A Proposal for Characterizing the Eigenspectra of the Power Recycling Cavity and Main Interferometer

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1 Experimental Proposal

This experiment would allow for characterization of the eigenspectrum of the power recycling cavity and the full interferometer. It is set up in a similar way as the input mode cleaner eigenspectrum measurements performed over the last year at LLO (see e.g. aLogs 9599, 9422, and 9095). It differs in that it uses a separate probe beam, phase locked with a tunable offset to the main beam, to probe the eigenspectrum of the PRC.

In order to make the measurements we need to be able to inject an axillary beam with a defined frequency relationship to the main beam. In the input mode cleaner this was done by adding an FM sideband on the main beam. These FM sidebands are unfortunately stripped off of the beam at the IMC for most frequencies, so this method will not work for the rest of the interferometer. Instead we propose to use inject an auxiliary beam, phase locked with a frequency offset to the main beam, into the PRC at PRM.

The IOT2R table has picked-off samples of the main beam with will be used for the phase locking. In addition, one of these ports allows for injection onto the interferometer by transmitting through IM4 which has a transmissivity of 2400 ppm. This allows us to deliver ~ 500 μW to the PRC which will be shown below to be enough for characterization of the HOM spectrum. The signal will be picked up on the BBPD on ISCT1 and carried over with a long BNC cable. This beam is attenuated before arriving at the PD by 5% and 50% beamsplitters on HAM 1 inside the vacuum envelope. On ISCT1 it is attenuated by two more 50% beamsplitters leading to a total transmissivity to the BBPD of $0.05 * 0.5 * 0.5 * 0.5 = 6300$ ppm of whatever power is reflected off of the interferometer.

1.1 Power Limitations

Most of the beam being injected into the backward going port will be reflected at IM4, which has a transmissivity of 2400 ppm. This power will mostly end up at the ISS photodiode array, so it is

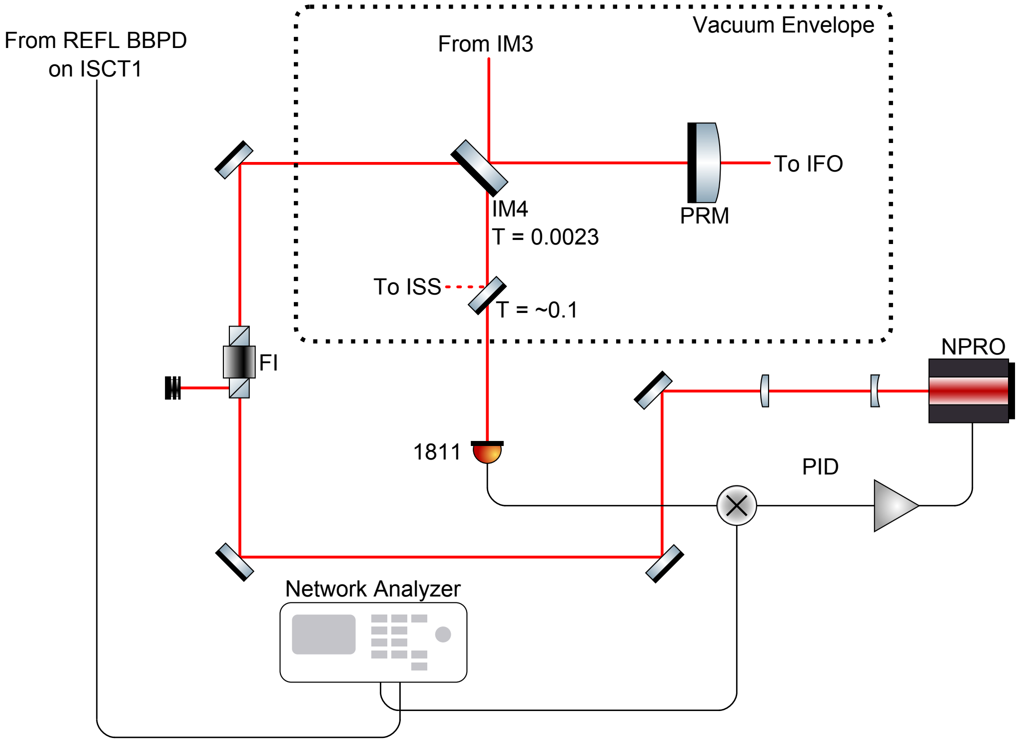

Figure 1: A rough optical layout of the proposed experiment. The experiment will use the forward going pickoff beam as a frequency reference for the PLL. The phase locked beam will be injected through the PRM REFL port where most of the power will reflect off The experiment will steal the ALS pickoff beam coming from the PSL (via HAM 1) with a flipper mirror. After passing it through an amplitude modulator and a Faraday isolator the beam will be mode matched and injected into the POP port of the interferometer. The transmitted probe light and the leakage from the interferometer will be picked up at the reflection port of the FI for analysis.

important to inject less power than this array can handle.

The interferometer was designed to be operated at 165 W of power injected into PRM. Using the transmissivity of IM4, this leads to a maximum power in this path of 400 mW. The maximum power output of the proposed laser is 210 mW, so staying below the safe operations level will not be a problem.

1.2 A Calculation of the Signal

Defining E_l as the field strength of the main beam at the input to the PRC, E_p as the power of the probe beam in the particular HOM of interest, $r(\omega)$ as the reflectivity of the PRC cavity at frequency ω, Ω as the offset frequency of the probe beam and the main beam, and ϕ as the phase difference between the two beams when $\Omega = 0$, the field in reflection of the interferometer is given by

$$E_r = r(\omega)E_l e^{-i\omega t} + r(\omega+\Omega)E_p e^{-i(\omega+\Omega)t-i\phi}. \tag{1}$$

Calculating the power in the usual way, $\langle P_r\rangle = E_r^*\, E_r$, gives

$$\begin{aligned}\langle P_r\rangle =& |r(\omega)|^2 E_l^2 + |r(\omega+\Omega)|^2 E_p^2 \\ &+ 2E_l E_p \left\{\Re[r^*(\omega)r(\omega+\Omega)]\cos(\Omega t+\phi) + \Im[r^*(\omega)r(\omega+\Omega)]\sin(\Omega t+\phi)\right\}.\end{aligned} \tag{2}$$

This expression contains the DC terms for the two beams involved as well as the beat note between the two which caries the signal of interest. Notice that noise in the path of the injection path of the probe beam, which shows up as noise in ϕ, will show up as noise in the readout. This should be mitigated by the fact that the two lasers are phase locked after the probe beam has traveled through its injection path. In the next section we will look at the strength of this signal compared to the shot noise at the PD and the readout noise of the spectrum analyzer.

1.3 Sensitivity

The expected RF power levels from the final term in (2) can be estimated with a few assumptions. If the main beam is on resonance, the reflectivity should drop to about 5%, $|r(\omega)|^2 \simeq 0.05$. If the probe beam is off resonance, then we may take the reflectivity to be 100%, $|r(\omega+\Omega)|^2 \simeq 1$. We may safely assume a higher order mode coupling coefficient or 1%, $C_{HOM} = 0.01$, a fact which is discussed in section 1.4 below. Finally, we will assume that the power of the two input beams is $E_l^2 = 10$ W and $E_p^2 = 500$ μW. Also, we will have to include separately the attenuation of the beam between reflection off of the interferometer and the photodiode, which was desciribed in section 1 to be $A = 6300$ ppm.

Putting all of this together, the RF power at the photodiode will be given by

$$\begin{aligned} P_{RF} &= 2AE_lE_pC_{HOM}|r(\omega)||r(\omega+\Omega)| && (3)\\ &= 2*6300\cdot 10^{-6}*\sqrt{10\text{ W}}\sqrt{500\ \mu\text{W}}0.01*\sqrt{0.05}*\sqrt{1.} = \boxed{2\ \mu\text{W}} && (4)\end{aligned}$$

We can also calculate the measurement noises of the experiment. The shot noise of the photodetector comes from the DC light which is incident at the same time as our RF signal. The DC power is given by

$$\begin{aligned} P_{DC} &= A(|r(\omega)|^2E_l^2 + |r(\omega+\Omega)|^2E_p^2 && (5)\\ &= 6300\cdot 10^{-6}(0.05*10W + 500\cdot 10^{-6}W) = \boxed{3.2\text{ mW}} && (6)\end{aligned}$$

The shot noise equivalent power at the photodiode input is given from the Schottky formula by

$$S_p(\omega) = \sqrt{\frac{2e}{\eta}\langle P_{DC}\rangle} = 45\ \frac{\text{pW}}{\sqrt{\text{Hz}}}, \qquad (7)$$

where e is the electron charge and $\eta = 0.25\ \frac{\text{A}}{\text{W}}$ is the photodiode responsivity for the Silicon detector. The noise floor of the Agilent spectrum analyzer is given in its manual as $S_V(\omega)$=17 $\frac{\text{nV}}{\sqrt{\text{Hz}}}$. Converting this to an equivalent input power noise at the PD using the specified transimpedence of the BBPD, $G = 1\cdot 10^3\ \frac{\text{V}}{\text{A}}$ gives

$$S_p(\omega) = \frac{S_n(\omega)}{\eta G} = 68\ \frac{\text{pW}}{\sqrt{\text{Hz}}}. \qquad (8)$$

Both of these sensing noises are significantly below the expected RF power given in (4). In particular, the analyzer noise limit gives an SNR of greater than 10,000 for 1 second of integration time.

1.4 Creating and Detecting the Higher Order Modes

In section 1.2 we calculated the signal at the photodetector, but this calculation did not include any discussion of how to detect this signal. In particular, it made no reference to the spatial distribution of the light at the photodetector or to the mode content of the beam incident upon the detector. To account for this in estimating the sensitivity of the experiment we included a HOM coupling coefficient, C_{HOM}. In this section we will estimate the size of this coefficient.

The strongest signal was shown to be the beat between the leakage from the cavity which will be in the TEM_{00} mode and the RF sidebands of the probe beam which will be in a TEM_{nm} mode with $n, m \neq 0$. The normalized Hermite Gaussian modes can be written *at the waist* as

$$HG_{mn} = \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{\omega_0^2 m!n!2^{m+n}}}H_n\left(\frac{\sqrt{2}x}{\omega_0}\right)H_m\left(\frac{\sqrt{2}y}{\omega_0}\right)e^{-\left(\frac{x^2+y^2}{\omega_0^2}\right)}, \qquad (9)$$

where ω_0 is the waist size and $H_n(x)$ is the nth order Hermite polynomial. The orthogonality of the Hermite-Gauss modes ensures that beating a signal composed of two different modes on a photodetector which detects the full beam will not detect anything.

In the IMC experiment higher order modes were created by sticking a 'toothscraper' into the beam which looked much like a large pin at the end except with an angle a few mm from the end. This method was used to create higher order modes on the input as well as to demodulate asymmetrically at the photodiode. We can approximate the effect of the toothscraper as cutting out a triangular portion of the beam which can be easily calculated by switching the orthogonality integral to spherical coordinates and integrating the angular coordinate from 0 to $2\pi - \phi$. Doing so and calculating the overlap integral, O, given by

$$O = \frac{2}{\pi\omega_0^2}\sqrt{\frac{1}{m!n!2^{m+n}}}\int_0^{2\pi-\phi} d\theta \int_0^{\infty} r\, dr H_n\left(\frac{\sqrt{2}r\cos\theta}{\omega_0}\right) H_m\left(\frac{\sqrt{2}r\sin\theta}{\omega_0}\right) e^{-2\left(\frac{r^2}{\omega_0^2}\right)} \tag{10}$$

with $\phi = 0.1\pi$ gives the results shown in table 1.

Units: $1{\cdot}10^{-3}$	0	1	2	3	4	5
0	950	9.8	33.1	11.4	26.8	12.1
1	61.6	15.2	39.4	17.4	30.9	18.2
2	33.1	12.8	18.9	14.2	13.0	14.4
3	2.4	5.0	4.6	4.9	6.0	4.2
4	11.4	1.1	8.8	2.0	8.0	2.8
5	0.1	1.8	0.5	2.3	0.9	2.6

Table 1: The mode overlap coefficients from equation (10) with $\phi = 0.1$ for modes up to order 5.

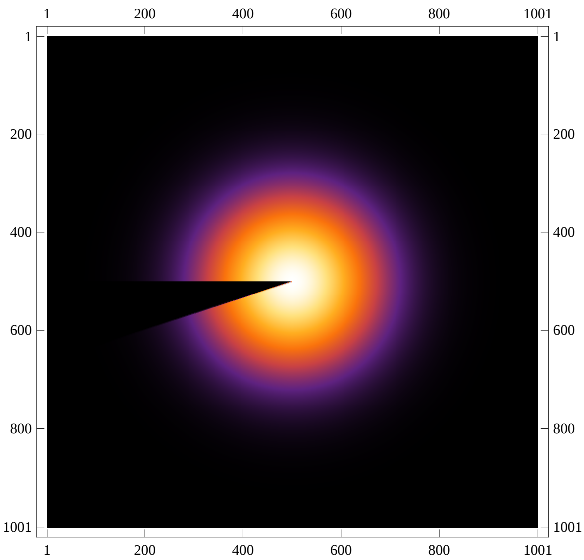

Figure 2: An image of the simulated occulting of the beam for the higher order mode overlap calculation.

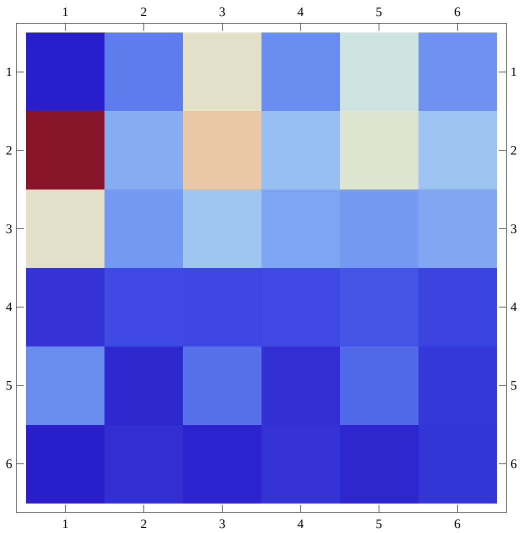

Figure 3: A visualization of table 1 with the 00 element set to 0.

Many of the coefficients shown in table 1 are larger than the assumed 0.1 in section 1.3, but this is deceptive because this the calculated overlap coefficients actually hit twice. This is because they overlap coefficient tells you how much higher order mode content is on the beam in the first place, and a similarly sized coefficient describes how sensitive the detection system will be to that higher order mode. Hence, C_{HOM} is more closely described by the square of the coefficients in table 1.

This makes some of the coefficients significantly smaller than 0.01, but this will hopefully be compensated by the fact that the other estimates of parameters in section 1.3 were conservative.

For instance, we can push the input power up to 10 W and work hard to get to modulation depths of 0.5.

2 What We Can Learn

- **Length of PRC:** The location of the peak at the FSR of the cavity gives a precise measurement of the length of the cavity. This method has been used to measure the length of the IMC at LLO (see e.g. aLog 9599 and was found to be accurate to 10s of Hz not including timing noise.
- **PRC Finesse/Linewidth:** One of the other advantages of using an amplitude modulator for the readout instead of a phase modulator is that the measurement will be first order sensitive to the lineshape at the FSR (with a phase modulator one needs to add an offset to the cavity servo).
- **g Factor of PRC X/Y as a function of TCS:** The spacing of the higher order modes is given to first order by the g factor of the cavity. This can be used in conjunction with TCS and the Hartmann sensors to characterize their effects on the stability of the PRC.
- **Absorption in the PRC:** Similarly to the IMC, the shift of the g factor of the cavity as a function of circulating power can be used to estimate the absorption of the mirrors in the cavity.
- **Identification of Deleterious Modes near Resonance:** Mapping out the higher order mode spectrum of the coupled cavity will allow us to identify low order modes near resonance which will be useful information to know for Advanced LIGO noise hunting.

3 Required Materials

All items should be available from the LLO optics lab of UF labs already without any new purchases.

1. 2 flipper mirrors
2. 2 fixed mount mirrors
3. 1 amplitude modulator
4. 1 RFPD
5. 1 Faraday isolator
6. 2 mode matching lenses
7. 1 network analyzer
8. 1(maybe 2) HWPs