Gravitational Wave Polarization and the Antenna Pattern

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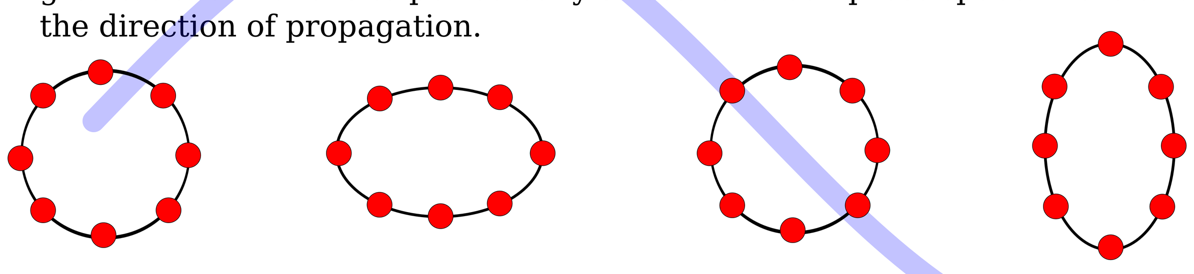
Gravitational waves

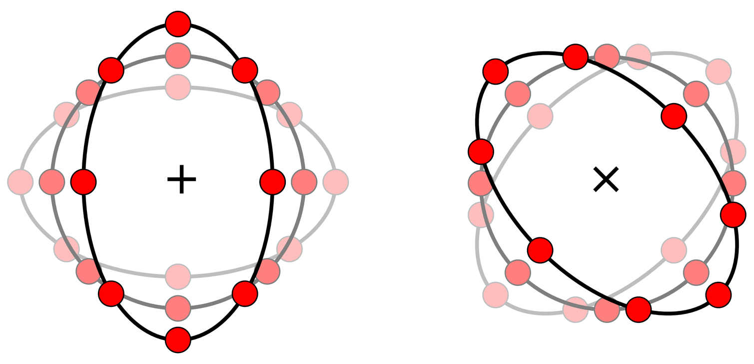
Gravitational waves are a propagating disturbance in the metric tensor. The effect of a passing gravitational wave is to periodically stretch and compress space in the two directions orthogonal to the direction of propagation.



Red dots indicate configurations of non-interacting inertial test masses.

This disturbance can be quantified as a strain of space. The expected strain at Earth due to astrophysical events is extremely small, making detection very challenging.

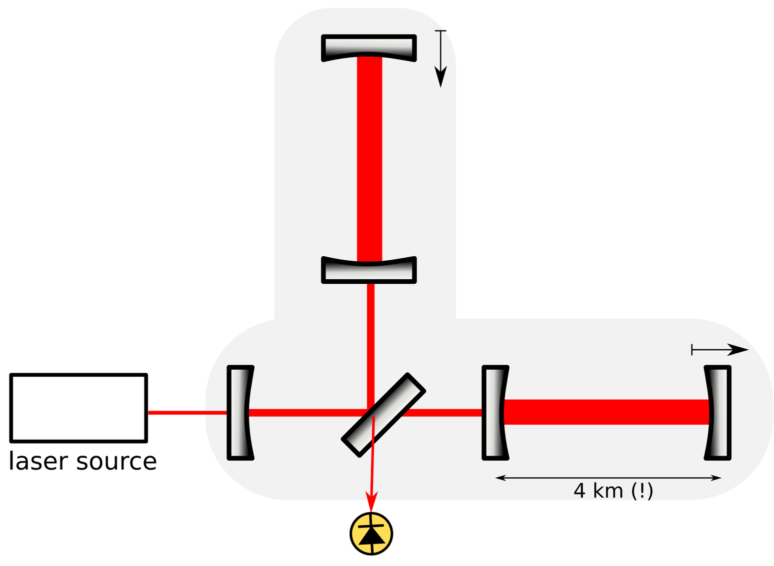
Polarization and the antenna pattern



Laser interferometric gravitational wave detectors use suspended mirrors as test-masses to detect the stretching and compressing of space due to a passing gravitational wave. The Fabry-Perot Michelson configuration is sensitive to gravitational waves that modulate the arm lengths *differentially*.

Like electromagnetic waves, gravitational waves have two polarizations. The two linear polarizations of gravitational waves are usually called plus (+) and cross (×), with reference to the pattern of stretching and compression caused by the wave.

These detectors are not omnidirectional, but exhibit an "**antenna pattern**" showing directional variations in sensitivity. This arises from the projection of the g.w. strain axes onto the interferometer arm axes.



Main points of this poster

It is tempting (and sometimes useful) to define "the" plus and cross polarizations over the entire sky, and to separate the antenna pattern into plus and cross components. However, there are both technical and pedagogical problems with this approach:

(1) it is **not possible** to define plus and cross polarizations in a consistent way over the entire sky. Ultimately this results from the "hairy ball theorem" which says that there is no smooth non-zero vector field over the sphere.

(2a) The most commonly depicted decomposition (using the "wave frame") has its coordinate singularities at the poles. The resulting antenna pattern is invalid for waves coming from directly above or below the detector. These directions should be the most intuitive.

(2b) In particular, this choice of polarization basis does not agree with the experimentalist's view that the "plus" polarization is the one that optimally excites the differential arm motion of the Michelson interferometer.

World-wide network of gravitational wave detectors

Several g.w. detectors are currently in operation and/or under construction around the world. Multiple detectors are operated together as a phased array in order to recover directional information from incoming gravitational waves. The Laser Interferometer Gravitational-wave Observatory (LIGO)'s installation in Livingston, Louisiana is pictured below. (Aerial photograph by Stefan Ballmer, 2008.) It is hoped that the next generation of g.w. detectors, currently undergoing initial commissioning, will make the first detection of a gravitational wave.

The "wave frame" coordinate system for polarization

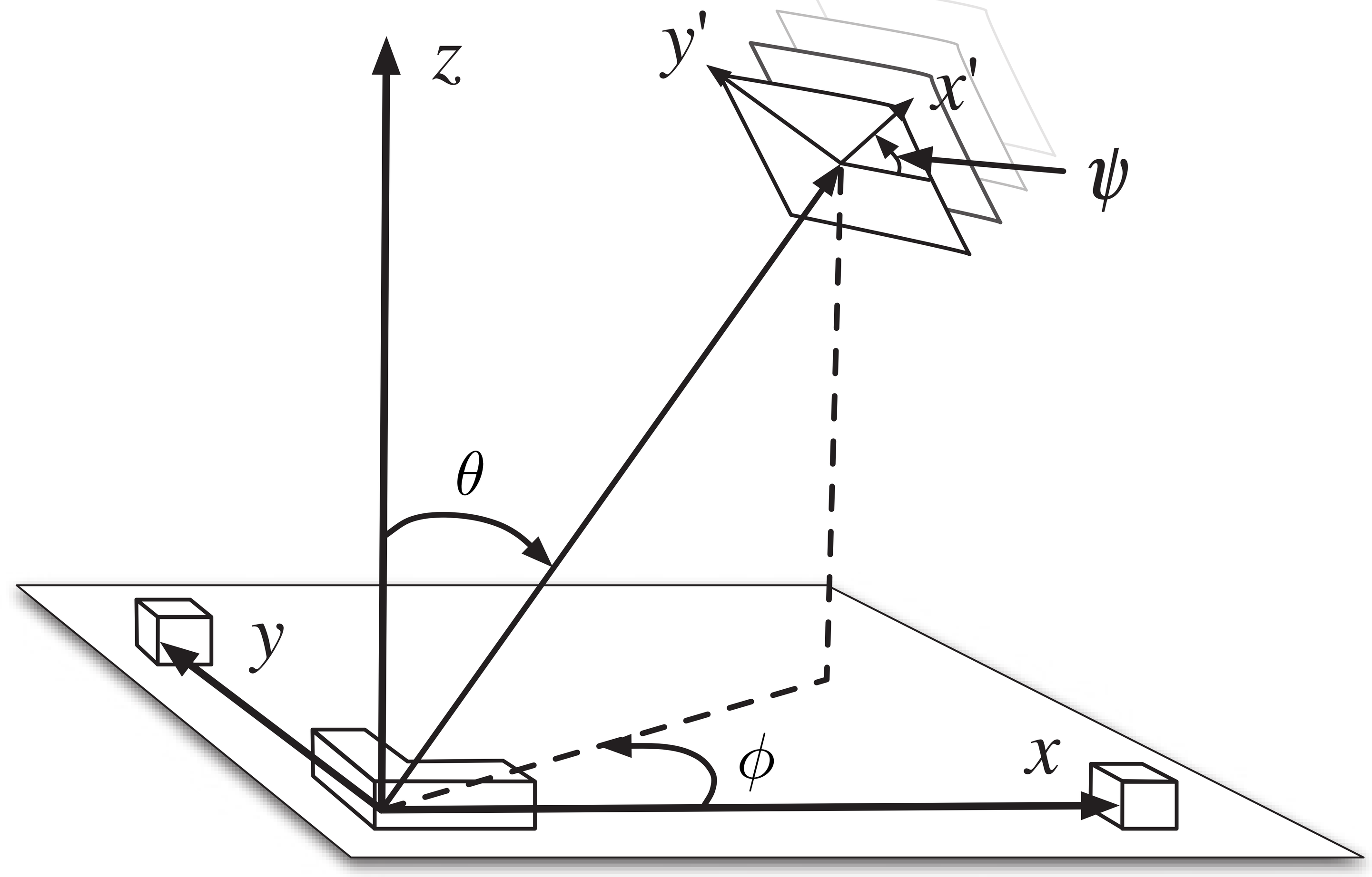


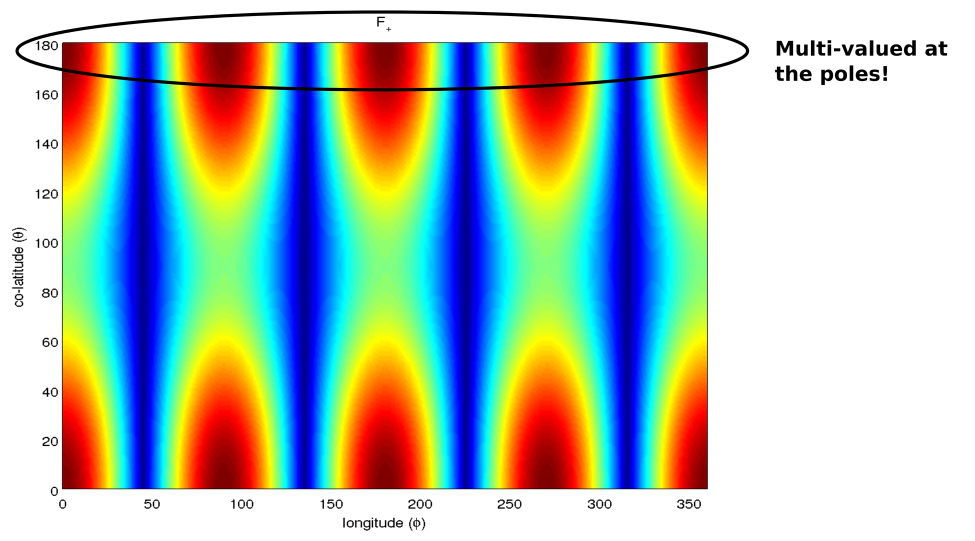
Figure by Jeff Kissel; appears in Abadie et al 2010, "Calibration of the LIGO gravitational wave detectors in the fifth science run," http://dx.doi.org/10.1016/j.nima.2010.07.089 .

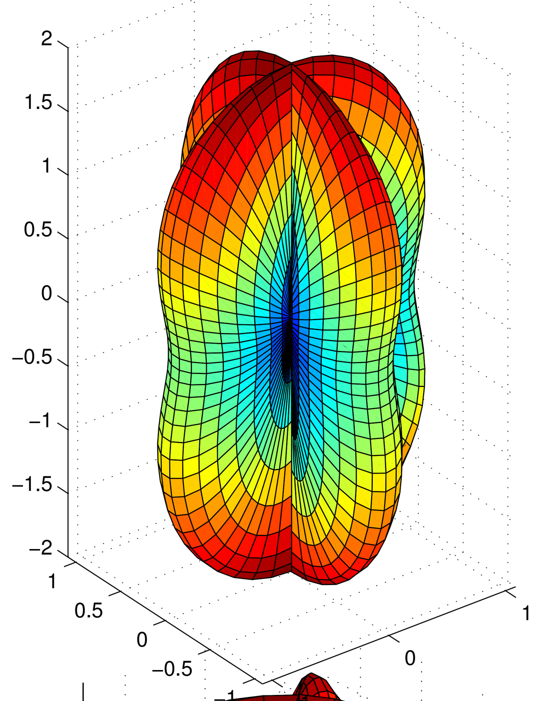
This coordinate system results in the following antenna patterns for the resulting plus and cross polarizations:

$$F_+(\theta,\phi) = -\cos^2\theta + 2\cos^2\phi - 1$$
$$F_\times(\theta,\phi) = 2\cos\theta\sin 2\phi$$
$$F_{\rm rms}(\theta,\phi) = \sqrt{{F_+}^2 + {F_\times}^2}$$

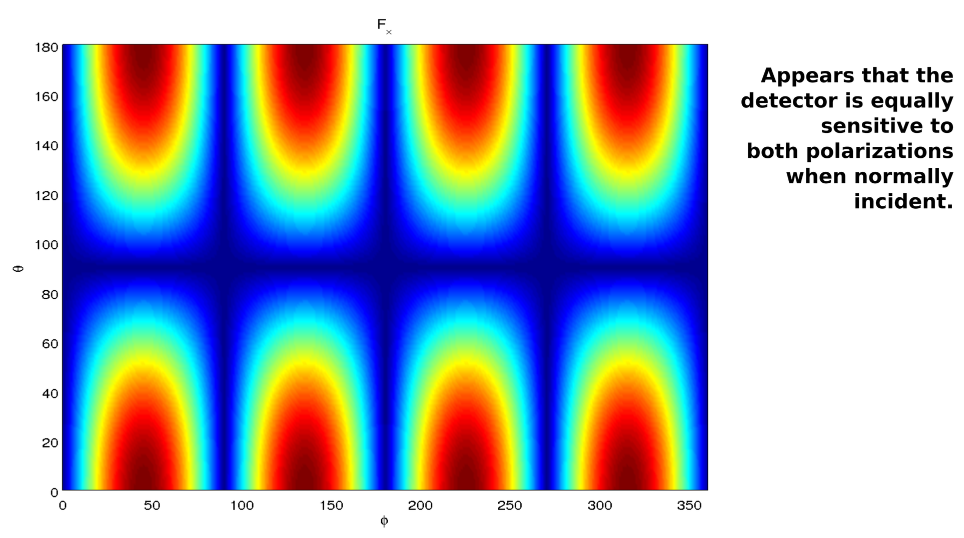
Unfortunately, these antenna patterns have some undesirable properties, especially in a pedagogical context.

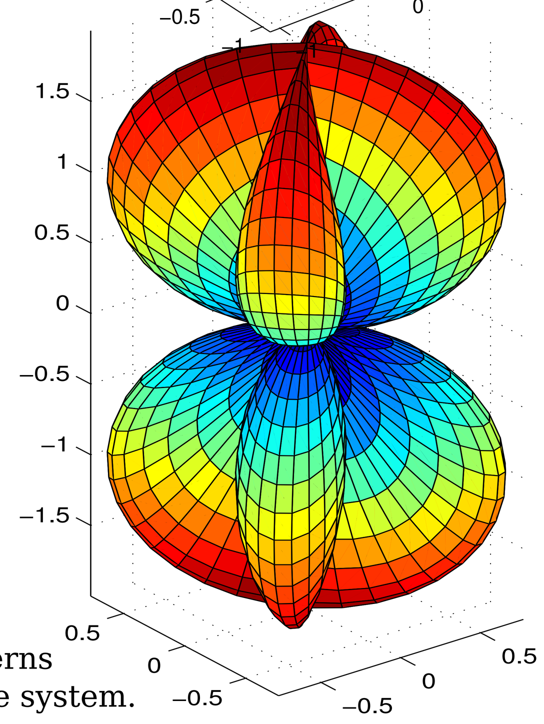
F_+





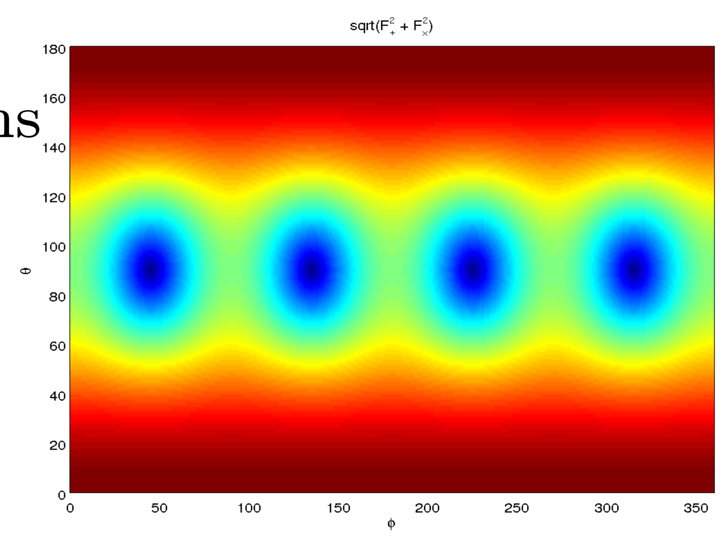
$F_\times$

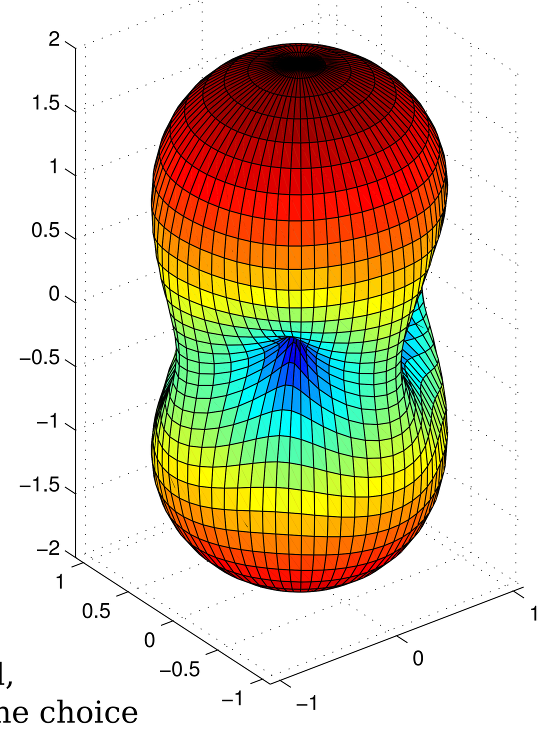




The resulting plus and cross antenna patterns are multi-valued at the poles (i.e. for waves coming from directly above or below the detector). Most of the complexity of these decomposed antenna patterns comes not from the detector physics but from the choice of coordinate system. **These figures look cool, but they are misleading.**

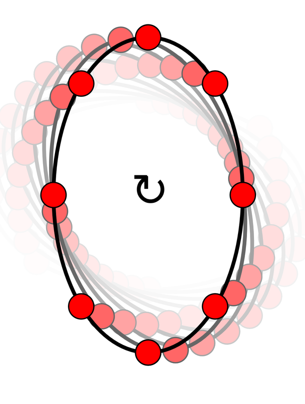
$F_{\rm rms}$





By contrast, the antenna pattern for "unpolarized" (or circularly polarized, or optimally-oriented linearly polarized) waves is completely physical. The choice of polarization basis cancels out. The unpolarized antenna pattern has pedagogical value. For instance, it shows:

- Optimally polarized waves from directly above (or below) produce twice as much signal as waves propagating along the arms, since these waves modulate the lengths of both arms, not just one.
- Waves arriving along the bisector between the arms produce no signal, since they can only produce common-mode modulation (both arms equally, not differentially).



Effect of **circularly polarized** gravitational wave on a ring of non-interacting test particles.

References

- Abadie *et al* 2010, "Calibration of the LIGO gravitational wave detectors in the fifth science run," `http://dx.doi.org/10.1016/j.nima.2010.07.089`
- Daniel Sigg 1997, "Strain Calibration in LIGO." LIGO Technical Document T970101-B, `https://dcc.ligo.org/public/0028/T970101/000/T970101-B.pdf`