



The Search for Gravitational Waves

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Outline

- 1 Gravitational Waves
 - Crash Course in Gravitational Wave Physics
 - Gravitational-Wave Sources & Signals
- 2 Initial and Advanced Gravitational-Wave Observations
 - Upper Limit Results from Initial Detectors
 - Prospects for Detections with Advanced Detectors
- 3 Periodic Gravitational Waves from Low-Mass X-Ray Binaries
 - Gravitational Wave Signal and Detection Problem
 - Cross-Correlation Search



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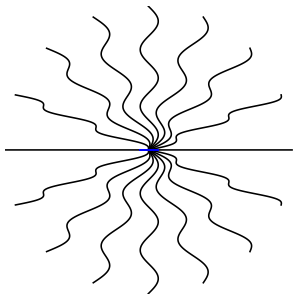
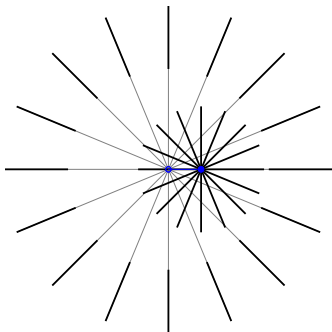
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Gravity + Causality = Gravitational Waves



- In **Newtonian gravity**, force dep on distance btwn objects
- If massive object suddenly moved, grav field **at a distance** would change **instantaneously**
- In relativity, **no** signal can travel faster than light
→ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime (Special Relativity):
 Invariant spacetime interval (all inertial observers agree):

$$\begin{aligned}
 ds^2 &= -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2 \\
 &= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \sum_{\mu=0}^3 \sum_{\nu=0}^3 \eta_{\mu\nu} dx^\mu dx^\nu
 \end{aligned}$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \sum_{\mu=0}^3 \sum_{\nu=0}^3 g_{\mu\nu} dx^\mu dx^\nu$$

Metric tensor $\{g_{\mu\nu}(\{x^\lambda\})\}$ determined by masses
 via Einstein's equations. (10 non-linear PDEs!)



Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to **1st order** in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

($h_{\mu\nu}$ “small” in weak-field regime, e.g. for GW detection)

- Small coord changes induce “**gauge transformation**” on $h_{\mu\nu}$
Convenient choice of gauge is **transverse-traceless**:

$$h_{0\mu} = h_{\mu 0} = 0 \quad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^\lambda} = 0 \quad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

In this gauge:

- Test particles w/constant coords are **freely falling**
- Vacuum Einstein eqns \implies **wave equation** for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$



Gravitational Wave Polarization States

Far from source, GW looks like plane wave prop along \vec{k}
TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $h_+ \left(t - \frac{x^3}{c}\right)$ and $h_\times \left(t - \frac{x^3}{c}\right)$ are components
in “plus” and “cross” polarization states



Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

| Plus (+) Polarization | Cross (\times) Polarization |
|-----------------------|---------------------------------|
| | |



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Generation of Gravitational Waves

- EM waves generated by **moving/oscillating** charges
- GW generated by **moving/oscillating** masses
- Lowest **multipole** is **quadrupole**
- Different types of signals:
 - Burst (transient, unmodelled)
 - Stochastic (long-lived, unmodelled)
 - **Binary coalescence** (transient, modelled)
 - **Periodic** (long-lived, modelled)



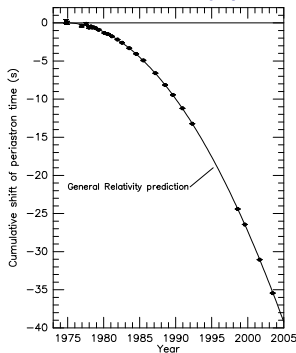
Gravitational Waves from Binary Orbit

- Orbital motion \rightarrow oscillating quadrupole moment \rightarrow GWs

Gravitational Waves from Binary Orbit

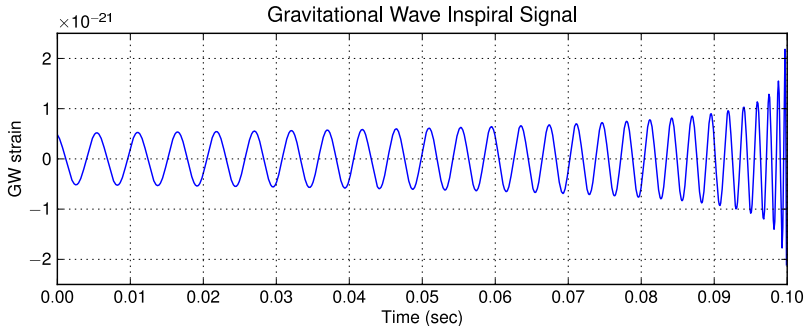
- Orbital motion \rightarrow oscillating quadrupole moment \rightarrow GWs
- GW emission removes energy \rightarrow orbit gets tighter
 \rightarrow amplitude & freq increase in “chirp”
- Hulse & Taylor saw this evolution in **binary pulsar 1913+16**
 1993 Nobel Prize

Weisberg, Nice & Taylor
ApJ **722**, 1030 (2010)



Gravitational Waves from Binary Orbit

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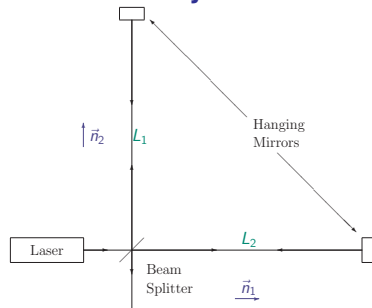
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Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



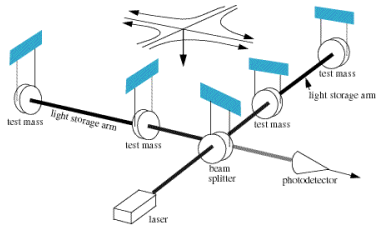
- Measure small change in

$$L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- Plausible signals: $h \lesssim 10^{-20}$
→ need L_0 very big!
- For LIGO, $L_0 = 4 \text{ km}$

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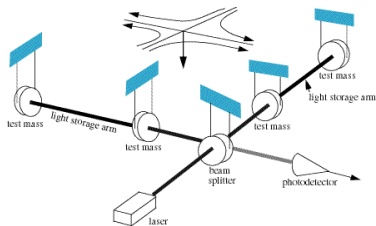
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Note: other detection methods include resonant bars, pulsar timing arrays & planned space-based interferometers

Rogues' Gallery of Ground-Based Interferometers



LIGO Hanford (Wash.)



LIGO Livingston (La.)



GEO-600 (Germany)



Virgo (Italy)

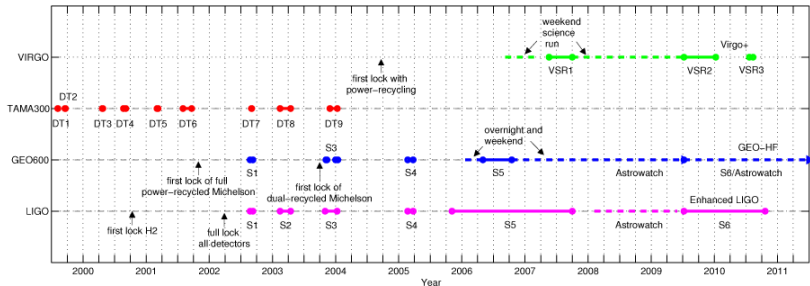


Initial Gravitational Wave Detector Network

- “1st generation” ground-based interferometric GW detectors (kilometer scale):
 - TAMA 300 (Tokyo, Japan) first online, late 90s; now offline
 - LSC (LIGO Scientific Collaboration) detectors conducting science runs since 2002
 - LIGO Hanford (4km H1 & 2km H2)
 - LIGO Livingston (4km L1)
 - GEO-600 (600m G1)
 - Virgo (3km V1) started science runs in 2007
 - LSC-Virgo long joint runs @ design sensitivity 2005-2010
- LIGO and Virgo being upgraded to 2nd generation “advanced” detectors (10× improvement in sensitivity)
- GEO-600 remains operational in “astrowatch” mode in case there’s a nearby supernova



Initial Gravitational Wave Detector Network



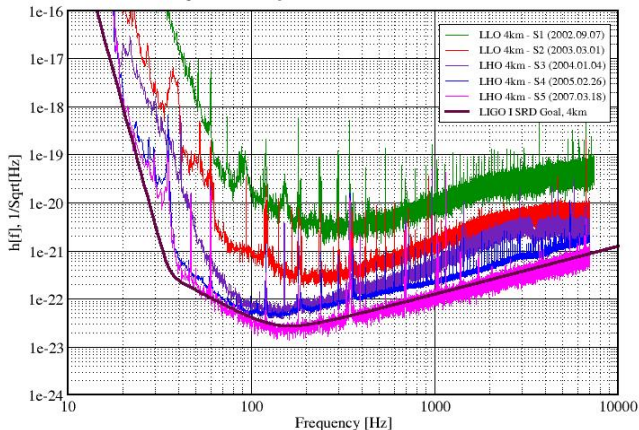
Living Reviews in Relativity 14, 5 (2011)



Evolution of LIGO Sensitivity S1-S5

Best Strain Sensivities for the LIGO Interferometers

Comparisons among S1 - S5 Runs LIGO-G060009-03-Z











Advanced Gravitational Wave Detector Network

Aasi et al (LSC & Virgo) [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)

- “2nd generation” ground-based interferometric GW detectors:
 - Adv LIGO expected to take science data from 2015
4km detectors in Livingston, La. & Hanford, Wa.
 - Advanced Virgo should be on comparable timescale
 - KAGRA (cryogenic detector in Kamioka mine, Japan)
uses 2.5-generation technology
 - Third advanced LIGO detector (4km)
to be installed in India, taking data 2019+
Big payoff for sky localization via triangulation
- Planning for 3rd generation already underway:
 - Einstein Telescope in Europe
 - USA 3G plans still under development



A Few Words About Collaborations

- LIGO Scientific Collaboration : hundreds of researchers at dozens of institutions world-wide working on instrument science, data analysis, astrophysics, etc.
 - LSC scientists operate  & GEO detectors
 -  and  consortium are LSC members
-  VIRGO Collaboration operates Virgo (Italy) and includes institutions in Italy, France, Netherlands & Hungary
 - LIGO & Virgo conduct data analysis jointly
-  KAGRA: Japanese collaboration constructing detector in Kamioka mine



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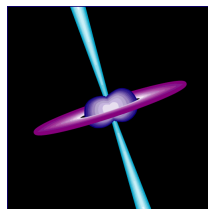
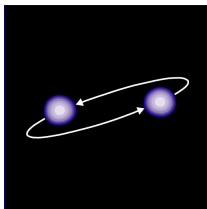
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Results of Initial Detector Observations

- 80+ Observational papers from initial LIGO/Virgo/GEO:
<https://www.lsc-group.phys.uwm.edu/ppcomm/Papers.html>
- No detections (although some analyses still trickling out)
- Assortment of null results and upper limits
- As sensitivity improved, some of these results gave new information to complement astronomical observations:
“Multi-Messenger Astronomy”
- Some highlights:
 - GW associated w/ γ -ray bursts (rule out nearby NS merger)
 - GW from known pulsars (beat spindown limit)
 - Stochastic background of GWs (beat nucleosynthesis limit)

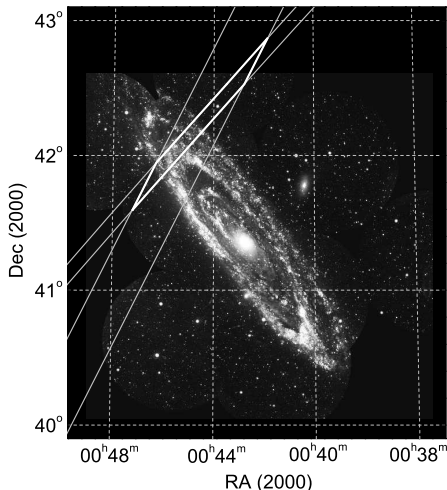
Gravitational Waves from Gamma-Ray Burst Events



- GRBs are bursts of high energy photons observed by orbiting satellites like **Swift** and **Fermi**
- One possible source is the merger of a neutron star w/another neutron star or a black hole
- Search for the GWs emitted by the neutron star as it inspirals; search is “triggered” by the GRB, so can compare data at GRB time to data at other times

GRB070201

- 2007 Feb 1: short GRB whose **error box** overlapped spiral arm of **M31** (770 kpc away)
- LHO **4 km** & **2 km** detectors operating & sensitive to inspiral out to **35.7** & **15.3** Mpc
- No GW seen; **rule out** binary progenitor in M31 w/ > **99%** conf
- *ApJ* **681**, 1419 (2008)



Similar result for GRB051103 & M81; *ApJ* **755**, 2 (2012)



Searching for Known Pulsars

- Pulsar=rapidly rotating neutron star emitting radio or X-ray “pulses” as it spins (pulse comes when magnetic pole points at Earth)
- Pulsars spin down mostly due to drag of magnetic field through nebula
- If pulsar has small bump, will emit GWs
- Can search for periodic GW signal modulated by Doppler effect as Earth rotates & orbits Sun
- Parameters like freq, sky position, etc known from pulsar
- Spindown produces **indirect upper limit**
 - GW emission above limit → more spindown than seen
 - Pulsars w/rapid spindown have “more room” for GW
 - LIGO/Virgo have **surpassed spindown** limit for **Crab** & **Vela**

Crab Pulsar Upper Limit



- Pulsar in Crab Nebula
- Created by SN 1054
- ~ 2 kpc away
- $f_{\text{rot}} = 29.7$ Hz
- $f_{\text{gw}} = 59.4$ Hz

Image credit: [Hubble](#)/[Chandra](#)

- Initial LIGO (S5 & S6) upper limits beat spindown limit
- Abbott et al (LSC) [ApJL 683, L45 \(2008\)](#)
- Abbott et al (LSC & Virgo) + Bégin et al [ApJ 713, 671 \(2010\)](#)
- No more than 2% of spindown energy loss can be in GW
- Similar limit set on Vela using Virgo data (lower freq)
 Abadie et al (LSC & Virgo) + Buchner et al [ApJ 737, 93 \(2011\)](#)

Searching for a Stochastic Background

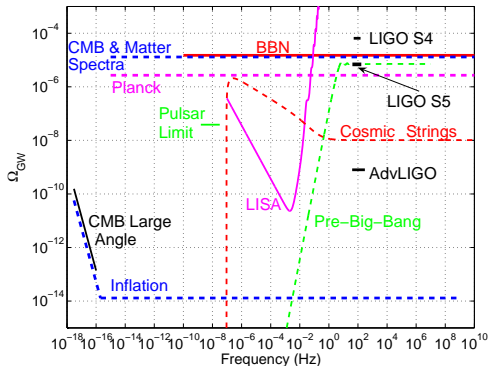
- Expect a random (stochastic) background of GWs left over from Big Bang (like the cosmic microwave background radiation) or from confusion of many faint sources
- Need to find a random signal in random noise!
- Noisy data from GW Detector:
 $x(t) = n(t) + h(t) = n(t) + \overset{\leftrightarrow}{h}(t) : \overset{\leftrightarrow}{d}$
- Look for correlations between detectors

$$\langle x_1 x_2 \rangle = \overbrace{\langle n_1 n_2 \rangle}^{\text{avgto0}} + \overbrace{\langle n_1 h_2 \rangle}^{\text{avgto0}} + \overbrace{\langle h_1 n_2 \rangle}^{\text{avgto0}} + \langle h_1 h_2 \rangle$$

- Details of expected correlation will depend on sky distribution of background

Allen & Romano *PRD* **59**, 102001 (1999)

Isotropic Stochastic Background Limit



$$S5 \text{ limit } \Omega_{\text{gw}}(f) < 6.9 \times 10^{-6} \left(\frac{72 \text{ km/s/Mpc}}{H_0} \right)^2$$
 [Abbott et al (LSC & Virgo) *Nature* **460**, 990 (2009)]
 surpasses indirect limit from Big-Bang Nucleosynthesis



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Improved Sensitivity w/Advanced Detectors

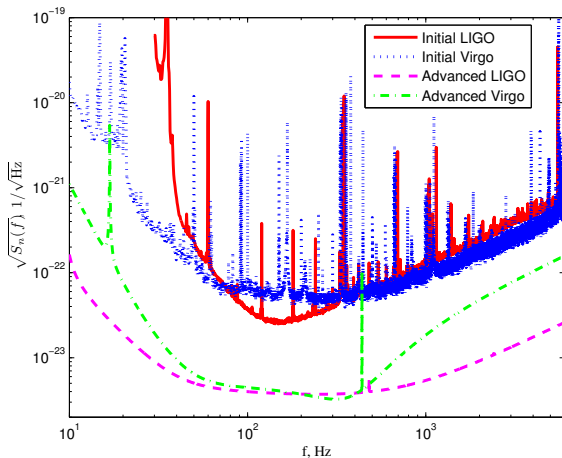
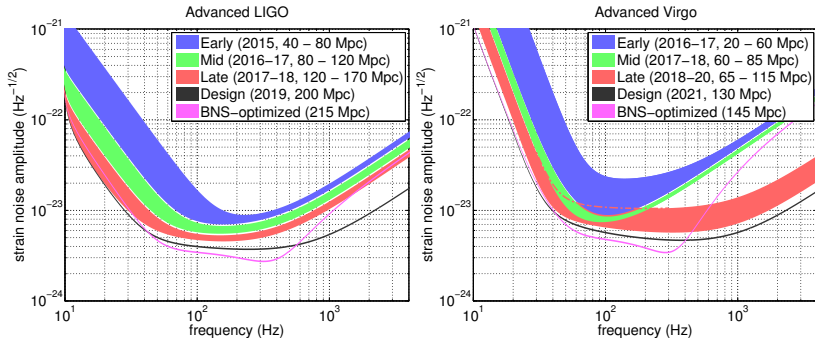


figure from [CQG 27, 173001 \(2010\)](#)



Anticipated Evolution of Advanced Detector Sensitivity



Figures from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)

Average BNS ranges are $\frac{\text{optimal range}}{2.26}$

Expansion of the GW Detector Network

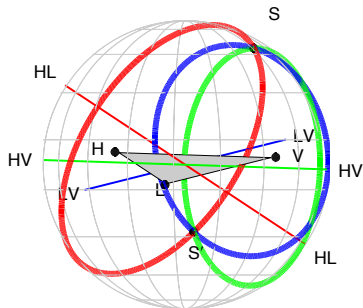


Figure from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)

- Sky loc for GW transients can be found by triangulation
- Spread detectors around globe to make this more accurate
- Put 3rd LIGO detector in India to improve sky localization and aid in identification of electromagnetic counterparts



Improvement in Triangulation with LIGO-India

Figures from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670)



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Gravitational Waves from Low-Mass X-Ray Binaries



- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides “hot spot”; rotating non-axisymmetric NS emits gravitational waves
Note: waves from NS rotation, NOT binary orbit
- Bildsten *ApJL* **501**, L89 (1998)
suggested GW spindown may balance accretion spinup
- Torque balance would give \approx constant GW freq
- Signal at solar system modulated by binary orbit



GW Signal from Periodic Source

GW signal arriving time τ at Solar System Barycenter

$$\vec{h}(\tau) = h_0 \left[\frac{1 + \cos^2 \iota}{2} \cos \Phi(\tau) \vec{e}_+ + \cos \iota \sin \Phi(\tau) \vec{e}_\times \right]$$

- Amplitude h_0 depends on distance, frequency, ellipticity
- Pol basis $\{\vec{e}_+, \vec{e}_\times\}$ depends on sky position $\{\alpha, \delta\}$ and polarization angle ψ
- Phase evolution e.g., $\Phi(\tau) = \phi_0 + 2\pi \left(f_0 \tau + \frac{f_1 \tau^2}{2} + \dots \right)$
(+Doppler mod if NS in binary; note constant Doppler shift OK)
- Signal $h(t) = \vec{h}(\tau(t)) : \vec{d}$ received in detector has $\{\alpha, \delta\}$ -dep Doppler shift $\tau(t)$ due to daily & yearly motion of detector
- Divide signal parameters into
 - **amplitude params:** $\{h_0, \iota, \psi, \phi_0\}$
 - **phase params:** $\{\alpha, \delta, f_0, f_1, \dots\}$ + orbital params for LMXB



Coherent Maximum-Likelihood Search (\mathcal{F} -statistic)

- Divide signal parameters into
 - **amplitude params:** $\{h_0, \iota, \psi, \phi_0\}$
 - **phase params:** $\lambda \equiv \{\alpha, \delta, f_0, f_1, \dots\}$ + orb params for LMXB
- Jaranowski, Królak, Schutz *PRD* **58**, 063001 (1998)
showed signal linear in $\{\mathcal{A}^\mu\}$, fcn's of amplitude params

$$h(t) = \mathcal{A}^\mu h_\mu(t) \quad (\text{assume } \sum_{\mu=1}^4)$$

template waveforms $h_\mu(t)$ depend on **phase params** λ

- Mismatch of obs data w/signal model quadratic in $\{\mathcal{A}^\mu\}$:

$$\chi^2(\mathcal{A}, \lambda) = \mathcal{A}^\mu \mathcal{M}_{\mu\nu}(\lambda) \mathcal{A}^\nu - 2\mathcal{A}^\mu x_\mu(\lambda) + \chi^2(\mathbf{0}, \lambda)$$

- \mathcal{F} -stat method uses best-fit amp params $\hat{\mathcal{A}}^\mu = \mathcal{M}^{\mu\nu}(\lambda) x_\nu(\lambda)$
($\mathcal{M}^{\mu\nu}$ is inv of $\mathcal{M}_{\mu\nu}$); detection statistic is max log-likelihood

$$\mathcal{F} = -\frac{\chi^2(\hat{\mathcal{A}}, \lambda) - \chi^2(\mathbf{0}, \lambda)}{2} = \frac{1}{2} x_\mu(\lambda) \mathcal{M}^{\mu\nu}(\lambda) x_\nu(\lambda)$$



Bayesian Interpretation (\mathcal{B} -statistic)

- Assume λ known; likelihood $P(x|\mathcal{A}) \propto e^{-\chi^2(\mathcal{A})/2}$
- Bayes's theorem says $P(\mathcal{H}|x) = \frac{P(x|\mathcal{H})P(\mathcal{H})}{P(x)}$
- Odds ratio $\frac{P(\mathcal{H}_1|x)}{P(\mathcal{H}_0|x)} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)} \frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}$; Bayes Factor $\mathcal{B}_{10} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)}$
- $\mathcal{H}_1 \equiv$ noise + signal w/some \mathcal{A} ; $\mathcal{H}_0 \equiv$ noise only
- \mathcal{F} -stat is maximized log-likelihood: $\max_{\mathcal{A}} \frac{P(x|\mathcal{A})}{P(x|0)} = e^{\mathcal{F}}$
- But \mathcal{H}_1 is composite hypoth. $P(x|\mathcal{H}_1) = \int P(x|\mathcal{A})P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Don't maximize; marginalize! \mathcal{B} -statistic (Prix): $\mathcal{B} = \int \frac{P(x|\mathcal{A})}{P(x|0)} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A} = \int e^{-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu} \mathcal{A}^\nu + \mathcal{A}^\mu x_\mu} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Prix & Krishnan [CQG 26, 204013 \(2009\)](#): If $P(\mathcal{A}|\mathcal{H}_1)$ uniform in $\{\mathcal{A}^\mu\}$, $\mathcal{B} = e^{\mathcal{F}}$ Unphysical; implies $P(h_0, \cos \iota, \psi, \phi_0|\mathcal{H}_1) \propto h_0^3(1 - \cos^2 \iota)^3$
- JTW, Prix, Cutler & Willis [arXiv:1311.0065](#): choice of coords $\{\mathcal{A}^{\check{\mu}}\}$ aids in approximate eval of \mathcal{B} -stat integral w/physical priors



Computational Costs & Phase Parameter Resolution

- If $\lambda \equiv \{\text{freq, sky pos etc}\}$ **known**, can do most sensitive **fully coherent search** (correlate **all data**)
- If some params **unknown**, have to search over them
- Long coherent observation \rightarrow **fine resolution** in freq etc \rightarrow need **too many templates** \rightarrow **computationally impossible**

e.g.
$$N_{\text{tplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta f} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches **semi-coherent**: deliberately limit **coherent integration time** & **param space resolution** to keep **number of templates** manageable



Brightest LMXB: Scorpius X-1

- Scorpius X-1
 - $1.4M_{\odot}$ NS w/ $0.4M_{\odot}$ companion
 - **unknown params** are f_0 , $a \sin i$, orbital phase
 - Parameters from Steeghs & Casares *ApJ* **568**, 273 (2002)
Update by Galloway et al *ApJ* **781**, 14 (2014)
- Promising source for **Advanced Detectors**
- Initial LSC/Virgo searches for **Sco X-1**:
 - **Coherent \mathcal{F} -stat search** w/6 hr of S2 data
Abbott et al (LSC) *PRD* **76**, 082001 (2007)
 - **Directed stochastic (“radiometer”) search** (unmodelled)
Abbott et al (LSC) *PRD* **76**, 082003 (2007)
Abbott et al (LSC) *PRL* **107**, 271102 (2011)
- **Mock data challenge** to compare Sco X-1 search methods
Poster by Messenger et al at GWPAW 2013
- One method: **Cross-corr** specialized to periodic signal
Dhurandhar et al *PRD* **77**, 082001 (2008)



Outline

- 1 Gravitational Waves
 - Crash Course in Gravitational Wave Physics
 - Gravitational-Wave Sources & Signals
- 2 Initial and Advanced Gravitational-Wave Observations
 - Upper Limit Results from Initial Detectors
 - Prospects for Detections with Advanced Detectors
- 3 Periodic Gravitational Waves from Low-Mass X-Ray Binaries
 - Gravitational Wave Signal and Detection Problem
 - Cross-Correlation Search



Basics of Cross-Correlation Method

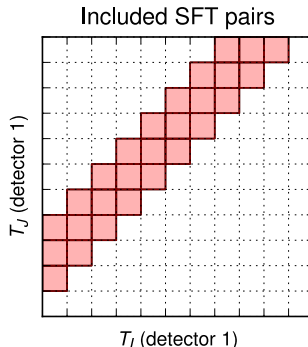
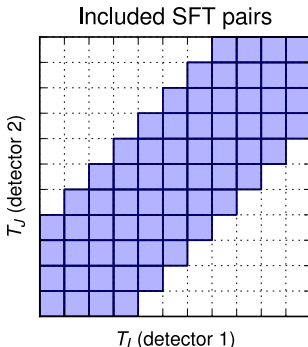
Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

- [BTW, other targets include SN1987A supernova remnant; see Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)]
- Divide data into segments of length T_{sft} & take “short Fourier transform” (SFT) $\tilde{x}_I(f)$
- Label SFTs by I, J, \dots and pairs by α, β, \dots
 - ☞ I & J can be same or different times or detectors
- Construct cross-correlation $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{k_I})\tilde{x}_J(f_{k_J})}{(T_{\text{sft}})^2}$
 - ☞ $f_{k_I} \approx$ signal freq @ time T_I Doppler shifted for detector I
- Use CW signal model to determine expected cross-correlation btwn SFTs & combine pairs into optimal statistic

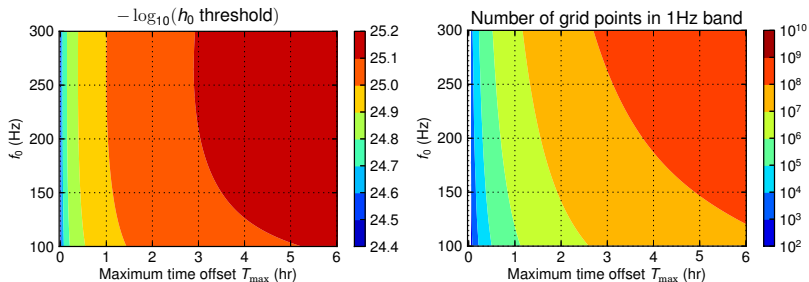
$$\rho = \sum_{\alpha} (u_{\alpha} \mathcal{Y}_{\alpha} + u_{\alpha}^* \mathcal{Y}_{\alpha}^*)$$

Tuning the Cross-Correlation Search

- **Computational considerations** limit **coherent integration time**
- Can make **tunable semi-coherent** search by **restricting** which SFT pairs α are included in $\rho = \sum_{\alpha} (u_{\alpha} \mathcal{Y}_{\alpha} + u_{\alpha}^* \mathcal{Y}_{\alpha}^*)$
- E.g., only include pairs where $|T_I - T_J| \equiv |T_{\alpha}| \leq T_{\max}$



Sensitivity and Computational Cost



- Can tune sensitivity vs # of param space points
- Methods paper detailing search projections plus assorted technical issues (param space metric, windowing & leakage, marginalization on ι & ψ , etc)
 JTW, Sundaresan, Zhang & Peiris **in preparation**



Summary

- Gravitational waves: predicted by Einstein confirmed indirectly (Hulse-Taylor binary pulsar)
- Advanced GW detectors in USA/Italy/Japan/India preparing to make first direct detections and initiate gravitational-wave astronomy
- Periodic GWs from rotating neutron stars require coherent or semi-coherent search techniques depending on knowledge of parameters
- Cross-correlation: one promising method to search for GW from Low-Mass X-Ray Binaries like Sco X-1