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Matrix calculations for the angular control of H1 OMC

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1 Introduction

This document describes Matrix calculations for the OMC angular control. The description is so far limited to the QPD alignment servo for H1 OMC.[1]

2 Transfer function measurement

The ISC alignment inputs of OM1/2/3 were excited at 3.9Hz and 2.9Hz for Pitch and Yaw, respectively[2] . For the excitations, LOCK filters in the suspension model were used. The ALIGNDRIVE matrix was already filled so that the pitch and yaw actuations are diagonalized[3]. The spot motion was read by QPDA and QPDB. Note that the QPD X/Y signals are normalized by the sum signal so they are supposed to span from -1 to +1. Measured transfer functions are

$$\begin{pmatrix} \texttt{H1:OMC-ASC\_QPD\_A\_PIT\_OUT} \\ \texttt{H1:OMC-ASC\_QPD\_B\_PIT\_OUT} \end{pmatrix} \equiv \begin{pmatrix} T_{\rm OM1P\_QAP} & T_{\rm OM2P\_QAP} & T_{\rm OM3P\_QAP} \\ T_{\rm OM1P\_QBP} & T_{\rm OM2P\_QBP} & T_{\rm OM3P\_QBP} \end{pmatrix} \begin{pmatrix} \texttt{H1:SUS-OM1\_M1\_LOCK\_P\_IN1} \\ \texttt{H1:SUS-OM2\_M1\_LOCK\_P\_IN1} \\ \texttt{H1:SUS-OM3\_M1\_LOCK\_P\_IN1} \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} -1.05\times10^{-3} & -1.64\times10^{-3} & -6.24\times10^{-4} \\ -6.38\times10^{-4} & -1.86\times10^{-3} & -9.55\times10^{-4} \end{pmatrix} \begin{pmatrix} \texttt{H1:SUSOM1\_M1\_LOCK\_P\_IN1} \\ \texttt{H1:SUSOM2\_M1\_LOCK\_P\_IN1} \\ \texttt{H1:SUSOM3\_M1\_LOCK\_P\_IN1} \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} \texttt{H1:OMC-ASC\_QPD\_A\_YAW\_OUT} \\ \texttt{H1:OMC-ASC\_QPD\_B\_YAW\_OUT} \end{pmatrix} \equiv \begin{pmatrix} T_{\rm OM1Y\_QAY} & T_{\rm OM2Y\_QAY} & T_{\rm OM3Y\_QAY} \\ T_{\rm OM1Y\_QBY} & T_{\rm OM2Y\_QBY} & T_{\rm OM3Y\_QBY} \end{pmatrix} \begin{pmatrix} \texttt{H1:SUS-OM1\_M1\_LOCK\_Y\_IN1} \\ \texttt{H1:SUS-OM2\_M1\_LOCK\_Y\_IN1} \\ \texttt{H1:SUS-OM3\_M1\_LOCK\_Y\_IN1} \end{pmatrix} \quad (3)$$

$$= \begin{pmatrix} -2.21\times10^{-3} & 3.05\times10^{-3} & -1.35\times10^{-3} \\ 1.11\times10^{-3} & -3.13\times10^{-3} & 1.84\times10^{-3} \end{pmatrix} \begin{pmatrix} \texttt{H1:SUSOM1\_M1\_LOCK\_Y\_IN1} \\ \texttt{H1:SUSOM2\_M1\_LOCK\_Y\_IN1} \\ \texttt{H1:SUSOM3\_M1\_LOCK\_Y\_IN1} \end{pmatrix} \quad (4)$$

Here we define the combined matrix $\mathcal{T}$:

$$\mathcal{T} \equiv \begin{pmatrix} T_{\rm OM1P\_QAP} & T_{\rm OM2P\_QAP} & T_{\rm OM3P\_QAP} & 0 & 0 & 0 \\ T_{\rm OM1P\_QBP} & T_{\rm OM2P\_QBP} & T_{\rm OM3P\_QBP} & 0 & 0 & 0 \\ 0 & 0 & 0 & T_{\rm OM1Y\_QAY} & T_{\rm OM2Y\_QAY} & T_{\rm OM3Y\_QAY} \\ 0 & 0 & 0 & T_{\rm OM1Y\_QBY} & T_{\rm OM2Y\_QBY} & T_{\rm OM3Y\_QBY} \end{pmatrix} \quad (5)$$

3 Suspension calibration

The measured transfer functions contain transfer function of the suspensions. Here we convert the transfer function to the DC response of the sensors.

The transfer function from the LOCK filters to the inputs of the damping filters are supposed to be calibrated to show the suspension responses in the unit of μrad. The measurement is shown in Figure 1.

Here is the summary of the measurement:

[1]We are hoping to include more measurements and calculations about the angular control of the OMC later. It should include the details of dither alignment.

[2]https://alog.ligo-wa.caltech.edu/aLOG/index.php?callRep=13655.

[3]https://alog.ligo-wa.caltech.edu/aLOG/index.php?callRep=13618.

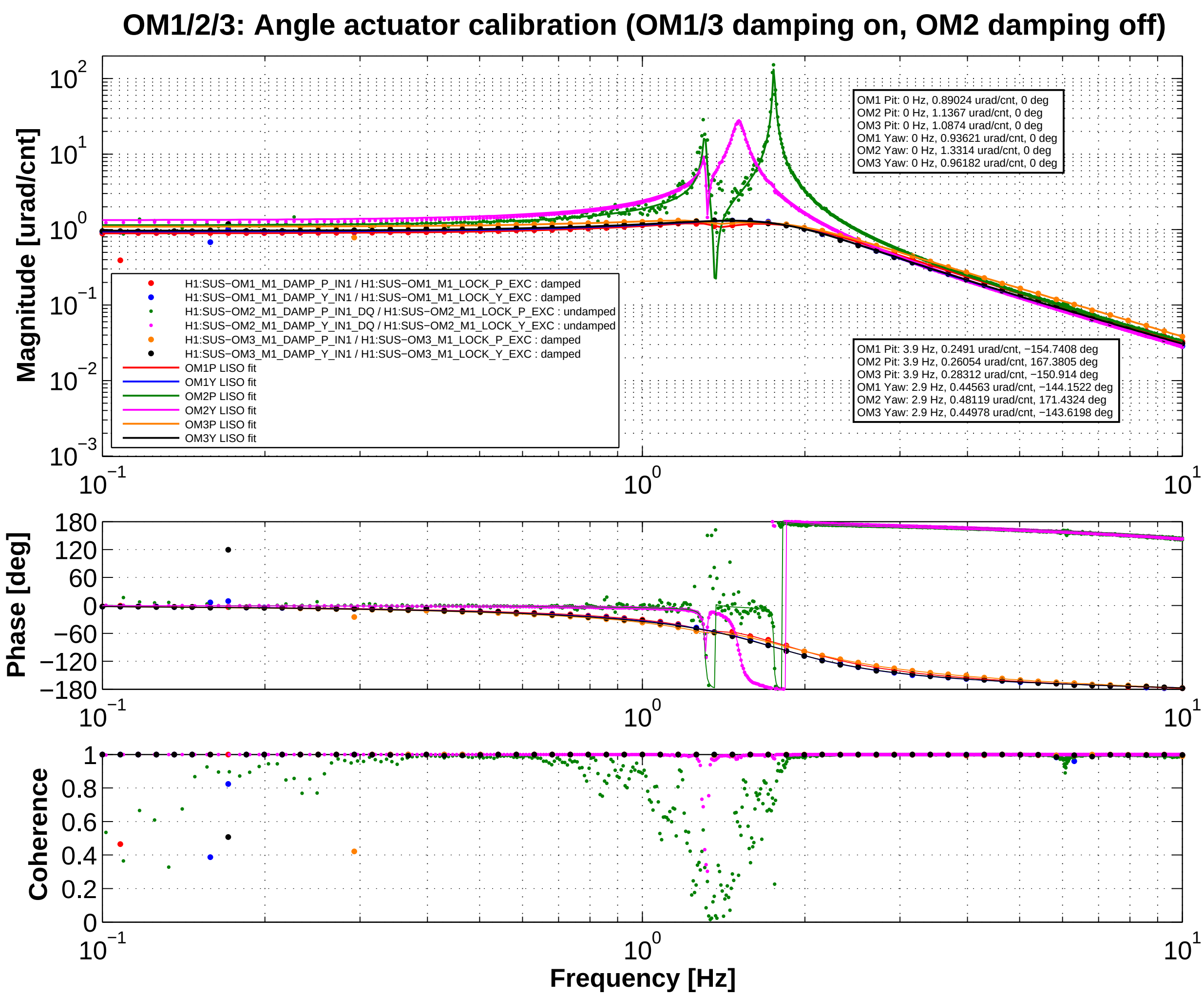

Figure 1: Transfer functions for OM suspension calibrations.

DC response:

(unit: [μrad/count])

- $R_{\mathrm{OM1\_PitchDC}} = 0.890$
- $R_{\mathrm{OM2\_PitchDC}} = 1.137$
- $R_{\mathrm{OM3\_PitchDC}} = 1.087$
- $R_{\mathrm{OM1\_YawDC}} = 0.936$
- $R_{\mathrm{OM2\_YawDC}} = 1.331$
- $R_{\mathrm{OM3\_YawDC}} = 0.962$

3.9Hz (Pitch) or 2.9Hz (Yaw) response:

(unit: [μrad/count])

- $R_{\mathrm{OM1\_Pitch3.9Hz}} = -0.249$
- $R_{\mathrm{OM2\_Pitch3.9Hz}} = -0.261$
- $R_{\mathrm{OM3\_Pitch3.9Hz}} = -0.283$
- $R_{\mathrm{OM1\_Yaw2.9Hz}} = -0.446$
- $R_{\mathrm{OM2\_Yaw2.9Hz}} = -0.481$
- $R_{\mathrm{OM3\_Yaw2.9Hz}} = -0.450$

Define a calibration matrix $\mathcal{C}$ as

$$\mathcal{C} = \begin{pmatrix} \frac{R_{\mathrm{OM1\_PitchDC}}}{R_{\mathrm{OM1\_Pitch3.9Hz}}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{R_{\mathrm{OM2\_PitchDC}}}{R_{\mathrm{OM2\_Pitch3.9Hz}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{\mathrm{OM3\_PitchDC}}}{R_{\mathrm{OM3\_Pitch3.9Hz}}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{R_{\mathrm{OM1\_YawDC}}}{R_{\mathrm{OM1\_Yaw2.9Hz}}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{\mathrm{OM2\_YawDC}}}{R_{\mathrm{OM2\_Yaw2.9Hz}}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{R_{\mathrm{OM3\_YawDC}}}{R_{\mathrm{OM3\_Yaw2.9Hz}}} \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} -3.57 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4.36 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.84 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2.10 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.77 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2.14 \end{pmatrix} \tag{7}$$

Then, a matrix product $\mathcal{T}.\mathcal{C}$ gives us expected response of the QPDs with regard to DC actuations.

4 Spot sizes on QPDs

Since OM3 is a flat mirror, it is straight forward to use it as a beam scanner to infer the beam size on the QPDs.

When the QPD signals are normalized by the sum, the pitch and yaw output signals becomes proportional to the spot displacement normalized by the spot size, as derived here.

A normalized intensity distribution $I(x)$ with the spot displacement and the spot radius of δ and w is expressed as

$$I(x) = \sqrt{\frac{2}{\pi w^2}} e^{-2(x-\delta)^2/w^2} \tag{8}$$

This gives us the QPD signal $s(\delta)$ as

$$s(\delta) \equiv \int_0^{\infty} I(x)\mathrm{d}x - \int_{-\infty}^{0} I(x)\mathrm{d}x \tag{9}$$

$$= \mathrm{Erf}\left(\frac{\sqrt{2}\delta}{w}\right) \quad , \tag{10}$$

where w is the beam radius on the QPD. This gives us the linear response of the following:

$$\left.\frac{\mathrm{d}s}{\mathrm{d}\delta}\right|_{\delta=0} = \sqrt{\frac{8}{\pi}}\frac{1}{w} \tag{11}$$

Therefore, the QPD signals upon OM3 actuation can be expressed as

$$T_{\mathrm{OM3P\_QAP}} = \sqrt{\frac{8}{\pi}}\frac{1}{w_{\mathrm{AV}}} \cdot 2L_{\mathrm{QPDA}} R_{\mathrm{OM3\_Pit3.9Hz}} \tag{12}$$

$$T_{\mathrm{OM3P\_QBP}} = \sqrt{\frac{8}{\pi}}\frac{1}{w_{\mathrm{BV}}} \cdot 2L_{\mathrm{QPDB}} R_{\mathrm{OM3\_Pit3.9Hz}} \tag{13}$$

$$T_{\mathrm{OM3Y\_QAY}} = \sqrt{\frac{8}{\pi}}\frac{1}{w_{\mathrm{AH}}} \cdot 2L_{\mathrm{QPDA}} R_{\mathrm{OM3\_Yaw2.9Hz}} \tag{14}$$

$$T_{\mathrm{OM3Y\_QBY}} = -\sqrt{\frac{8}{\pi}}\frac{1}{w_{\mathrm{BH}}} \cdot 2L_{\mathrm{QPDA}} R_{\mathrm{OM3\_Yaw2.9Hz}} \tag{15}$$

Here $w_{\mathbf{xy}}$ is the spot size at QPDx in y (horiz. or vert.) direction. L_{QPDx} is the lever length from OM3 to QPDx. Note that the negative sign for the last formula comes due to odd number of reflecting optics in the OMC QPDB path. $L_{\mathrm{QPDA}} = 0.520$[m] and $L_{\mathrm{QPDB}} = 0.962$[m] can be read from Figure 2.

These numbers and formulae provide us the following numbers for the beam sizes:

Spot radius on QPDA

Horiz: $w_{\mathrm{AH}} = 0.55$mm

Vert: $w_{\mathrm{AV}} = 0.66$mm

Nominal: $w_{\mathrm{AN}} = 0.40$mm (nominal design)

Spot radius on QPDB

Horiz: $w_{\mathrm{BH}} = 0.75\mathrm{mm}$

Vert: $w_{\mathrm{BV}} = 0.80\mathrm{mm}$

Nominal: $w_{\mathrm{BN}} = 0.61\mathrm{mm}$ (nominal design)

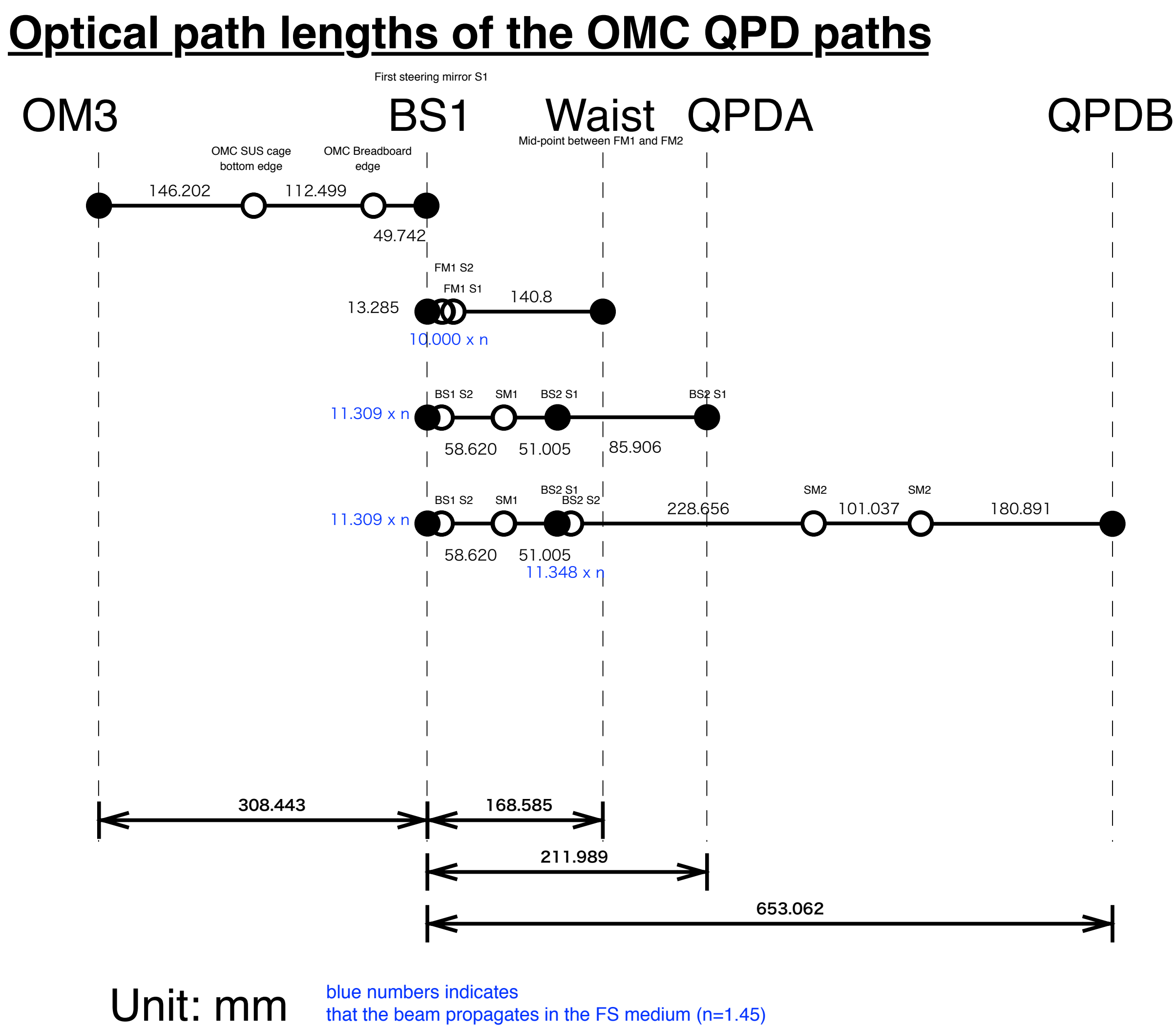

Figure 2: Diagram to indicate lever length of the QPD paths

5 Input matrix

Now we wonder what spot sizes we should use. We are not sure how the beam sizes are different with the arms and the intra-IFO power. If we use the nominal beam parameters, this may deviate from the reality. Here the measured values are used in the equation for the spot displacement on the QPDs.

$$\begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_PIT\_OUT)}\, w_{\rm AV}\sqrt{\pi/8} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_PIT\_OUT)}\, w_{\rm BV}\sqrt{\pi/8} \end{pmatrix} = \begin{pmatrix} 1 & L_{\rm QPDA\_WAIST} \\ 1 & L_{\rm QPDB\_WAIST} \end{pmatrix} \begin{pmatrix} V_{\rm POS} \\ V_{\rm ANG} \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_YAW\_OUT)}\, w_{\rm AH}\sqrt{\pi/8} \\ -\texttt{(H1:OMC-ASC\_QPD\_B\_YAW\_OUT)}\, w_{\rm BH}\sqrt{\pi/8} \end{pmatrix} = \begin{pmatrix} 1 & L_{\rm QPDA\_WAIST} \\ 1 & L_{\rm QPDB\_WAIST} \end{pmatrix} \begin{pmatrix} H_{\rm POS} \\ H_{\rm ANG} \end{pmatrix} \tag{17}$$

Again, the last negative sign comes due to odd number of reflecting optics in the OMC QPDB path.

Looking at Figure 2 again and the distances from the QPDs to the waist position are given as $L_{\rm QPDA\_WAIST} = 0.0434[\rm m]$ and $L_{\rm QPDA\_WAIST} = 0.484[\rm m]$.

Taking the inverse matrices of the right hand side, we obtain the calibrated beam axis motion with regard to the waist.

$$\begin{pmatrix} V_{\rm POS\_m} \\ V_{\rm ANG\_rad} \end{pmatrix} = \begin{pmatrix} 4.56\times10^{-4} & -4.94\times10^{-5} \\ -9.42\times10^{-4} & 1.139\times10^{-3} \end{pmatrix} \begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_PIT\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_PIT\_OUT)} \end{pmatrix} \tag{18}$$

$$\begin{pmatrix} H_{\rm POS\_m} \\ H_{\rm ANG\_rad} \end{pmatrix} = \begin{pmatrix} 3.81\times10^{-4} & 4.63\times10^{-5} \\ -7.86\times10^{-4} & -1.067\times10^{-3} \end{pmatrix} \begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_YAW\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_YAW\_OUT)} \end{pmatrix} \tag{19}$$

$$\tag{20}$$

$$\begin{pmatrix} V_{\rm POS\_um} \\ V_{\rm ANG\_urad} \end{pmatrix} = \begin{pmatrix} 456 & -49.4 \\ -942 & 1139 \end{pmatrix} \begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_PIT\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_PIT\_OUT)} \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} H_{\rm POS\_um} \\ H_{\rm ANG\_urad} \end{pmatrix} = \begin{pmatrix} 381 & 46.3 \\ -786 & -1067 \end{pmatrix} \begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_YAW\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_YAW\_OUT)} \end{pmatrix} \tag{22}$$

After all, the input matrix is

$$\begin{pmatrix} H_{\rm POS\_um} \\ V_{\rm POS\_um} \\ H_{\rm ANG\_urad} \\ V_{\rm ANG\_urad} \end{pmatrix} \equiv \mathcal{M}_{\rm in} \begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_PIT\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_PIT\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_A\_YAW\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_YAW\_OUT)} \end{pmatrix} \tag{23}$$

$$= \begin{pmatrix} 0 & 0 & 381 & 46.3 \\ 456 & -49.4 & 0 & 0 \\ 0 & 0 & -786 & -1067 \\ -942 & 1139 & 0 & 0 \end{pmatrix} \begin{pmatrix} \texttt{(H1:OMC-ASC\_QPD\_A\_PIT\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_PIT\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_A\_YAW\_OUT)} \\ \texttt{(H1:OMC-ASC\_QPD\_B\_YAW\_OUT)} \end{pmatrix} \tag{24}$$

This $\mathcal{M}_{\rm in}$ is the matrix to be loaded on the MEDM screen.

6 Actuator selection

In order to know which mirror combination we should use as an actuator, Gouy phases of the beam at the location of the OM mirrors were checked[4]. It was suggested that OM2 and

[4]Refer zipped PDF files on https://dcc.ligo.org/LIGO-D1000342

OM3 has about 70deg separation with regard to the Gouy phase at OM1. Therefore we decided to use OM1 and OM3. This matrix $\mathcal{A}$ describes how the actuators are combined for each d.o.f.

$$\mathcal{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{25}$$

7 Output matrix

Now we want to set the output matrix to make the roundtrip open loop matrix diagonal. That means we want find $\mathcal{M}_{\text{out}}$ to fullfil the condition

$$\mathcal{M}_{\text{in}}\mathcal{T}\mathcal{C}\mathcal{A}\mathcal{M}_{\text{out}} = \mathcal{I} \tag{26}$$

Therefore the output matrix (6×4 version) is

$$\mathcal{A}\mathcal{M}_{\text{out}} = \mathcal{A}(\mathcal{M}_{\text{in}}\mathcal{T}\mathcal{C}\mathcal{A})^{-1} \tag{27}$$

$$= \begin{pmatrix} 0 & -1019 & 0 & 410 \\ 0 & 0 & 0 & 0 \\ 0 & -369 & 0 & -781 \\ -405 & 0 & 214 & 0 \\ 0 & 0 & 0 & 0 \\ -301 & 0 & -392 & 0 \end{pmatrix} \tag{28}$$

This $\mathcal{A}\mathcal{M}_{\text{out}}$ is the matrix to be loaded on the MEDM screen.