Quantum noise of white light cavity using double-gain medium

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and Yanbei Chen

For more details: DCC-P1400162

Outline

- **Background**
  - Quantum noise
  - Radiation-pressure noise and shot noise
- **Mizuno theorem and white-light cavity**
  - gain (peak sensitivity) and bandwidth product
  - double-gain medium with negative dispersion
- **Stability condition and sensitivity gain**
  - Nyquist theorem
  - Resulting sensitivity gain

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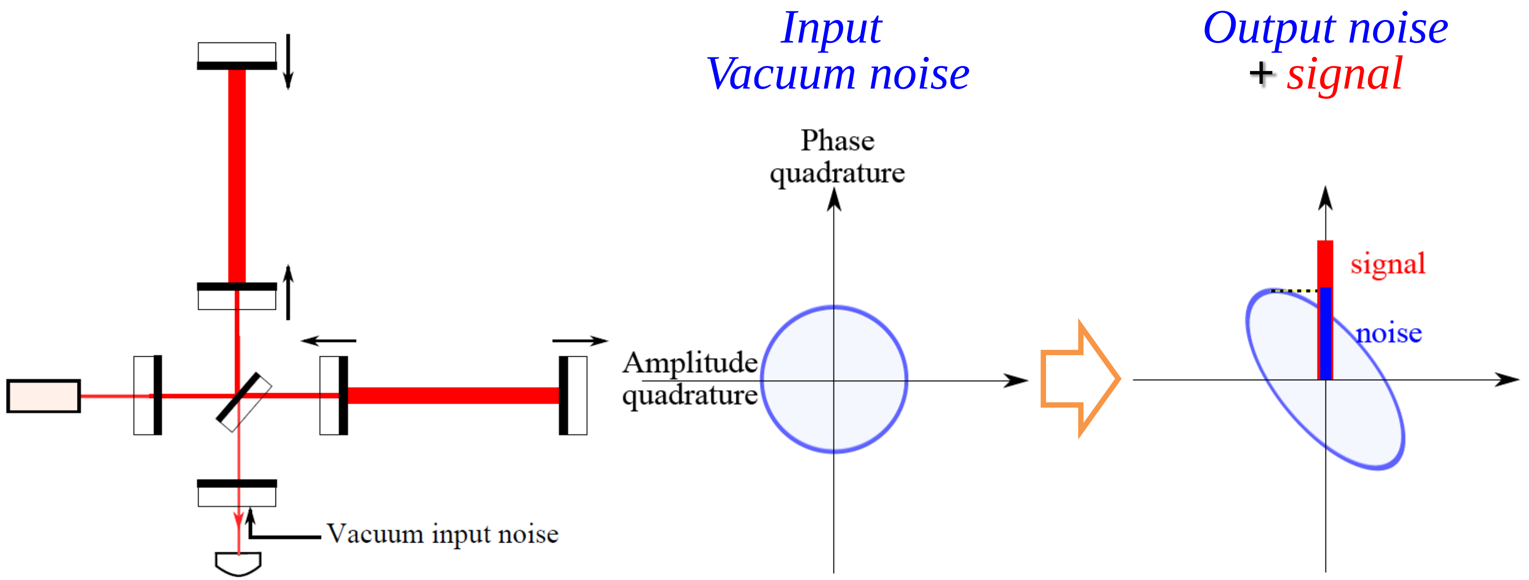

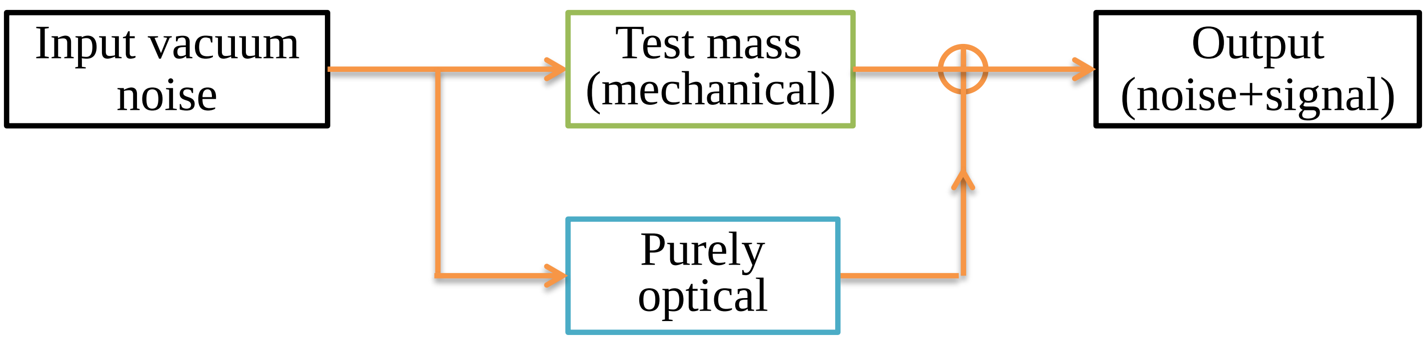

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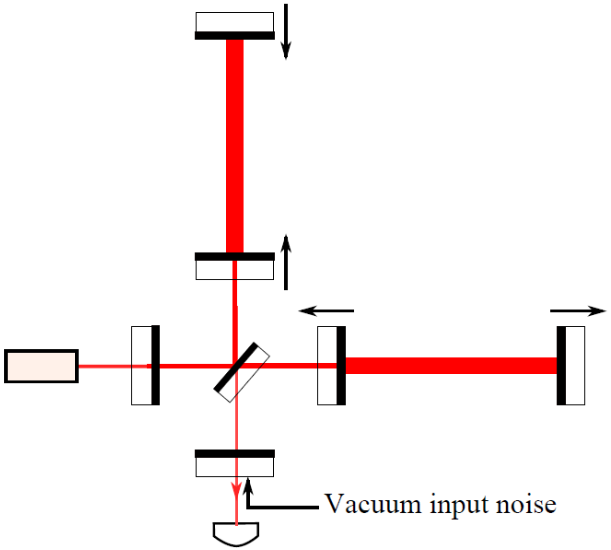

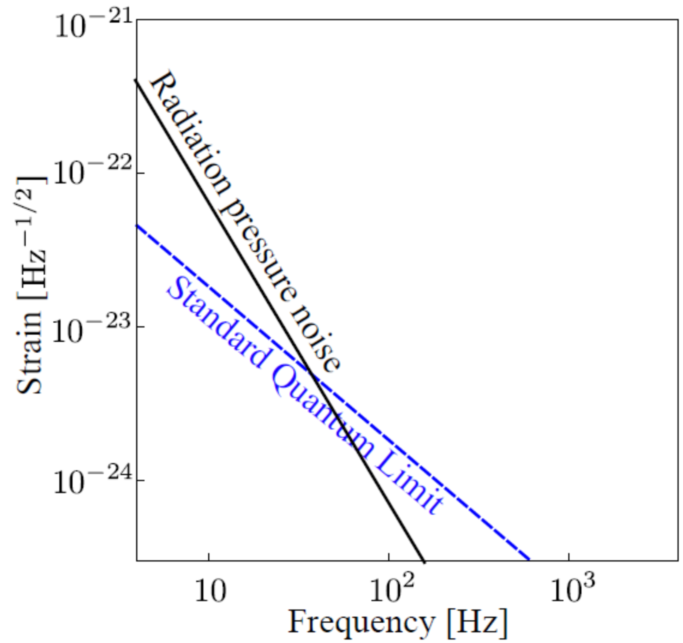

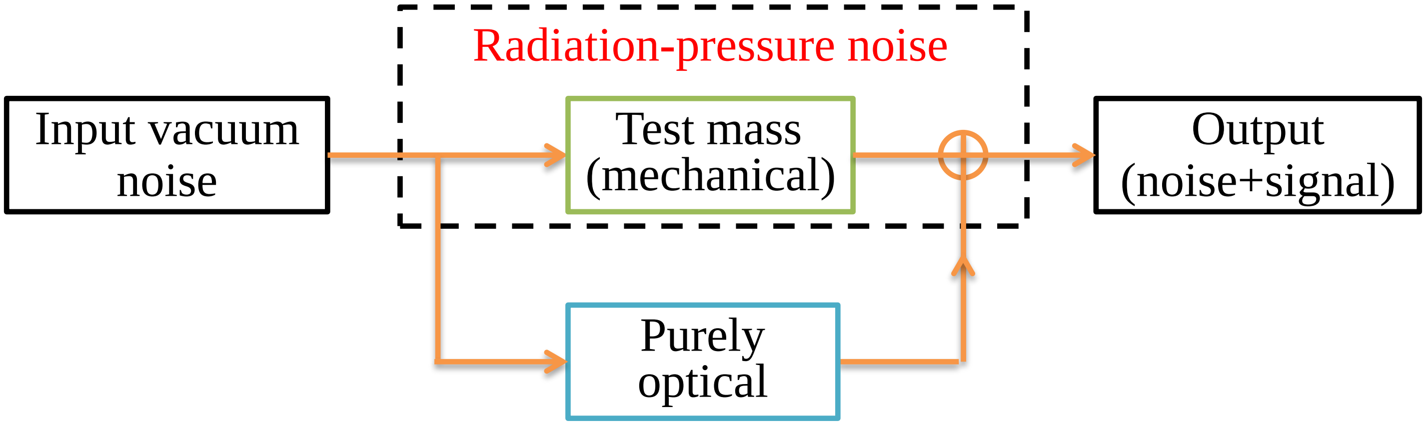

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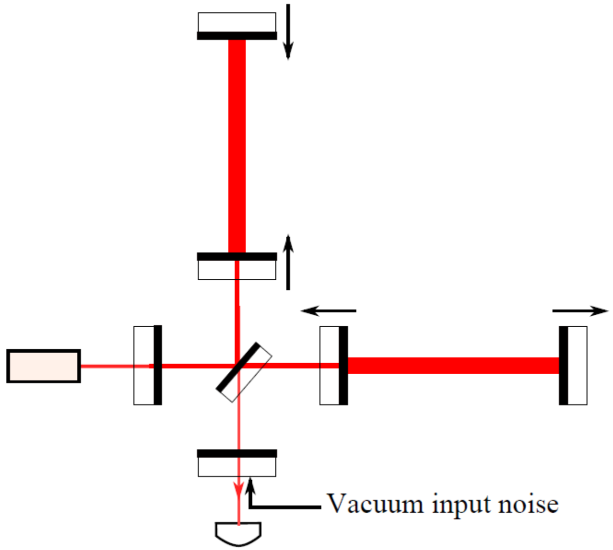

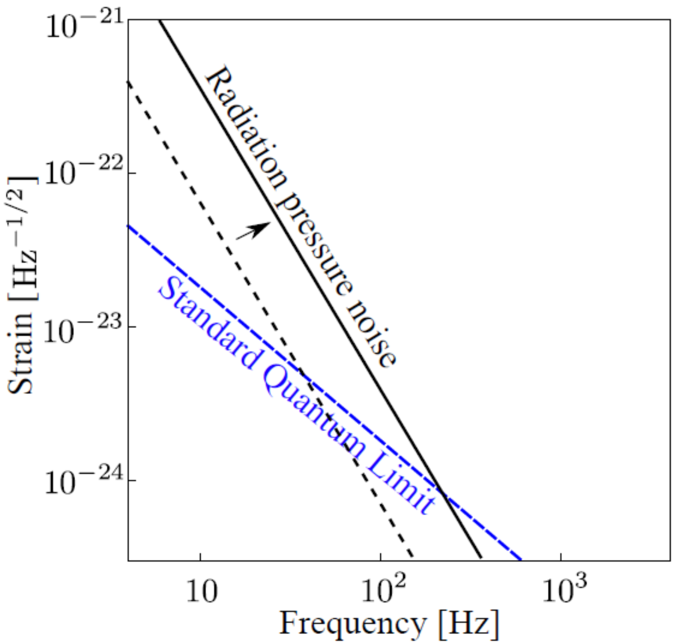

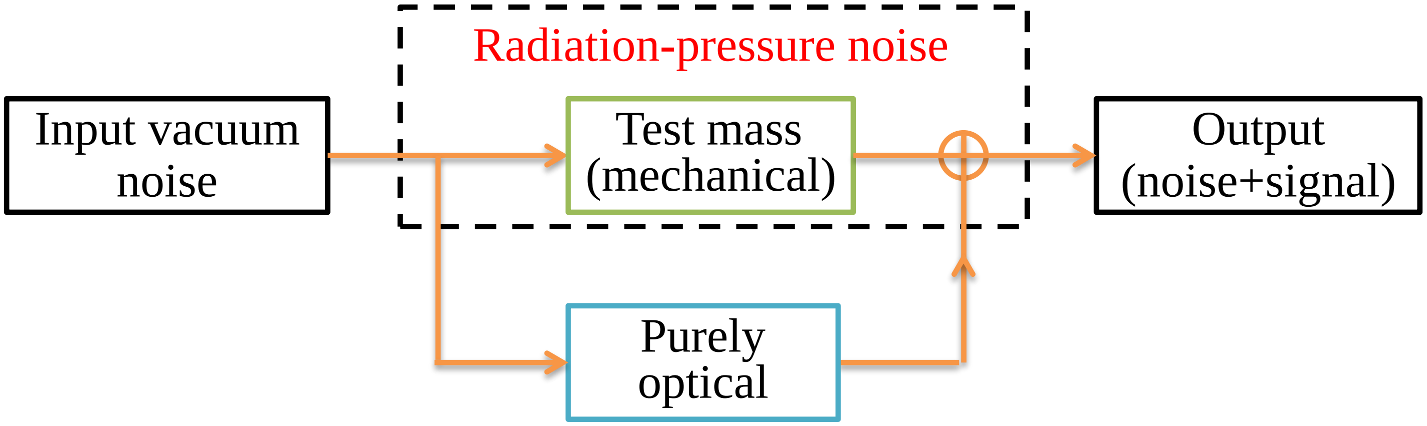

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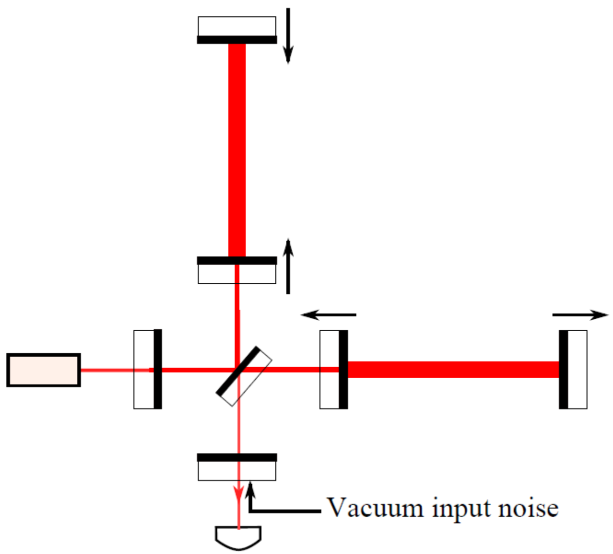

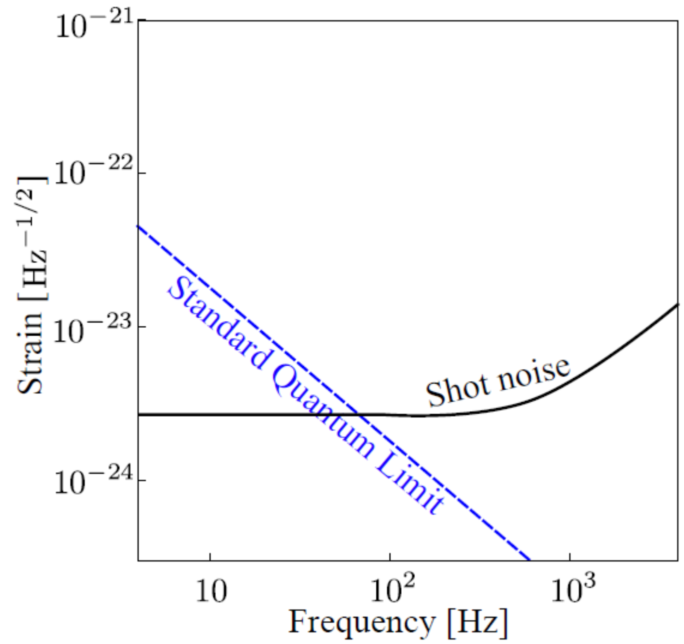

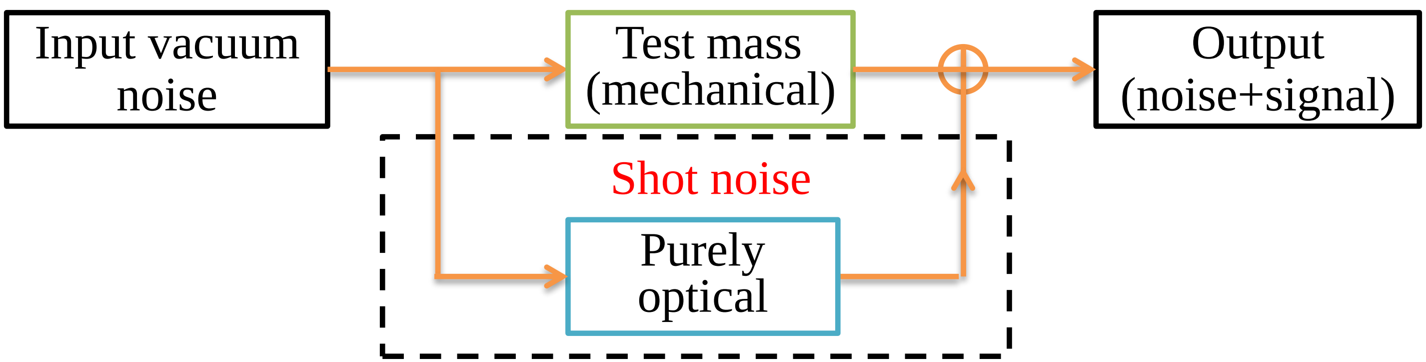

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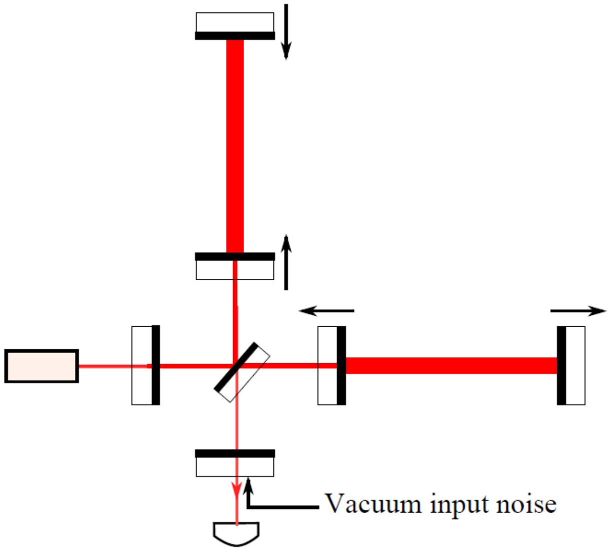

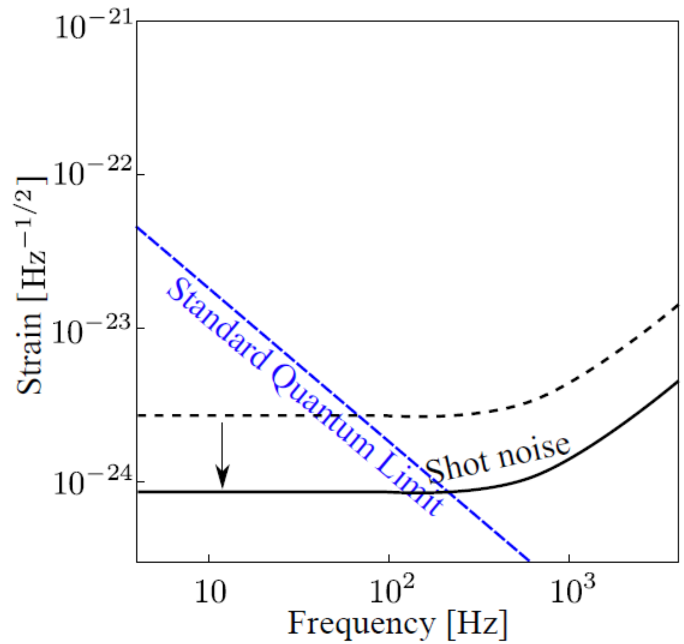

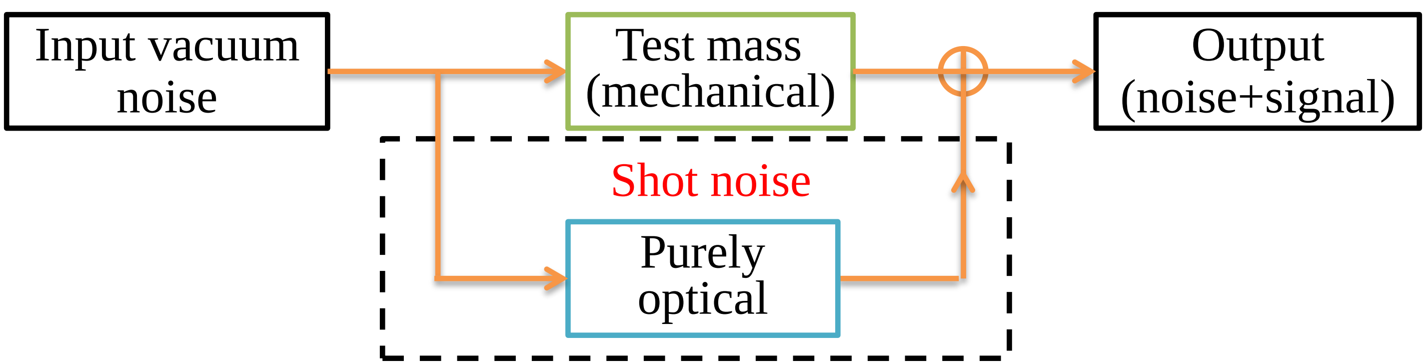

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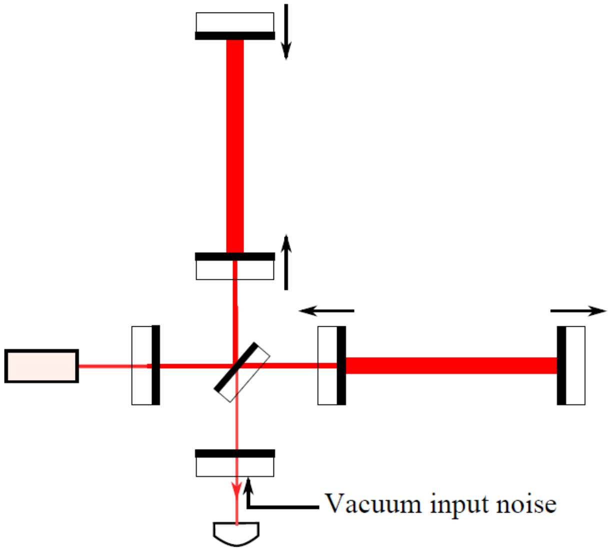
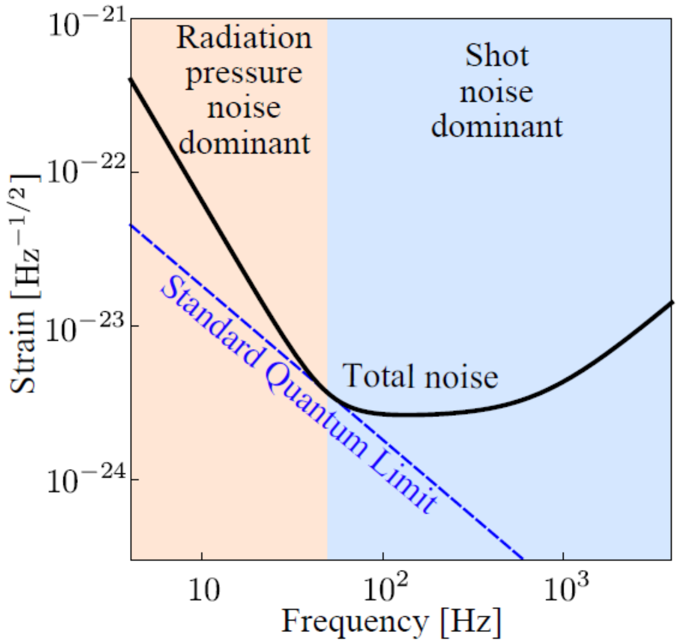
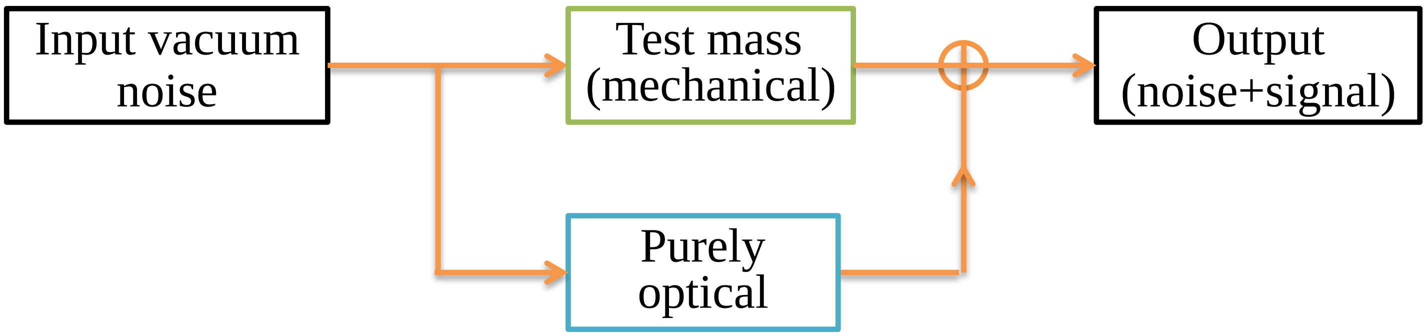
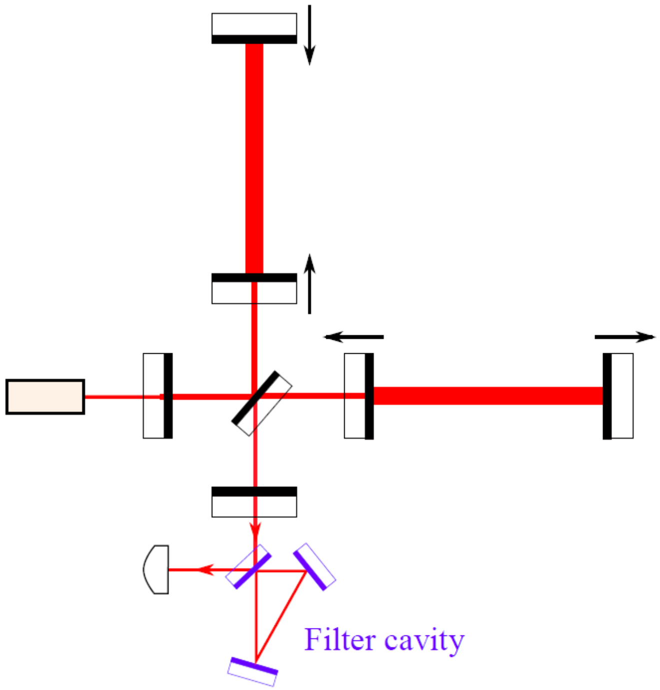


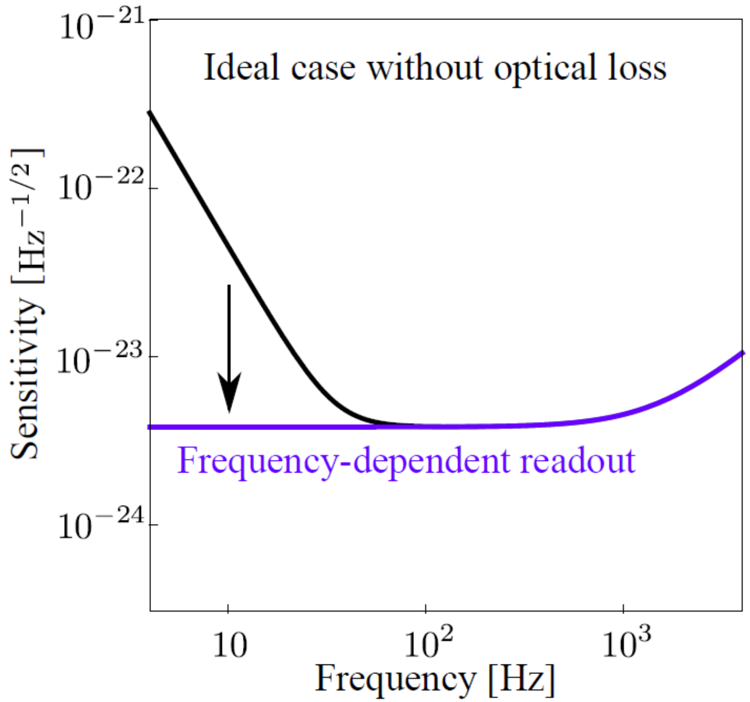




Cancelling radiation pressure noise

Frequency-dependent readout





In principle, this allows for a shot-noise only sensitivity.

Reference: H. Kimble, Y. Levin, A. Matsko, K. Thorne, and S. Vyatchanin, PRD **65**, 022002 (2001).

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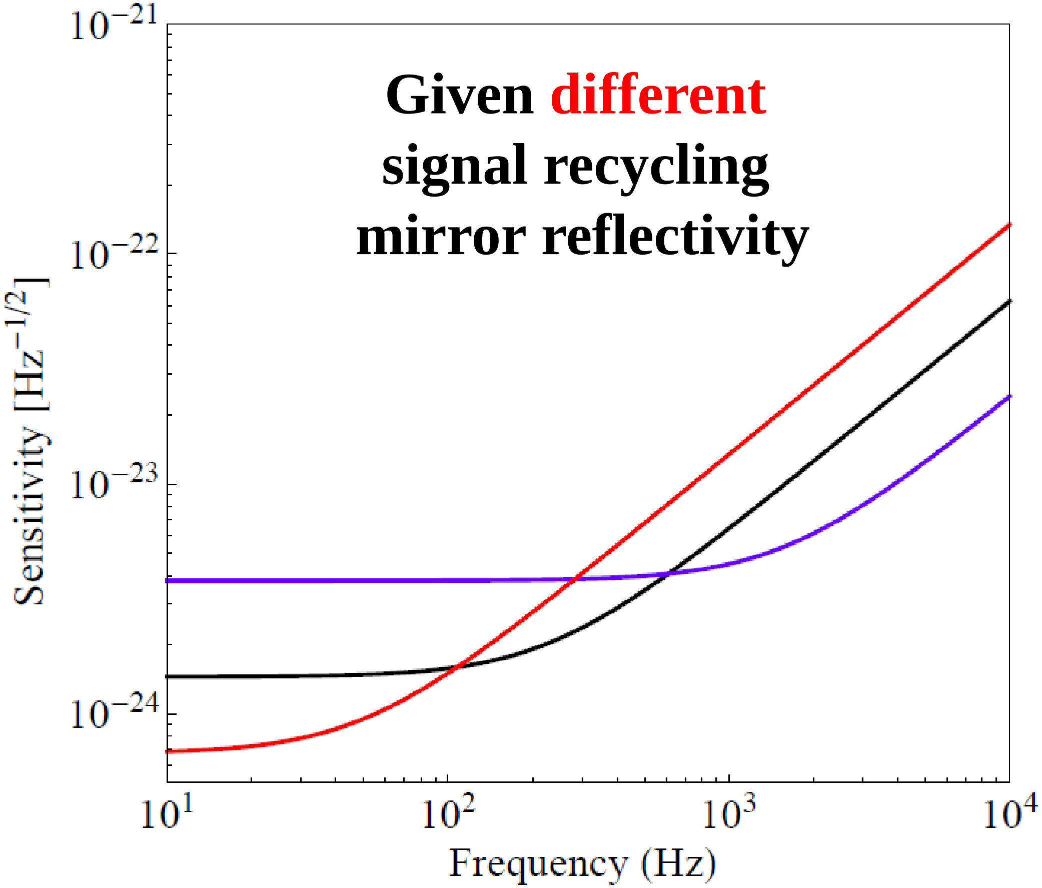
Mizuno theorem

Shot-noise-only sensitivity



Increasing peak-sensitivity

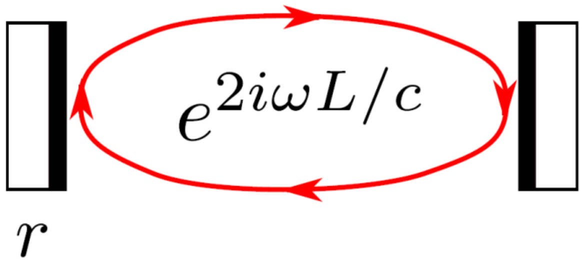
Decreasing detection bandwidth

$$\int_0^{f_{\rm FSR}} \frac{\mathrm{d}f}{S_h^{\rm shot}(f)} = \mathrm{const}$$

Only depends on power & arm cavity length

Mizuno theorem

For a single cavity (free space):



A positive feedback but with a phase delay: $\phi = 2\omega L/c$

Resonant condition:

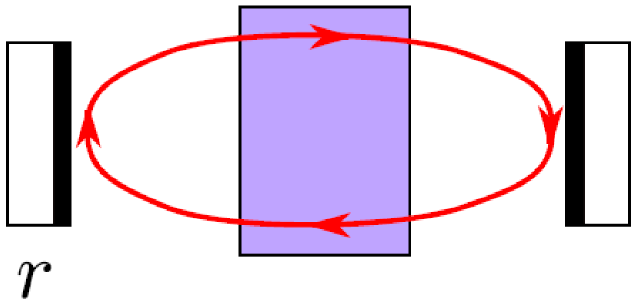
$$2\omega_0 L/c = 2n\pi$$

Only valid at one single frequency: $\omega = \omega_0 + \Omega$

$\Omega L/c = \pi/2$ ⇨ Feedback changes sign

White-light-cavity idea

Introducing a medium:



$$2\omega L/c + \phi_m(\omega) = 2n\pi$$

$$\Rightarrow \quad \frac{\mathrm{d}\phi_m(\omega)}{\mathrm{d}\omega} = -2\frac{L}{c} < 0$$

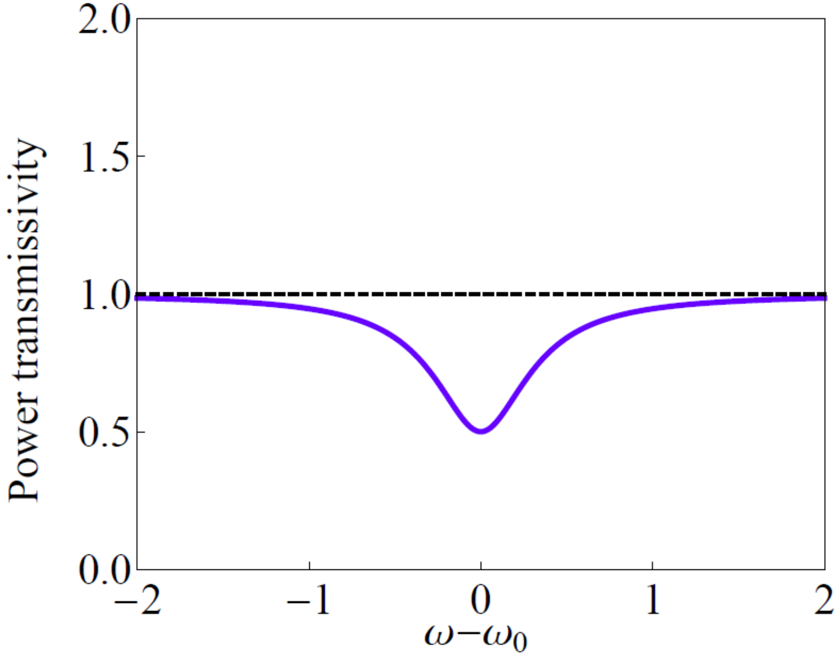
Negative-dispersion medium

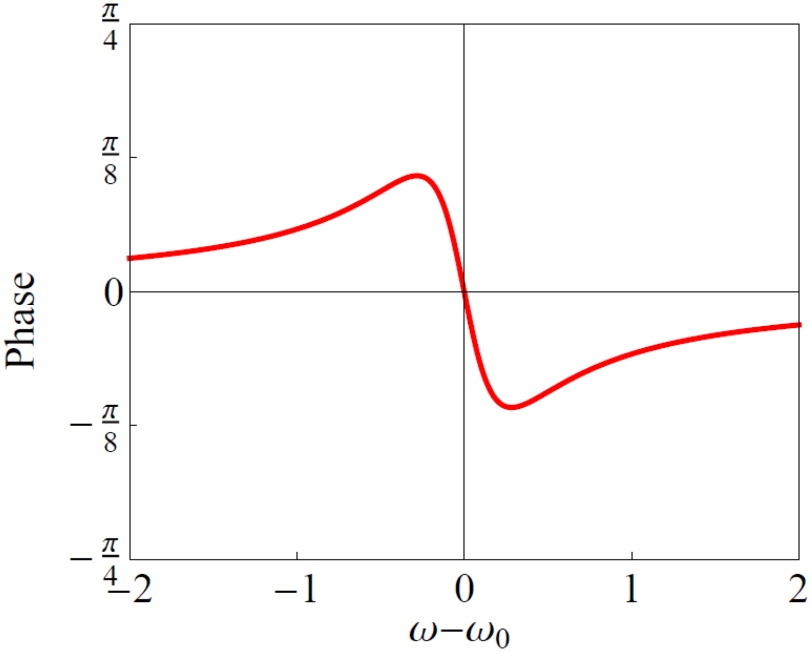
Negative dispersion

For usual mediums at low frequencies:

$$\left.\frac{d\phi}{d\omega}\right|_{\text{low freq}} > 0 \quad \Rightarrow \quad \text{positive (normal) dispersion}$$

Around absorption (attenuation) line:





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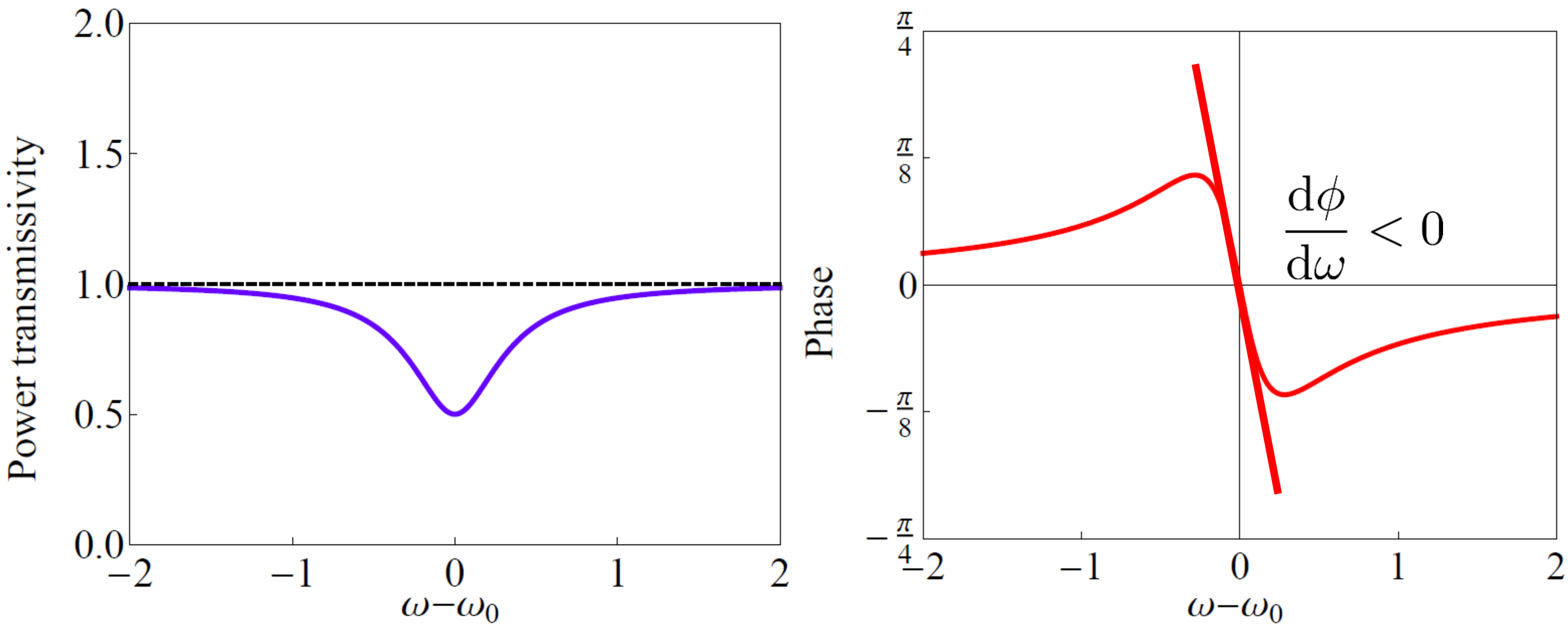
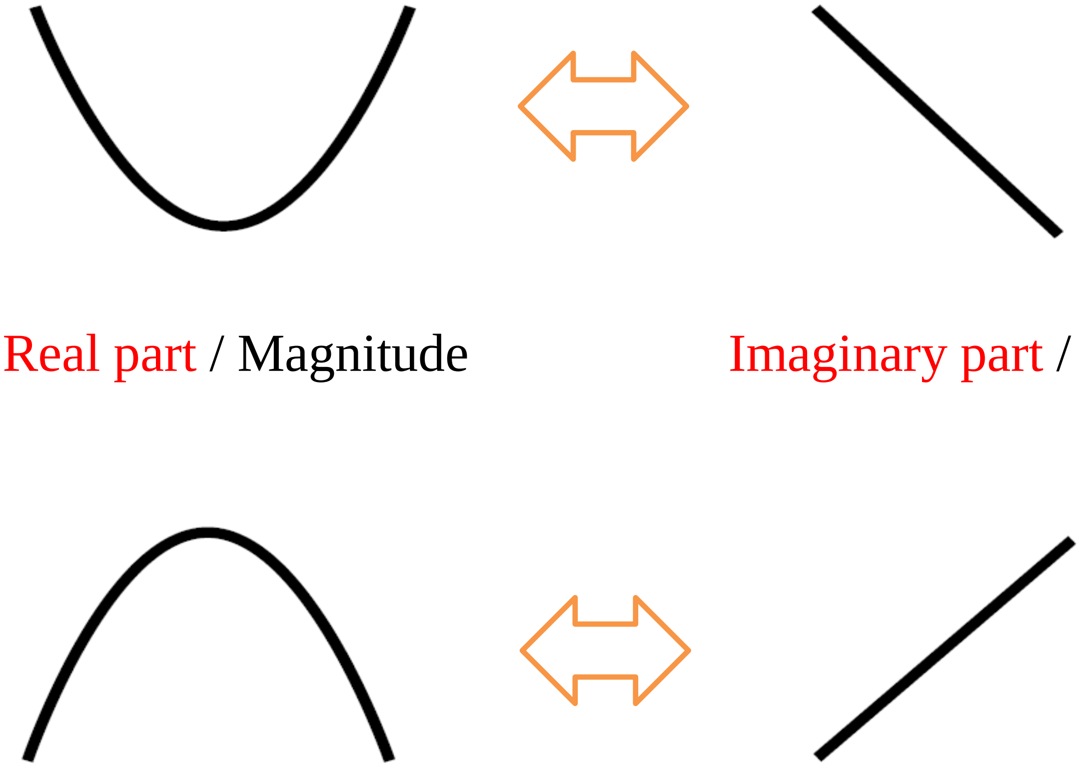


Stable linear response: KK relation

Rule of thumb for Kramers-Kronig (KK) relation (global):



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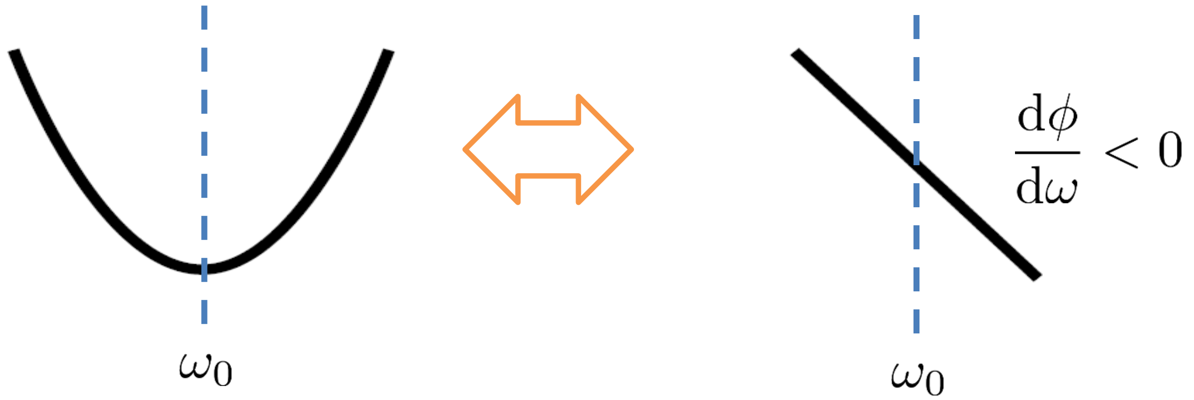


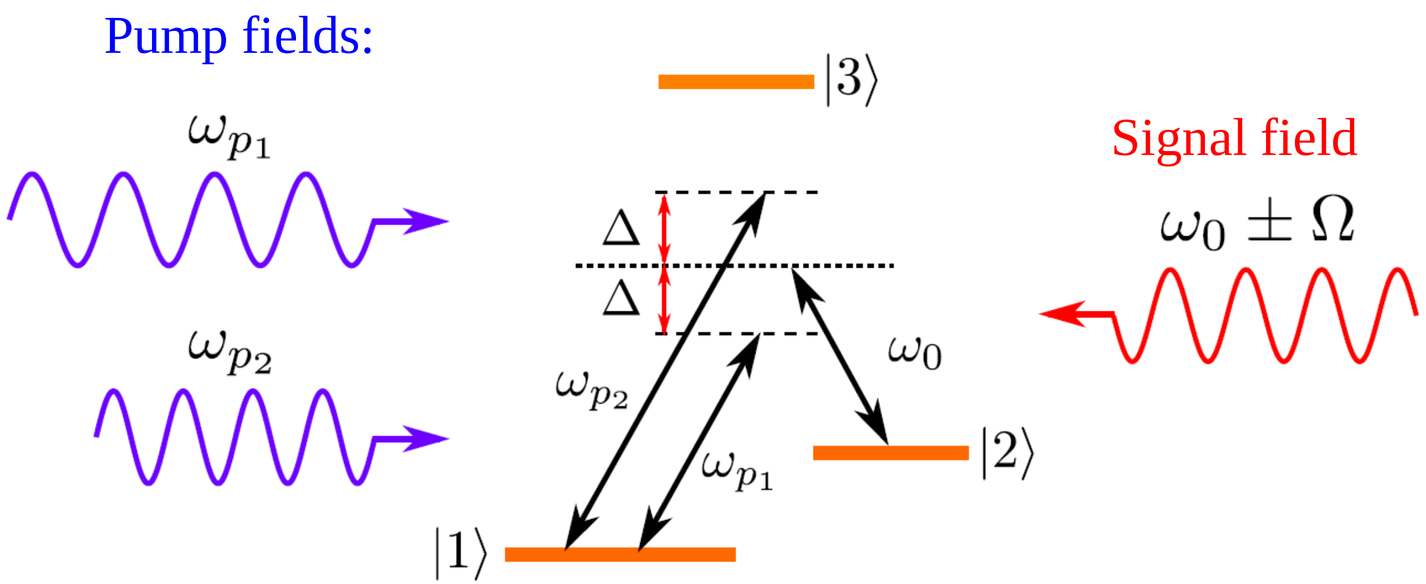
Lossless: Transmissivity around ω_0 = 1

⇩

Gain medium with amplified “wing”

Double gain medium

Three-level atomic system with double pumps:

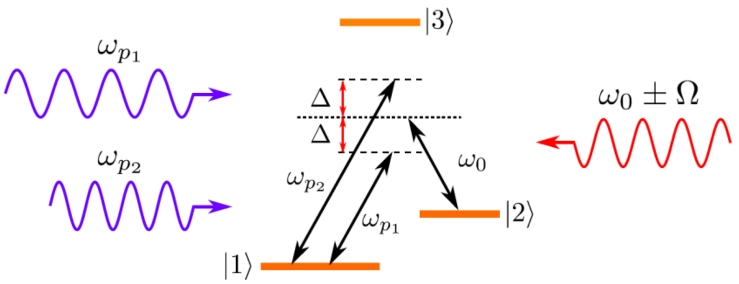


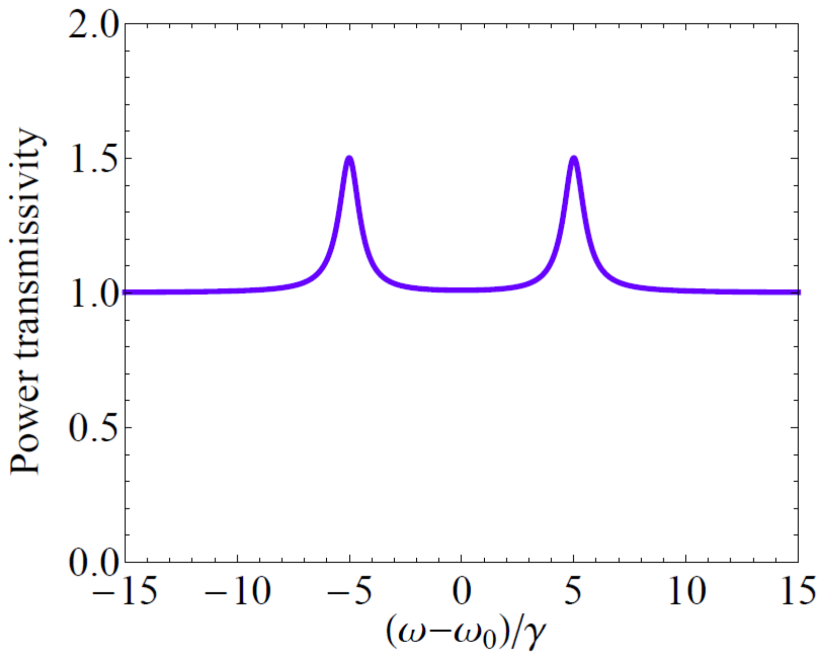
1. Virtually populated level 3

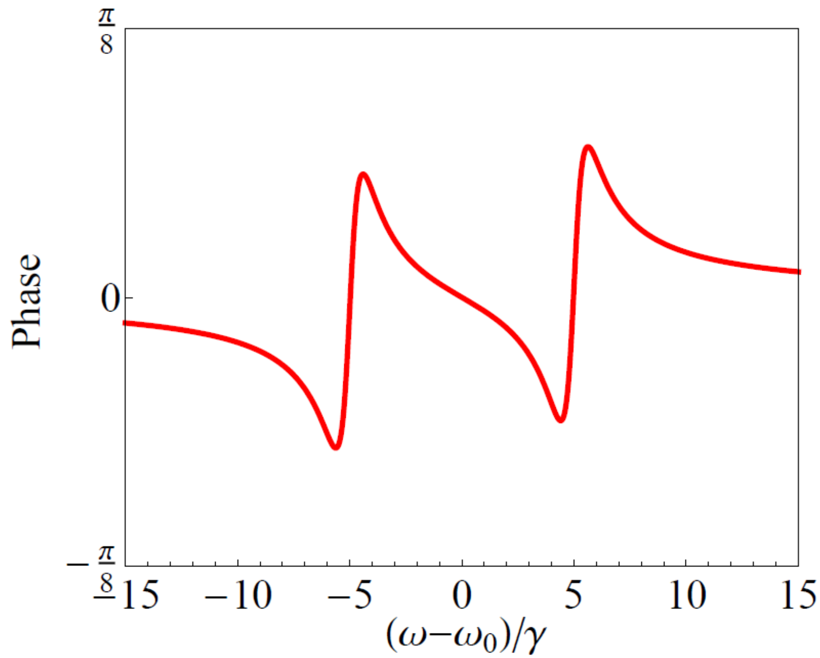
2. Two frequencies at which there is gain for signal

Double gain medium

Three-level atomic system with double pumps:







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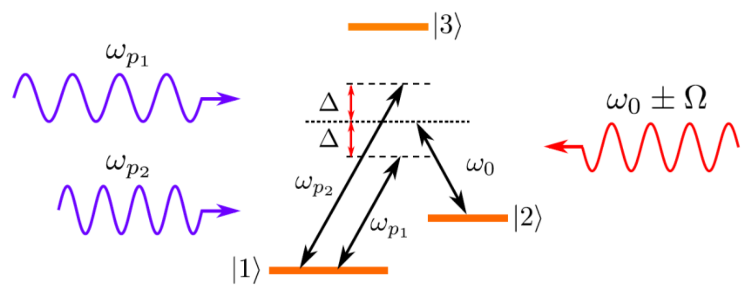
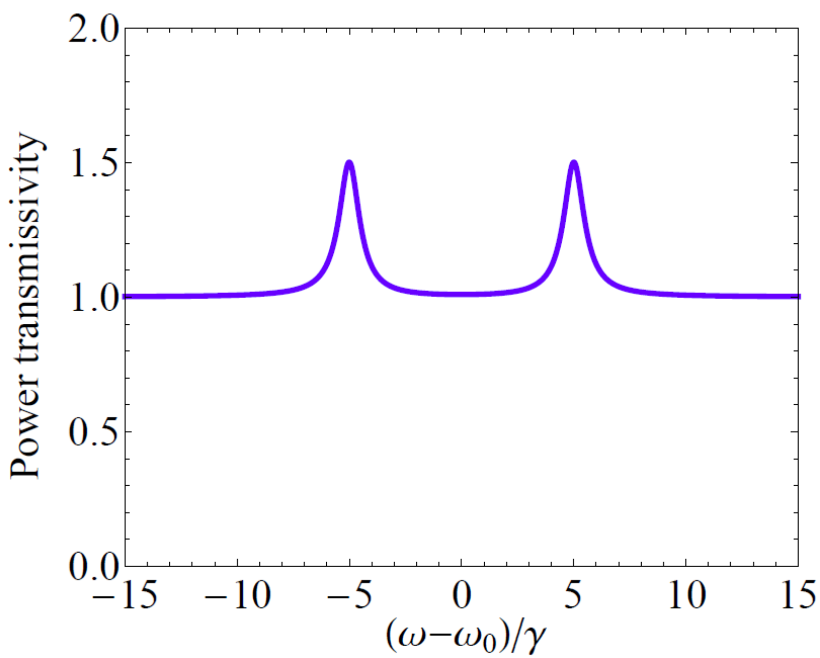
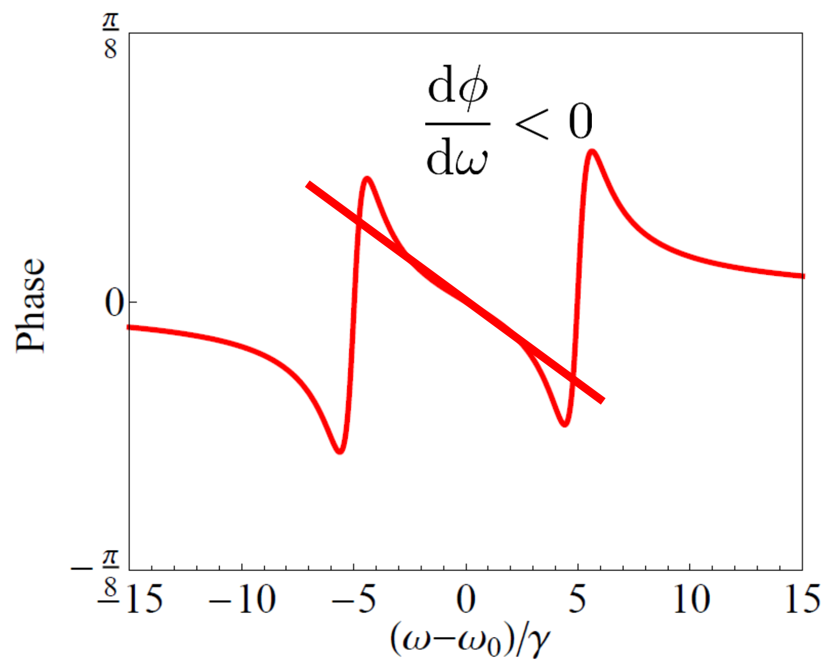
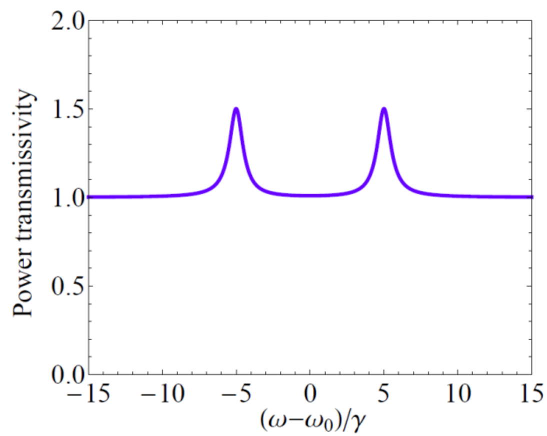


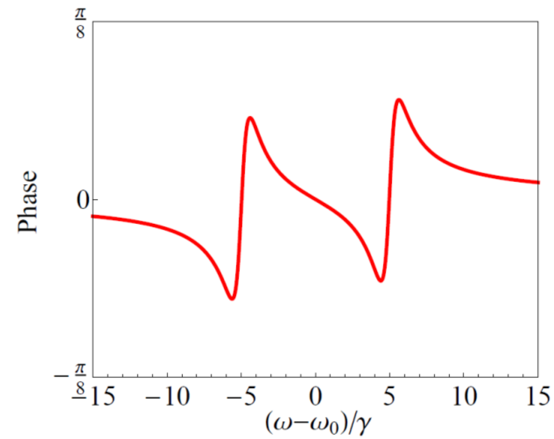




Double-gain medium

Some math:





$$\hat{a}_{\rm out} = \left[1 + \frac{i\gamma_{\rm opt}}{(\Delta+\Omega)+i(\gamma_{12}-\gamma_{\rm opt})} + \frac{i\gamma_{\rm opt}}{(\Delta-\Omega)+i(\gamma_{12}-\gamma_{\rm opt})}\right]\hat{a}_{\rm in}$$

$$+ \text{ additional noise term}$$

$\gamma_{\rm opt} \propto$ pumping power

γ_{12} : Line-width

Phase cancellation: $\left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_0} = -2\frac{L}{c}$ ⇒ $\gamma_{\rm opt} \approx \Delta^2 L/c$

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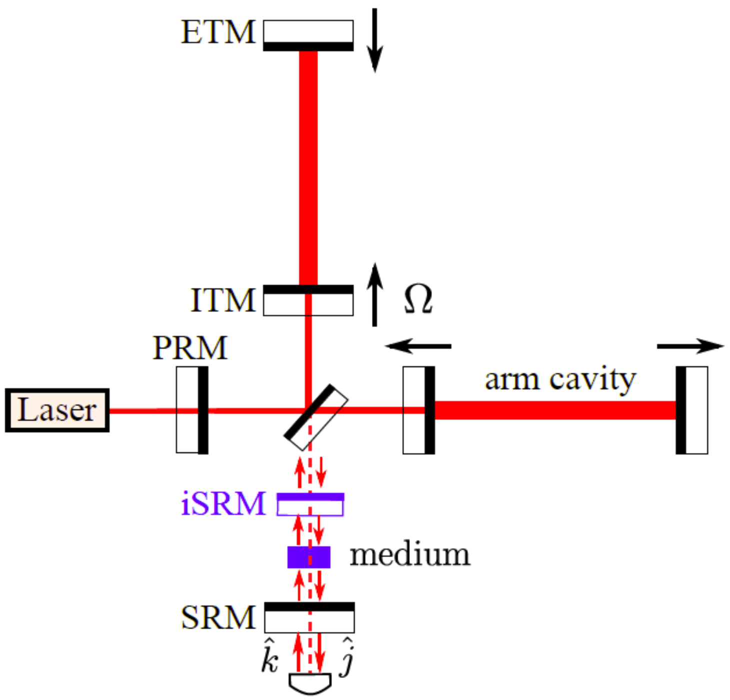
Stability condition

$$\hat{a}_{\rm out} = \left[1 + \frac{i\gamma_{\rm opt}}{(\Delta+\Omega)+i(\gamma_{12}-\gamma_{\rm opt})} + \frac{i\gamma_{\rm opt}}{(\Delta-\Omega)+i(\gamma_{12}-\gamma_{\rm opt})}\right]\hat{a}_{\rm in}$$

$$\hat{a}_{\rm out}(\Omega) \equiv [1+\chi(\Omega)]\,\hat{a}_{\rm in}(\Omega)$$



Open-loop transfer function:

$$H_{\rm OL}(\Omega) = r_{\rm sr}[1+\chi(\Omega)]e^{2i\Omega L/c}$$

Close-loop transfer function:

$$\frac{1}{1-H_{\rm OL}(\Omega)}$$

Stability in terms of poles:

$$1 - H_{\rm OL}(\Omega) = 0$$

Stability condition: Nyquist theorem

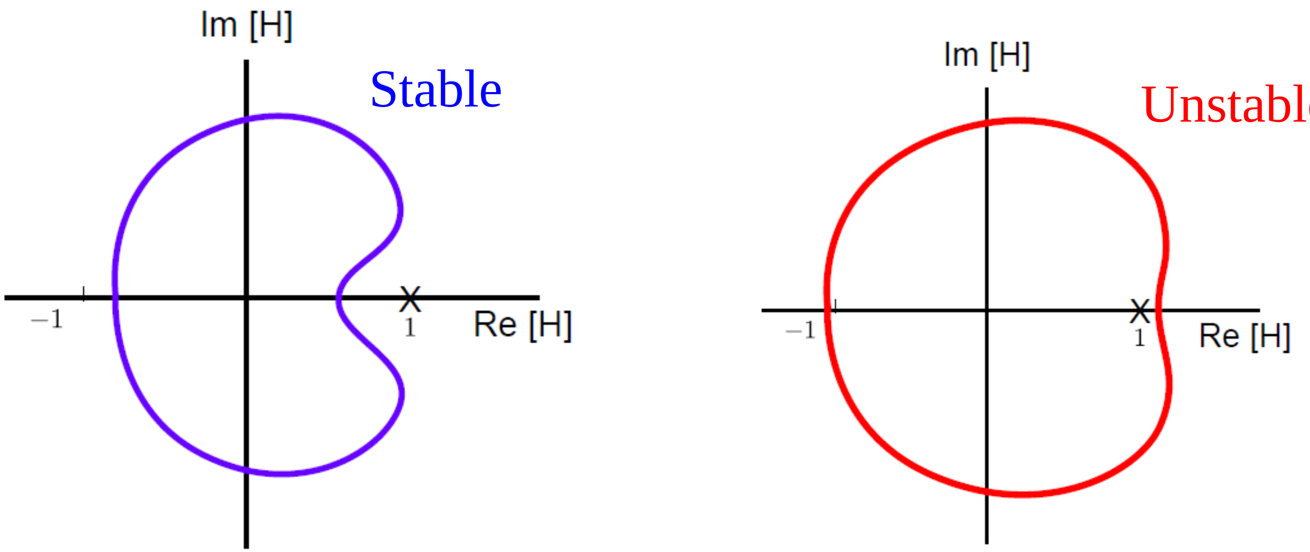
Open-loop transfer function: $H_{\rm OL}(\Omega) = r_{\rm sr}[1+\chi(\Omega)]e^{2i\Omega L/c}$

Gain and phase margin: positive feedback

$$|H_{\rm OL}(\Omega_0)| = 1 \quad \Rightarrow \quad \arg[H_{\rm OL}(\Omega_0)] > 0$$

$$\arg[H_{\rm OL}(\Omega_0)] = 0 \quad \Rightarrow \quad |H_{\rm OL}(\Omega_0)| < 1$$

Nyquist plot:



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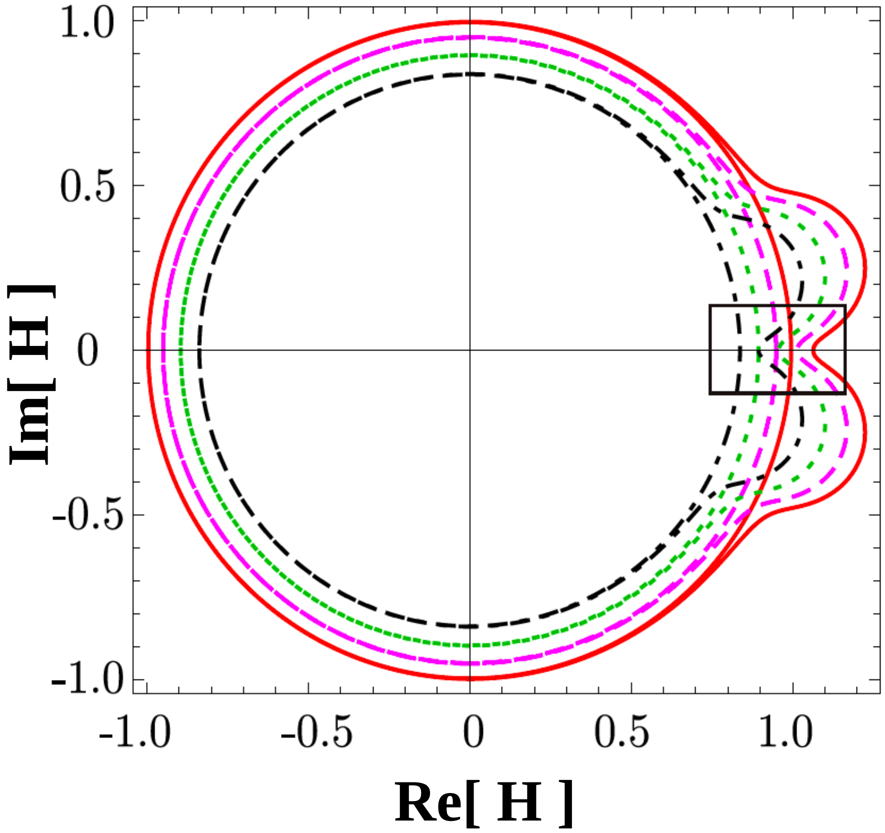

Fixing other parameters while changing $r_{\rm sr}$

Stability condition: Nyquist theorem

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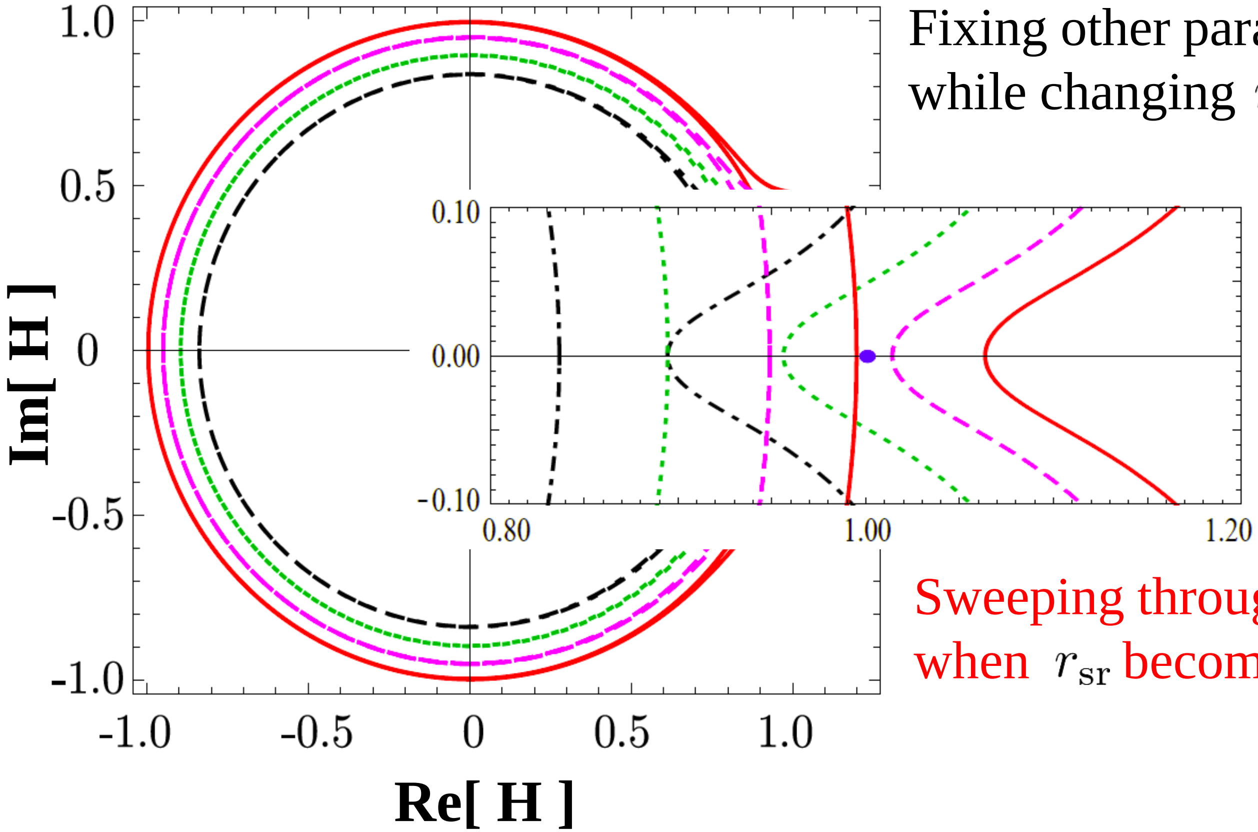


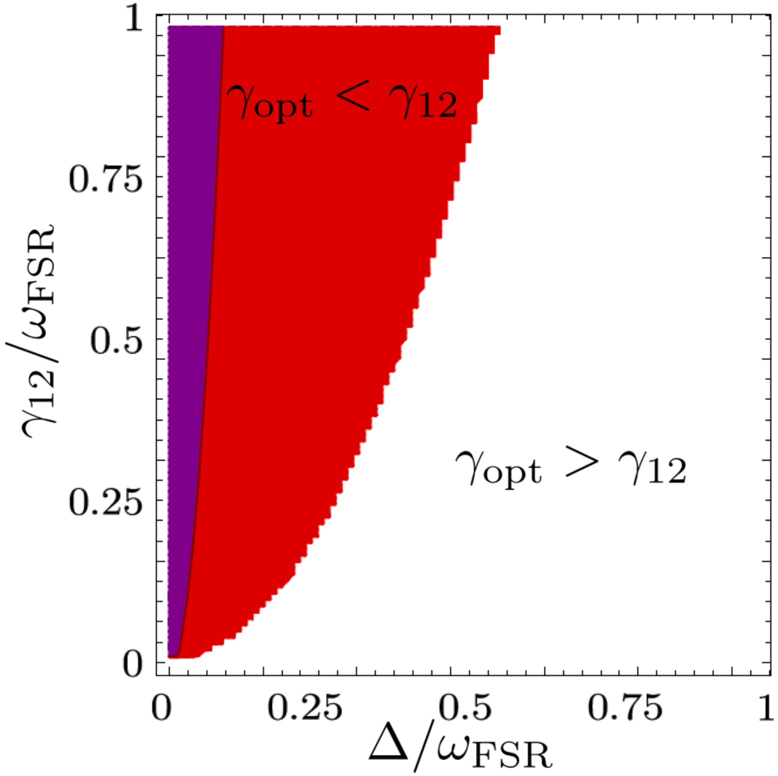
Fixing other parameters while changing $r_{\rm sr}$

Sweeping through 1 when $r_{\rm sr}$ becomes larger

Stability condition: Nyquist theorem

Open-loop transfer function: $H_{\rm OL}(\Omega) = r_{\rm sr}[1+\chi(\Omega)]e^{2i\Omega L/c}$

$$\chi(\Omega) = \frac{i\gamma_{\rm opt}}{(\Delta+\Omega)+i(\gamma_{12}-\gamma_{\rm opt})} + \frac{i\gamma_{\rm opt}}{(\Delta-\Omega)+i(\gamma_{12}-\gamma_{\rm opt})}$$



Purple: Stable

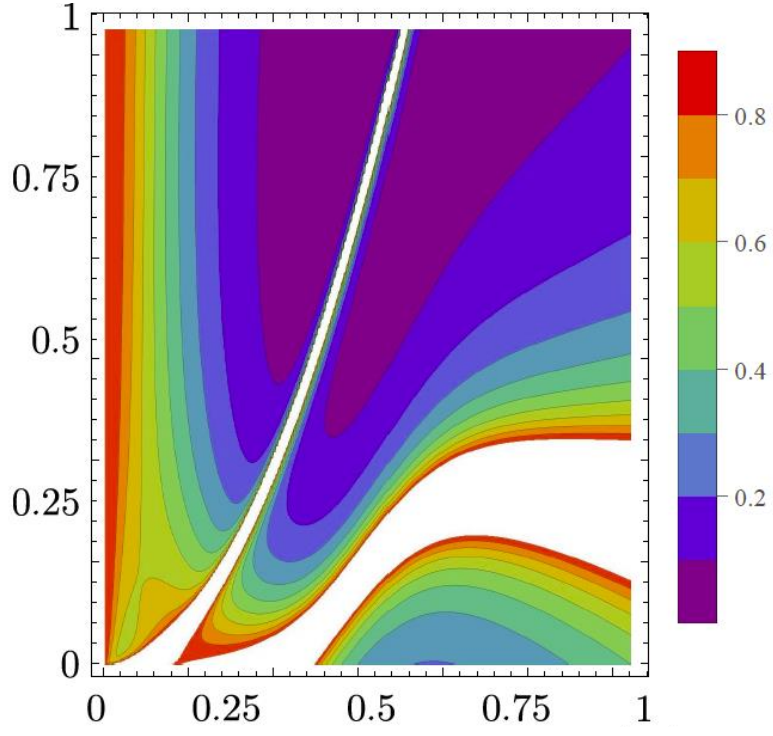
Red: Unstable (Lasing) with SR

White: Unstable by the medium itself

Sensitivity gain

$$\text{gain} \equiv \int_0^{f_{\rm FSR}} \frac{\mathrm{d}f}{S_h^{\rm shot}(f)|_{\rm WLC}} \Big/ \int_0^{f_{\rm FSR}} \frac{\mathrm{d}f}{S_h^{\rm shot}(f)|_{\rm con}}$$

No additional noise



White regime:

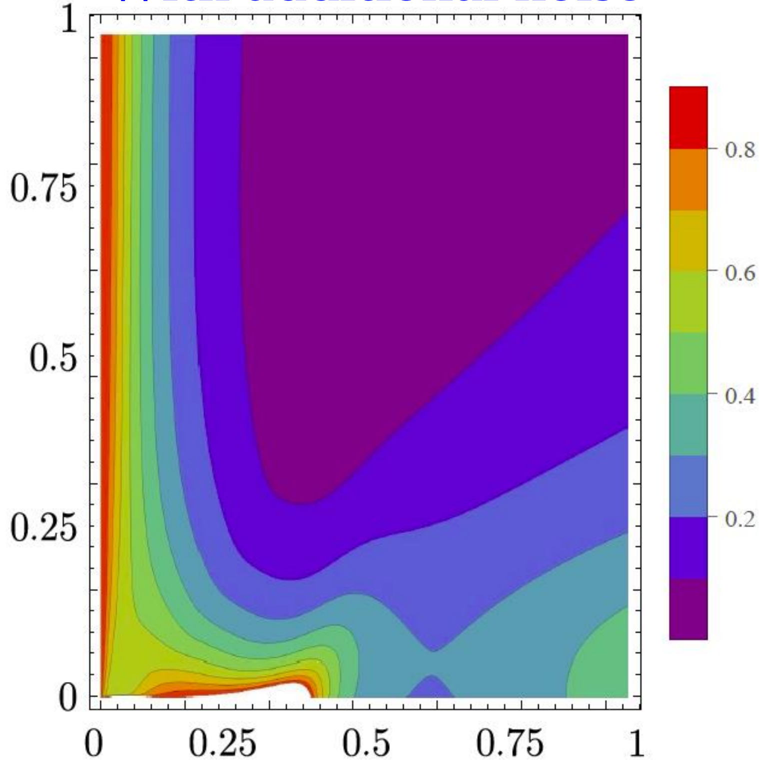
gain > 1

Also in the unstable regime

Sensitivity gain

$$\text{gain} \equiv \int_0^{f_{\text{FSR}}} \frac{\mathrm{d}f}{S_h^{\text{shot}}(f)|_{\text{WLC}}} \Big/ \int_0^{f_{\text{FSR}}} \frac{\mathrm{d}f}{S_h^{\text{shot}}(f)|_{\text{con}}}$$

With additional noise



White regime:

gain > 1

Also in the unstable regime

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