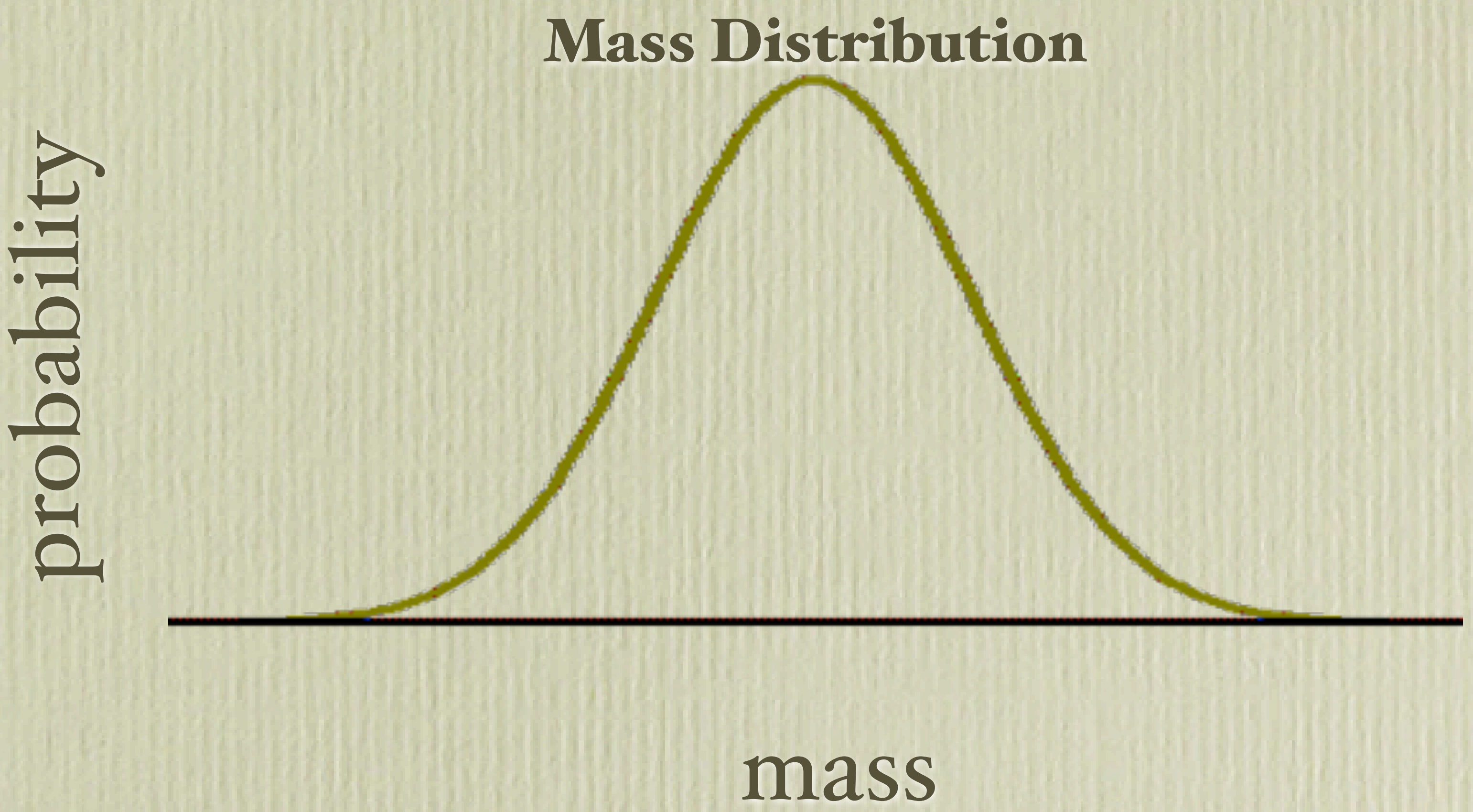
Inference on Binary Neutron Star Populations

Naomi Gendler
Mentors: Larry Price and Vivien Raymond

The Problem



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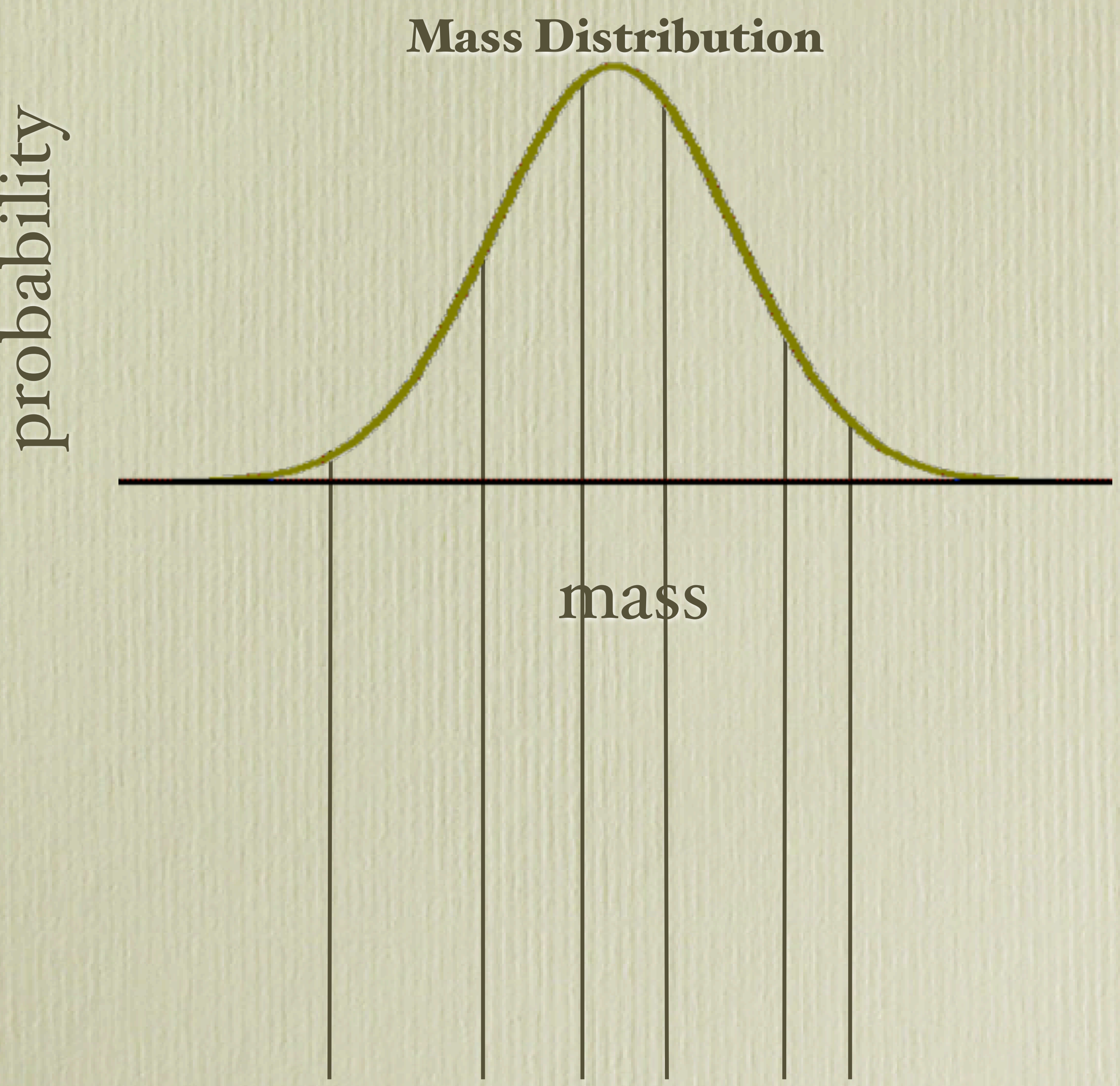

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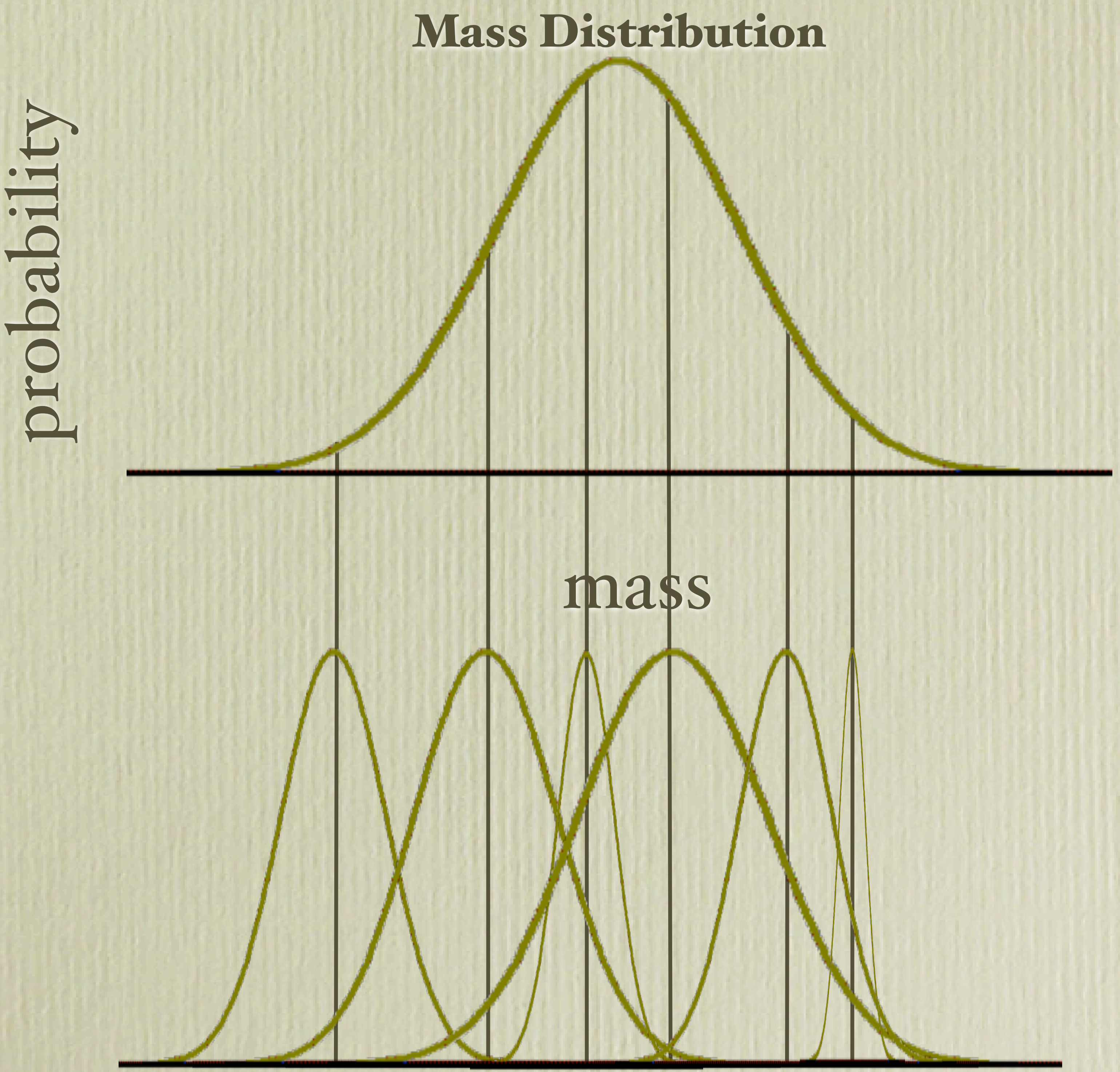

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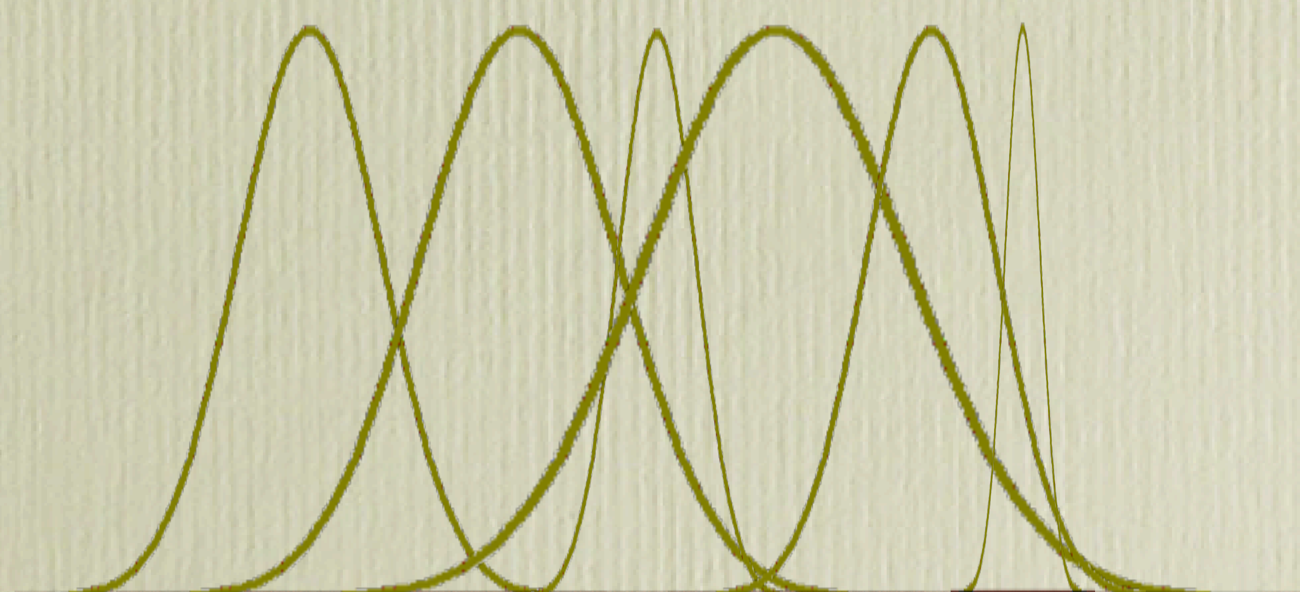
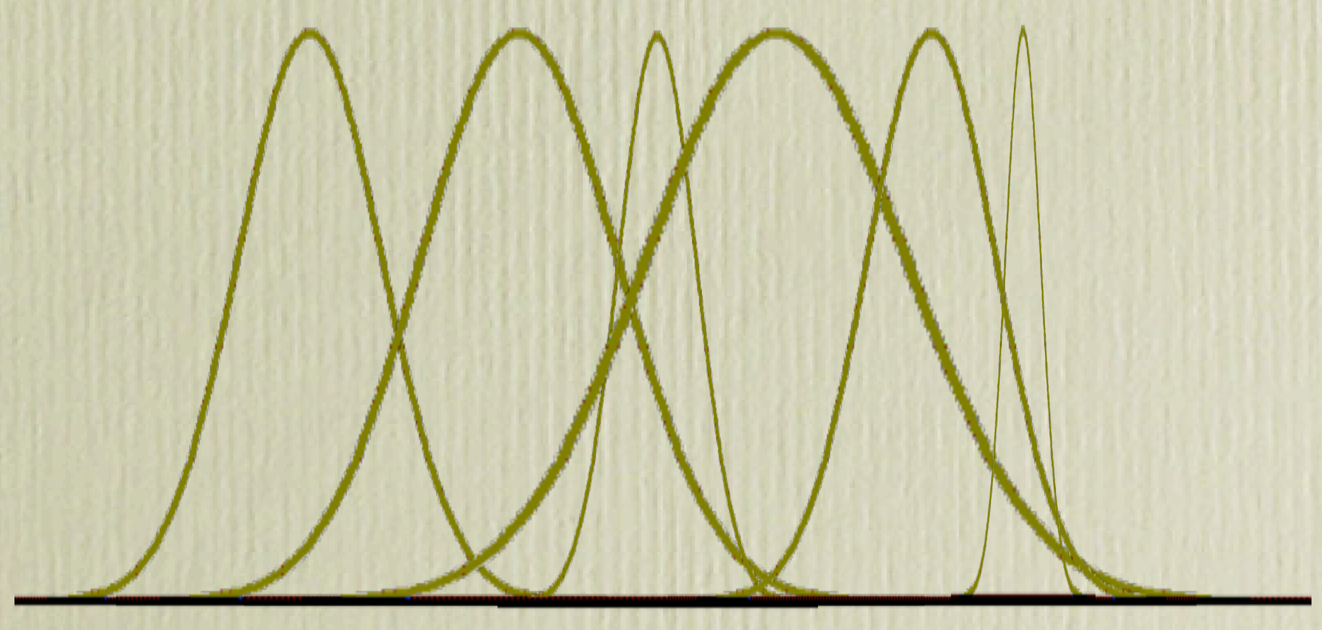


The Problem

Can we estimate the parameters of the original distribution?



Measuring the Chirp Mass

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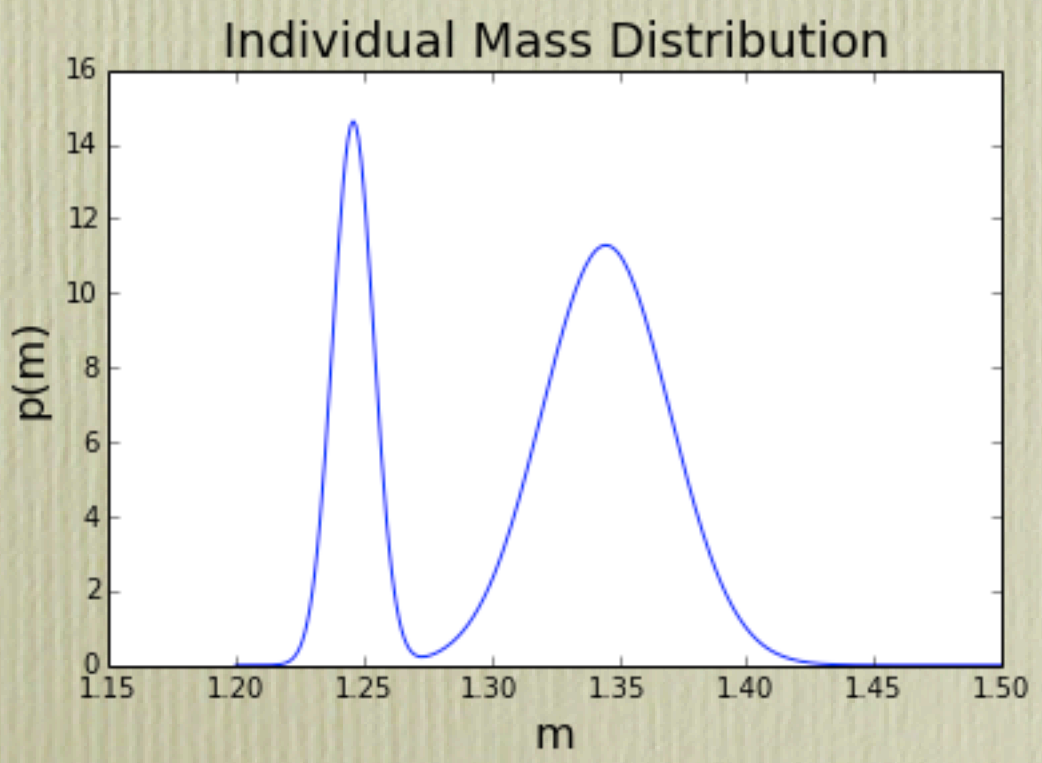

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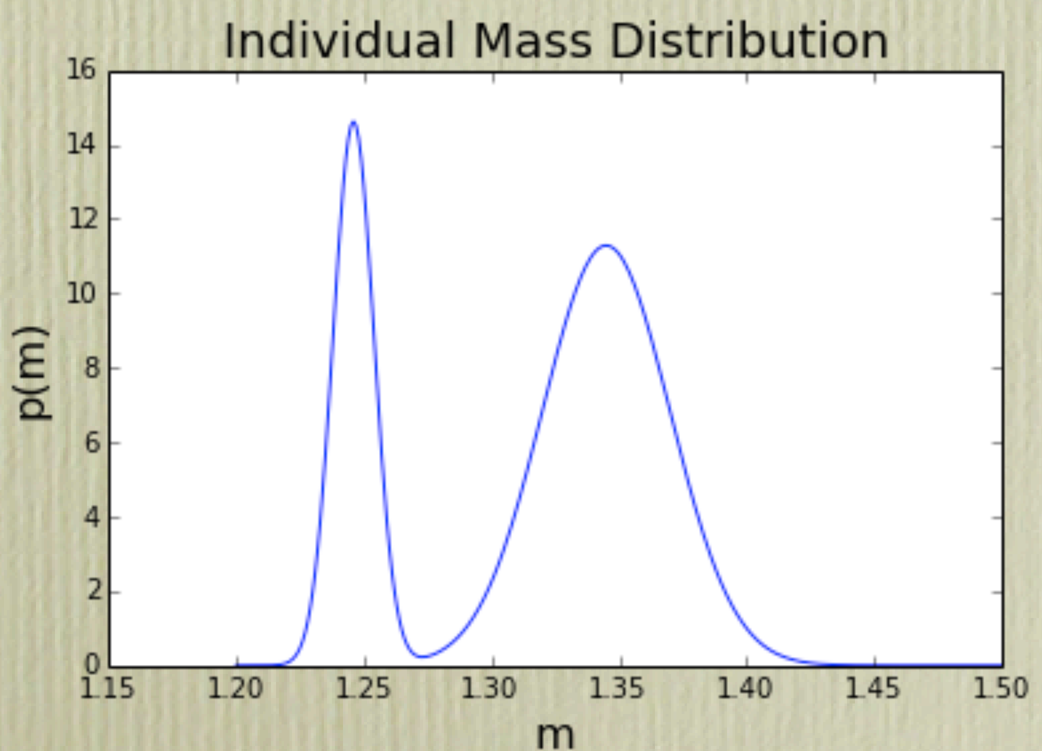

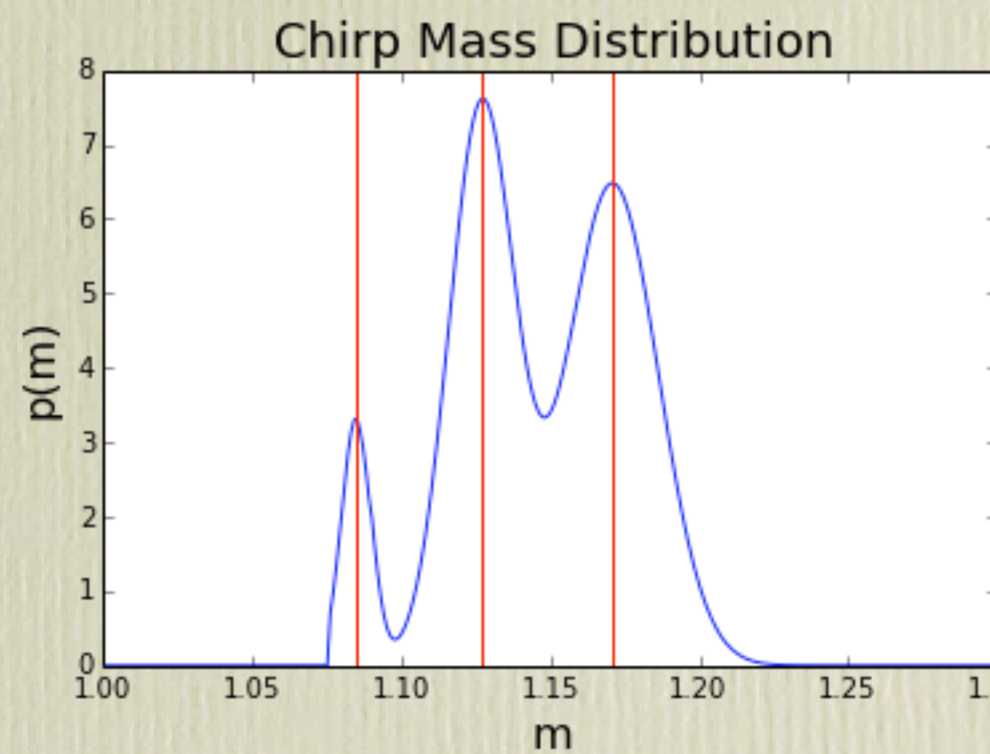

Selection Bias

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- Distant or dim objects may not be taken into account
- Weak signals may not produce signal-to-noise ratios above LIGO's threshold
- If we don't take into account these weak signals, we are leaving out a portion of the population whenever we make inferences

Theory: Hierarchical Modeling

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Global Parameters (α)

$\mu, \sigma, ...$

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Local Parameters (θ)

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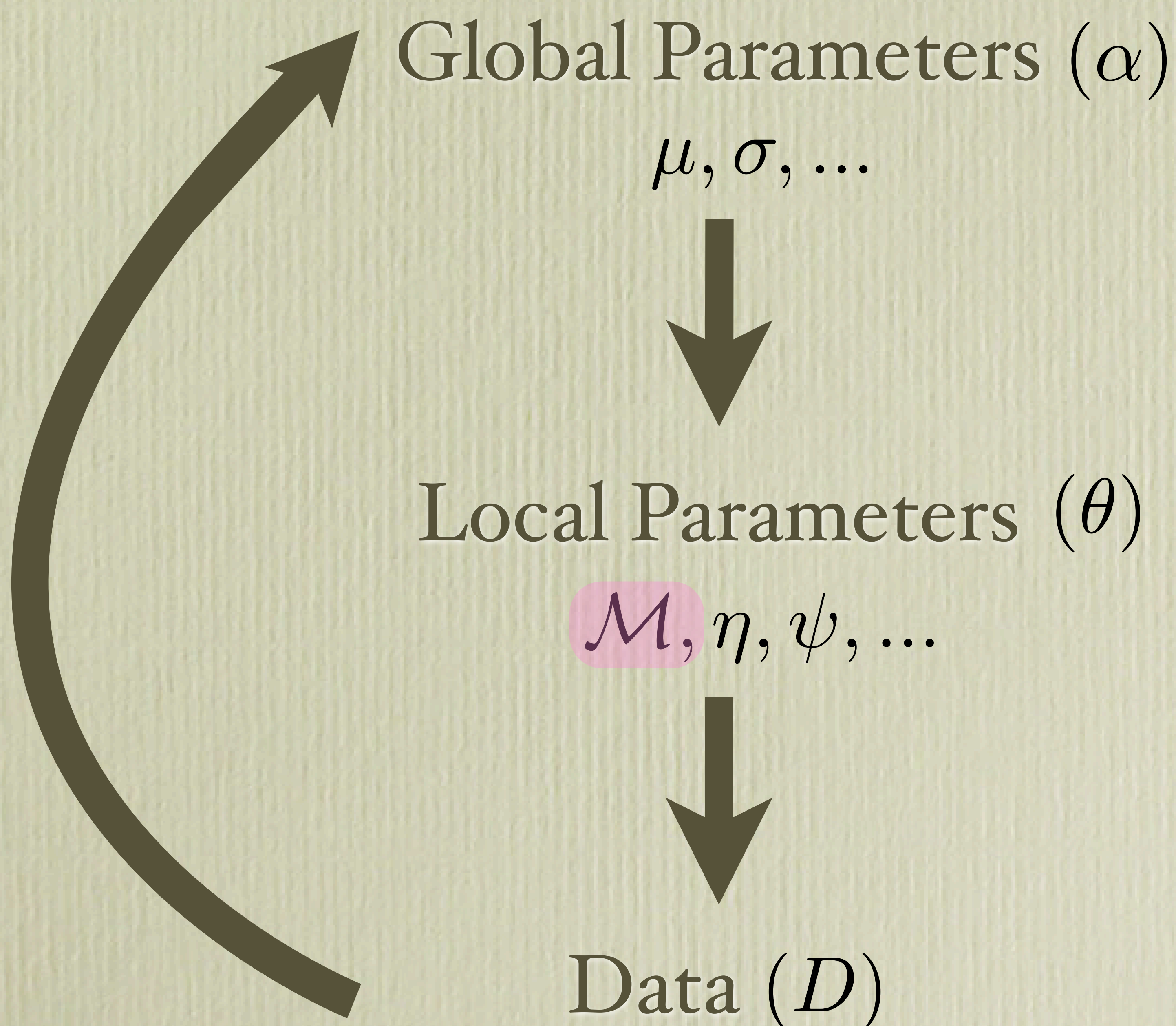
Data (D)

Theory: Hierarchical Modeling



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explain likelihood
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Bayes' Theorem for an event n out of N

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Assuming event independence and parameter separability:

$$p(\boldsymbol{\alpha}|\{\mathbf{D_n}\}_{n=1}^{N}) = p(\boldsymbol{\alpha})\prod_{n=1}^{N}\int p(\boldsymbol{\theta}|\mathbf{D_n})\frac{p(\mathcal{M}_n|\boldsymbol{\alpha})}{p(\mathcal{M}_n)}\mathrm{d}\boldsymbol{\theta}$$

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Standard Monte Carlo integral approximation:

$$p(\boldsymbol{\alpha}|\{\mathbf{D_n}\}_{n=1}^{N}) = p(\boldsymbol{\alpha}) \prod_{n=1}^{N} \frac{1}{K_n} \sum_{k=1}^{K_n} \frac{p(\mathcal{M}_n^{(k)}|\boldsymbol{\alpha})}{p(\mathcal{M}_n^{(k)})}$$

Toy Model: Gaussian

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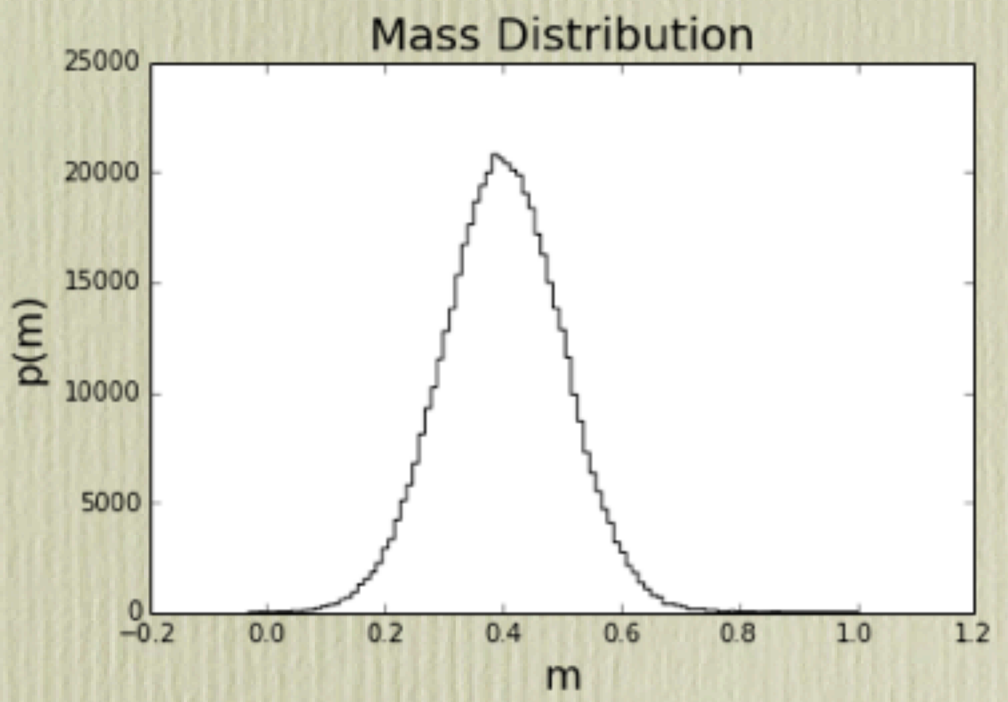
- We start by sampling a normal gaussian with arbitrarily chosen parameters:

$$\mu = 0.4 \qquad \sigma = 0.1$$



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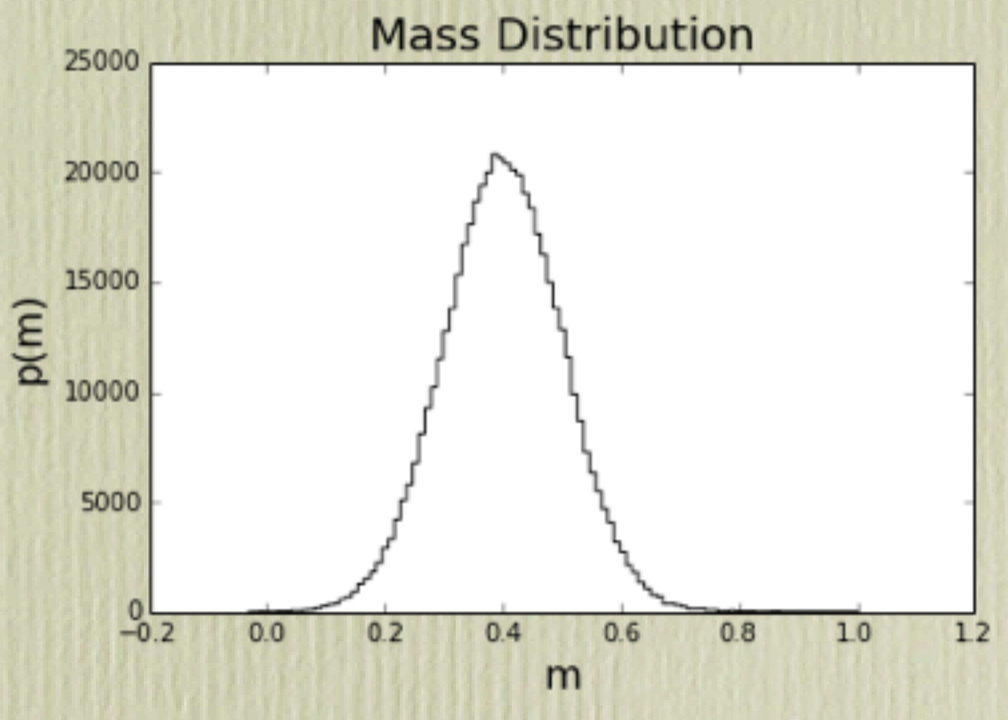

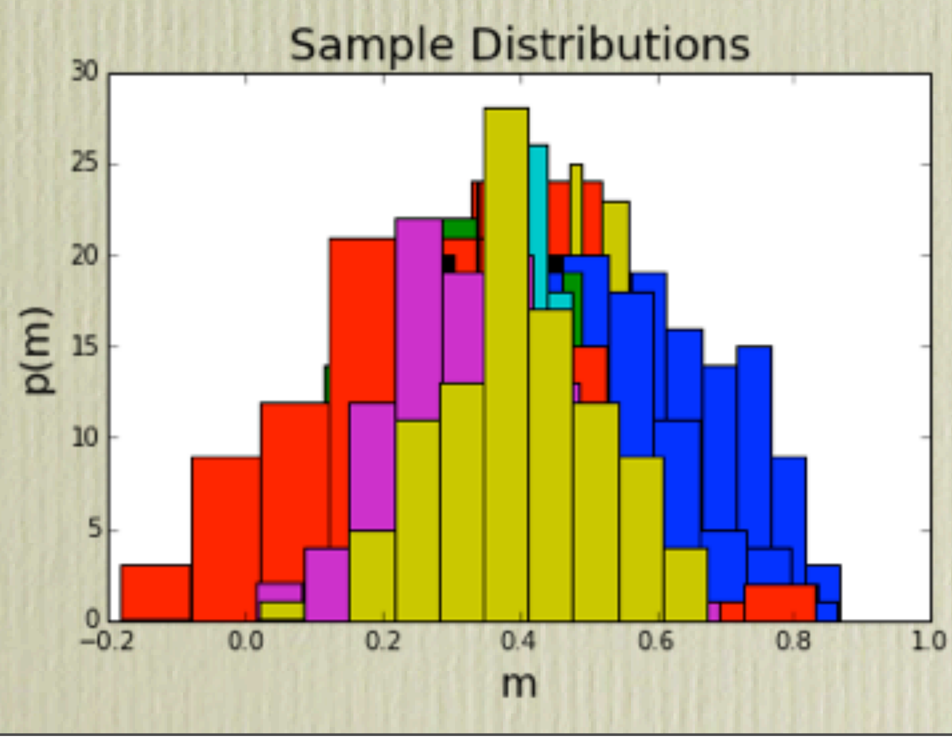

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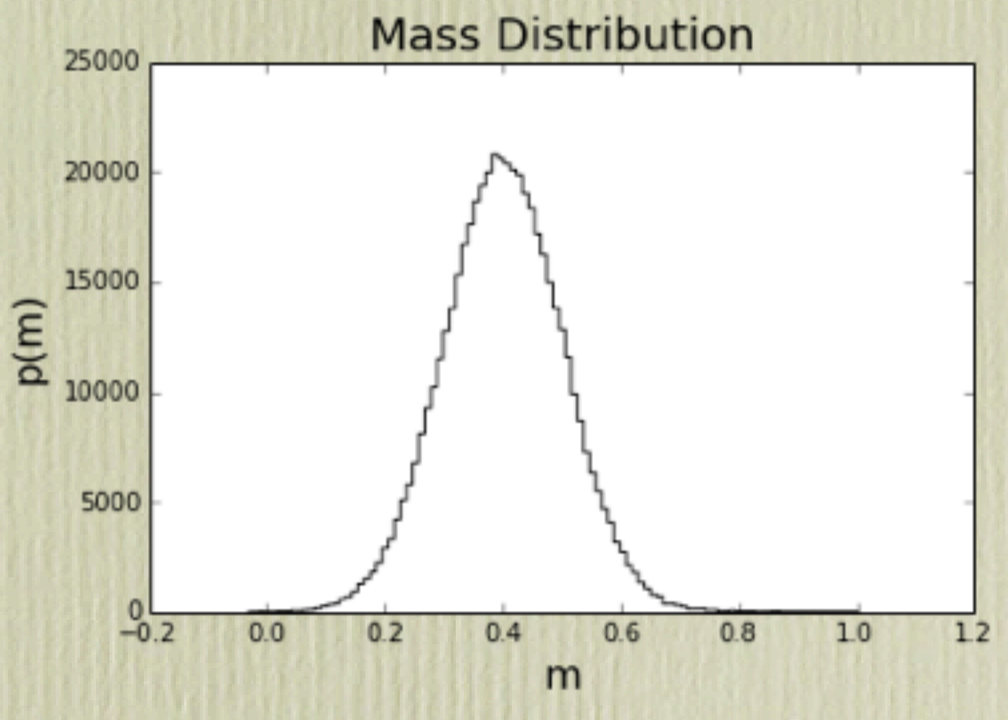

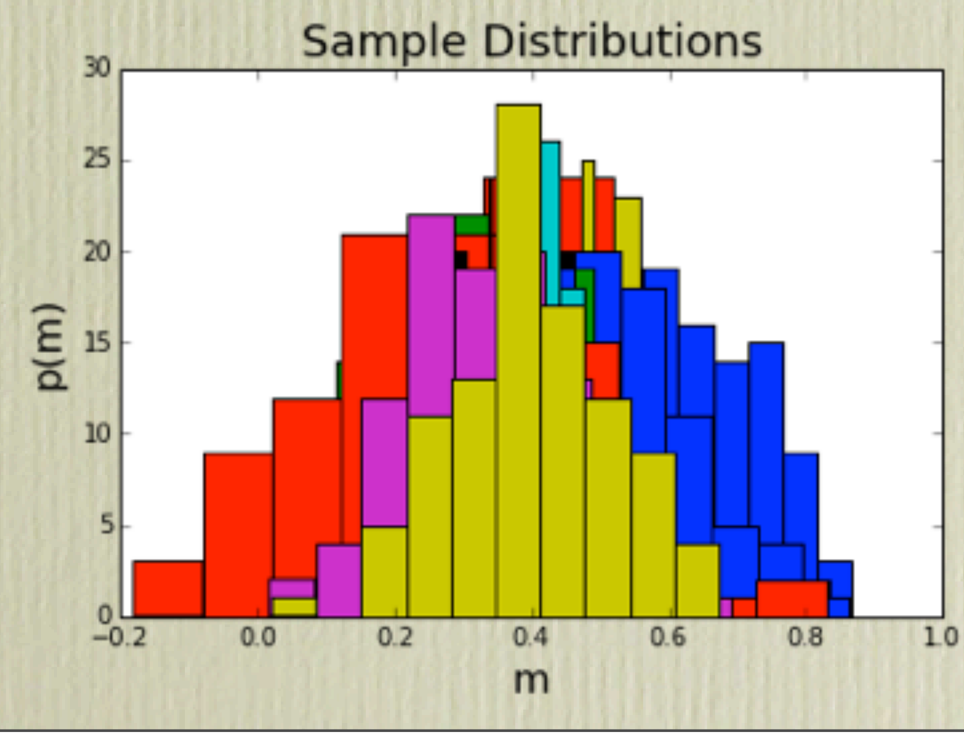

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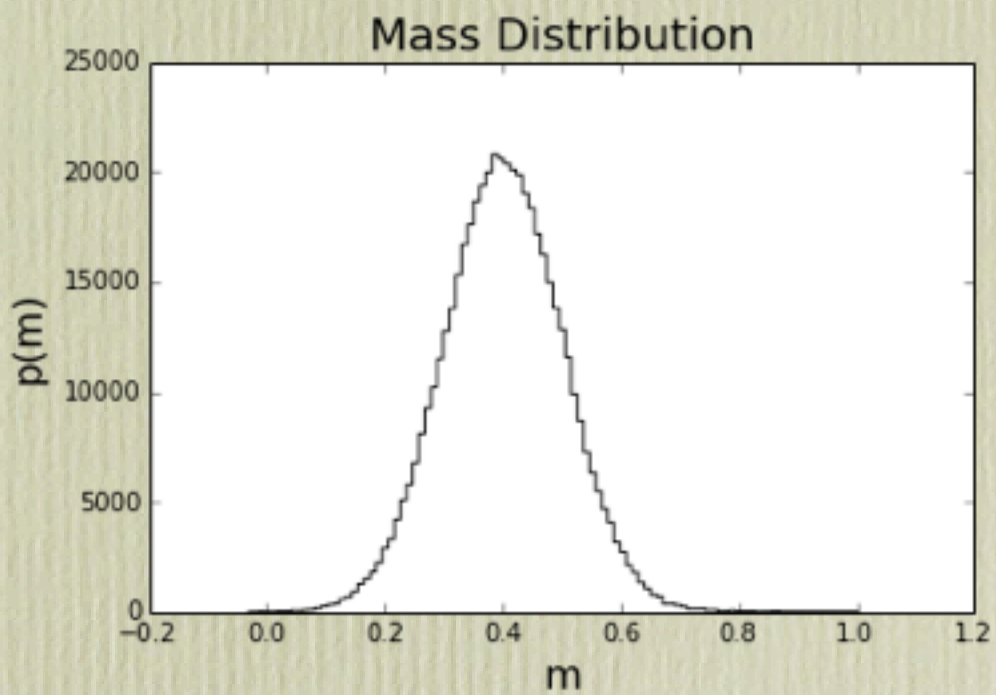

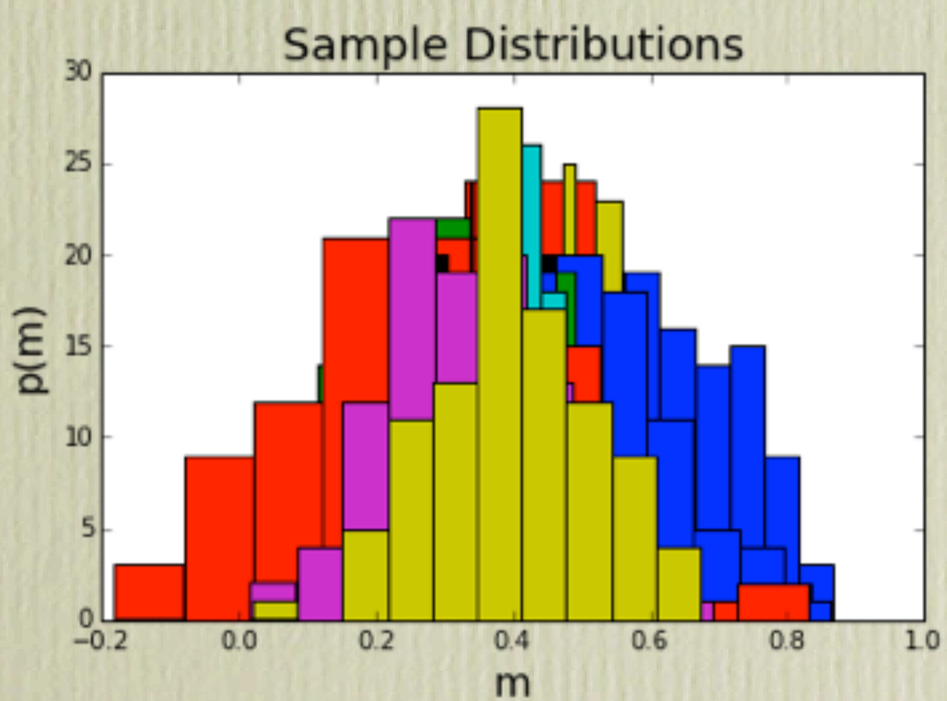

- sampled more heavily at higher probabilities
- distributions given random standard deviation between 1×10^{-50} and 0.2

Results

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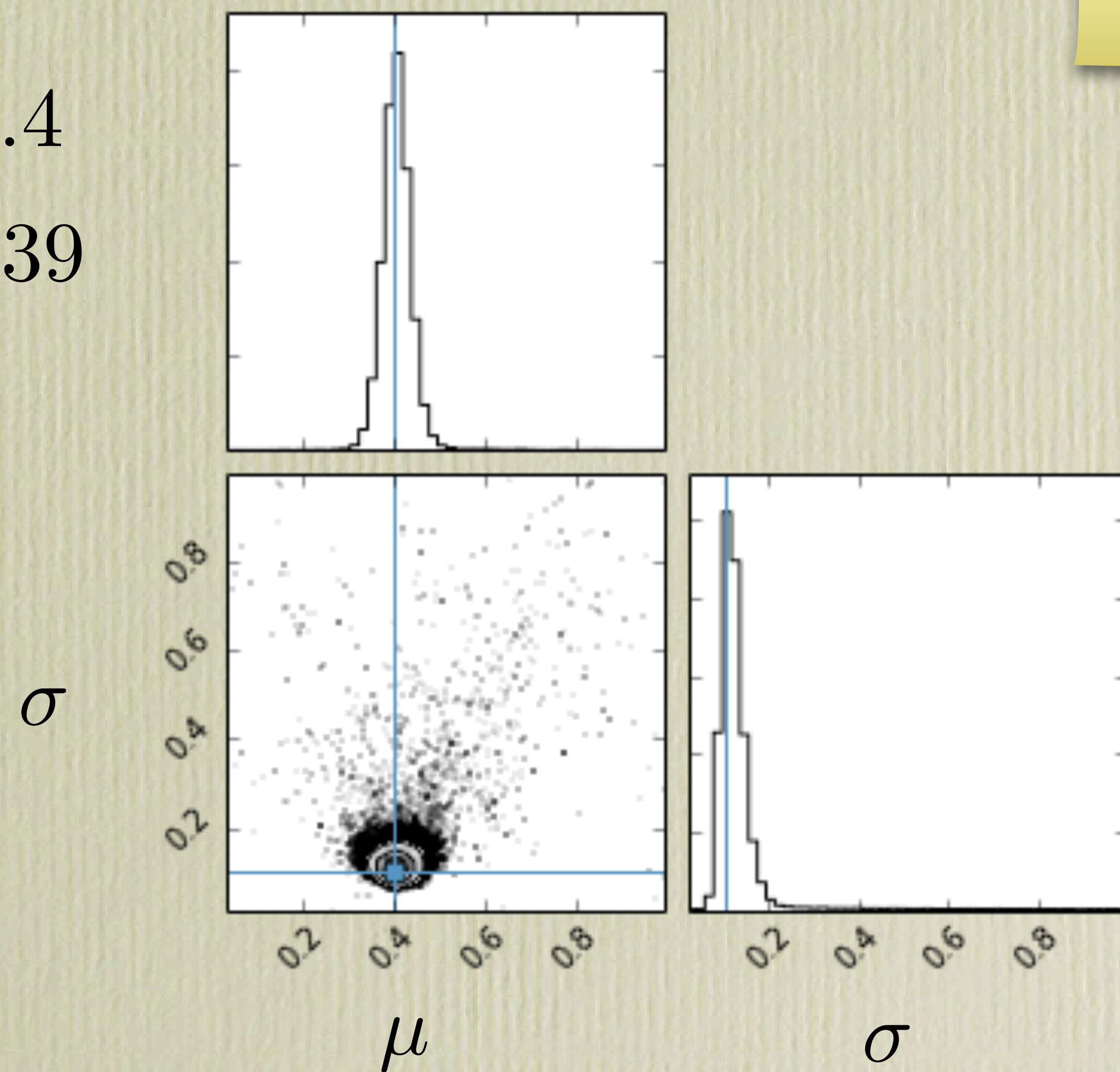
$$\bar{\mu} = 0.39$$



$$\sigma = 0.01$$

$$\bar{\sigma} = 0.056$$

Toy Model: Chirp Mass

- We use Parallel Tempering method within our Markov-Chain Monte Carlo module
- Walkers explore different “energy levels” with altered likelihoods
- Allows for easier sampling of distributions with multiple peaks

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$$\mu_1 = 1.246$$
$$\mu_2 = 1.345$$
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$$\sigma_2 = 0.025$$
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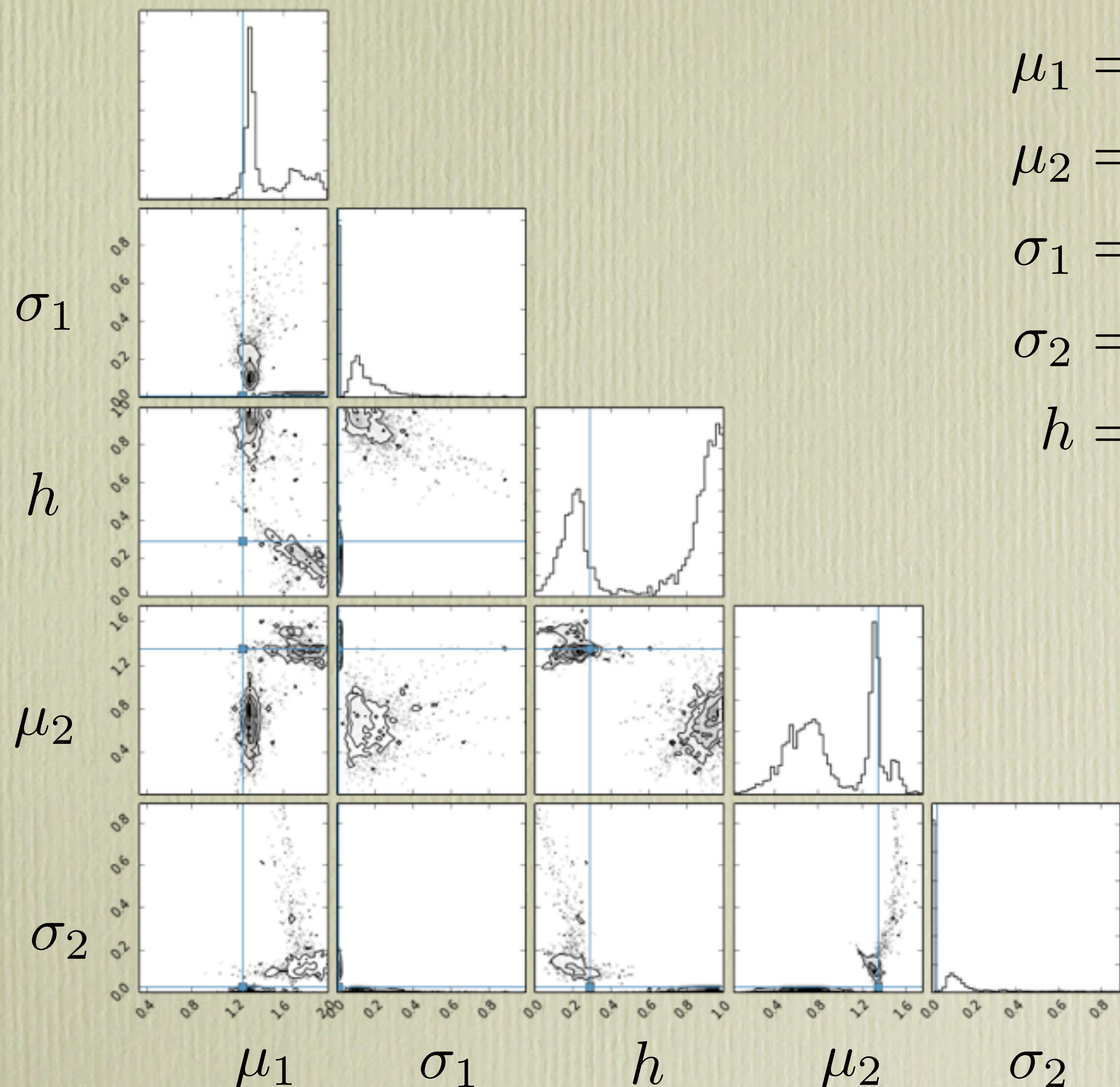

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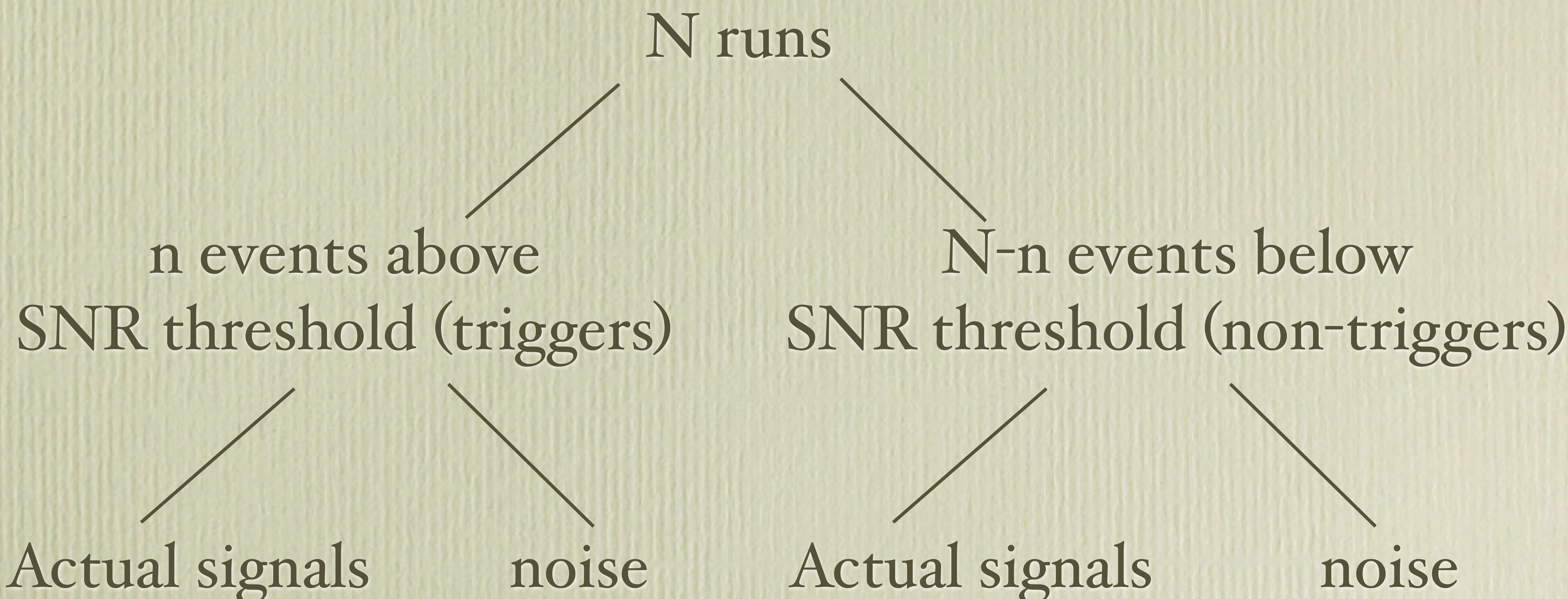
n events above SNR threshold (triggers)

N-n events below SNR threshold (non-triggers)

Theory: Selection Bias



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$$\text{Altered Likelihood} =$$

$$\prod^{\text{triggers}} [p(\text{trigger}|\text{signal}) + p(\text{trigger}|\text{no signal})]$$

$$\times$$

$$\prod^{\text{non-triggers}} [p(\text{no trigger}|\text{signal}) + p(\text{no trigger}|\text{no signal})]$$

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- Published version of the paper was missing parameter-dependent factors in the altered likelihood

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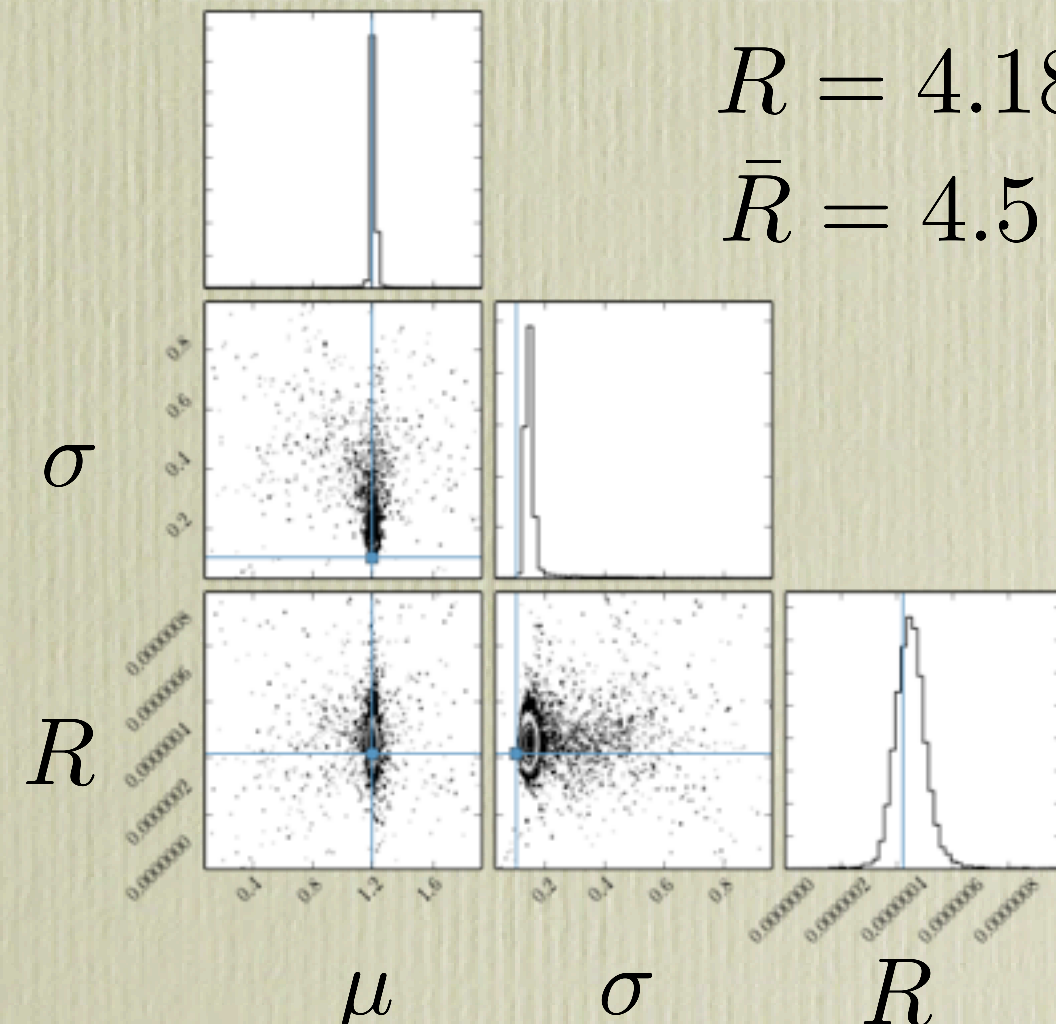
$$\bar{\mu} = 1.2 M_\odot$$

$$\sigma = 0.1 M_\odot$$

$$\bar{\sigma} = 0.14 M_\odot$$



$$R = 4.18 \times 10^{-7} \mathrm{Mpc}^{-3} \mathrm{yr}^{-1}$$

$$\bar{R} = 4.5 \times 10^{-7} \mathrm{Mpc}^{-3} \mathrm{yr}^{-1}$$

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- Selection bias effects

Future Work

- Calculate number of events we need to make accurate inferences
- Full 10-dimensional parameter estimation
- Bayesian model selection
- Explore distributions on other parameters-- gravitational-wave astronomy!

Acknowledgments

- My mentors, Larry Price and Vivien Raymond
- Everyone at LIGO
- California Institute of Technology
- The National Science Foundation

Mass Measurement

- Gravitational waveforms depend explicitly on the mass of the source

$$\tilde{h}(f) = \left(\frac{1\mathrm{Mpc}}{D_{\mathrm{eff}}}\right) \mathcal{A}_{1\mathrm{Mpc}}(M, \mu) f^{-7/6} \mathrm{e}^{-i\Psi(f;M,\mu)}$$

where

$$\mathcal{A}_{1\mathrm{Mpc}} = -\left(\frac{5}{24\pi}\right)^{1/2} \left(\frac{GM_{\odot}/c^2}{1\mathrm{Mpc}}\right) \left(\frac{\pi G M_{\odot}}{c^3}\right)^{-1/6} \left(\frac{\mathcal{M}}{M_{\odot}}\right)^{-5/6}$$

$D_{\mathrm{eff}} =$ distance from detector

$f =$ frequency of gravitational wave

$\Psi =$ polarization

$M =$ total mass

$\mu =$ reduced mass

$\mathcal{M} =$ chirp mass

We measure the chirp mass.