Extracting Physics from the Stochastic Gravitational Wave Background

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Overview

What is the Stochastic Gravitational Wave Background?

- Composed of many independent and unresolved gravitational wave sources (too weak to be detected on their own)
- Every model can be described by gravitational wave energy density:

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df}$$

- $\frac{d\rho_{GW}}{df}$ = energy density of GWs in f to $f + df$
- ρ_c = critical energy density of the universe

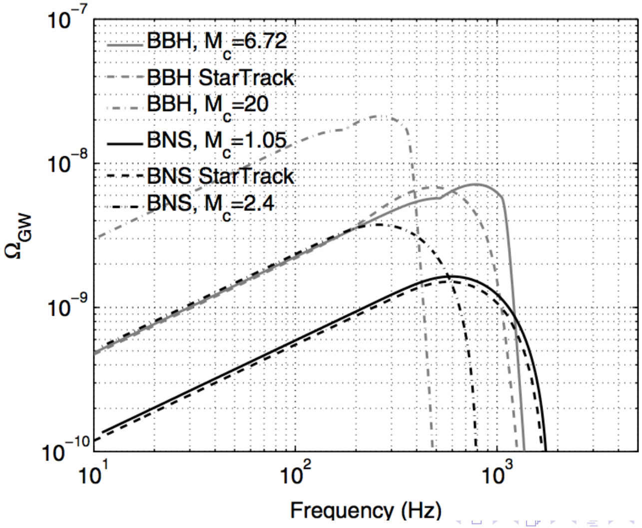
GW Source Model: Coalescing Binary Neutron Stars

We can write $\boldsymbol{\Omega_{GW}}$ as an integral over redshift $\boldsymbol{z}$ and chirp mass, approximating by using only the average chirp mass $\boldsymbol{M_c}$, where

$$\Omega_{GW}(f; M_c, \lambda) \approx \underbrace{\frac{8\lambda(\pi G M_c)^{5/3}}{9 H_0^3 c^2} f^{2/3}}_{\textbf{Signal strength}} \int_0^{Z_{sup}(M_c)} \underbrace{\frac{\overbrace{R_v(z; r_0, W, Q, R)}^{\text{star formation rate density}} \, dz}{(1+z)^{1/3} E(\Omega_M, \Omega_\Lambda, z)}}_{\textbf{Location}}$$

- $r_0 \implies$ local star formation rate in $Mpc^{-3} yr^{-1}$
- $W, Q, R \implies$ phenomenological parameters

Plot of Ω_{GW}



Why Do Parameter Estimation on BNS Sources?

Using data from gravitational-wave detectors, we can estimate the parameters r_0, W, R, Q to infer star formation rate density.

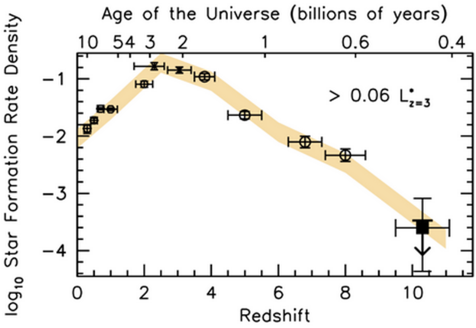


Figure 2: The data points show different observational measurements.

Parameter Estimation Using Bayesian Inference

Bayes' formula:

$$\overbrace{\Pr(\hat{\theta}|D)}^{\text{Posterior}} = \frac{\overbrace{\Pr(D|\hat{\theta})}^{\text{Likelihood}}\overbrace{\Pr(\hat{\theta})}^{\text{Prior}}}{\underbrace{\Pr(D)}_{\text{Evidence}}}$$

- $\hat{\theta} \Longrightarrow$ Parameter(s) to be estimated
- $D \Longrightarrow$ Observational data

Inference Example for Star Formation Rate Parameter

Estimate r_0 for a compact binary coalescence stochastic background model:

$$\overbrace{\Pr(r_0|\hat{\Omega})}^{\text{Posterior}} = \frac{\overbrace{\Pr(\hat{\Omega}|r_0)}^{\text{Likelihood}}\ \overbrace{\Pr(r_0)}^{\text{Prior}}}{\underbrace{\Pr(\hat{\Omega})}_{\text{Evidence}}}$$

- r_0 : local star formation rate to be estimated $(Mpc^{-3}yr^{-1})$
- $\hat{\Omega}$: cross correlation estimator: correlated output of two gravitational wave detectors
  - Simulated $\hat{\Omega}$ with "true" values of parameters taken from a research source (Coward, 2012): $r_0 = 5 \times 10^{-12}, W = 45, Q = 3.4, R = 3.8$
- Take prior to be flat, evidence is a normalization constant.

Likelihood Function

$$\overbrace{\Pr(\hat{\Omega}_i, \sigma_i|\vec{\theta})}^{\text{Likelihood}} \propto \exp\left[-\frac{1}{2}\sum_i \frac{[\hat{\Omega}_i - \Omega_{GW}(f_i;\vec{\theta})]^2}{\sigma_i^2}\right]$$

- $\vec{\theta}$: The parameter(s) that we wish to estimate
- $\hat{\Omega}$: Simulated cross correlated output of two gravitational wave detectors (LLO,LHO)
- σ_i : varience of $\hat{\Omega}$
- Summed over all frequency bins f_i.

Sampling the Likelihood Distribution

Problem: *Sampling the likelihood is difficult.*

- Parameter estimation requires us to sample the values of $\vec{\theta}$ with the highest likelihoods.
- Evaluating the likelihood everywhere is costly, especially in high dimensional spaces.
- To save time, we need a way to sample the likelihood without iterating over most or all of the possible states of $\vec{\theta}$.

Sampling the Likelihood Distribution

Solution: Generate samples using a Markov Chain Monte Carlo Method

- Markov chain: sequence of random variables that randomly moves from state to state over discrete units of time, t.
- Monte Carlo: refers to a computer-driven algorithm that generates the Markov chain.
- **Key idea:** construct a Markov Chain that converges to the desired distribution (the likelihood distribution in this case).

MCMC Parameter Estimation Result for r_0 Alone (a.LIGO)

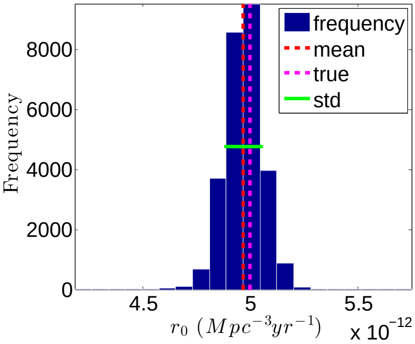
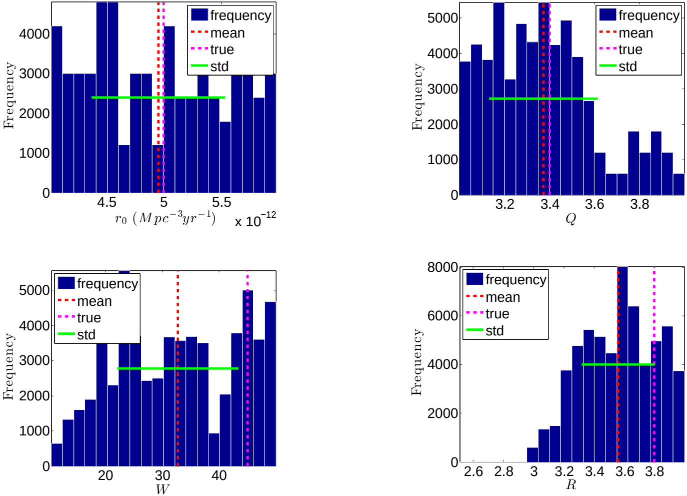


Figure 3: True value of $r_0 = 5 \times 10^{-12}$

Results for r_0, Q, W, R (A.Ligo)



Star Formation Density Comparison

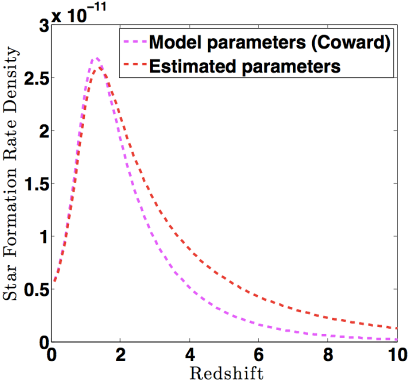


Figure 4: Plots of star formation rate density for true parameter values vs. MCMC estimated parameter values.

Conclusion and Future Plans

- Tune Metropolis algorithm to be more sensitive - current version only produced good results when noise was taken out of the signal.
- Test updated algorithm on real data collected from the detectors rather than simulated data, to see real parameter estimates.

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Simulating Detector Data $\hat{\Omega}$

1. Calculate a "true" gravitational wave signal $\Omega_{GW_{true}}$ using expected values for the parameters $\vec{\theta}$:
   - $r_0 = 5 \times 10^{-12}, W = 45, Q = 3.4, R = 3.8$
2. Calculate a varience σ (noise):

$$\sigma_\Omega(f) = \frac{P(f)}{\frac{1}{5}\gamma(f)} f^3 \frac{2\pi^2}{3H_0^2} \sqrt{\frac{1}{T\delta f}}$$

   - $P(f) =$ cross correlated power spectral density
   - $\gamma(f) =$ overlap reduction function: from the overlap of antenna patterns of GW detectors at different locations and with different orientations
3. $\hat{\Omega} =$ array of random numbers with mean $\Omega_{GW_{true}}$, and varience σ_Ω.

Metropolis Algorithm

Metropolis Algorithm

- Generates a sequence of random $\vec{\theta}$ samples from a probability distribution for which direct sampling is too difficult.

Pick a symmetric proposal density Q which depends on the current state $\vec{\theta}(t)$. Given previous state θ_0, proposal density Q, and target density P, iterate over the following:

1. Generate a new state $\theta^{'}$ with probability density $Q(\theta^{'}|\theta_0)$
2. Compute the quantity $a = \frac{P(\theta^{'})}{P(\theta_0)}$
3. If $a \geq 1$ then the new state $\theta^{'}$ is accepted. Otherwise, the new state is accepted with probability a.
4. If $\theta^{'}$ was accepted, then add it to the chain. Otherwise, add θ_0 to the chain again.

Star Formation Rate Density Function

$$\text{SFR Density} \propto \frac{r_0(1+W)e^{Qz}}{e^{Rz}+W}$$