Noise Cancellation for Gravitational Wave Detectors

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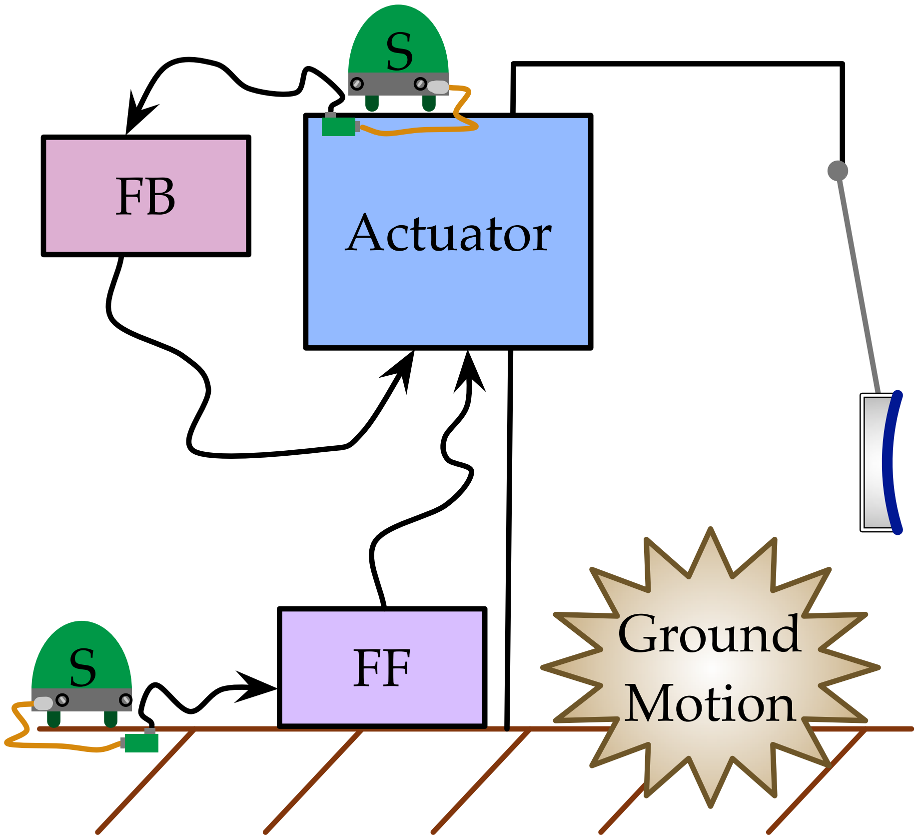
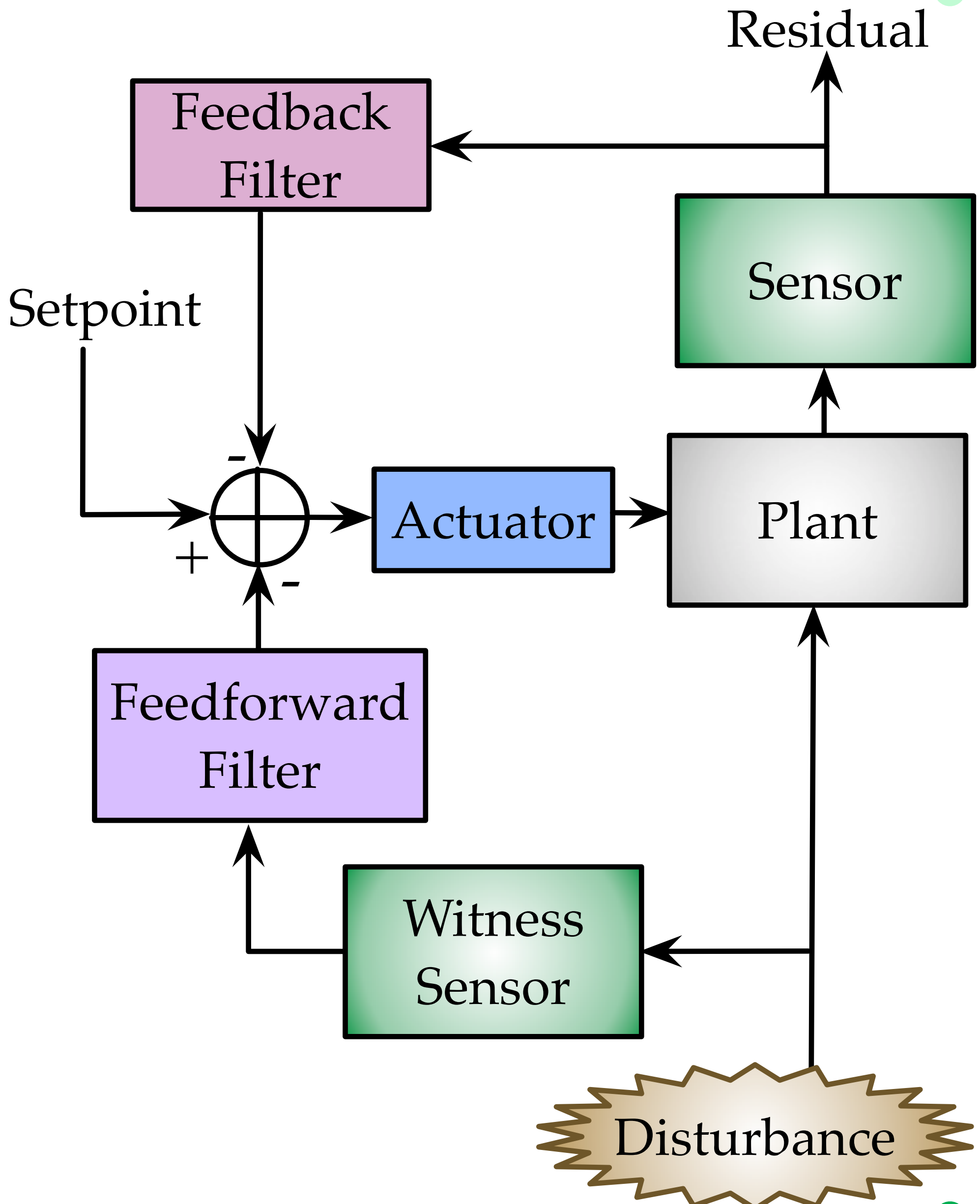
Noise Sources and Cancellation

Noise Sources

- Seismic noise
- Alignment and angular noise coupling to gravitational wave channel
- Newtonian gravitational noise
- Auxiliary length control coupling to gravitational wave channel

Feedback vs. Feedforward





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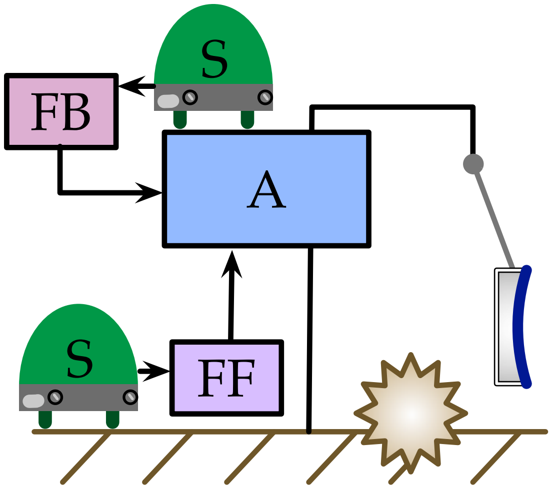

Feedback

- Pros:
  - Can handle small variations in the plant
  - Only need rough model of plant
- Cons:
  - Time lag
  - Disturbances must pass through system

Feedforward

- Pros:
  - Does not require disturbance to propagate through system
  - Predicts incoming disturbances
- Cons:
  - Requires very accurate model of system
  - Can only handle disturbances that are externally witnessed

Global Seismic Noise Cancellation

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Global seismic cancellation has been done before, but most of the focus in recent years has been on local seismic cancellation

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- Static noise cancellation simulations, and adaptive implementation at the 40m:
  - **J. C. Driggers**, M. Evans, K. Pepper, R. Adhikari "Active noise cancellation in a suspended interferometer" Rev. Sci. Instrum. 83, 024501 (2012) Go to paper

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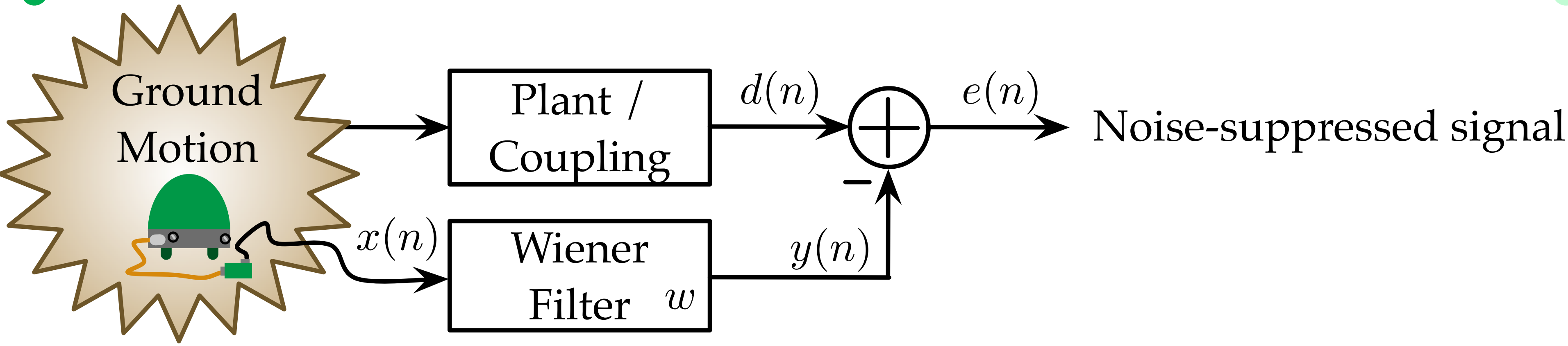
- Static noise cancellation implementation during Enhanced LIGO:
  - R. DeRosa, **J. C. Driggers**, D. Atkinson, H. Miao, V. Frolov, M. Landry, J. A. Giame, R. Adhikari "Global feed-forward vibration isolation in a km scale interferometer" Class. Quantum Grav. 29, 215008 (2012) Go to paper

Optimal Wiener Filters



R is auto-correlation matrix - how is sensor self-correlated?

$\vec{p}$ is cross-correlation vector - how are the sensor and the desired signal related?

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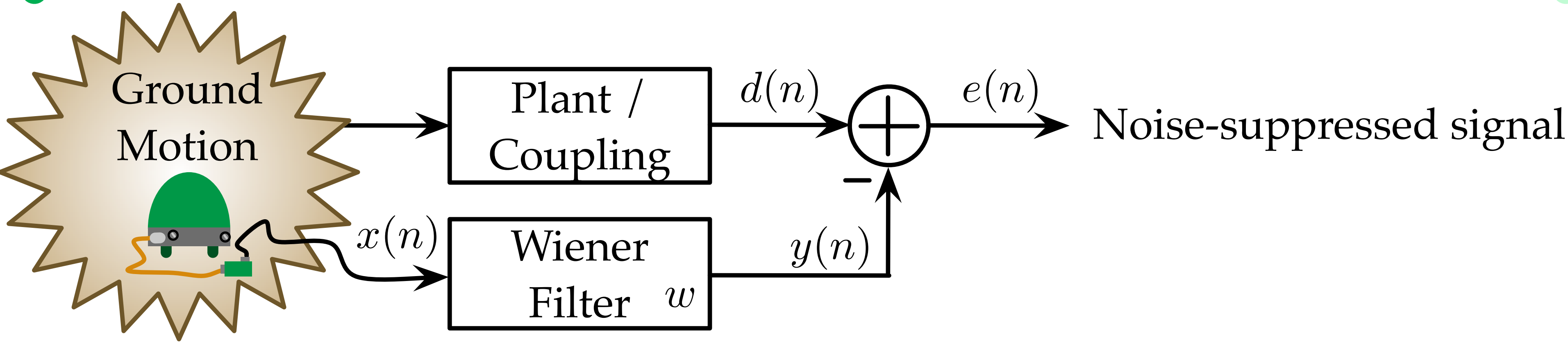

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Define a cost function, set derivative equal to zero: $\xi = E[e(n)^2]$

Solve for w: $\vec{w}_{\text{optimum}} = R^{-1}\vec{p}$

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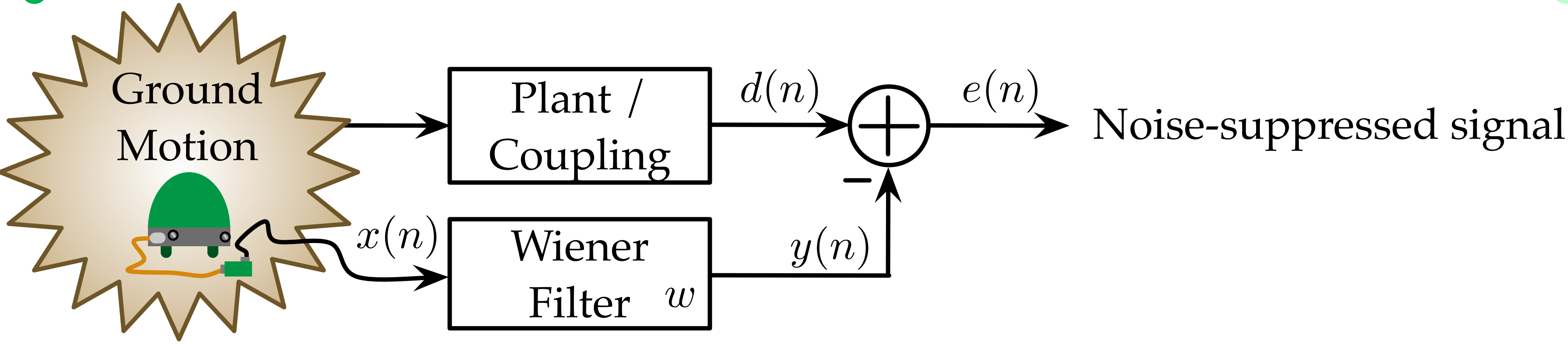

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Numerical precision problems arise for matrix inversion and witness pre-filtering

Wiener Filters

Cost function is the mean square error between the target signal and the estimate of the target

$$\text{MSE} = \frac{1}{2}\sum_i \left[d_i - \sum_{j=0}^{N} w_j x_{i-j} \right]^2$$

$x =$ witness sensor

$d =$ target signal

$w =$ Wiener coefficients

$N =$ filter order

To find the extrema, we set $\dfrac{\partial \text{MSE}}{\partial w_j} = 0$

This gives us the Wiener-Hopf equations $p_i = \sum_{j=0}^{N} h_j R_{(j-i)}$

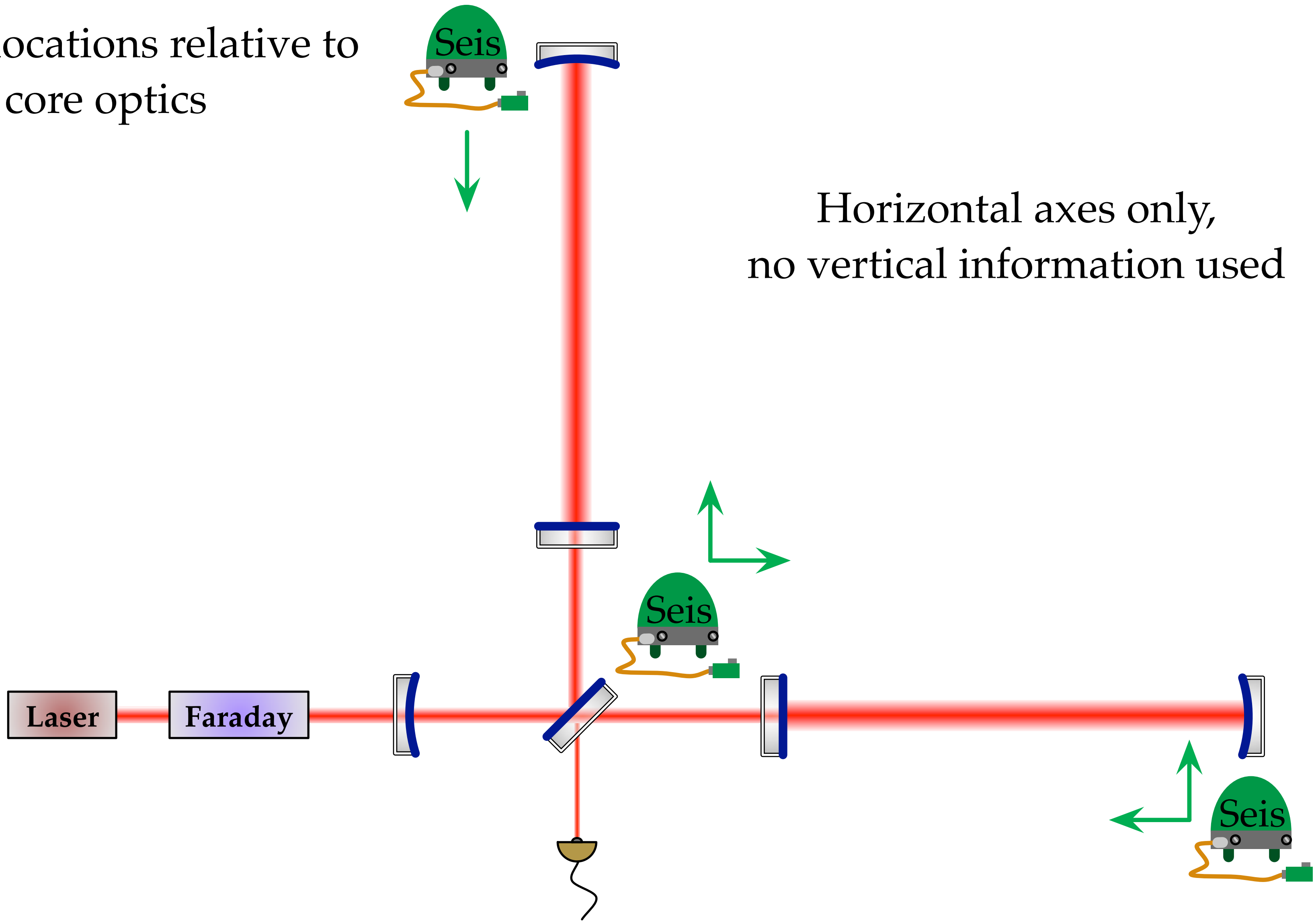
R is a symmetric Toeplitz matrix

$$R_{(j-i)} = \begin{pmatrix} R[0] & R[1] & \cdots & R[N] \\ R[1] & R[0] & \cdots & R[N-1] \\ \vdots & \vdots & \ddots & \vdots \\ R[N] & R[N-1] & \cdots & R[0] \end{pmatrix}$$

Static Seismic Noise Cancellation

Sensor locations relative to core optics

Horizontal axes only, no vertical information used



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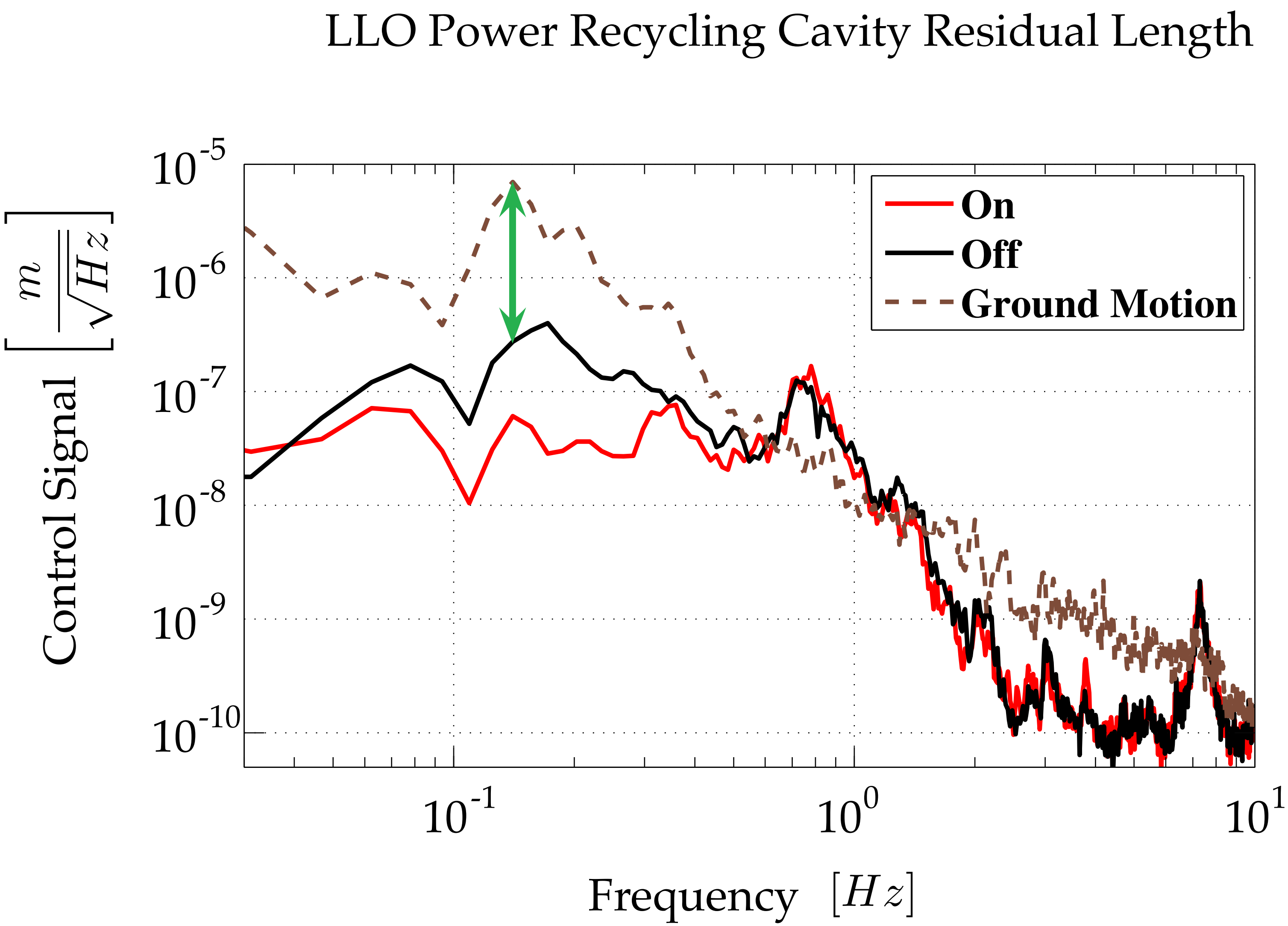
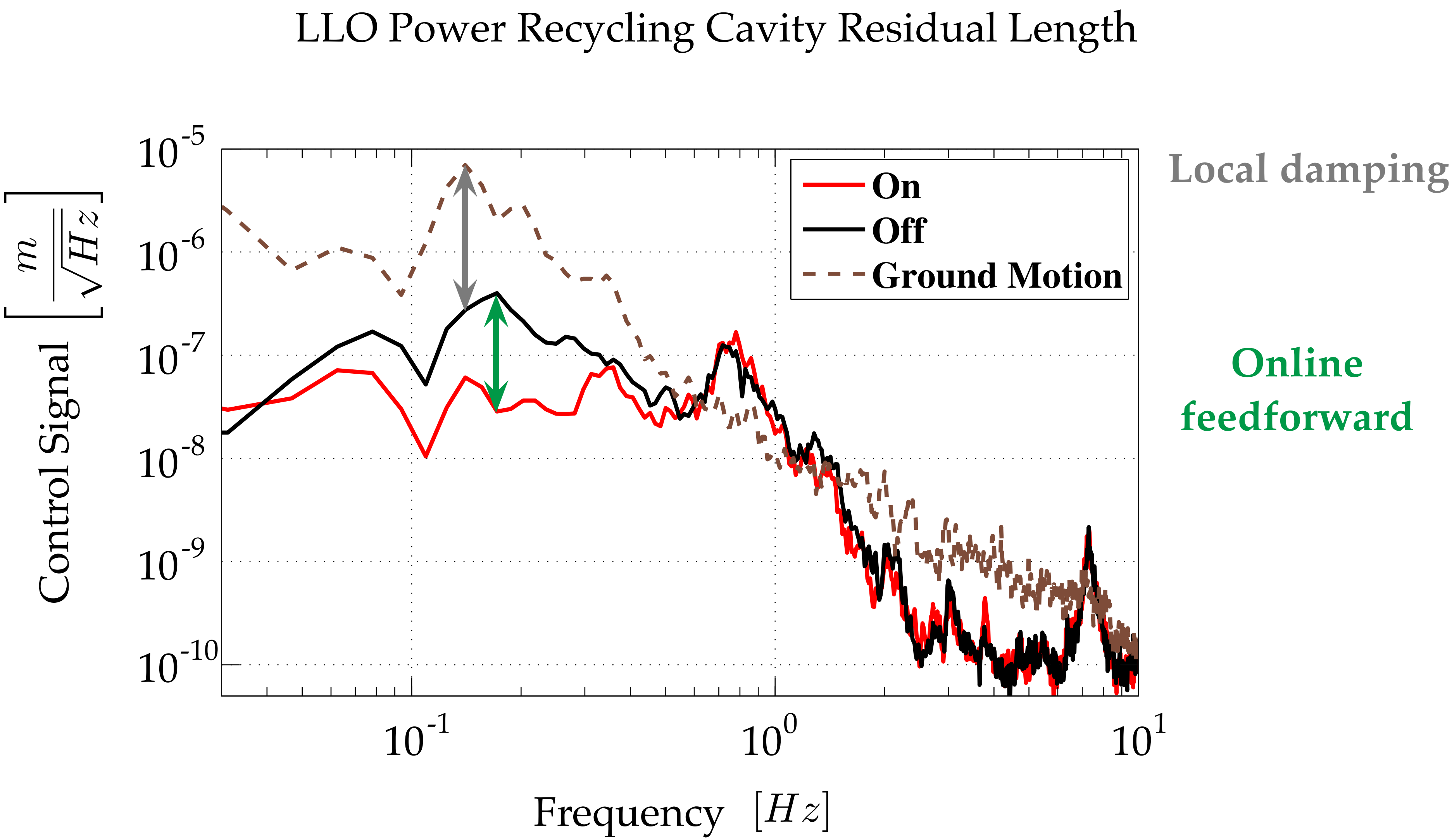


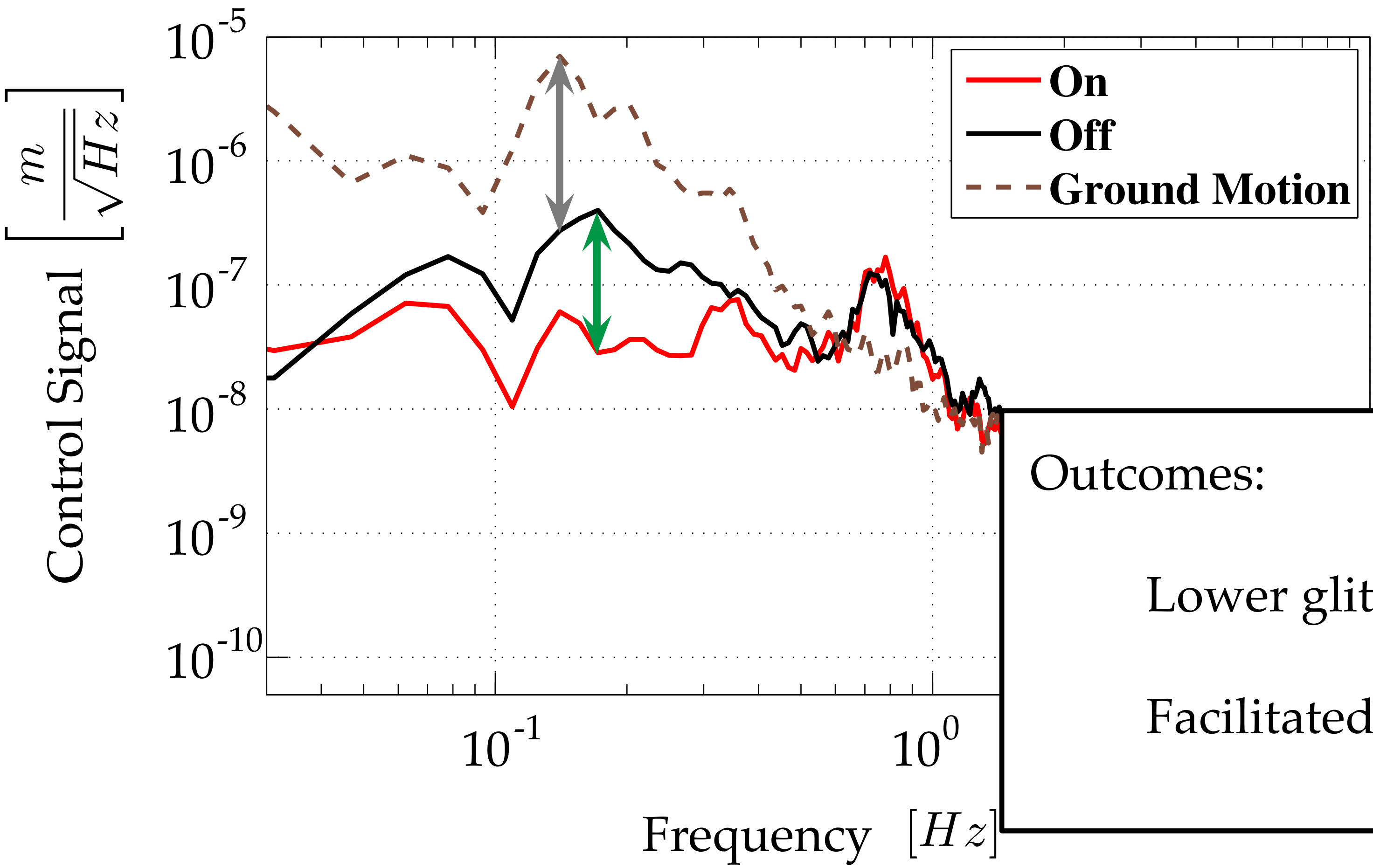
Local damping

Static Seismic Noise Cancellation



Static Seismic Noise Cancellation

LLO Power Recycling Cavity Residual Length



Local damping

Online feedforward

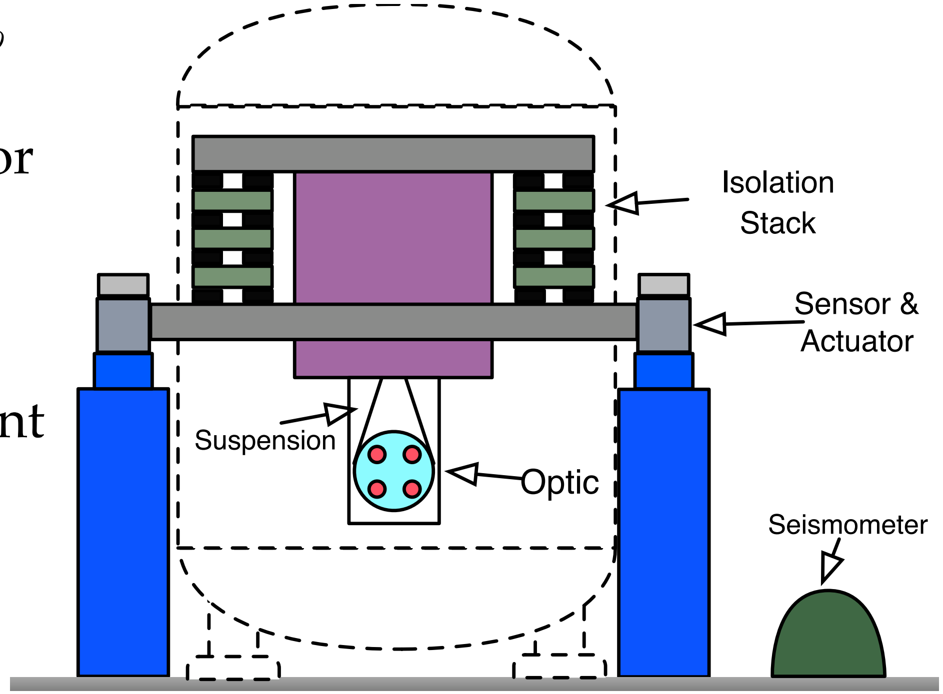
Outcomes:

Lower glitch rate

Facilitated S6 high power

Static Seismic Noise Cancellation

- Challenges:
  - Measure transfer function between actuator and desired signal to 1%
  - Fit measured transfer function, for pre-filtering use, before Wiener filter was calculated
  - Then fit Wiener filter to implement in digital real-time system



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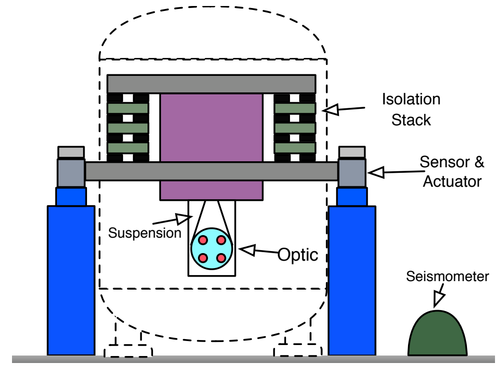

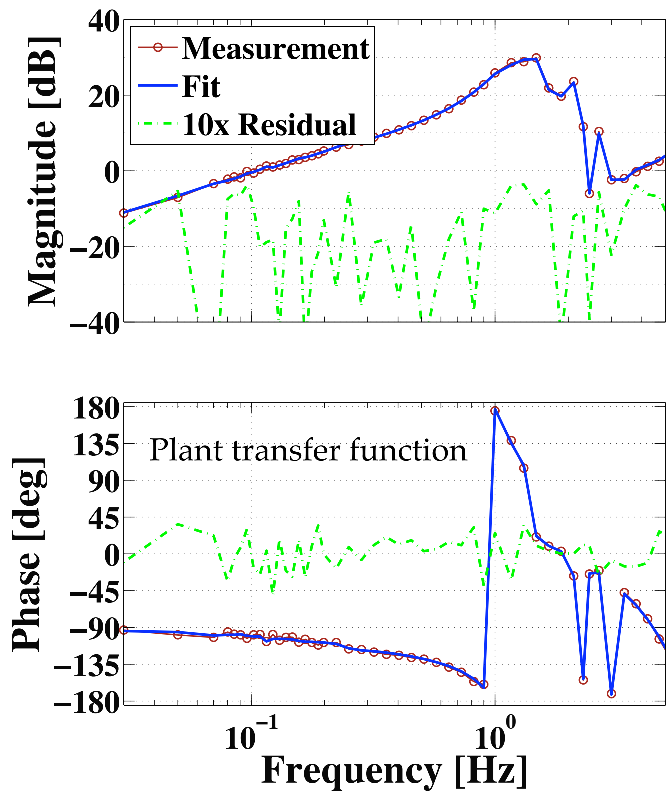

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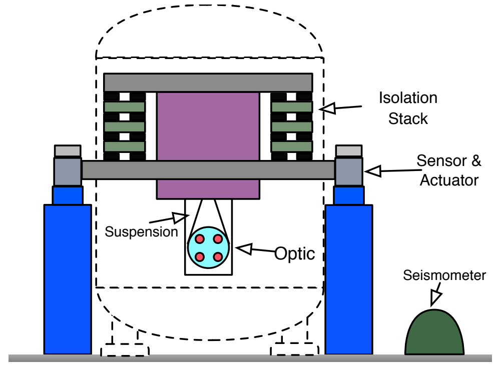
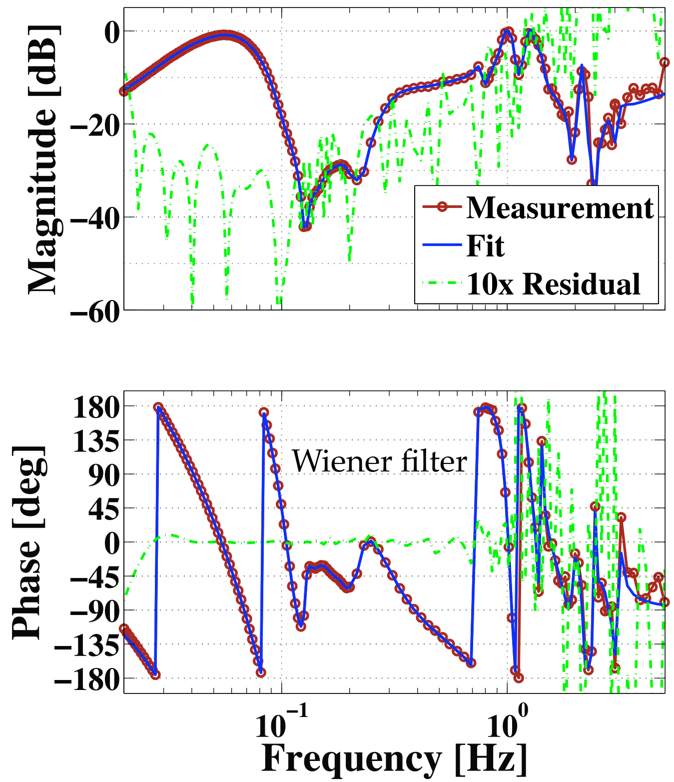
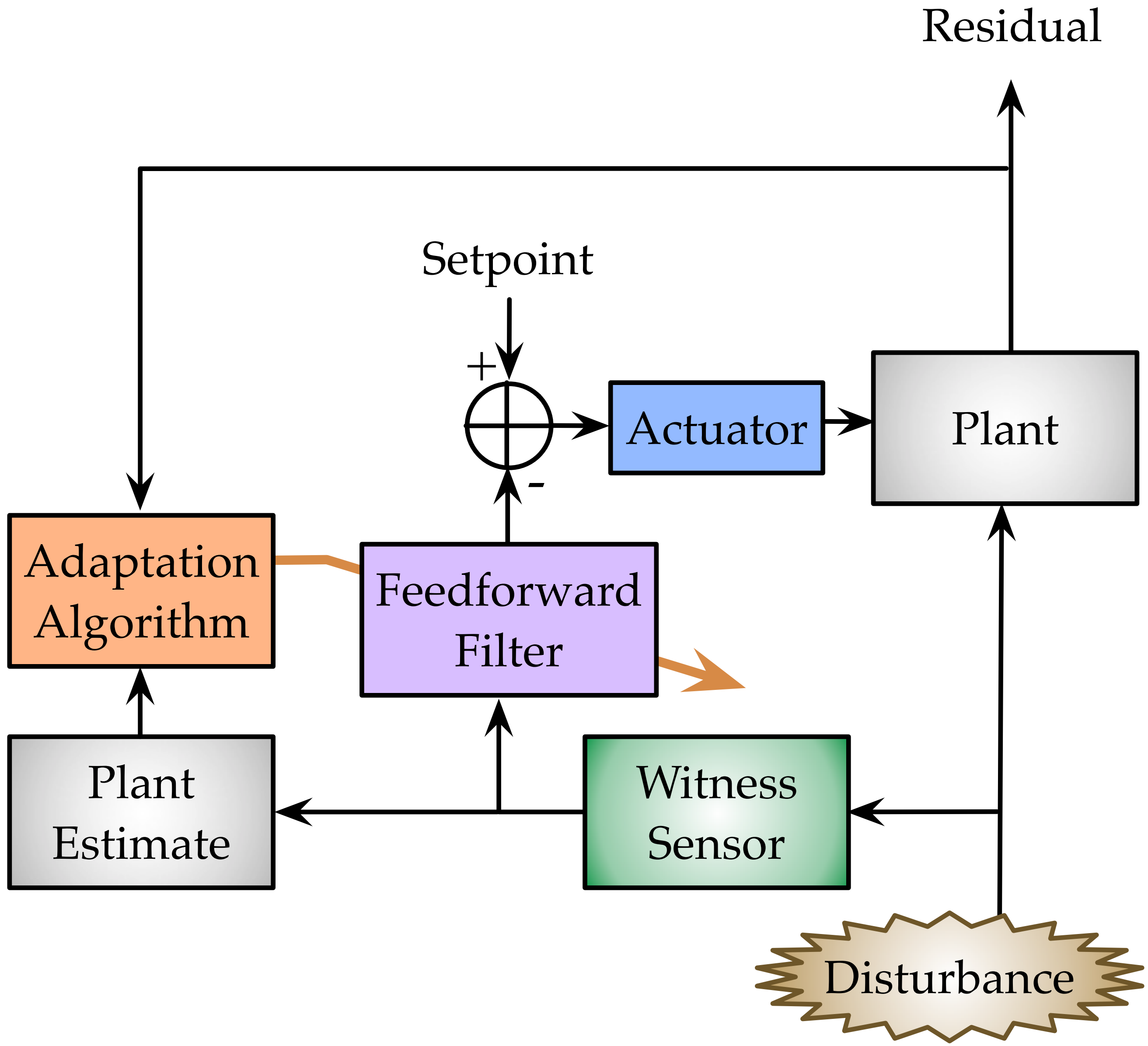




Adaptive Noise Cancellation

Similar to static technique, but can follow changes in transfer function

Adjust the feedforward filter in realtime for optimal cancellation



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We use a **leaky normalized filtered-x least mean squared** (LMS) algorithm

Combination of 3 modifications to the simple LMS algorithm

$$w(n+1) = w(n) + \mu(1-\tau)x(n)e(n)$$

$\vec{w}$ is the vector of filter weights

μ is the step size

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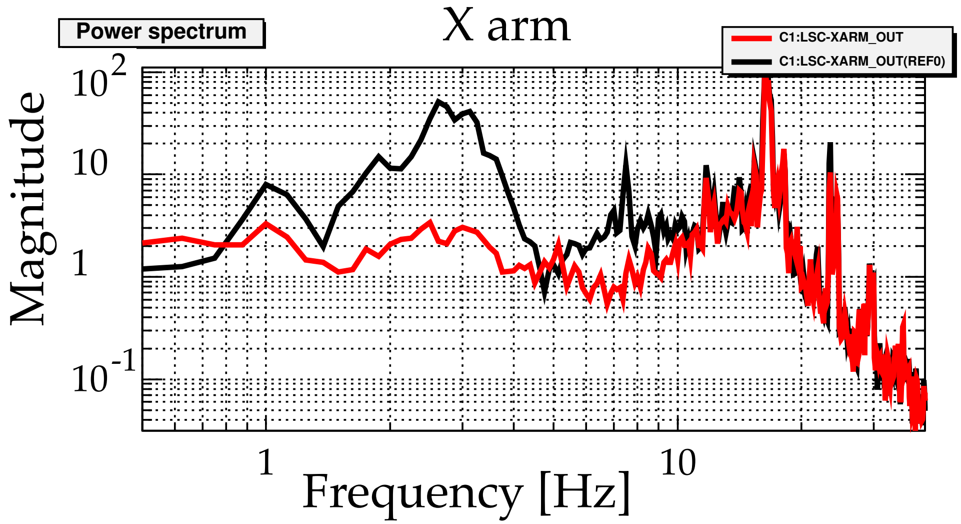
Leaky: allow response to decay using $(1-\tau)$

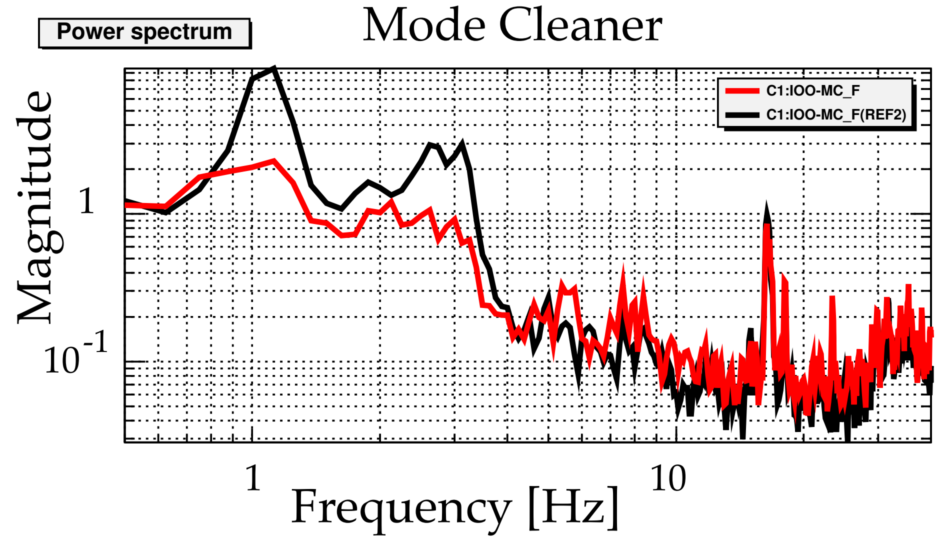
Normalized: μ is a function of time $\mu(n) = \frac{\mu}{\vec{x}^T(n)\vec{x}(n)}$

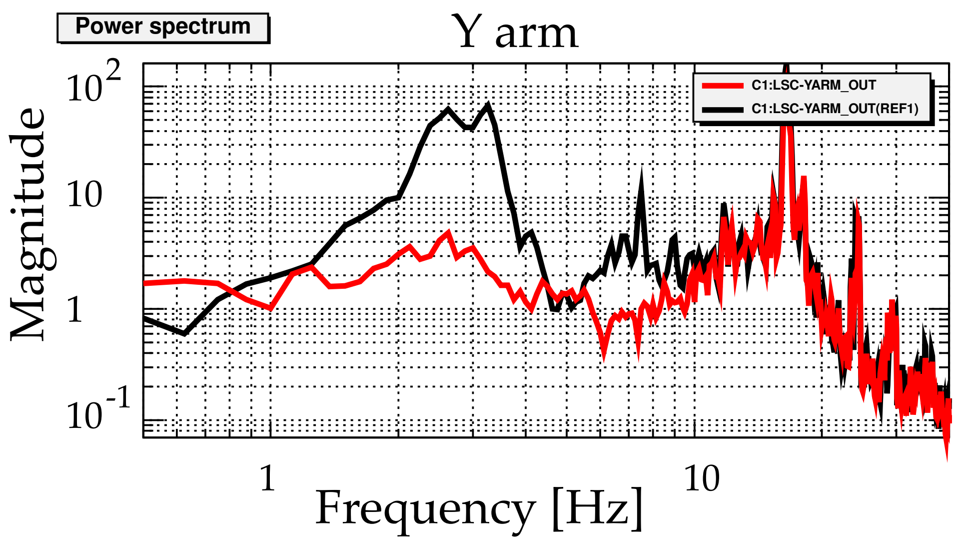
Filtered-x: pre-filter $\vec{x}(n)$ with an estimate of the plant transfer function

Adaptive Noise Cancellation

Implemented at the 40m







No seismic noise cancellation

Adaptive noise cancellation ON

μ(Mode Cleaner) = 0.2

μ(Arms) ~ 0.04

Future Extensions

- Offline analysis of S5 H1/H2 to potentially improve stochastic searches
- Other external sensors
  - Laser power monitors
  - Microphones
  - Magnetometers
- Offline analysis on One Arm Test data to see aLIGO potential
- Remove auxiliary length degrees of freedom from gravitational wave channel
- Implement online (work already begun at LLO by others)