SILICON CANTILEVERS REPORT

MARIE LU, NIC SMITH, ZACH KORTH

Introduction

Originally proposed in 1916 as a part of the theory of general relativity, gravitational waves and their detection have been a major investment for many scientists. LIGO, the Laser Interfermeter Gravitational-Wave Observatory, measures ripples in the force of gravity emitted from major cosmic events. The LIGO detector is a Michelson interferometer, composed of a series of fixed silica mirrors suspended from fibers in a pendulum. Because gravitational waves are incredibly weak, it is necessary to reduce noise sources as much as possible in order to obtain accurate results. Currently, the material for the wires–fused silica– still contributes significant thermal noise uniformly across the noise floor, preventing the detectors from reaching their quantum mechanical sensitivity limits. Our research will focus an alternative material, silicon, for the fibers that has lower thermal fluctuations. This material will likely operate under cryogenic temperatures to minimize the mechanical dissipation of atoms vibrating from thermal fluctuations.

In reality, thermal fluctuations are incredibly small and hard to measure accurately. So instead of measuring fluctuations, the fluctuation dissipation theorem allows us to measure the mechanical dissipation of a material, which is directly related to the amount of thermal fluctuations. To derive one from the other, we start with the equation of motion of the particles inside a material modeled by

$$m\ddot{x} = -k(1 + i\phi)(x - x_g) + F \tag{1}$$

where ϕ is the loss angle:

$$\phi = \Delta \frac{\omega\tau}{1 + \omega^2\tau^2} \tag{2}$$

The vibration transfer function is

$$\frac{x}{x_g} = \frac{\omega_0^2(1 + i\phi)}{\omega_0^2 - \omega^2 + i\phi\omega_0^2} \tag{3}$$

and a quality factor given by

$$Q = \frac{1}{\phi(\omega_0)} \tag{4}$$

To solve this, one starts with the power spectrum of the motion of the mass, given by

$$x^2(\omega) = \frac{4k_B T\sigma(\omega)}{\omega^2} \tag{5}$$

with $\sigma(\omega)$ denoting the mechanical conductance, which is also the real part of the admittance:

$$Y(\omega) = \frac{\omega\phi + i(\omega - m\omega^3)}{(k - m\omega^2)^2 + k^2\phi^2} \tag{6}$$

Wolving for $\sigma(\omega)$ and substituting into Equation (5) produces the power spectral density of the position of the mass:

$$x^2(\omega) = \frac{4k_B T k\phi(\omega)}{\omega[(k - m\omega^2)^2 + k^2\phi^2]} \tag{7}$$

Clamp Models

Our goal is to make measurements of the quality factor Q of silicon, a value inversely proportional to the mechanical dissipation. However, before we make any measurements of the thermal dissipation of our silicon wafer, we must design an experimental set-up. In particular, we need to find a clamp structure to hold the silicon test mass that will minimize the amount of clamping loss. The silicon cantilevers are etched out of whole silicon wafer (See Fig 1 for reference). The clamp will hold the wafer on either side, at the center, secured by a screw that will pass through the center hole. We hypothesized that reducing the area of contact between the silicon wafer and the steel clamp components would lessen the clamp loss. Thus we played with model shapes that would test this factor. Here are the three major clamp designs that we focused on:

- Model 1: The clamp is composed of two solid cylinders on either side of the wafer held in place by a screw. See Figures 2and 3 for reference.
- Model 2: Two cylindrical disks, one on either side of the wafer, are supported on four hemispheres, making contact with the silicon surface at only four points on each side of the wafer. The four contact points form a square. See Figures 4 and 5.
- Model 3: Two solid cylinders, one on either side of the wafer, with the center of the cylinders are hollowed out such that there is only a ring of contact between the steel clamp and the wafer. See Figures 6.

Method

We used COMSOL to simulate each of the clamp models and calculated the strain energies within the two different materials at individual eigenmodes. Our objective is to minimize the ratio $\frac{E_{steel}}{E_{Si}+E_{steel}}$ for each eigenmode where E_i is the strain energy for a specific material. For Model 1, we varied the clamp radius from 6 mm up to 12 mm. We analyzed Model 2 by changing the side length of the square formed by the four hemispheres from 6 mm to 12 mm. We varied the ring thickness from 1mm to 3 mm, while keeping the inner radius at 8 mm. We found that a 2mm thick ring produced the best strain energy ratio. From there, we simulated the clamp with the inner radius from 6 mm to 10 mm.

Analysis of Results

We found that a Model 1 clamp with a 6 mm radius produced the best strain energy ratio. Models 1 and 3 produced strain energy ratios that were 2 orders of magnitude better than that produced by Model 2. Thus we will focus on Models 1 and 3 in this discussion. Shortening the radii allowed for smaller clamp loss in both designs (Refer to Figure 7 for a plot of strain energies over the eigenfrequencies). Although Model 3 has lower strain energy ratios at eigenfrequencies below approximately 245 Hz, our experiment mainly pertains to eigenmodes of the rectangular cantilevers on the wafer, which are from 251 to 298 Hz. This is where Model 1 consistently produced lower strain energy than Model 3, although the two are close.

Planning Ahead

In the upcomming weeks, we plan to design the clamp with SolidWorks and make the clamp in the machine shop while we wait for the wafers to arrive from the manufacturers. We are planning to place the clamp in a cryostat where it will be excited into oscillatory motion by an electrostatic driver. An infrared laser will be directed to the cantilevers and back to a photodiode to record the motion of the cantilever, which can then be related to the thermal noise through the Fluctuation dissipation theorem described above. Afterbuilding this assemblyl, we will then run experiments to test the quality factor of the silicon cantilevers as a function of temperature, frequency, and surface treatments.

Figures

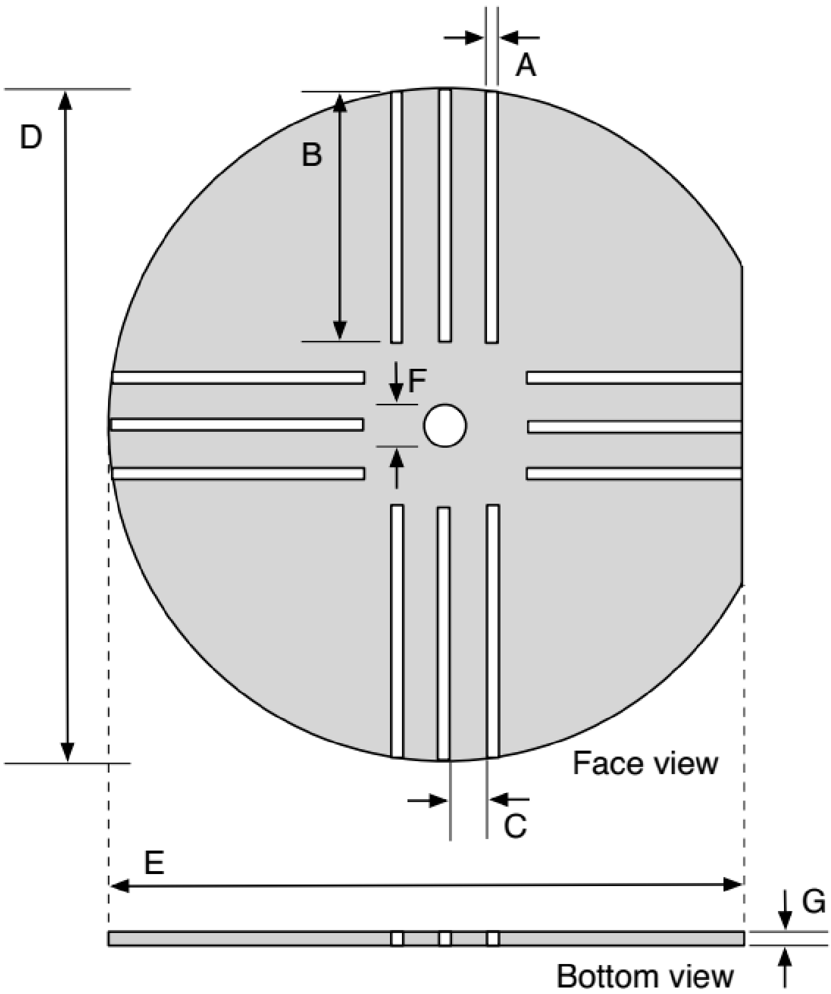

FIGURE 1. A = 1 mm, B = 35 mm, C = 5 mm, D = 100 mm (as per their info), E = (100 - X) mm, where X = flat sagitta (yet to be specified), F = 10 mm, G = 0.3 mm

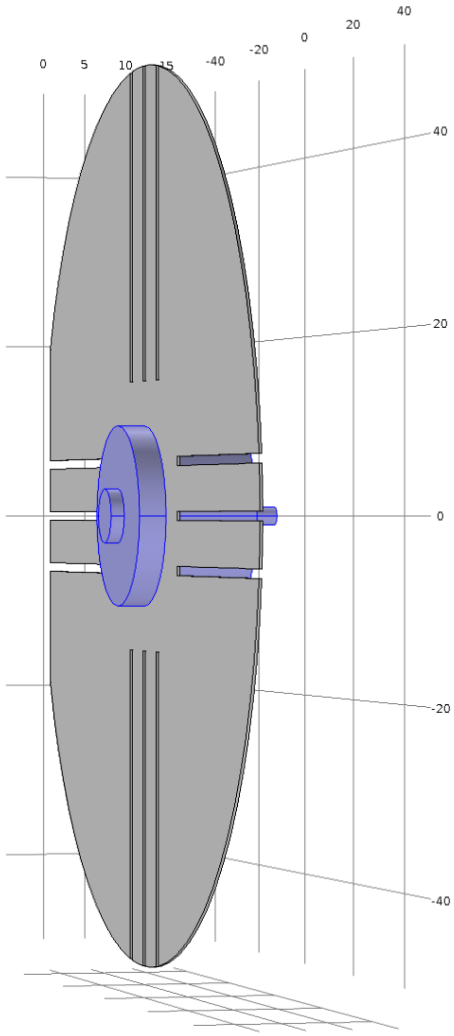

FIGURE 2. Model 1, front view

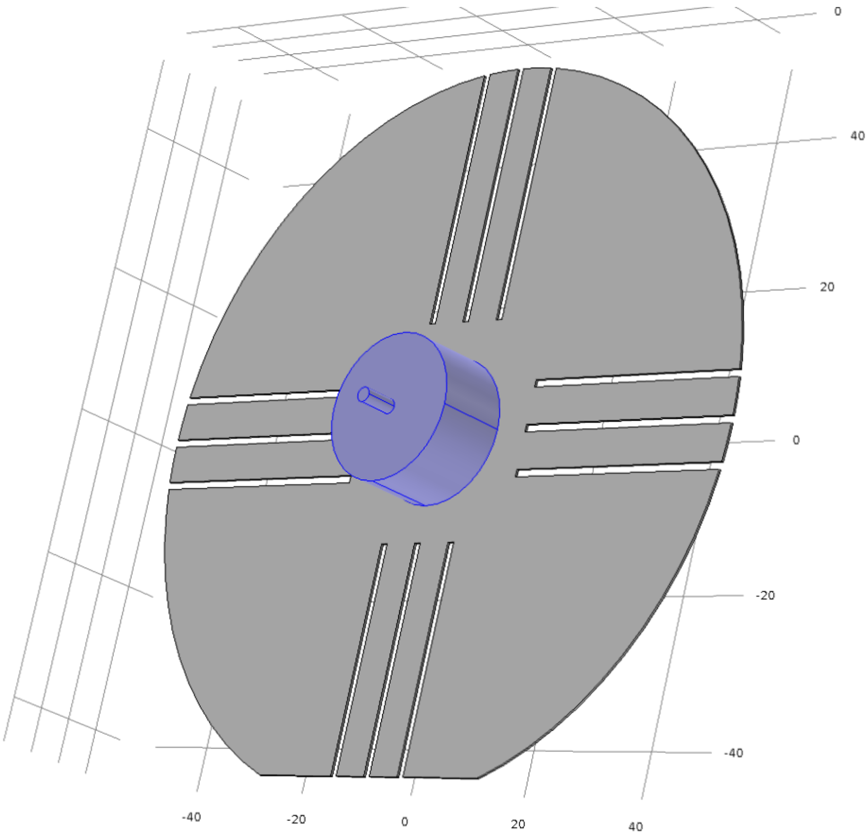

Figure 3. Model 1, back view

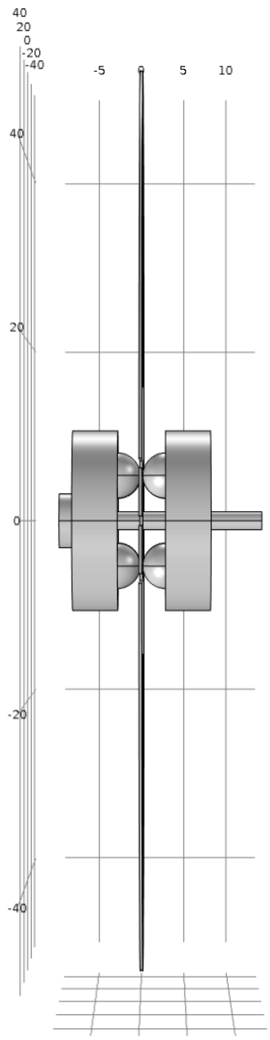

FIGURE 4. Model 2, side view

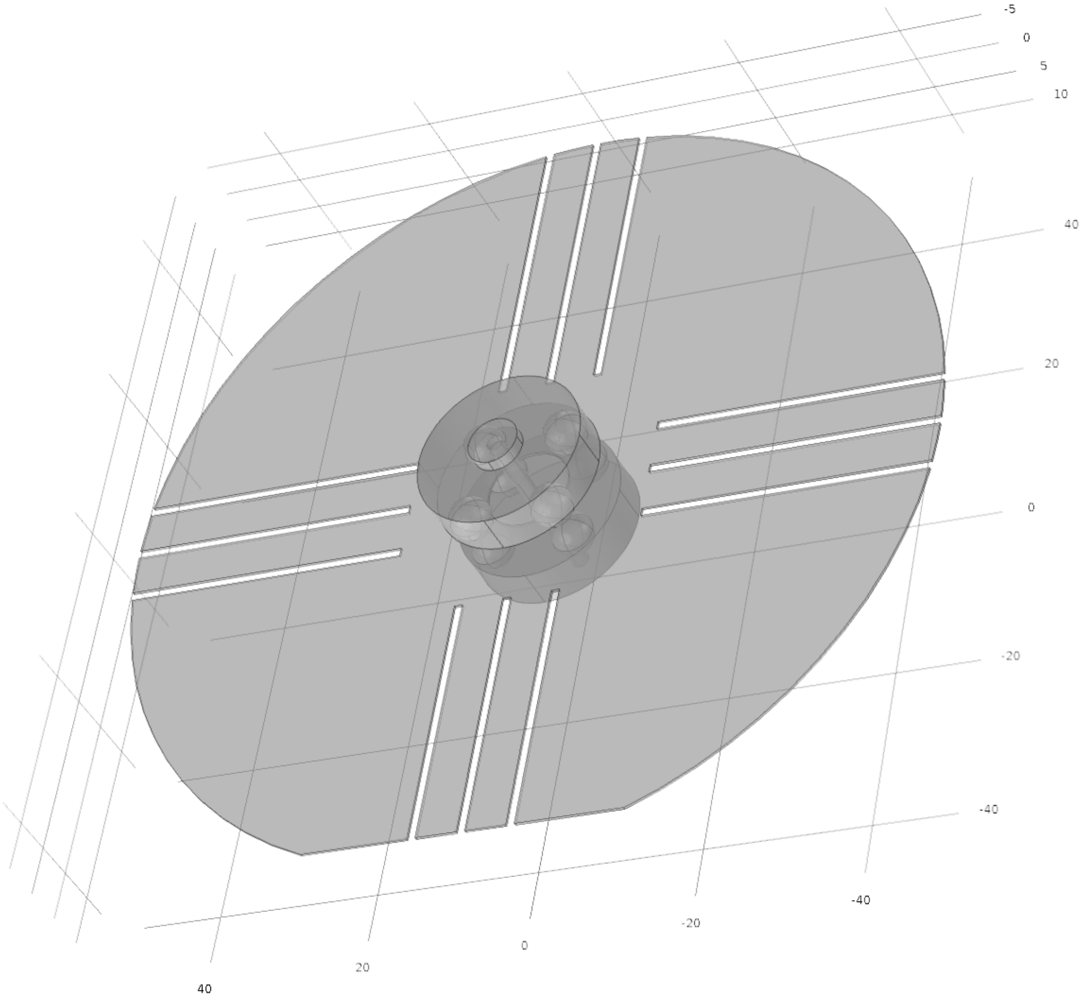

FIGURE 5. Model 2, front view

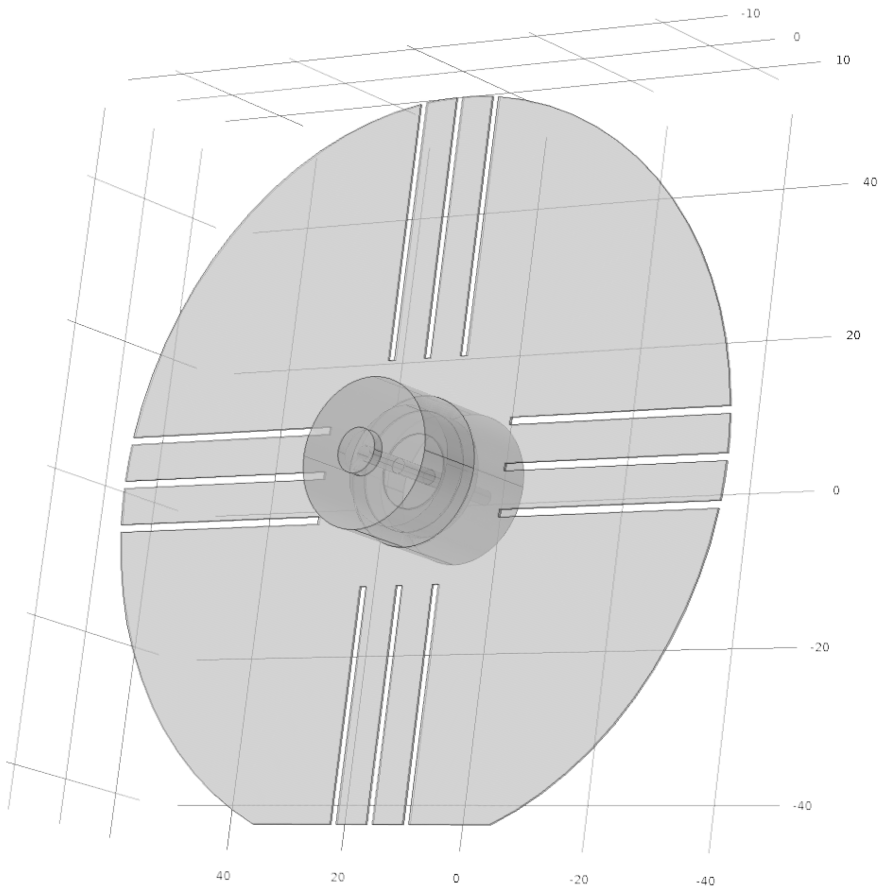

Figure 6. Model 3, front view

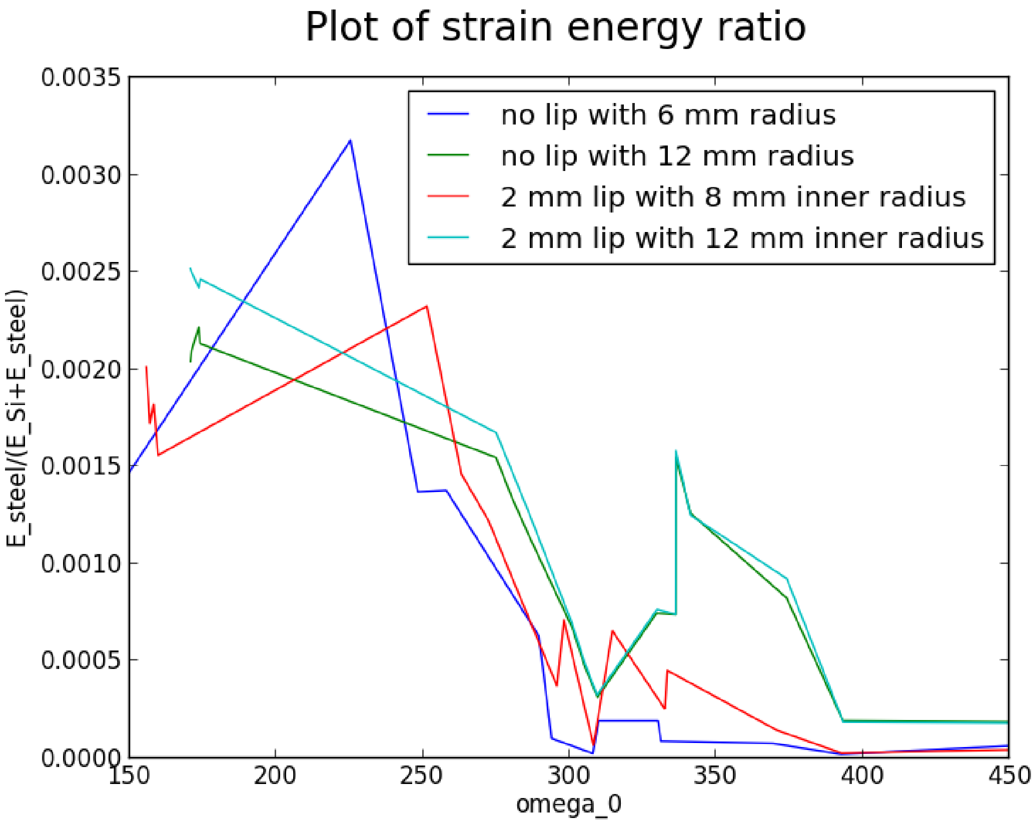

FIGURE 7. Comparison of strain energy ratio between Models 1 and 3 with varying radii

References

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