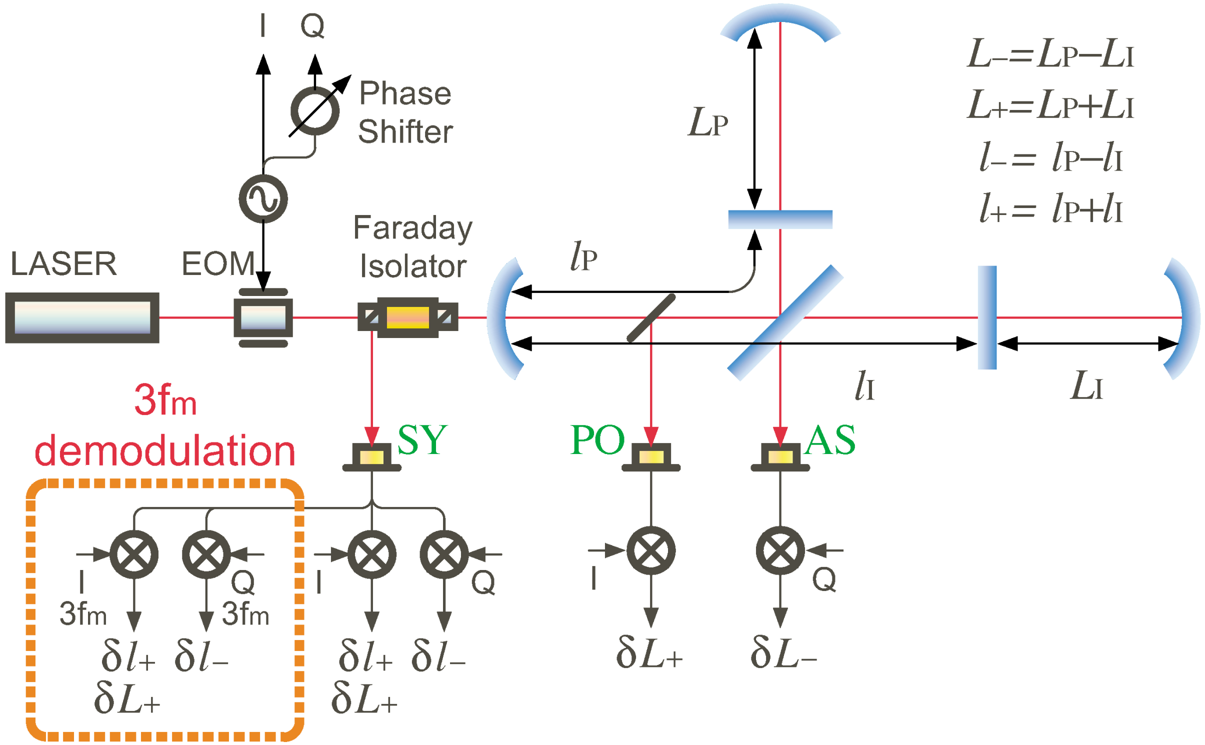
R. Ward PhD thesis (2009) California Institute of Technology
https://dcc.ligo.org/LIGO-P1400105

Lock acquisition strategy

- **3f locking (aka Arai technique!)**

Harmonic Demodulation Scheme

Demodulating reflected light at the 3rd harmonic frequency



K. Arai PhD thesis (2001) Univ. of Tokyo
http://t-munu.phys.s.u-tokyo.ac.jp/theses/arai_d.pdf

Lock acquisition strategy

Problems of conventional scheme

- Difficulty in separating δl_+ from δL_+

  ~ **because of the phase enhancement by the arm cavities**

  ~ **signal ports are ~100 times more sensitive to $\delta L+$ than to $\delta l+$**

- The δl_+ and δl_- signals can disappear

  The $\delta l+$ signal vanishes when G0 = G1 .

  The $\delta l-$ signal vanishes when G0 is maximized.

Recycling gain

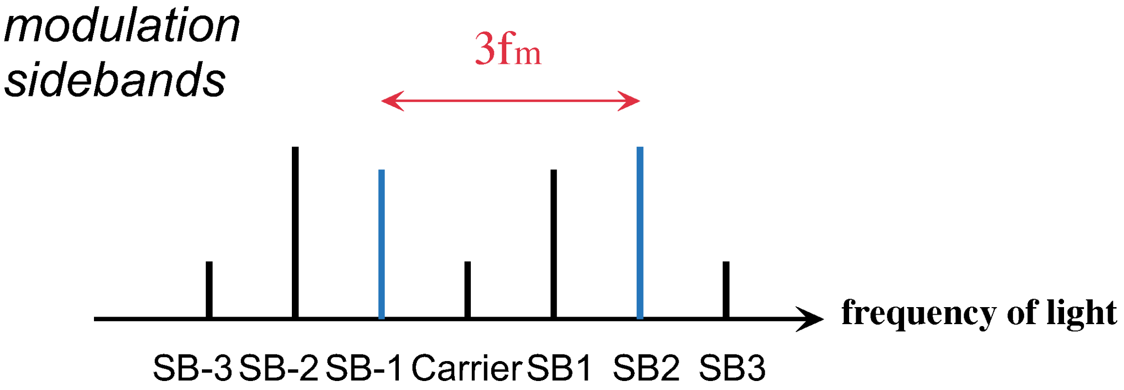
G_0 : for the carrier

G_1 : for the 1st order sidebands

Lock acquisition strategy

Principle

- Photocurrent at the $3f_m$ ~ beating of SB2 and SB-1



- SB2s are not resonant with the IFO

  → Effect of SB2

  Emphasized at the reflection port

  → The amplitudes and the signs

  Less dependent on the couplings of CA and SB1

Lock acquisition strategy

Advantages

- Easy to implement

  ~ no addtional modulation

- Contribution of δL_+ is reduced

  ~ in comparison with the fm demodulation

- Robust extraction of δl_+ and δl_-

  Amplitudes of the signals

  ~ less dependent on the optical parameters

  Signs of the signals

  ~ do not depend on the optical parameters
  ~ never change during lock acquisition

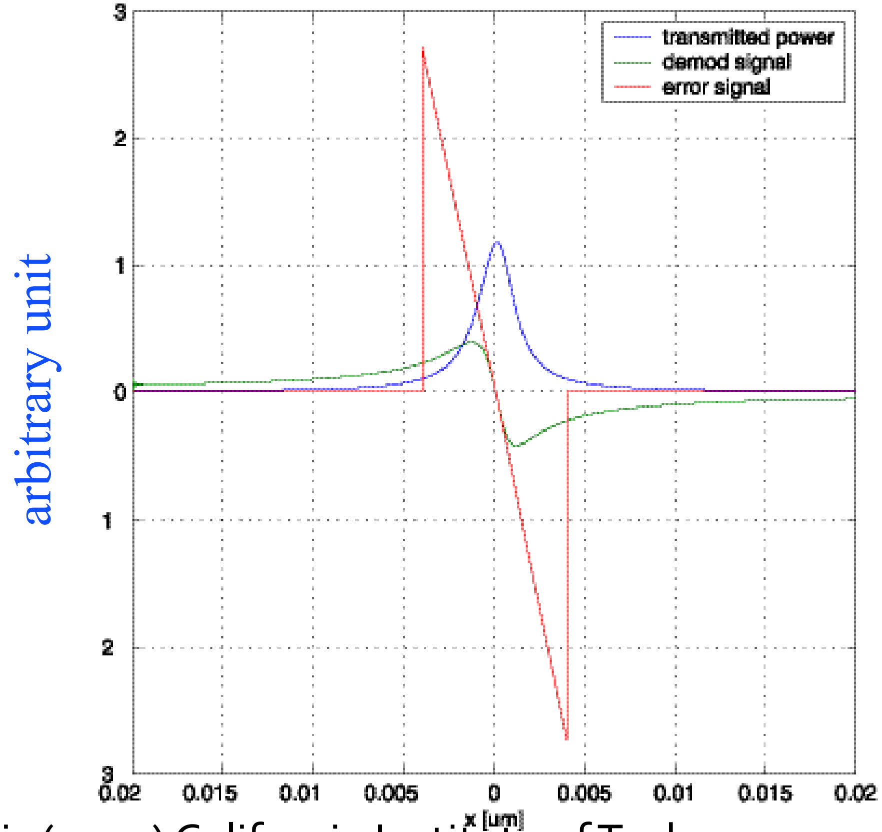
- Operating the IFO without the pick-off mirror

  ~ the δL_+ signal is extracted at the reflection port

Lock acquisition strategy

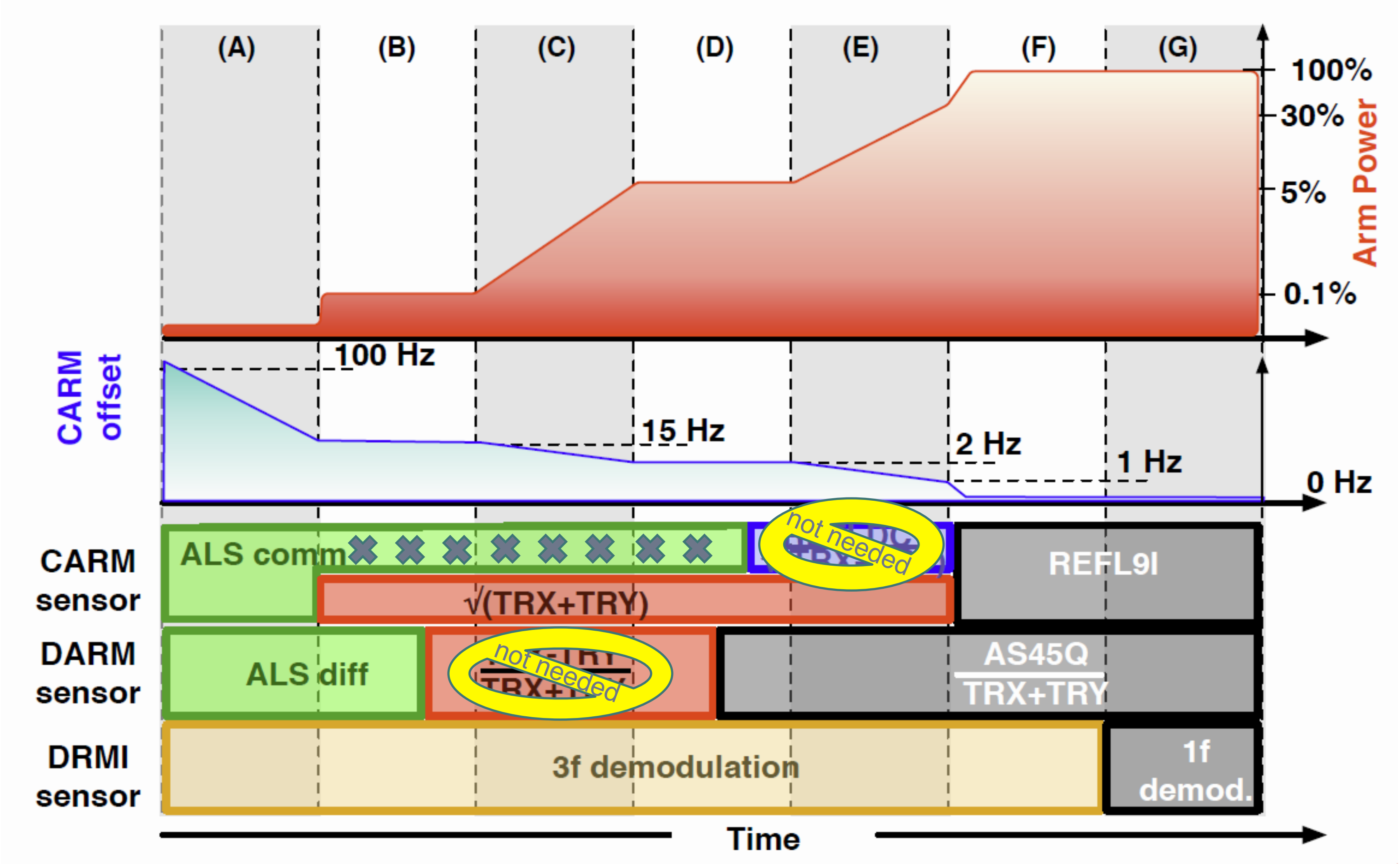
- **PDH linearization**

$$S_{err} = \frac{S_{demod}}{\left|A_{cav}\right|^2} \approx g_{FP}x, \text{ for } x << \lambda$$

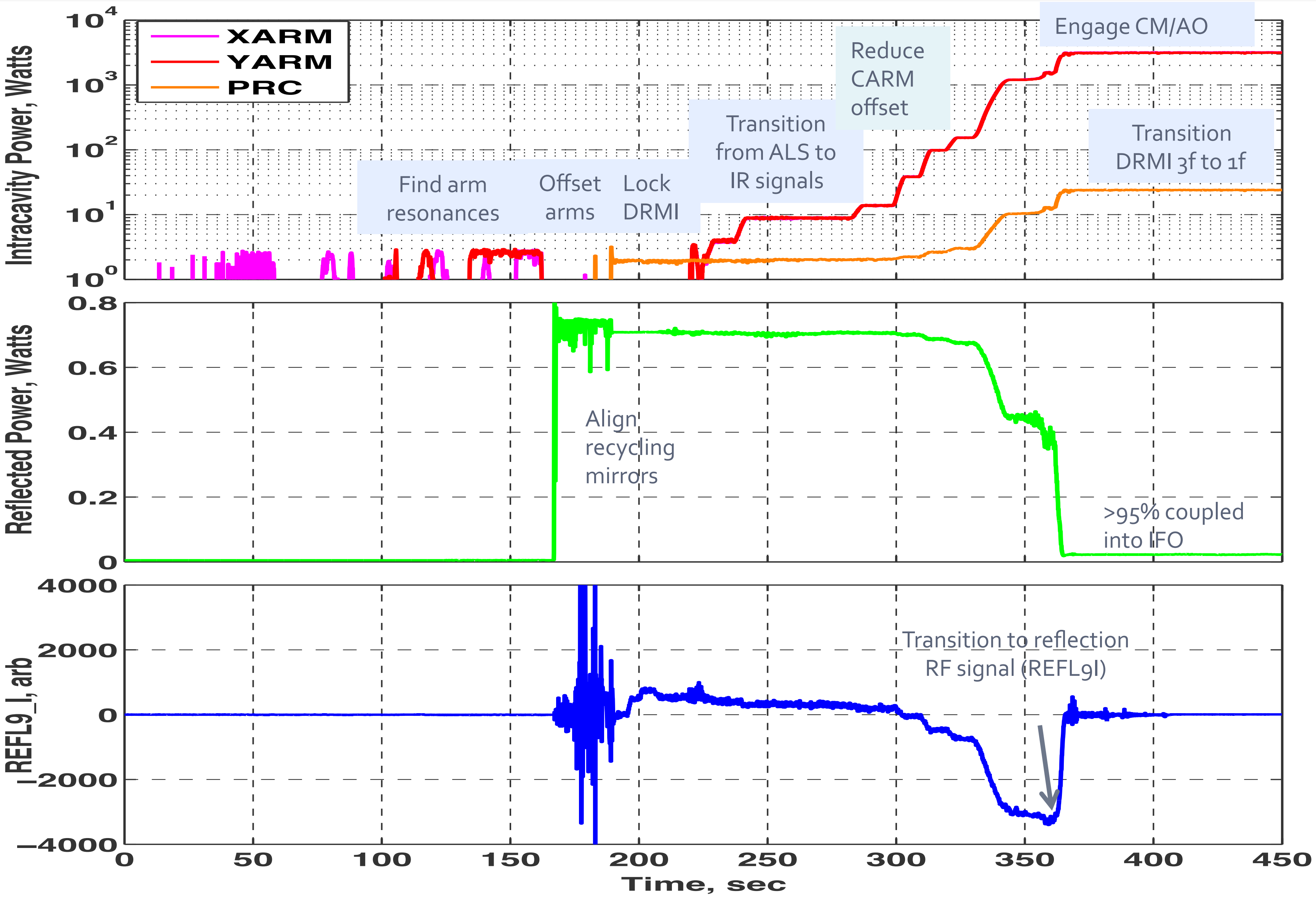


M. Evans PhD thesis (2002) California Institute of Tech
http://www.ligo.caltech.edu/~mevans/lockAcq/thesis.pdf

Design vs. realization



Locking Process: realized



Find arm resonances

Offset arms

Lock DRMI

Transition from ALS to IR signals

Reduce CARM offset

Engage CM/AO

Transition DRMI 3f to 1f

Align recycling mirrors

>95% coupled into IFO

Transition to reflection RF signal (REFL9I)