Estimating a likelihood-ratio Threshold for Renormalizing the Noise Likelihood Ratio Background Distribution

Cody Messick

Pennsylvania State University
University Park, PA

Current Process

- Currently behavior of noise likeilihood ratio background distribution calculated by Monte Carlo
- Though background behavior is believed to be modeled correctly, the normalization constant is unknown
- Background distribution normalized by stitching to observed distribution at $\ln \Lambda = 2$

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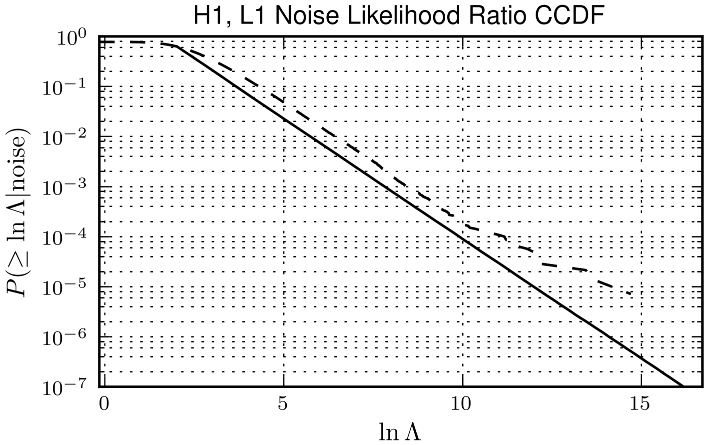


- Curvature of dashed line caused by low Λ clustering extinction

Hypothesis

- There is a threshold value of Λ, Λ^*, over which the probability of clustered events is small enough that the clustering process does not distort the distribution
- To test this, wrote a simulation where likelihood threshold values were pulled from a power law distribution and timings were pulled from a poisson distribution

Derivation

- Assumed probability of clustering follows Poisson stats, so probability of two events in a time interval T occurring within a clustering window ω_c

$$P\left(k \geq 2 \mid \lambda_c\right) = \sum_{i=2}^{\infty} \frac{\lambda_c^i \exp\left(-\lambda_c\right)}{i!} = 1 - \exp\left(-\lambda_c\right)\left(1 + \lambda_c\right)$$

- Definitions
  - $\lambda_c \equiv \frac{n(\geq \Lambda)}{T}\omega_c$
  - $n\left(\geq \Lambda\right)$: Unnormalized observed distribution ccdf
- Assuming small probability of clustering $\Rightarrow$ small λ_c

$$P\left(k \geq 2 \mid \lambda_c\right) \approx \frac{1}{2}\lambda_c^2$$

Renormalizing the Background

$$P_N\left(\geq \ln \Lambda\right) = \frac{P_O\left(\geq \ln \Lambda^*\right)}{P_B\left(\geq \ln \Lambda^*\right)} P_B\left(\geq \ln \Lambda\right)$$

where

- $P_B\left(\geq \ln \Lambda\right) \equiv$ Background ccdf (Monte Carlo generated)
- $P_O\left(\geq \ln \Lambda\right) \equiv$ Observed ccdf
- $P_R\left(\geq \ln \Lambda\right) \equiv$ Renormalized background ccdf

Single Distribution Simulation

Summary of single distribution process

- Draw n events from likelihood ratio distribution

$$P_B(\Lambda) = \frac{2}{25}\Lambda^{-3}$$

- Draw time intervals between events over a total time of T seconds from poisson distribution

$$P(t) = \frac{n}{T}\exp\left(-\frac{nt}{T}\right)$$

- Cluster data with clustering window ω_c
- Generate ccdf from clustered data, $P_O(\geq \Lambda)$
- Calculate Λ^*
- Renormalize $P_B(\geq \Lambda)$ via

$$P_R(\geq \Lambda) = \frac{P_O(\geq \Lambda^*)}{P_B(\geq \Lambda^*)} P_B(\geq \Lambda)$$

Multiple Distributions Simulation

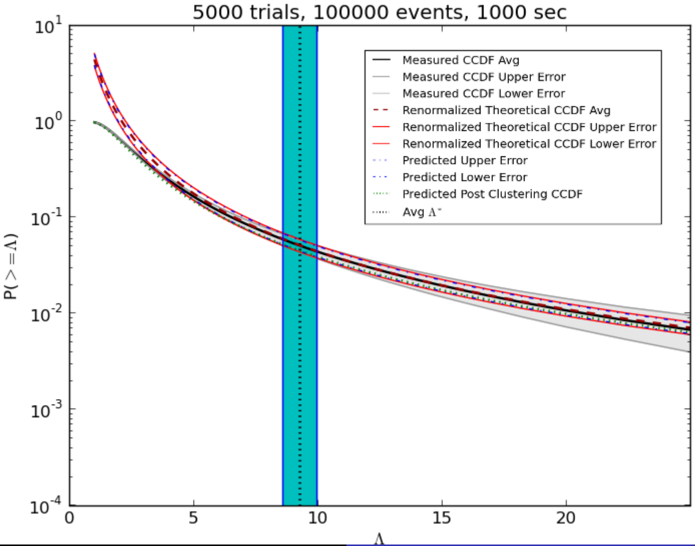
Summary of multiple distribution process

- After single distribution process, save observed ccdf, renormalized background ccdf, and likelihood threshold
- Run single distribution process several times ($\sim 10^3 - 10^4$)
- Plot ensemble averages of distributions and threshold, as well as standard deviation
- Calculate predicted errors and add to plot (more info on next slide)

Error Estimation

- We expect the error in this process to be dominated by the counting error in the observed ccdf, thus we assume we can ignore the errors introduced bythe background distribution and the threshold value
- $\sigma_{P_R} = \frac{P_B(\geq \ln \Lambda)}{P_B(\geq \ln \Lambda^*)} \sigma_{P_o(\geq \Lambda^*)}$
- To test this, analytically calculated the threshold value and the distribution after clustering in simulation and plotted against ensemble standard deviation in renormalized background distribution

Simulation Plot Result



Topics for Further Investigation

- Calculating uncertainty in $P_B(\geq \Lambda^*)$ and Λ^* to model higher order error in the renormalization process
- It may be possible to predict the behavior the of the monte carlo generated background after clustering, which would remove the need to renormalize the distribution as a specific point
- Simulation showed that clustering process lowered calculated value of Λ^*, but we ignored it because the effect was minimal. Accounting for this slight change in the calculated value could improve the threshold estimate (but may not be worth it)