Numerical Simulations of Superkick Binary Black Holes

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Overview

- How are numerical simulations useful to LIGO?
- Background: Parameter space and why Superkicks are a special case
- Simulation process, outcomes, ongoing work
- Eccentricity Reduction: how the existing process works
- Problems with this model, improvements, and results
- Conclusion

Purpose of Simulations for LIGO

- Generate reliable templates for detection and parameter estimation
- Better understand these systems (ie compact binaries)
- Needed accuracy: Less than 3% mismatch between template and measured waveform for detection

Challenge: 7 Parameters

- Mass ratio, 3 components of spin for each black hole
- Interpolation using reduced basis method
- We want to know: when is this effective/reliable?

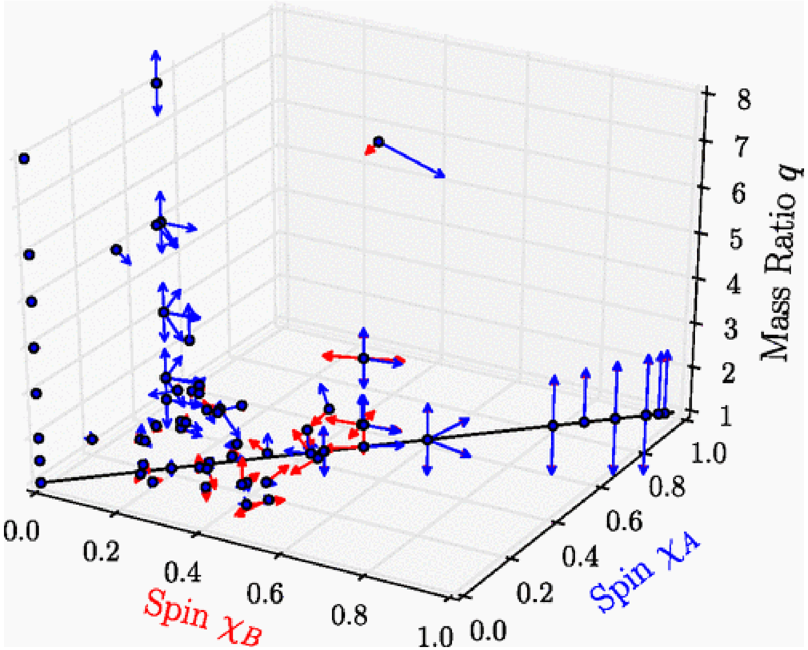


Figure 1 of Phys.Rev.Lett. 111, 241104, 2013

Special Case: Superkicks

- Opposite spin in plane of orbit maximizes radiated linear momentum; final black hole gets a “kick”
- If spins are large enough, the black hole can be “ejected” from its galaxy
- Magnitude of kick depends sinusoidally on initial angle of spins
- Goal: find out how quickly waveforms change when this initial angle is varied

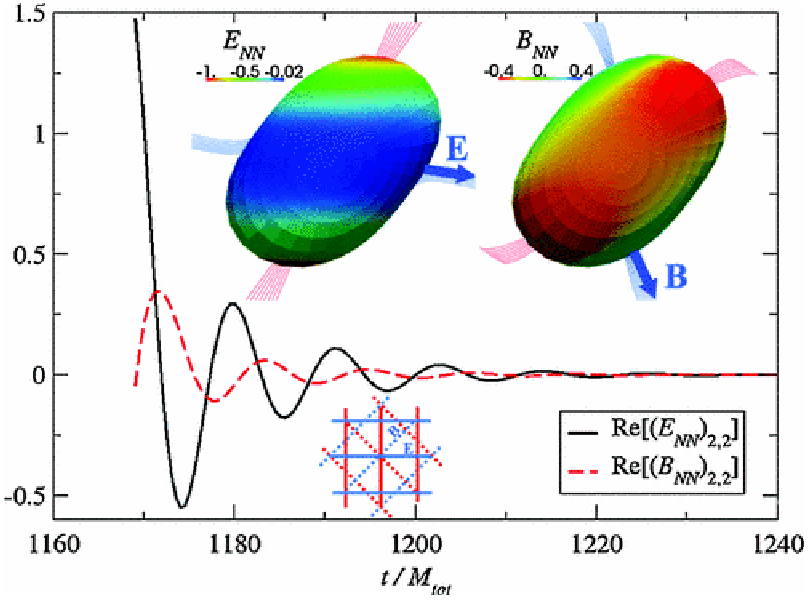


Figure 5 of Phys.Rev.Lett.106:151101,2011

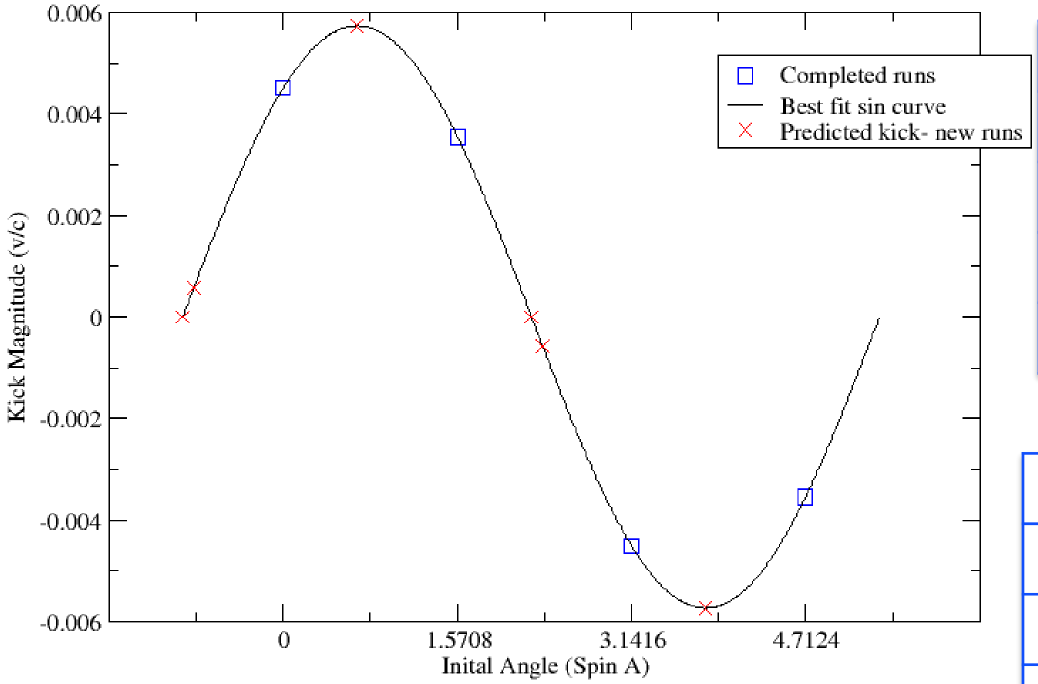
Simulation Steps

- Eccentricity Reduction
- Full Run: inspiral, merger, & ringdown
- Use multiple resolutions to ensure convergence
- Extract and examine data: extrapolation, overlaps, hybridization

Run Parameters

- Completed
  - 4 different initial separations: 14.94-14.97 M
  - 4 initial spin angles: every π/2
  - These are different ways of changing the same thing, since spin angle changes as distance decreases
- In Progress
  - Use results from first set and sinusoidal dependence to choose next round
  - Maximum/minimum kick: compare to get worst-case total overlap
  - Maximum/minimum derivative: run 2 very close together to get worst-case rate of change

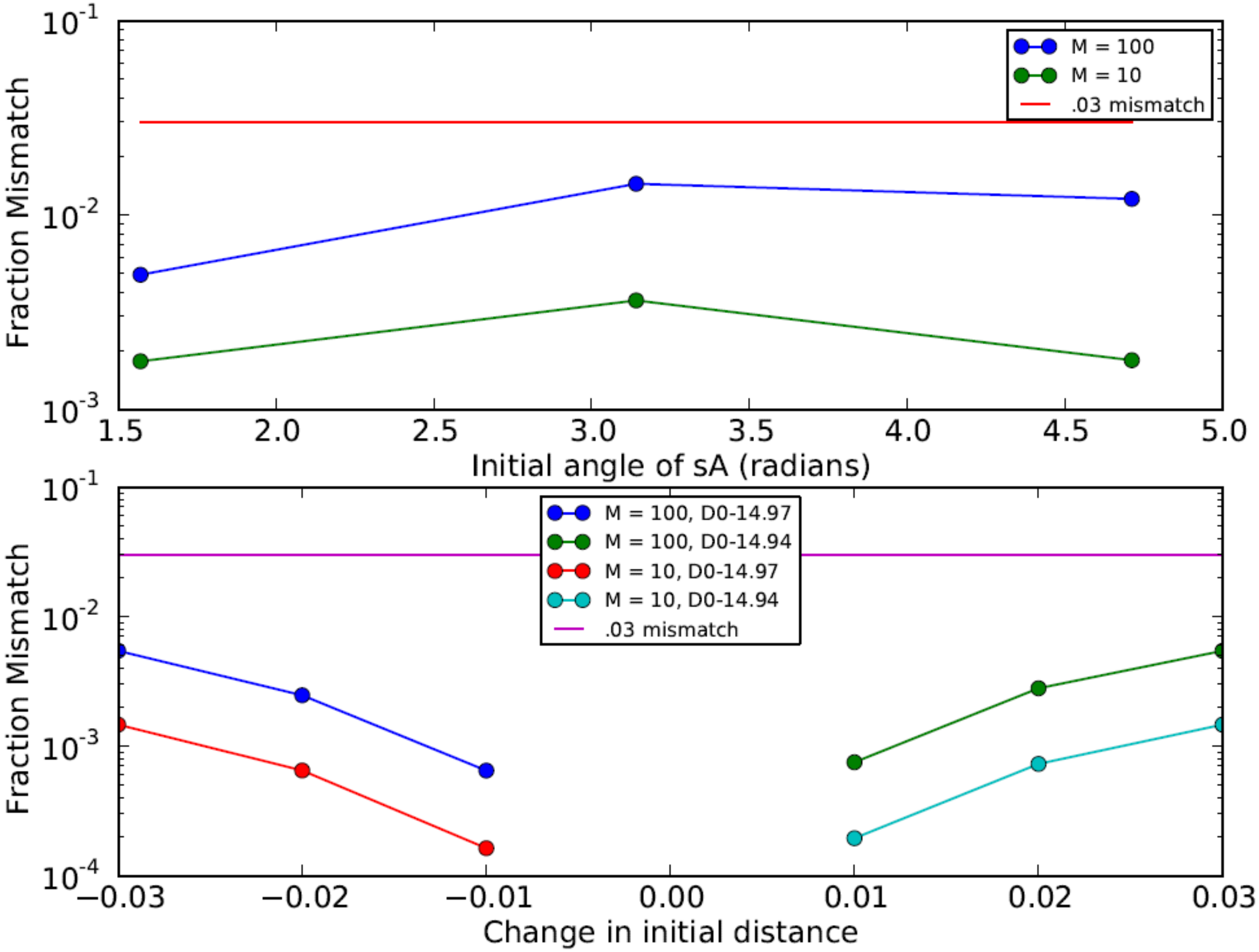
Kick vs. Initial Angle of Spin A



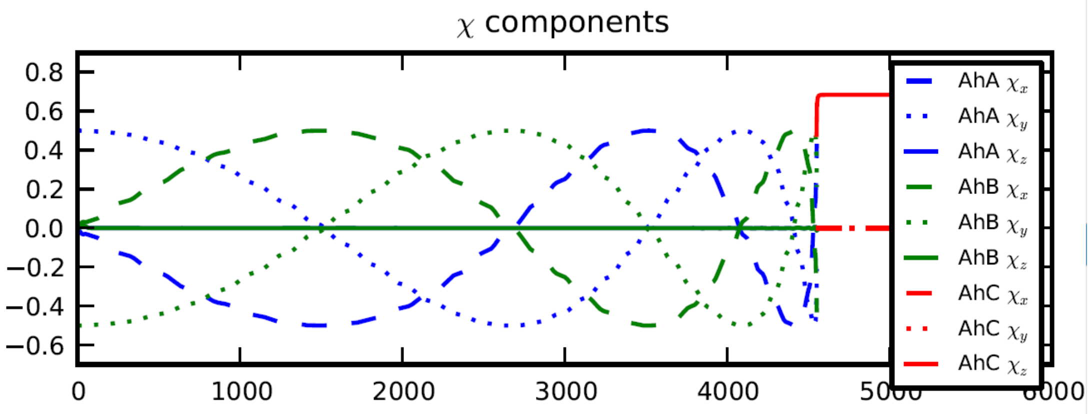
Angle	Kick (v/c)	Kick (km/s)
0	0.004499	1349
π/2	0.003546	1063
π	-0.004504	-1350
3π/2	-0.003541	-1061

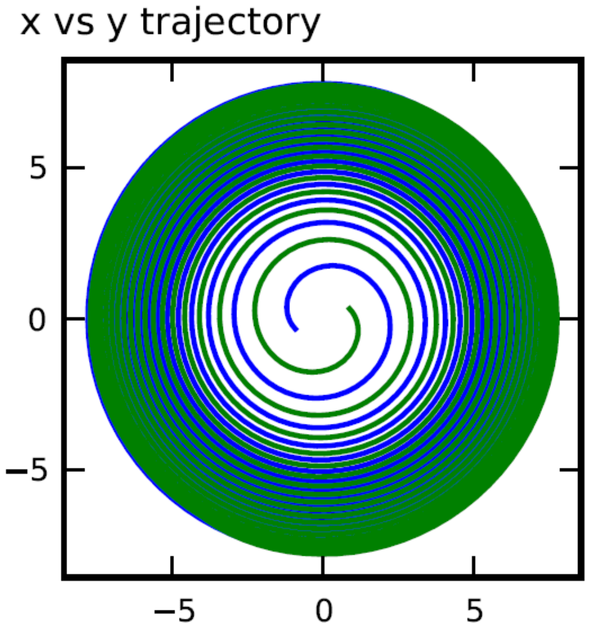
Angle	Kick (v/c)	Kick (km/s)
-0.9039	0	0
-.8039	0.000572	171
0.6668	0.00573	1717
2.2376	0	0
2.3376	-.000572	-171
3.8084	-.00573	-1717

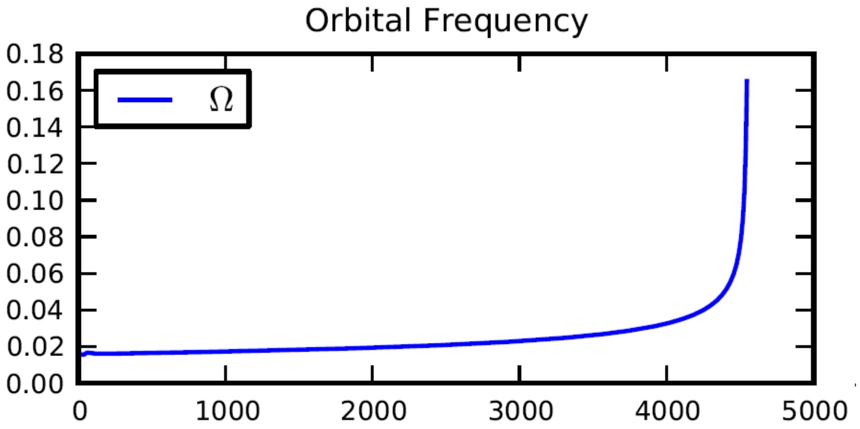
Mismatch vs. Change in Angle; Mismatch vs. Change in Distance



Example Trajectory Results

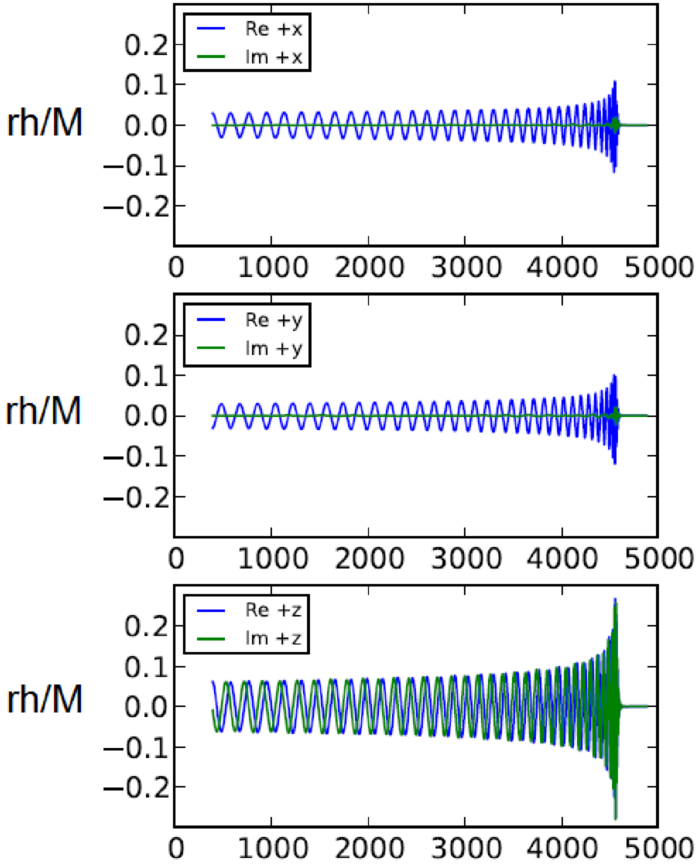




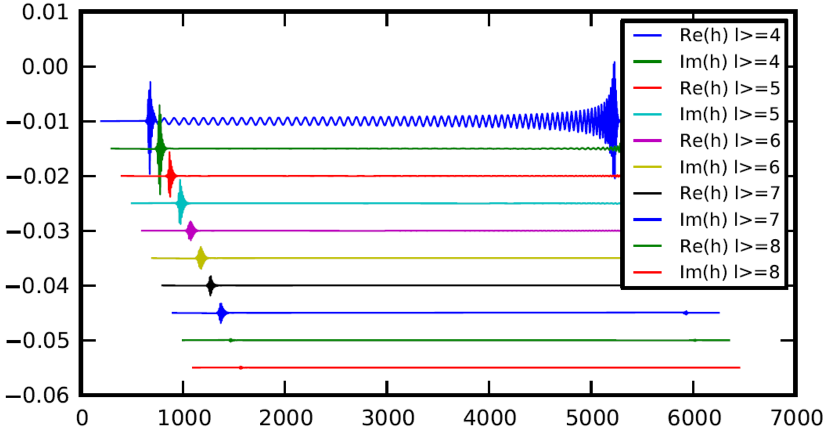


Example Waveforms

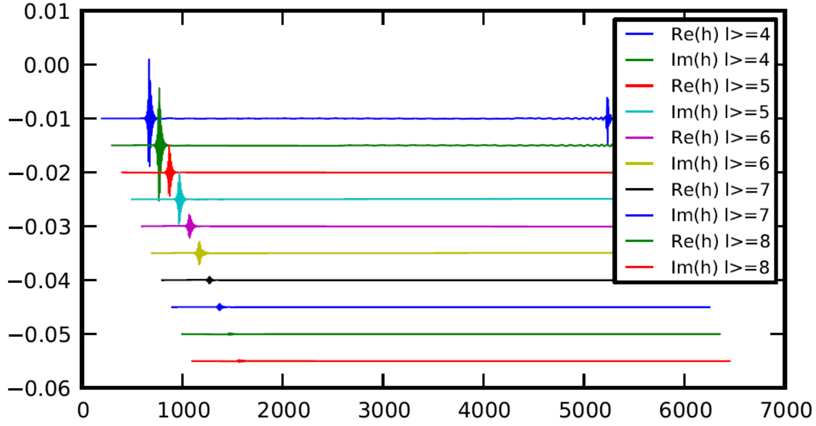
D = 14.97M, initial sA in +y direction



Impact of modes l>=4 (Theta=90, Phi=0)



Impact of modes l>=4 (Theta=0, Phi=0)



Simulation Results Summary

- All mismatches < 0.03: A template from one of these runs would be sufficient to detect any of them
- Trends in degree of mismatch agreed with expectations
- Outcomes of second round will determine worst possible match, most quickly changing waveforms
- Future work: higher spin magnitude (too slow for this project)

Eccentricity Reduction Overview

- Motivation: we expect actual events to have low eccentricity in LIGO band
- Alternate between solving constraint equations, evolving approx. 2 orbits
- Use fits to trajectory info to choose new initial parameters
- Issues: Slow, fitting algorithm sometimes unreliable
- No measurement of eccentricity during a run, so we have to stop everything

Fitting Algorithm

- Fit to oscillations in time derivative of ang. velocity Ω
- Levenberg-Marquadt least squares minimization:
  - Requires reasonably good initial guesses
  - Terms for non-oscillating, oscillating parts
  - Each successive fit uses 1-2 more terms than the previous one

$$\frac{d\Omega}{dt} = P_1(t - P_0)^{-\frac{11}{8}} + P_2(t - P_0)^{-\frac{13}{8}} + P_3\cos(P_4 t + P_5 + P_6 t^2)\ \ldots$$

Green = used for initial data updates   Red = optional, included in later fits

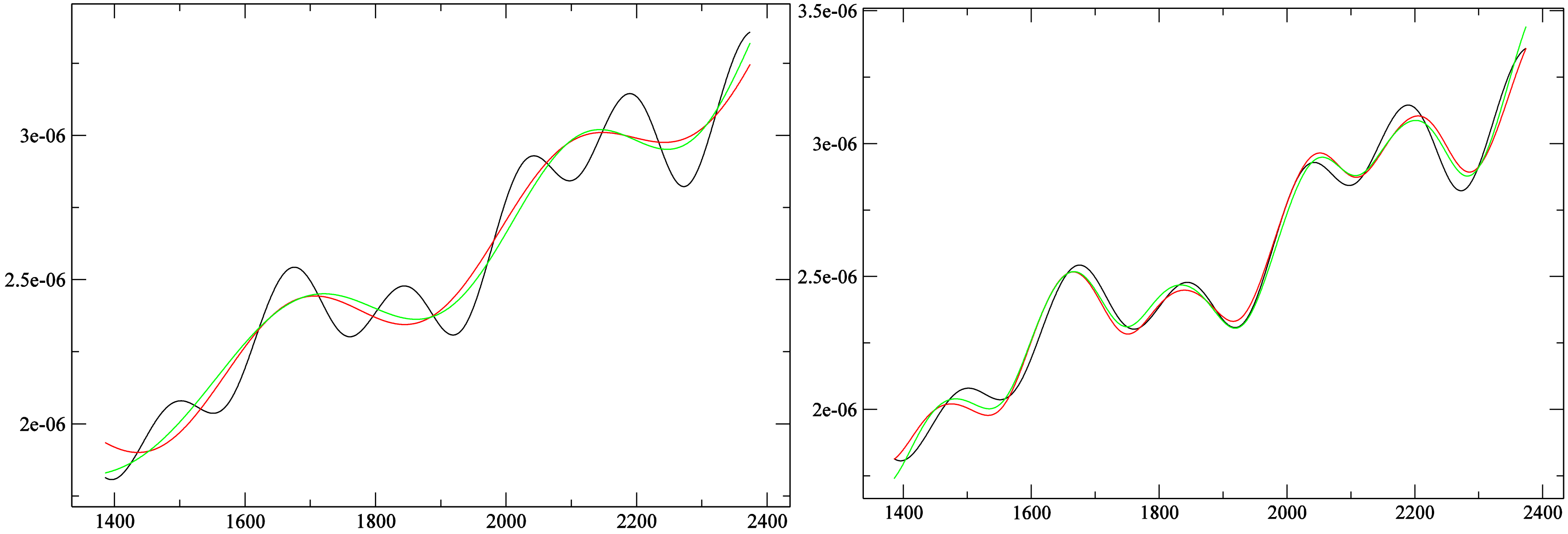
- Post-Newtonian approximations with adjustable parameters used to fit to oscillations from spin-spin terms

$$\ldots - P_7\ \sin(\alpha(t) + \ P_8)$$

Improvement Goals

- Want to improve estimates, get faster reduction
- Measurement during run- don't proceed with evolution any longer than necessary to get good measurement
- Develop C++ code based on existing Python implementation
- Challenges:
  - » Bugs in the initial script (i.e. reading wrong data)
  - » Unnecessary parameters make important ones unreliable
  - » Initial guess for spin-spin term amplitude too small for superkicks

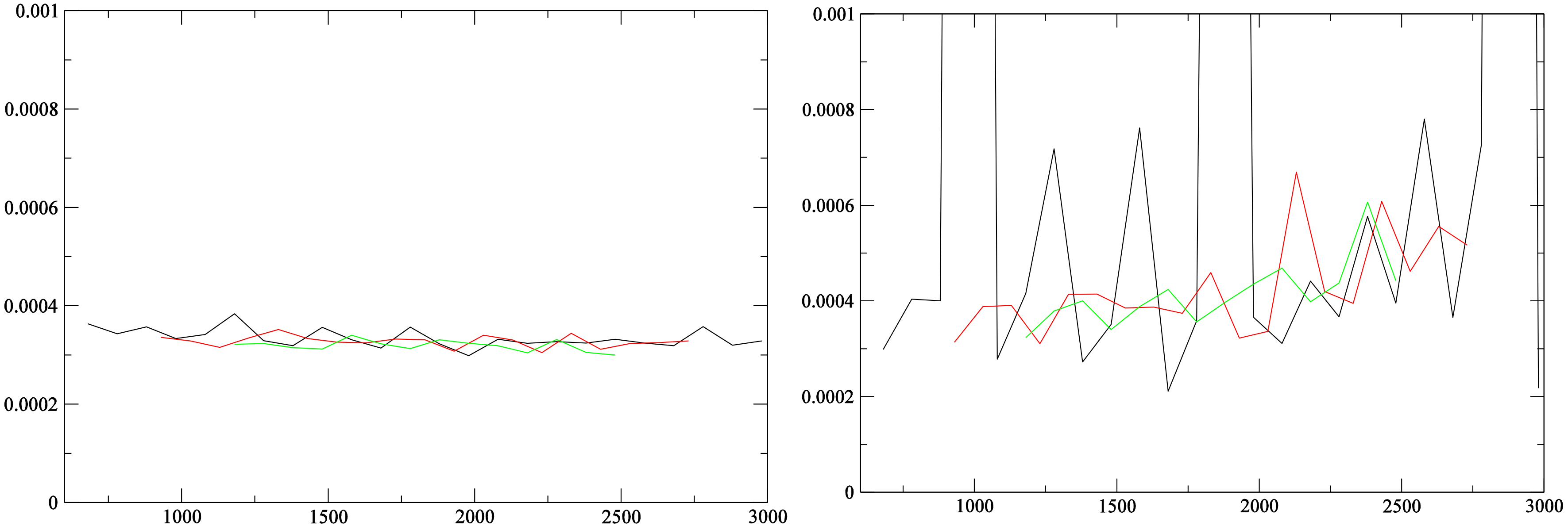
Comparison of Fit Quality



On the right, spin-spin terms are included, while on the left they are not included. The spin-spin terms greatly improve the fit quality, while only a small improvement in the error is made by including the t^2 term (green) instead of omitting it (red)

Eccentricity vs. t:

Comparison of values found with and without t^2 term



$$\frac{d\Omega}{dt} = P_1(t-P_0)^{-\frac{11}{8}} + P_2(t-P_0)^{-\frac{13}{8}} + P_3\cos(P_4 t + P_5 + P_6 t^2) - P_7\ \sin(\alpha(t) + P_8)$$

Inclusion of P_6 in fit makes P_4 value inconsistent and leads to unreliable eccentricity and updates, even though it slightly decreases the rms error.

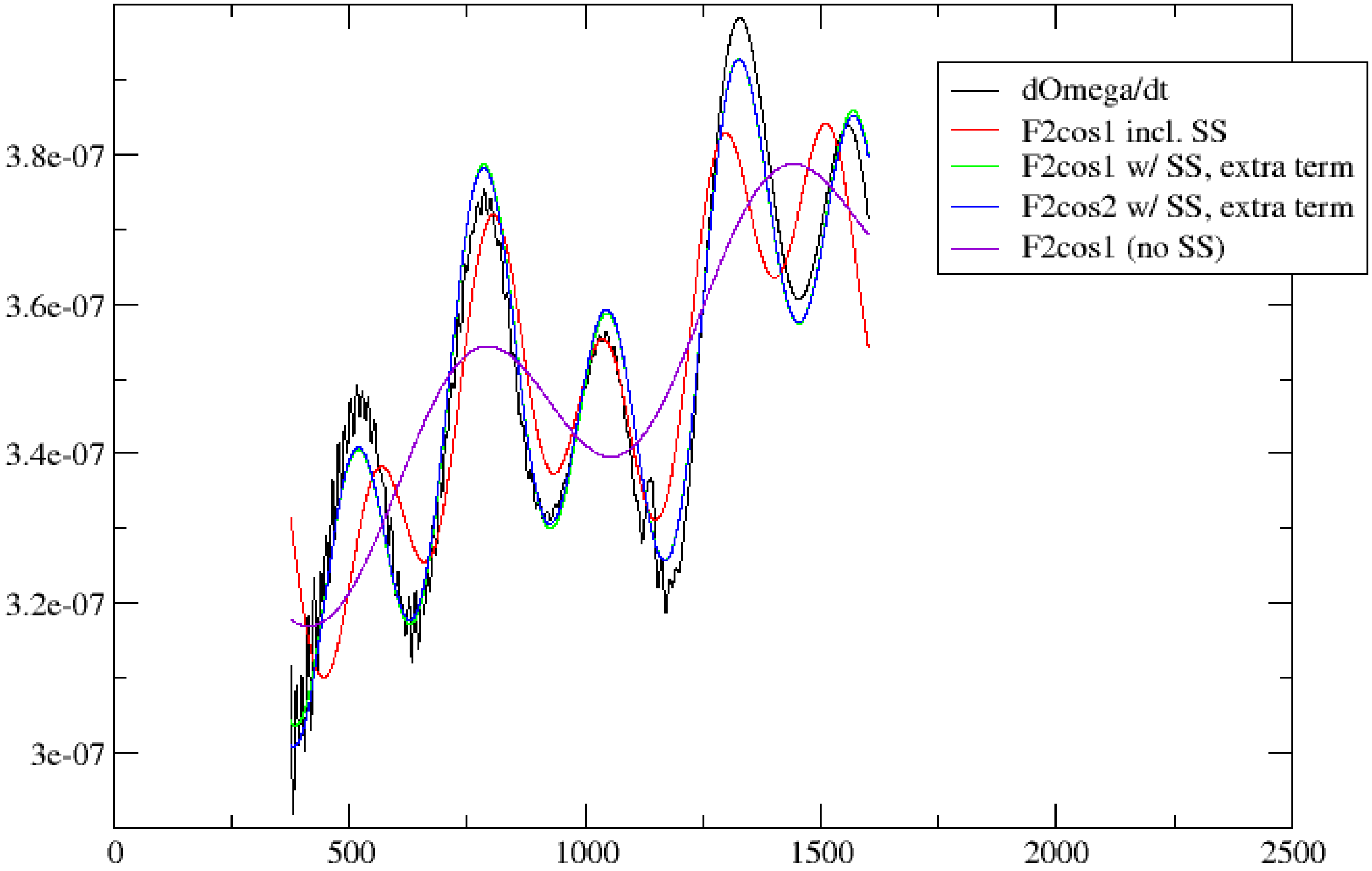
Changes to Fitting Strategy

- An additional parameter:
  - Sometimes, the spin terms don't fit very well
  - The time-dependent parameter α is based only on order 1.5, 2 post-Newtonian analysis

$$\ldots - P_7 \sin(\alpha(t) + P_8) \rightarrow \ldots - P_7 \sin(\alpha(t) + P_8 + P_9 t)$$

- How to know when to include these 'optional' parameters?
  - Additional parameters always reduce the error at least by a small amount (or they would be fit to be 0)
  - However, if they represent effects that are not present or not significant, they can make essential parameter values unreliable
  - Include them in fit used for updates only if they reduce error by more than 10%

Example of Significant Additional Term



Eccentricity Reduction: Results Summary

- Some terms are redundant, should be included only if they significantly improve the fit
- Completed: a new, faster, more reliable program to update initial data
- Ran this on 217 eccentricity reduction runs to compare results of different fits
- Future work: using this more compatible code to measure eccentricity during a run; drafted but not tested

Acknowledgements

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