



# The Search for Gravitational Waves

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# Outline

- 1 **Gravitational Waves**
  - Crash Course in Gravitational Wave Physics
  - Gravitational-Wave Sources & Signals
- 2 **Initial and Advanced Gravitational-Wave Observations**
  - Upper Limit Results from Initial Detectors
  - Prospects for Detections with Advanced Detectors
- 3 **Periodic Gravitational Waves from Low-Mass X-Ray Binaries**
  - Gravitational Wave Signal and Detection Problem
  - Cross-Correlation Search



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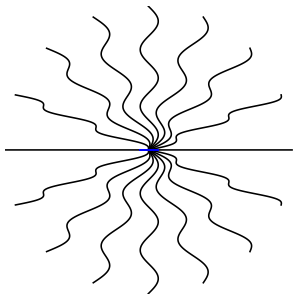
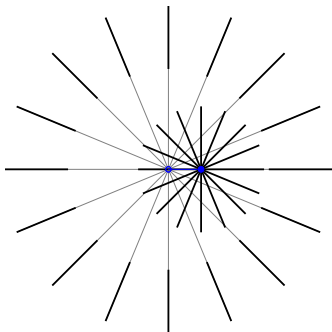
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# Gravity + Causality = Gravitational Waves



- In **Newtonian gravity**, force dep on distance btwn objects
- If massive object suddenly moved, grav field **at a distance** would change **instantaneously**
- In relativity, **no** signal can travel faster than light  
 → time-dep grav fields must propagate like light waves

# Gravity as Geometry

- Minkowski Spacetime (Special Relativity):  
 Invariant spacetime interval (all inertial observers agree):

$$\begin{aligned}
 ds^2 &= -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2 \\
 &= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \sum_{\mu=0}^3 \sum_{\nu=0}^3 \eta_{\mu\nu} dx^\mu dx^\nu
 \end{aligned}$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \sum_{\mu=0}^3 \sum_{\nu=0}^3 g_{\mu\nu} dx^\mu dx^\nu$$

Metric tensor  $\{g_{\mu\nu}(\{x^\lambda\})\}$  determined by masses  
 via Einstein's equations. (10 non-linear PDEs!)



# Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to **1st order** in  $h_{\mu\nu} \equiv$  difference btwn actual metric  $g_{\mu\nu}$  & flat metric  $\eta_{\mu\nu}$ :

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

( $h_{\mu\nu}$  “small” in weak-field regime, e.g. for GW detection)

- $h_{\mu\nu}$  is like electromagnetic potentials  $\varphi, \vec{A}$
- Small coord changes induce “**gauge transformation**” on  $h_{\mu\nu}$   
Convenient choice of gauge is **transverse-traceless**:  
In this gauge:
  - Vacuum Einstein eqns  $\implies$  **wave equation** for  $\{h_{ij}\}$ :

$$\left( -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$

- Test particles w/constant coords are **freely falling**



# Gravitational Wave Polarization States

- Far from source, GW looks like plane wave prop along  $\vec{k}$
- TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

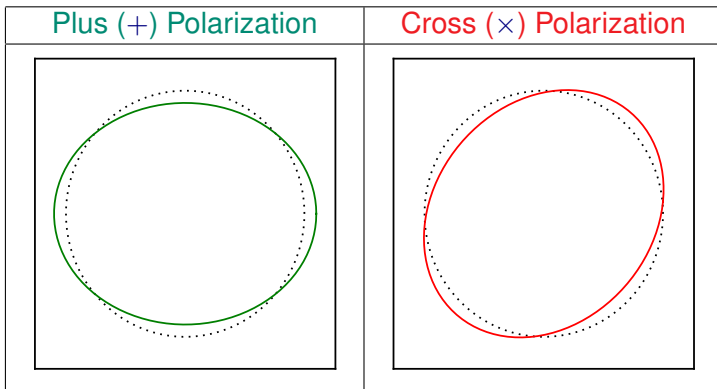
where  $h_+ \left(t - \frac{x^3}{c}\right)$  and  $h_\times \left(t - \frac{x^3}{c}\right)$  are components in “plus” and “cross” polarization states

- EM (spin-1 massless photon) & grav (spin-2 massless “graviton”) waves both have two polarization states



# Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:



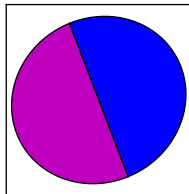
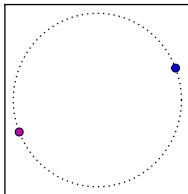


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# Generation of Gravitational Waves

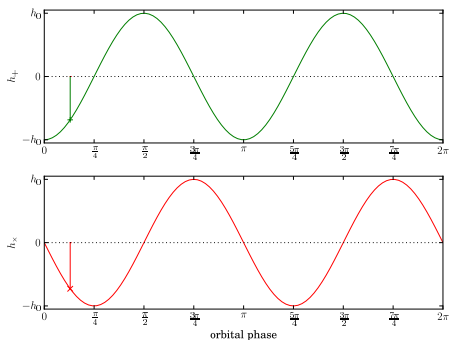
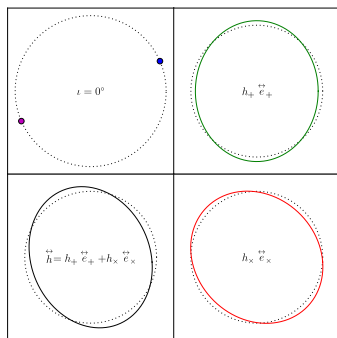
- EM waves generated by **moving/oscillating** charges
- GW generated by **moving/oscillating** masses
- Lowest **multipole** is **quadrupole**
- Different types of signals:
  - Burst (transient, unmodelled)
  - Stochastic (long-lived, unmodelled)
  - **Binary coalescence** (transient, modelled)
  - **Periodic** (long-lived, modelled)





# Gravitational Waves from Binary Orbit

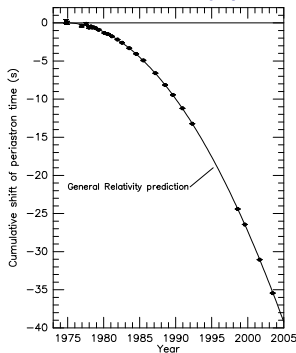
- Orbital motion  $\rightarrow$  oscillating quadrupole moment  $\rightarrow$  GWs



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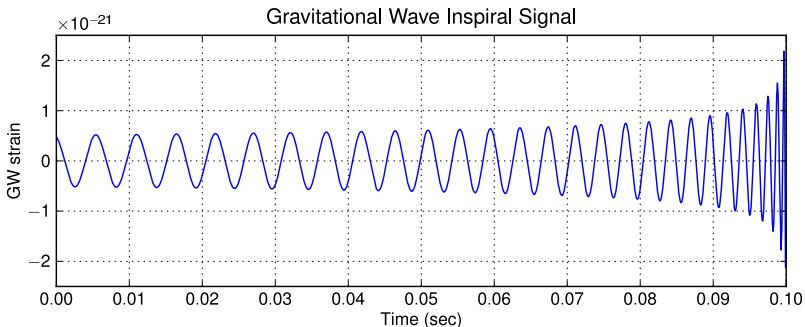
- Orbital motion  $\rightarrow$  oscillating quadrupole moment  $\rightarrow$  GWs
- GW emission removes energy  $\rightarrow$  orbit gets tighter  
 $\rightarrow$  amplitude & freq increase in “chirp”
- Hulse & Taylor saw this evolution in **binary pulsar 1913+16**  
 1993 Nobel Prize

Weisberg, Nice & Taylor  
*ApJ* **722**, 1030 (2010)



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# Methods for Measuring Gravitational Waves

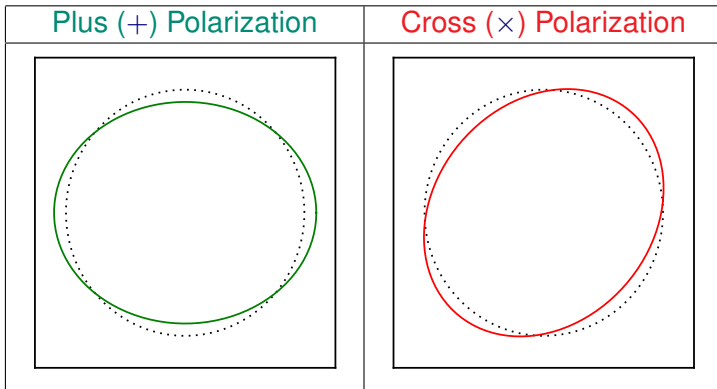
- Cosmic Microwave Background Perturbations ( $f_{\text{gw}} \sim H_0 \sim 10^{-18}$  Hz)
- Pulsar Timing Arrays ( $10^{-9}$  Hz  $\lesssim f_{\text{gw}} \lesssim 10^{-7}$  Hz)
- Laser Interferometers
  - Space-Based ( $10^{-3}$  Hz  $\lesssim f_{\text{gw}} \lesssim 10^{-1}$  Hz)
  - ⇒ Ground-Based ( $10^1$  Hz  $\lesssim f_{\text{gw}} \lesssim 10^3$  Hz)
- Resonant-Mass Detectors (narrowband,  $f_{\text{gw}} \sim 10^3$  Hz)

Note, observable GW freq cover **20** orders of magnitude, similar to EM radiation, but the frequencies are much lower ( $10^3$  Hz  $\lesssim f_{\text{em}} \lesssim 10^{23}$  Hz)



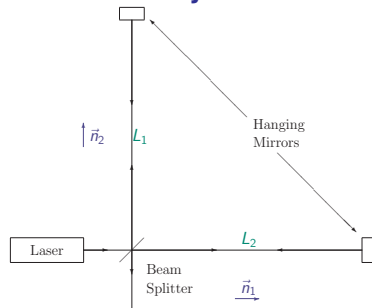
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# Measuring GWs w/Laser Interferometry

**Interferometry:** Measure GW-induced distance changes



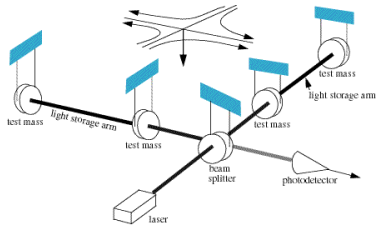
- Measure small change in

$$L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- Plausible signals:  $h \lesssim 10^{-20}$   
→ need  $L_0$  very big!
- For LIGO,  $L_0 = 4 \text{ km}$

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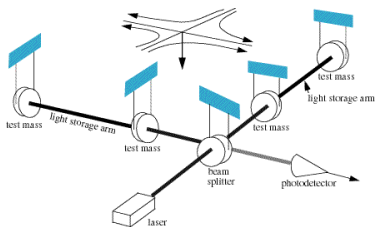
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Note: other detection methods include resonant bars, pulsar timing arrays & planned space-based interferometers (space-based ifos measure low-freq GWs, PTA very low-freq)

# Rogues' Gallery of Ground-Based Interferometers



LIGO Hanford (Wash.)



LIGO Livingston (La.)



GEO-600 (Germany)



Virgo (Italy)

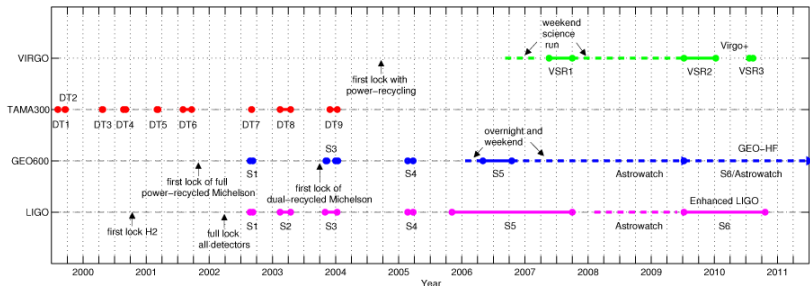


# Initial Gravitational Wave Detector Network

- “1st generation” ground-based interferometric GW detectors (kilometer scale):
  - TAMA 300 (Tokyo, Japan) first online, late 90s; now offline
  - LSC (LIGO Scientific Collaboration) detectors conducting science runs since 2002
    - LIGO Hanford (4km H1 & 2km H2)
    - LIGO Livingston (4km L1)
    - GEO-600 (600m G1)
  - Virgo (3km V1) started science runs in 2007
  - LSC-Virgo long joint runs @ design sensitivity 2005-2010
- LIGO and Virgo being upgraded to 2nd generation “advanced” detectors (10× improvement in sensitivity)
- GEO-600 remains operational in “astrowatch” mode in case there’s a nearby supernova



# Initial Gravitational Wave Detector Network



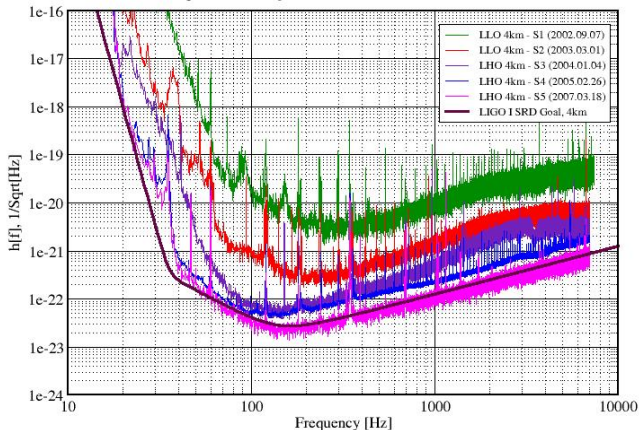
*Living Reviews in Relativity 14, 5 (2011)*



# Evolution of LIGO Sensitivity S1-S5

## Best Strain Sensivities for the LIGO Interferometers

Comparisons among S1 - S5 Runs LIGO-G060009-03-Z







# Advanced Gravitational Wave Detector Network

Aasi et al (LSC & Virgo) [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)

- “2nd generation” ground-based interferometric GW detectors:
  - **Adv LIGO** expected to take **science data** from **2015**  
4km detectors in **Livingston, La.** & **Hanford, Wa.**
  - **Advanced Virgo** should be on comparable timescale
  - **KAGRA** (cryogenic detector in Kamioka mine, Japan)  
uses 2.5-generation technology
  - Third advanced LIGO detector (4km)  
to be installed in **India**, taking data **2019+**  
Big payoff for **sky localization** via triangulation
- Planning for 3rd generation already underway:
  - Einstein Telescope in Europe
  - USA 3G plans still under development

## A Few Words About Collaborations

- LIGO Scientific Collaboration : hundreds of researchers at dozens of institutions world-wide working on instrument science, data analysis, astrophysics, etc.
  - LSC scientists operate  & GEO detectors
  -  and  consortium are LSC members
-  VIRGO Collaboration operates Virgo (Italy) and includes institutions in Italy, France, Netherlands, Poland & Hungary
  - LIGO & Virgo conduct data analysis jointly
-  KAGRA: Japanese collaboration constructing detector in Kamioka mine



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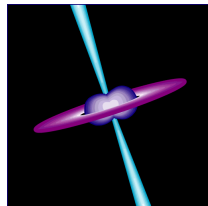
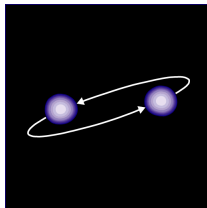
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# Results of Initial Detector Observations

- 80+ Observational papers from initial LIGO/Virgo/GEO:  
<https://www.lsc-group.phys.uwm.edu/ppcomm/Papers.html>
- No detections (although some analyses still trickling out)
- Assortment of **null results** and **upper limits**
- As sensitivity improved, some results gave **new information** to complement other **astronomical observations**:  
**“Multi-Messenger Astronomy”**
- Some highlights:
  - GW associated w/ $\gamma$ -ray bursts (rule out nearby NS merger)
  - GW from known pulsars (beat spindown limit)
  - Stochastic background of GWs (beat nucleosynthesis limit)

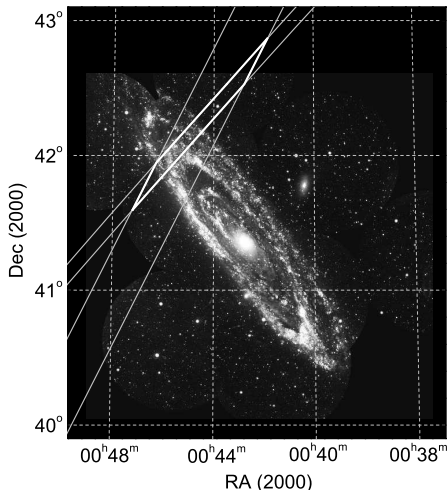
# Gravitational Waves from Gamma-Ray Burst Events



- GRBs are bursts of **high energy photons** observed by orbiting satellites like **Swift** and **Fermi**
- One possible source is the **merger** of a **neutron star** w/another neutron star or a black hole
- Search for GWs emitted by neutron star as it inspirals; search is “triggered” by the GRB, so can compare data at GRB time to data at other times

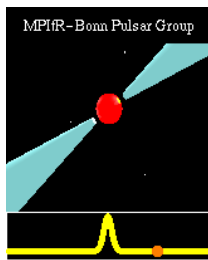
# GRB070201

- 2007 Feb 1: short GRB whose **error box** overlapped spiral arm of **M31** (770 kpc away)
- LHO **4 km** & **2 km** detectors operating & sensitive to inspiral out to **35.7** & **15.3 Mpc**
- No GW seen; **rule out** binary progenitor in M31 w/ > **99%** conf
- *ApJ* **681**, 1419 (2008)



Similar result for GRB051103 & M81; *ApJ* **755**, 2 (2012)

# Searching for Known Pulsars



- Pulsar=rapidly rotating neutron star emitting radio or X-ray “pulses” as it spins (pulse comes when magnetic pole points at Earth)
- Pulsars spin down mostly due to drag of magnetic field through nebula
- If pulsar has small bump, will emit GWs
- Can search for periodic GW signal modulated by Doppler effect as Earth rotates & orbits Sun
- Parameters like freq, sky position, etc known from pulsar
- Spindown produces **indirect upper limit**
  - GW emission above limit → more spindown than seen
  - LIGO/Virgo have **surpassed spindown** limit for **Crab** & **Vela**

# Crab Pulsar Upper Limit



- Pulsar in Crab Nebula
- Created by SN 1054
- $\sim 2$  kpc away
- $f_{\text{rot}} = 29.7$  Hz
- $f_{\text{gw}} = 59.4$  Hz

Image credit: [Hubble](#)/[Chandra](#)

- Initial LIGO (S5 & S6) **upper limits** beat **spindown limit**
- Abbott et al (LSC) *ApJL* **683**, L45 (2008)
- Abbott et al (LSC & Virgo) + Bégin et al *ApJ* **713**, 671 (2010)
- **No more than 2%** of spindown energy loss can be in GW
- Similar limit set on **Vela** using Virgo data (lower freq)  
 Abadie et al (LSC & Virgo) + Buchner et al *ApJ* **737**, 93 (2011)





# Searching for a Stochastic Background

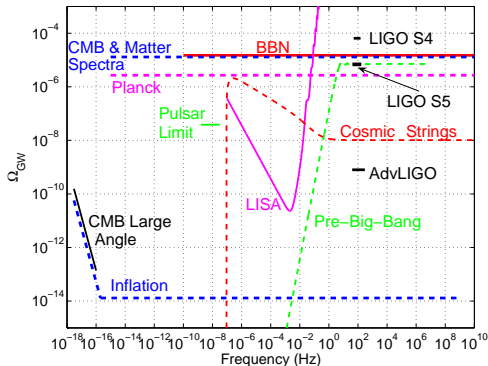
- Expect **stochastic background** of GWs left over from Big Bang (like **cosmic microwave background radiation**) or from confusion of **many faint sources**
- Need to find a **random signal** in **random noise**!
- Noisy data from GW Detector:  
 $x(t) = n(t) + h(t) = n(t) + \overset{\leftrightarrow}{h}(t) : \overset{\leftrightarrow}{d}$
- Look for correlations between detectors

$$\langle x_1 x_2 \rangle = \overbrace{\langle n_1 n_2 \rangle}^{\text{avgto0}} + \overbrace{\langle n_1 h_2 \rangle}^{\text{avgto0}} + \overbrace{\langle h_1 n_2 \rangle}^{\text{avgto0}} + \langle h_1 h_2 \rangle$$

- Details of **expected correlation** will depend on sky distribution of background

Allen & Romano *PRD* **59**, 102001 (1999)

# Isotropic Stochastic Background Limit



$$S5 \text{ limit } \Omega_{\text{gw}}(f) < 6.9 \times 10^{-6} \left( \frac{72 \text{ km/s/Mpc}}{H_0} \right)^2$$
 [Abbott et al (LSC & Virgo) *Nature* **460**, 990 (2009)]  
 surpasses indirect limit from Big-Bang Nucleosynthesis



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# Improved Sensitivity w/Advanced Detectors

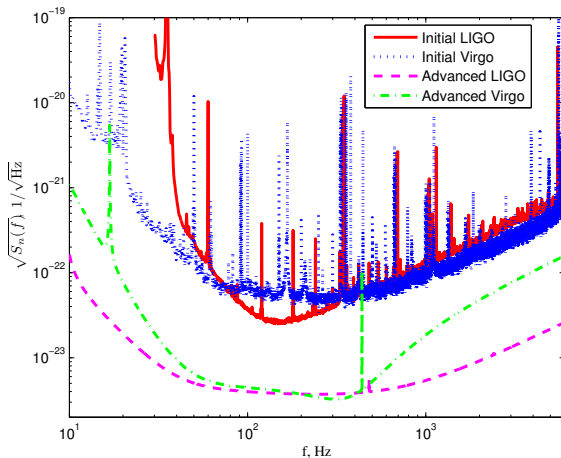


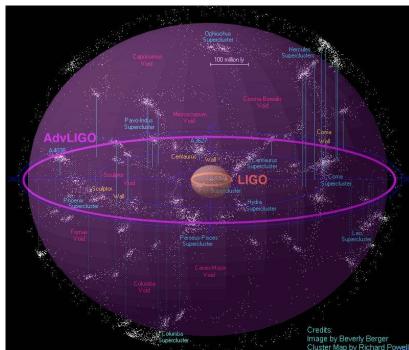
figure from [CQG 27, 173001 \(2010\)](#)

# Expected Event Rates w/Advanced Detectors

## CQG 27, 173001 (2010)

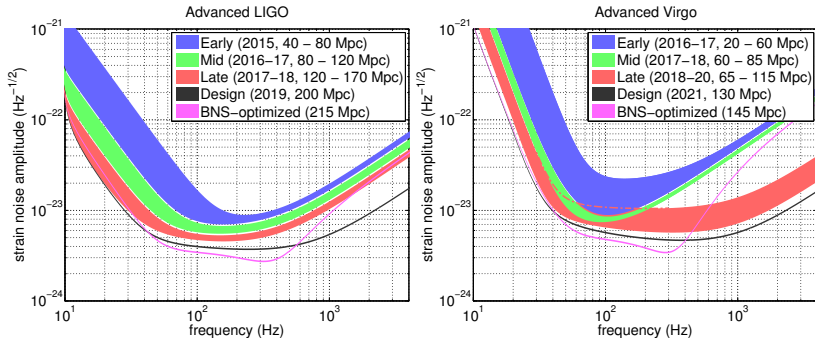
- Advanced detectors should see **NS binary inspiral** up to 400 Mpc & **BH binary coalescence** up to 2 Gpc away

⇒ Expect between a **few** and **hundreds** of events/year





# Anticipated Evolution of Advanced Detector Sensitivity



Figures from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)

Average BNS ranges are  $\frac{\text{optimal range}}{2.26}$

# Expansion of the GW Detector Network

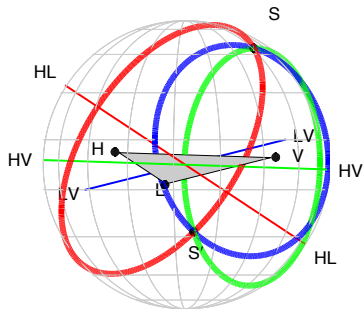
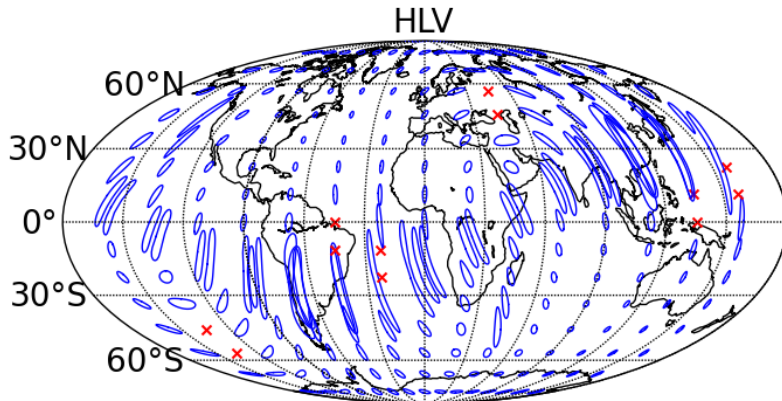


Figure from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)

- Sky loc for **GW transients** can be found by **triangulation**
- Spread detectors around globe to make this **more accurate**
- Put 3rd LIGO detector in **India** to improve sky localization and aid in identification of **electromagnetic counterparts**

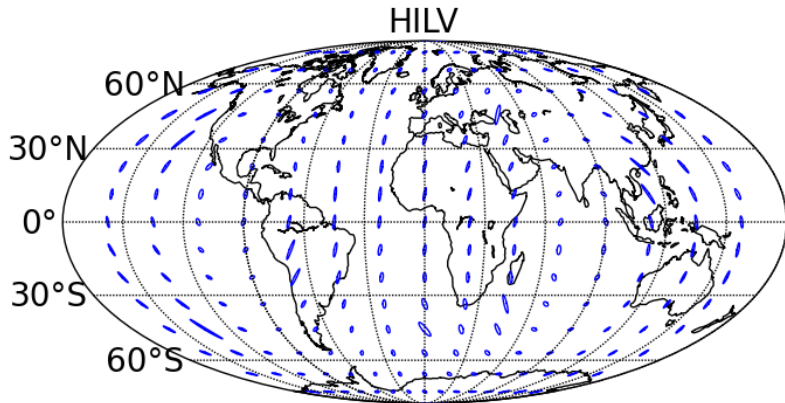
# Improvement in Triangulation with LIGO-India



Figures from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)



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# Gravitational Waves from Low-Mass X-Ray Binaries



- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides “hot spot”; rotating non-axisymmetric NS emits gravitational waves  
Note: waves from NS rotation, NOT binary orbit
- Bildsten *ApJL* **501**, L89 (1998)  
suggested GW spindown may balance accretion spinup
- Torque balance would give  $\approx$  constant GW freq
- Signal at solar system modulated by binary orbit



# GW Signal from Periodic Source

GW signal arriving time  $\tau$  at Solar System Barycenter

$$\vec{h}(\tau) = h_0 \left[ \frac{1 + \cos^2 \iota}{2} \cos \Phi(\tau) \vec{e}_+ + \cos \iota \sin \Phi(\tau) \vec{e}_\times \right]$$

- Amplitude  $h_0$  depends on distance, frequency, ellipticity
- Pol basis  $\{\vec{e}_+, \vec{e}_\times\}$  depends on sky position  $\{\alpha, \delta\}$  and polarization angle  $\psi$
- Phase evolution e.g.,  $\Phi(\tau) = \phi_0 + 2\pi \left( f_0 \tau + \frac{f_1 \tau^2}{2} + \dots \right)$   
(+Doppler mod if NS in binary; note constant Doppler shift OK)
- Signal  $h(t) = \vec{h}(\tau(t)) : \vec{d}$  received in detector has  $\{\alpha, \delta\}$ -dep Doppler shift  $\tau(t)$  due to daily & yearly motion of detector
- Divide signal parameters into
  - **amplitude params:**  $\{h_0, \iota, \psi, \phi_0\}$
  - **phase params:**  $\{\alpha, \delta, f_0, f_1, \dots\}$  + orbital params for LMXB



# Coherent Maximum-Likelihood Search ( $\mathcal{F}$ -statistic)

- Divide signal parameters into
  - **amplitude params:**  $\{h_0, \iota, \psi, \phi_0\}$
  - **phase params:**  $\lambda \equiv \{\alpha, \delta, f_0, f_1, \dots\}$  + orb params for LMXB
- Jaranowski, Królak, Schutz *PRD* **58**, 063001 (1998) showed signal linear in  $\{\mathcal{A}^\mu\}$ , fcns of amplitude params

$$h(t) = \mathcal{A}^\mu h_\mu(t) \quad (\text{assume } \sum_{\mu=1}^4)$$

template waveforms  $h_\mu(t)$  depend on **phase params**  $\lambda$

- Mismatch of obs data w/signal model quadratic in  $\{\mathcal{A}^\mu\}$ :

$$\chi^2(\mathcal{A}, \lambda) = \mathcal{A}^\mu \mathcal{M}_{\mu\nu}(\lambda) \mathcal{A}^\nu - 2\mathcal{A}^\mu x_\mu(\lambda) + \chi^2(0, \lambda)$$

- $\mathcal{F}$ -stat method uses best-fit amp params  $\hat{\mathcal{A}}^\mu = \mathcal{M}^{\mu\nu}(\lambda) x_\nu(\lambda)$  ( $\mathcal{M}^{\mu\nu}$  is inv of  $\mathcal{M}_{\mu\nu}$ ); detection statistic is max log-likelihood

$$\mathcal{F} = -\frac{\chi^2(\hat{\mathcal{A}}, \lambda) - \chi^2(0, \lambda)}{2} = \frac{1}{2} x_\mu(\lambda) \mathcal{M}^{\mu\nu}(\lambda) x_\nu(\lambda)$$



## Bayesian Interpretation ( $\mathcal{B}$ -statistic)

- Assume  $\lambda$  known; likelihood  $P(x|\mathcal{A}) \propto e^{-\chi^2(\mathcal{A})/2}$
- Bayes's theorem says  $P(\mathcal{H}|x) = \frac{P(x|\mathcal{H})P(\mathcal{H})}{P(x)}$
- Odds ratio  $\frac{P(\mathcal{H}_1|x)}{P(\mathcal{H}_0|x)} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)} \frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}$ ; Bayes Factor  $\mathcal{B}_{10} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)}$
- $\mathcal{H}_1 \equiv$  noise + signal w/some  $\mathcal{A}$ ;  $\mathcal{H}_0 \equiv$  noise only
- $\mathcal{F}$ -stat is maximized log-likelihood:  $\max_{\mathcal{A}} \frac{P(x|\mathcal{A})}{P(x|0)} = e^{\mathcal{F}}$
- But  $\mathcal{H}_1$  is composite hypoth.  $P(x|\mathcal{H}_1) = \int P(x|\mathcal{A})P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Don't maximize; marginalize!  $\mathcal{B}$ -statistic (Prix):  $\mathcal{B} = \int \frac{P(x|\mathcal{A})}{P(x|0)} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A} = \int e^{-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu} \mathcal{A}^\nu + \mathcal{A}^\mu x_\mu} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Prix & Krishnan [CQG 26, 204013 \(2009\)](#): If  $P(\mathcal{A}|\mathcal{H}_1)$  uniform in  $\{\mathcal{A}^\mu\}$ ,  $\mathcal{B} = e^{\mathcal{F}}$  Unphysical; implies  $P(h_0, \cos \iota, \psi, \phi_0|\mathcal{H}_1) \propto h_0^3(1 - \cos^2 \iota)^3$
- JTW, Prix, Cutler & Willis [arXiv:1311.0065](#): choice of coords  $\{\mathcal{A}^{\check{\mu}}\}$  aids in approximate eval of  $\mathcal{B}$ -stat integral w/physical priors



# Computational Costs & Phase Parameter Resolution

- If  $\lambda \equiv \{\text{freq, sky pos etc}\}$  **known**, can do most sensitive **fully coherent search** (correlate **all data**)
- If some params **unknown**, have to search over them
- Long coherent observation  $\rightarrow$  **fine resolution** in freq etc  $\rightarrow$  need **too many templates**  $\rightarrow$  **computationally impossible**

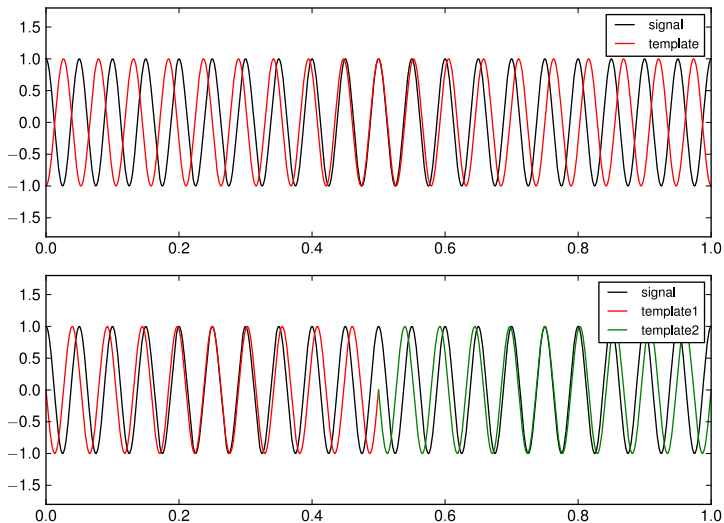
e.g. 
$$N_{\text{tplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta f} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches **semi-coherent**: deliberately limit **coherent integration time** & **param space resolution** to keep **number of templates** manageable





# Coherent vs Semicoherent Searches





# Brightest LMXB: Scorpius X-1

- Scorpius X-1
  - $1.4M_{\odot}$  NS w/ $0.4M_{\odot}$  companion
  - **unknown params** are  $f_0$ ,  $a \sin i$ , orbital phase
  - Parameters from Steeghs & Casares *ApJ* **568**, 273 (2002)  
Update by Galloway et al *ApJ* **781**, 14 (2014)
- Promising source for **Advanced Detectors**
- Initial LSC/Virgo searches for **Sco X-1**:
  - **Coherent  $\mathcal{F}$ -stat search** w/6 hr of S2 data  
Abbott et al (LSC) *PRD* **76**, 082001 (2007)
  - **Directed stochastic (“radiometer”) search** (unmodelled)  
Abbott et al (LSC) *PRD* **76**, 082003 (2007)  
Abbott et al (LSC) *PRL* **107**, 271102 (2011)
- **Mock data challenge** to compare **Sco X-1** search methods  
Poster by Messenger et al at **GWPAW 2013**
- One method: **Cross-corr** specialized to periodic signal  
Dhurandhar et al *PRD* **77**, 082001 (2008)



# Outline

- 1 Gravitational Waves
  - Crash Course in Gravitational Wave Physics
  - Gravitational-Wave Sources & Signals
- 2 Initial and Advanced Gravitational-Wave Observations
  - Upper Limit Results from Initial Detectors
  - Prospects for Detections with Advanced Detectors
- 3 Periodic Gravitational Waves from Low-Mass X-Ray Binaries
  - Gravitational Wave Signal and Detection Problem
  - Cross-Correlation Search



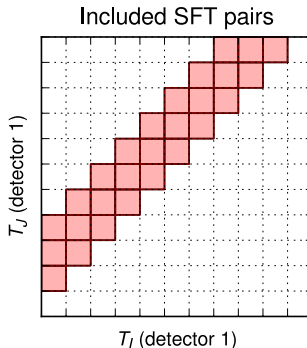
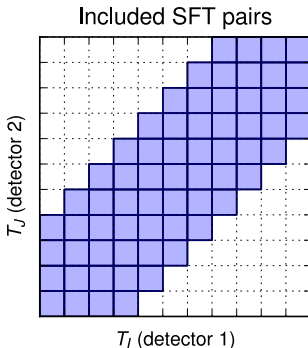
# Basics of Cross-Correlation Method

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

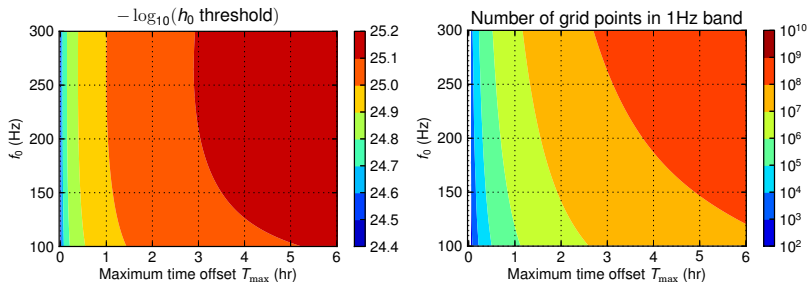
- [BTW, other targets include SN1987A supernova remnant; see Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)]
- Divide data into segments of length  $T_{\text{sft}}$  & take “short Fourier transform” (SFT)  $\tilde{x}_I(f)$
- Label SFTs by  $I, J, \dots$  and pairs by  $\alpha, \beta, \dots$ 
  - ☞  $I$  &  $J$  can be same or different times or detectors
- Construct cross-correlation  $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{k_I})\tilde{x}_J(f_{k_J})}{(T_{\text{sft}})^2}$ 
  - ☞  $f_{k_I} \approx$  signal freq @ time  $T_I$  Doppler shifted for detector  $I$
- Use CW signal model to determine expected cross-correlation btwn SFTs & combine pairs into optimal statistic
$$\rho = \sum_{\alpha} (u_{\alpha} \mathcal{Y}_{\alpha} + u_{\alpha}^* \mathcal{Y}_{\alpha}^*)$$

# Tuning the Cross-Correlation Search

- **Computational considerations** limit **coherent integration time**
- Can make **tunable semi-coherent** search by **restricting** which SFT pairs  $\alpha$  are included in  $\rho = \sum_{\alpha} (u_{\alpha} \mathcal{Y}_{\alpha} + u_{\alpha}^* \mathcal{Y}_{\alpha}^*)$
- E.g., only include pairs where  $|T_I - T_J| \equiv |T_{\alpha}| \leq T_{\max}$



# Sensitivity and Computational Cost



- Can tune **sensitivity** vs **# of param space points**
- **Methods paper** detailing search projections plus assorted technical issues (**param space metric**, **windowing & leakage**, **marginalization on  $\iota$  &  $\psi$** , etc)  
 JTW, Sundaresan, Zhang & Peiris **in preparation**



## Summary

- Gravitational waves: **predicted** by Einstein **confirmed indirectly** (Hulse-Taylor binary pulsar)
- Advanced GW detectors in USA/Italy/Japan/India preparing to make first **direct detections** and initiate **gravitational-wave astronomy**
- Periodic GWs from rotating neutron stars require **coherent** or **semi-coherent** search techniques depending on **knowledge of parameters**
- **Cross-correlation**: one promising method to search for GW from **Low-Mass X-Ray Binaries** like **Sco X-1**