Minimising Quantum Back-Action Noise in Quantum State Tomography of Cavities

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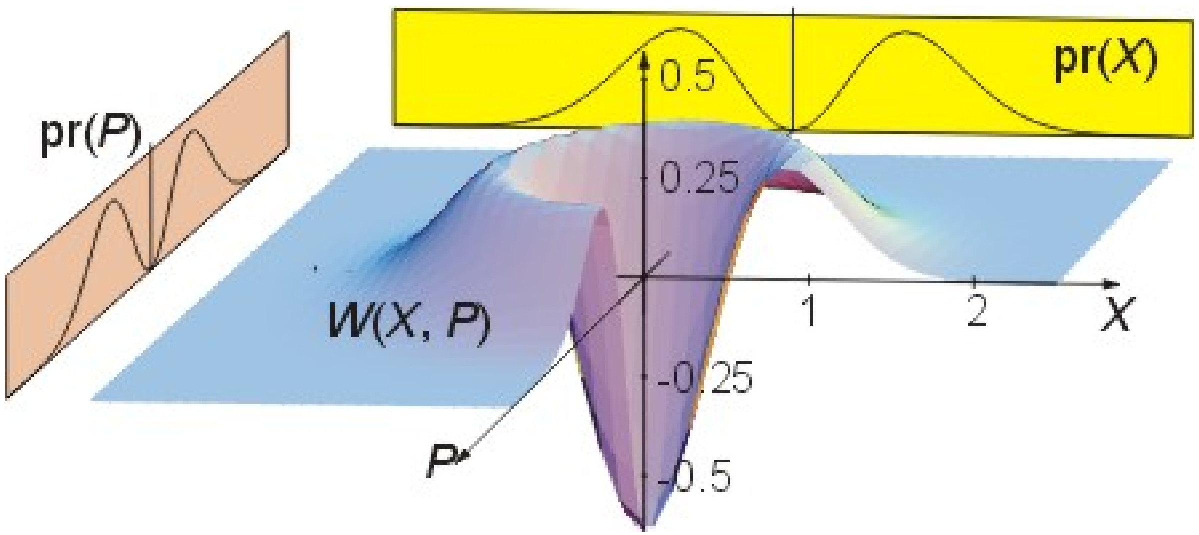
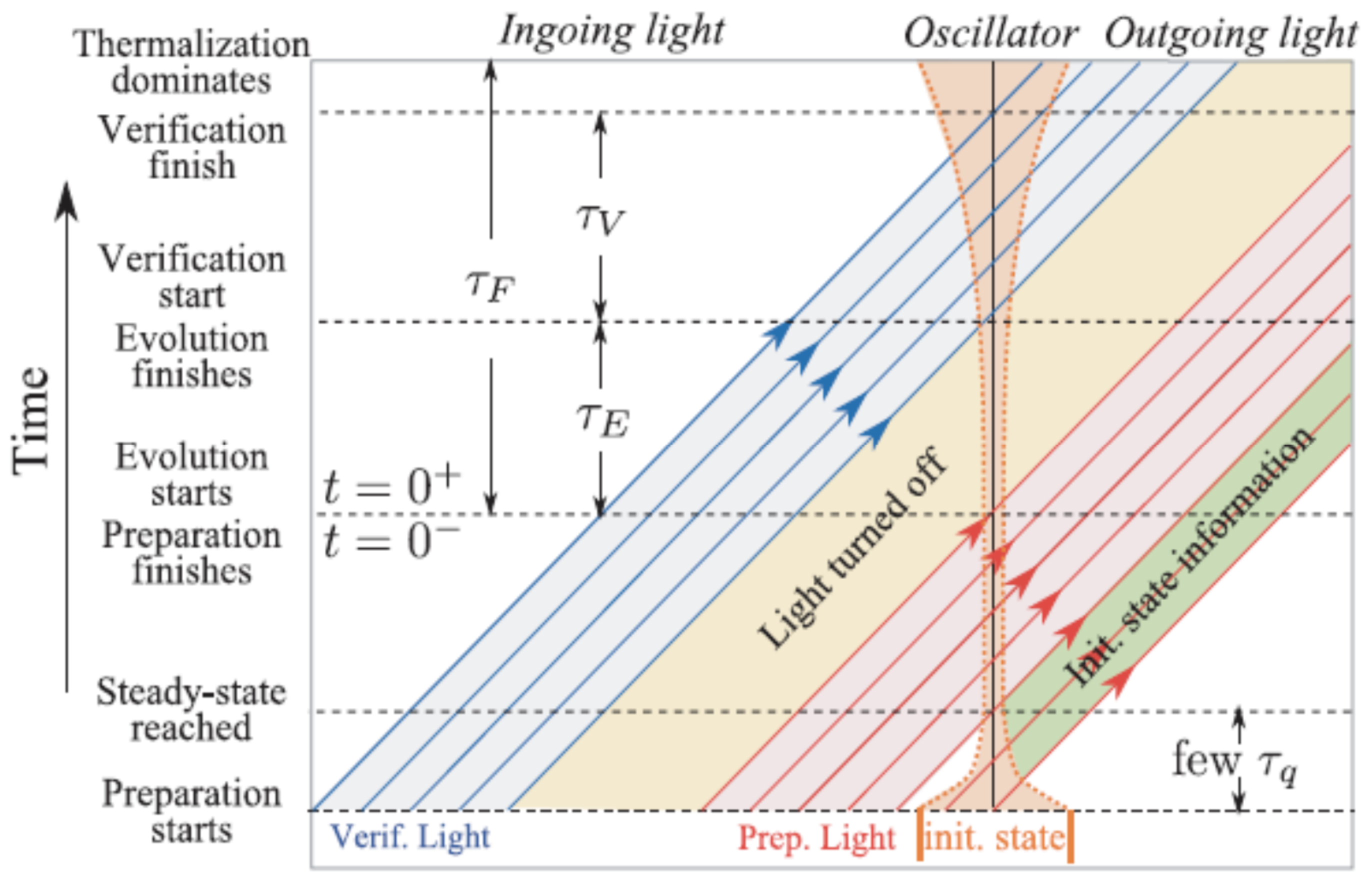
Quantum Tomography

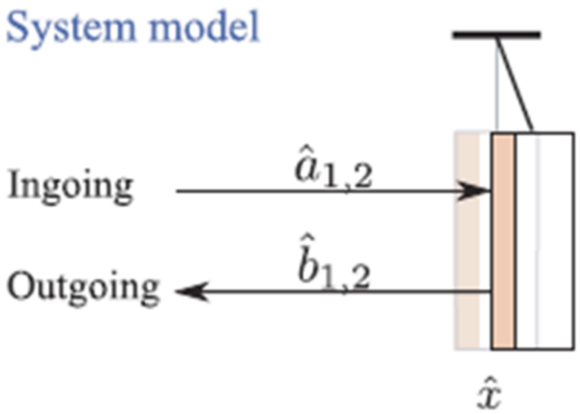
- Reconstructing the quantum state of the system
- Deduce the Wigner-function





Thermalization dominates
Ingoing light
Oscillator
Outgoing light
Verification finish
Verification start
Evolution finishes
Evolution starts
Preparation finishes
Steady-state reached
Preparation starts
Time
τ_F
τ_V
τ_E
$t = 0^+$
$t = 0^-$
Light turned off
Init. state information
few τ_q
Verif. Light
Prep. Light
init. state

Broadband cavity



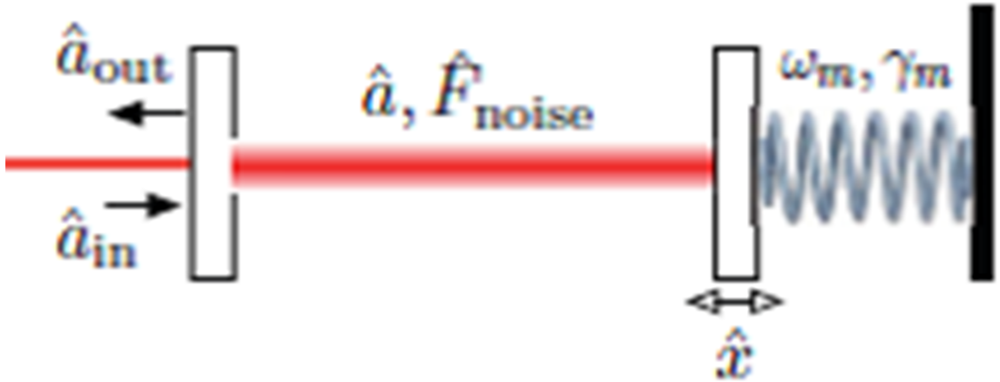
$$\hat{b}_1(t) = \sqrt{\eta}\hat{n}_1(t) + \sqrt{1-\eta}\hat{a}_1(t)$$

$$\hat{b}_2(t) = \sqrt{\eta}\hat{n}_2(t) + \sqrt{1-\eta}[\hat{a}_2(t) + \frac{\alpha}{\hbar}\hat{x}(t) + \frac{\alpha}{\hbar}\hat{\xi}_x(t)]$$

where

$$\hat{x}(t) = \hat{x}_q(t) + \int_0^\infty \mathrm{d}t' G_x(t-t')[\alpha\hat{a}_1(t') + \hat{\xi}_F(t')]$$

Cavity with finite bandwidth

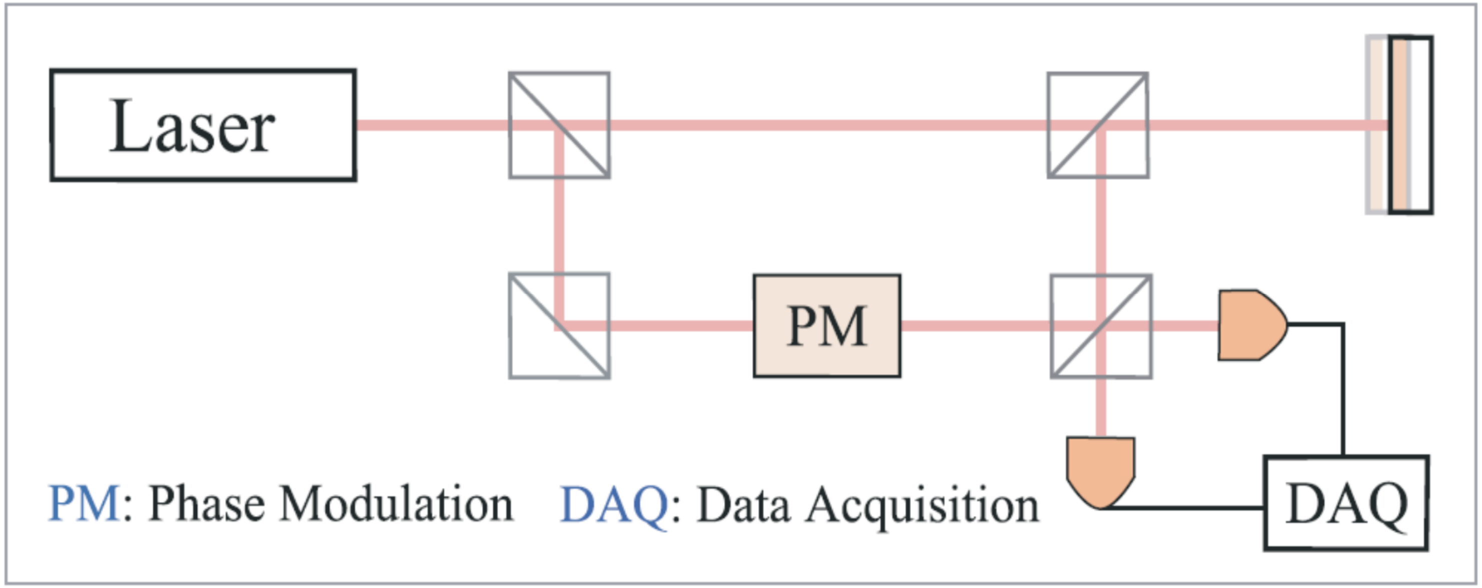


Quadratures:

$$\hat{c}_1(t) = \sqrt{2\gamma}\hat{a}_1(0)e^{-\gamma t} - \hat{b}_1(t) + 2\gamma \int_0^t \mathrm{d}t_1 e^{-\gamma(t-t1)}\hat{b}_1(t_1)$$

$$\hat{c}_2(t) = \sqrt{2\gamma}\hat{a}_2(0)e^{-\gamma t} - \hat{b}_2(t) + 2\gamma \int_0^t \mathrm{d}t_1 e^{-\gamma(t-t_1)}\hat{b}_2(t_1) - 2Ga\sqrt{\gamma}\int_0^t \mathrm{d}t_1 e^{-\gamma(t-t_1)}\hat{x}(t_1)$$

Homodyne detection



Homodyne detection

Outgoing optical field:

$$\hat{C}_{out}(t) = \hat{c}_1(t)\mathrm{cos}\omega_0 t + \hat{c}_2(t)\mathrm{sin}\omega_0 t$$

Local oscillator:

$$L(t) = L_0\mathrm{cos}[\omega_0 t - \phi(t)]$$

Photocurrent:

$$\hat{i}(t) \propto L_0\hat{c}_1(t)\mathrm{cos}\phi(t) + L_0\hat{c}_2\mathrm{sin}\phi(t)$$

Signal:

$$\hat{Y} = \int_0^T W(t)\hat{i}(t)\mathrm{d}t = (g_1|\hat{c}_1) + (g_2|\hat{c}_2)$$

Normalisation

We want:

$$\hat{Y}_s \propto [(x_o \cos\zeta + p_o \sin\zeta)\sin\psi + a_{1o}\cos\psi]\sin\phi + a_{2o}\cos\phi$$

Constraints:

$$(g_2|f_1) = \cos\phi$$

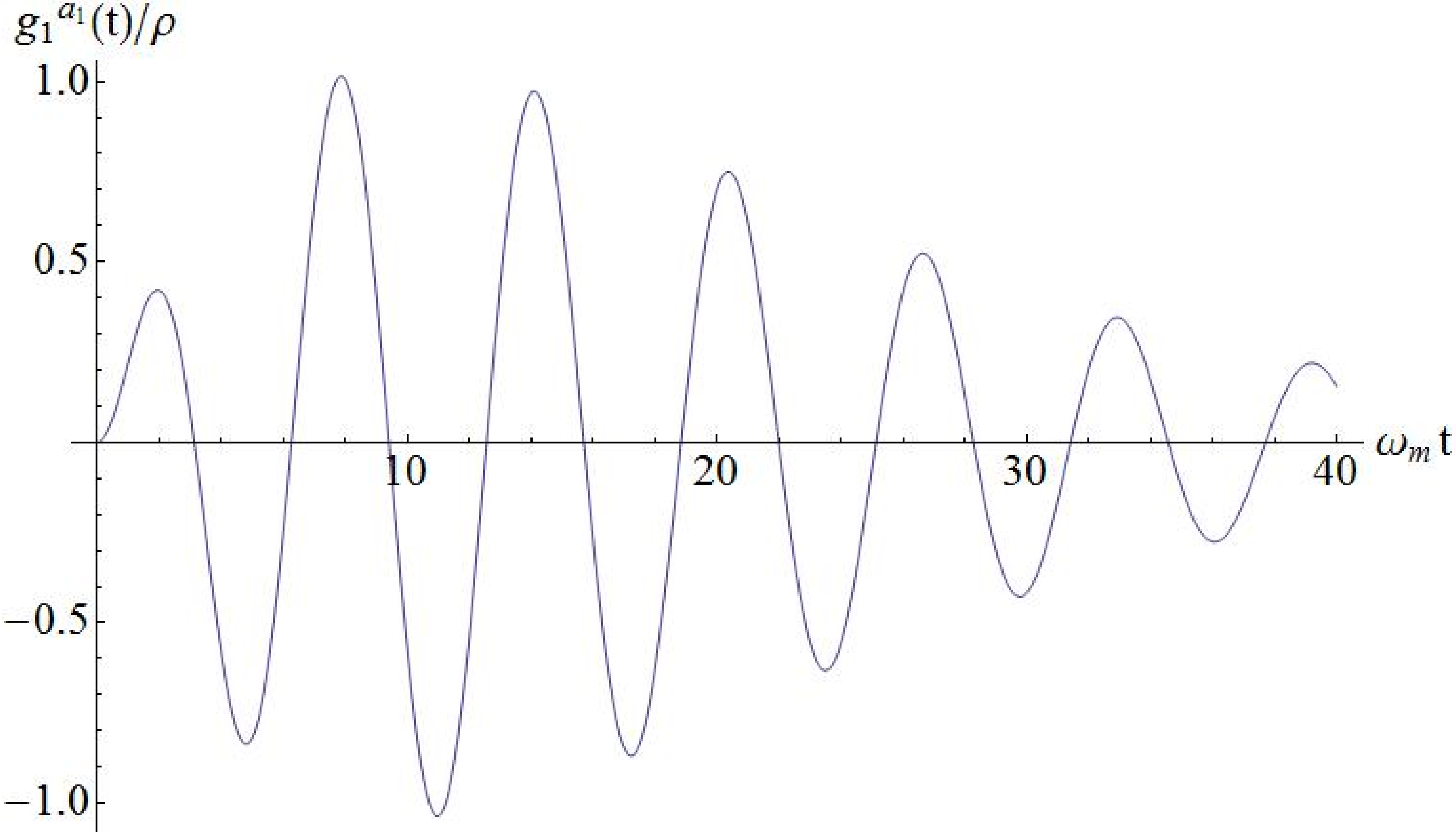
$$(g_1|f_2) = \sin\phi\cos\psi$$

$$(g_2|f_3) = \sin\phi\sin\psi\cos\zeta$$

$$(g_2|f_4) = \sin\phi\sin\psi\sin\zeta$$

Result so far

$$g_i(t) = [(g_i^x \cos\zeta + g_i^p \sin\zeta)\sin\psi + g_i^{a_2}\cos\psi]\sin\phi + g_i^{a_1}\cos\phi$$



Further work

- Calculating the full filtering function for both quadratures
- Obtaining the covariance matrix
- Comparing results with actual experiments

Acknowledgements

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- Michael Coughlin
- Alan Weinstein

General method

$$\delta c_i(t) = \int_0^t \mathrm{d}t_1 G_{ij}(t-t_1)\hat{b_j}(t_1)$$

$$\delta c_i(t) = \int_{-\infty}^t \mathrm{d}t_1 G_{ij}(t-t_1)\hat{b_j}(t_1) - \int_{-\infty}^0 \mathrm{d}t_1 \sum_n M_{ij}^n e^{-i\Omega_n(t-t_1)}\hat{b_j}(t_1)$$

$$\sigma^2 = \sum_{i,j} \langle g_i(t)\delta\hat{c}_i(t) g_j(t')\delta\hat{c}_j(t')\rangle$$

$$I[g_i] = (g_i|C_{ij}|g_j) + M_{i\alpha}^n M_{j\alpha'}^m \langle \xi_\alpha \xi_{\alpha'}\rangle (g_i|e^{-i\Omega_n t + i\Omega_m t'}|g_j) - \mu_i(g_i|f_i)$$

$$C_{ij}|g_j) + M_{i\alpha}^n M_{j\alpha'}^m \langle \xi_\alpha \xi_{\alpha'}\rangle e^{-i\Omega_n t} g_j^{(m)} - \mu_i|f_i) = 0$$

where

$$g_j^{(m)} = \int_0^\infty \mathrm{d}t e^{i\Omega_m t} g_j(t)$$

Solution in the frequency domain

$$[S_{ij}(\Omega)g_j(\Omega) + \frac{M_{i\alpha}^{(n)} M_{j\alpha'}^{(m)} \langle \xi_\alpha \xi_{\alpha'} \rangle}{\Omega - \Omega_m} g_j^{(m)} - \mu_i f_i(\Omega)]_+ = 0$$

$$\mathsf{S} = \psi_+ \psi_-$$

$$g_l(\Omega) = [\psi_+^{-1}]_{lk} \{ [\psi_-^{-1}]_{ki} [\mu_i f_i(\Omega) - \frac{M_{i\alpha}^{(n)} M_{j\alpha'}^{(m)} \langle \xi_\alpha \xi_{\alpha'} \rangle}{\Omega - \Omega_m} g_j^{(m)}]\}_+$$