Surfing through the universe: Bayesian inference of gravitational waves

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Gravitational waves and supernovae

- information about astrophysical mechanisms, such as core collapse supernovae, is transported across the universe via gravitational waves;
- gravitational waves are notoriously difficult to detect, but with improvements to the network of ground-based detectors (LIGO and Virgo), the chances of detecting them have never been better;
- we use waveform catalogues from recent numerical simulations in general relativity and Bayesian methods to extract information about a progenitor and the resultant neutron star or black hole.

Principal component regression

The waveform catalogue from [1] is decomposed into principal components (PCs) using singular value decomposition. The first k PCs are explanatory variables under the linear model framework.

The data analysed is a time series vector $\mathbf{y}$ of length N and decomposes into signal and noise components. Let $\tilde{\mathbf{y}}$ be the Fourier transformed data vector of length N and let $\tilde{\mathbf{X}}_T$ be the $N \times k$ design matrix, whose columns are the Fourier transformed PC vectors, and T is a cyclical time shift. Following from [2], the frequency domain linear model is

$$\tilde{\mathbf{y}} = \tilde{\mathbf{X}}_T \boldsymbol{\beta} + \tilde{\boldsymbol{\epsilon}}, \tag{1}$$

where $\boldsymbol{\beta}$ are the PC regression coefficients and $\tilde{\boldsymbol{\epsilon}}$ is the Fourier transformed coloured Gaussian noise vector whose variance terms are proportional to the one-sided power spectral density $S_1(f)$ (estimated *a priori* using the advanced LIGO noise curve).

Bayesian signal reconstruction

We build on the Bayesian signal reconstruction model presented in [2]. For a given time shift T, the conditional posterior distribution for the PC coefficients $\boldsymbol{\beta}|T$ is

$$\mathrm{P}(\boldsymbol{\beta}|T, \tilde{\mathbf{y}}) = \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{2}$$

where

$$\boldsymbol{\Sigma} = (\tilde{\mathbf{X}}_T' \mathbf{D}^{-1} \tilde{\mathbf{X}}_T)^{-1}, \quad \boldsymbol{\mu} = \boldsymbol{\Sigma} \tilde{\mathbf{X}}_T' \mathbf{D}^{-1} \tilde{\mathbf{y}}, \tag{3}$$

and $\mathbf{D}$ is the covariance matrix for the noise component.

To estimate $\boldsymbol{\beta}$ and T, we construct a Markov chain whose stationary distribution is the posterior distribution of interest using Metropolis-Hastings MCMC simulation [3]. The algorithm is:

1. propose T from a symmetric (Student-t) distribution;
2. approximate the conditional posterior distribution of $\boldsymbol{\beta}|T$ using equation 2;
3. accept proposal with the usual Metropolis-Hastings acceptance probability.

We assume flat (non-informative) priors on $\boldsymbol{\beta}$ and T, and likelihood

$$p(\tilde{\mathbf{y}}|\boldsymbol{\beta}, T) \propto \exp\left(-2 \sum_{j=1}^{N} \frac{\frac{\Delta_t}{N}\left(\tilde{y}_j - \left(\tilde{\mathbf{X}}_T \boldsymbol{\beta}\right)_j\right)^2}{S_1(f_j)}\right), \tag{4}$$

where Δ_t is distance between two consecutive time points and $f_j, j \in \{1, 2, \ldots, N\}$, are the Fourier frequencies.

The gravitational wave signal is then reconstructed using the posterior means of the PC regression coefficients and time shift parameter from the MCMC simulation.

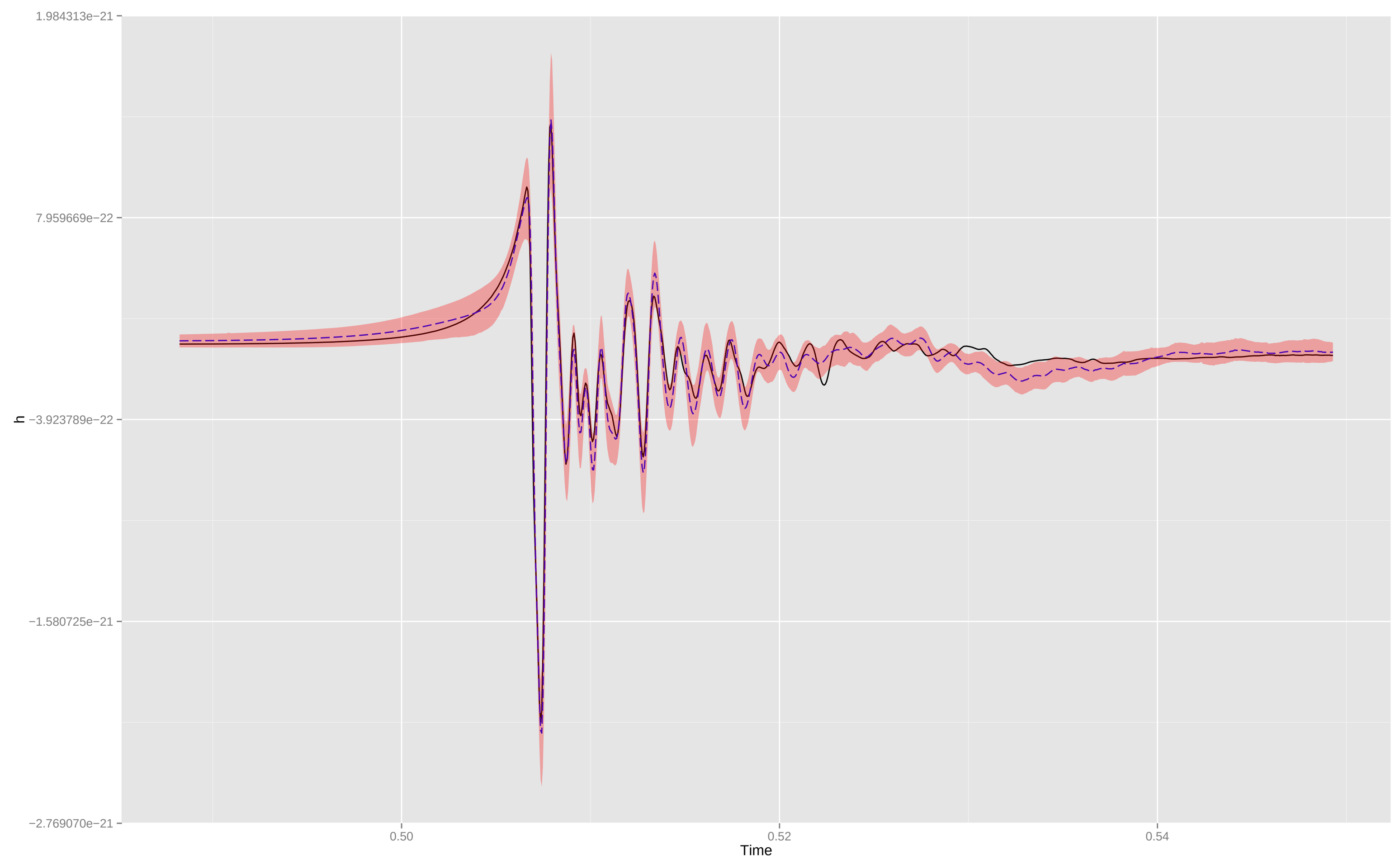

Figure 1: A typical rotating stellar core collapse and bounce gravitational wave test signal reconstructed using $k = 20$ PCs, a signal-to-noise ratio (SNR) of 20, and advanced LIGO noise curve. The black line is the injected signal, the dashed blue line is the posterior reconstruction, and the shaded region is the 90% credible interval.

The reconstruction in figure 1 is reasonable at collapse and bounce (peaks of the time series) and for most of the ring-down oscillations at the end. The extended model in [2] incorporated a random effect which we found to improve the reconstruction in some cases but not in others. Rather than arbitrarily choosing $k = 20$ PCs, we plan to implement a reversible jump MCMC algorithm to select the best model dimensionality. This should improve reconstruction in the ring-down period.

References

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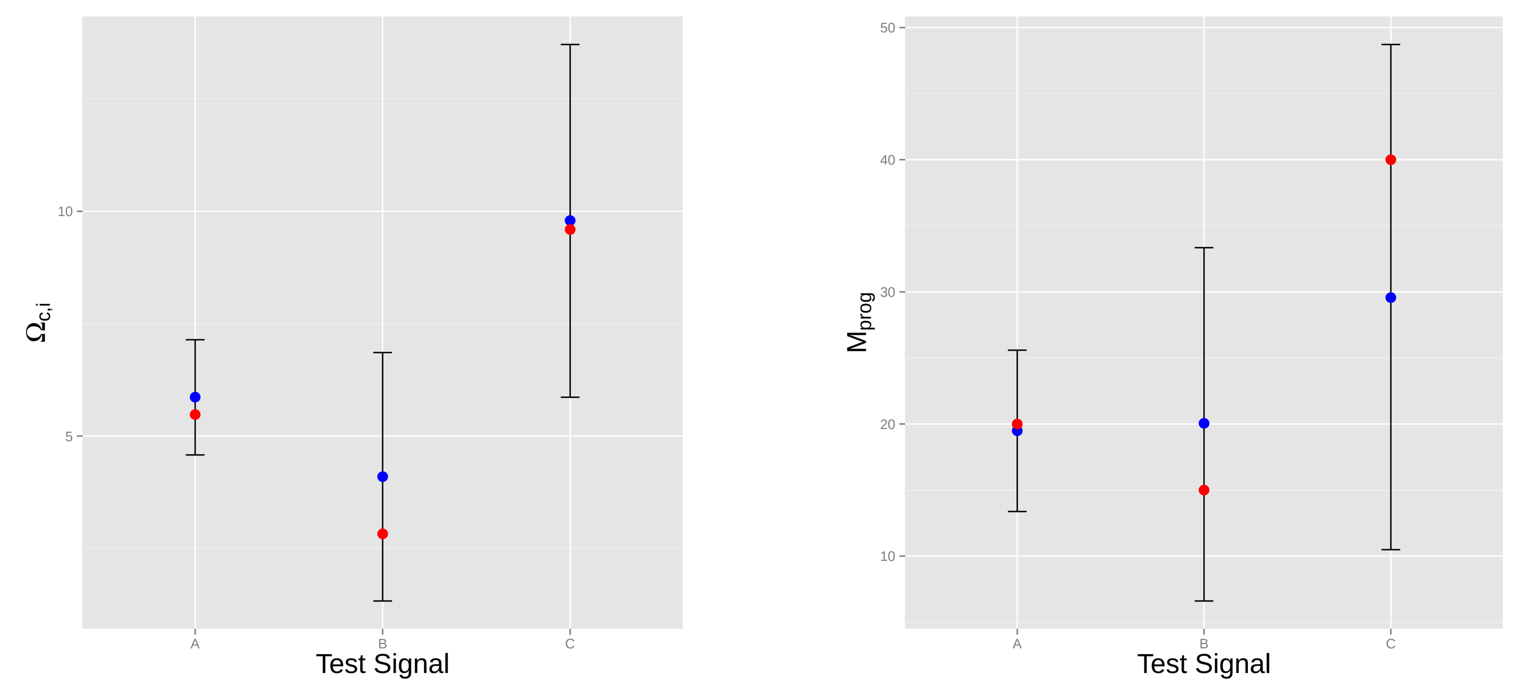
Inference of astrophysical parameters

For core collapse supernovae, we are interested in inferring precollapse central angular velocity $\Omega_{c,i}$, progenitor mass M_{prog}, differential rotation A, and nuclear equation of state (EOS). [4] applied matched filtering and Bayes factors to infer angular momentum to within $\pm 20\%$ for rapidly spinning cores. We use an alternative approach to infer $\Omega_{c,i}$ and M_{prog}:

1. perform MCMC reconstruction on each of the $M = 128$ waveforms from [1] and construct the $M \times (k+1)$ design matrix whose rows are the posterior means of the PC coefficients;
2. fit a Bayesian linear regression model and predict $\Omega_{c,i}$ and M_{prog} for test signals.



(a) Precollapse central angular velocity $\Omega_{c,i}$ **(b)** Progenitor mass M_{prog}

Figure 2: 95% credible intervals for three test signals not in the original [1] catalogue. The red dot is the known physical parameter and the blue dot is the predicted value. We used $k = 10$ PCs and SNR = 20.

[1] demonstrated that $\Omega_{c,i}$ has a very pronounced influence on the gravitational waveform. Unsurprisingly, the prediction for this parameter in figure 2 looks reasonable. The true M_{prog} is also within confidence. However, the large credible intervals for M_{prog} likely reflects a poor statistical model.

For fixed angular velocity, [1] found differential rotation A to have weak influence on the gravitational wave signal, and our inferences have been unsuccessful so far. However, initial exploratory analysis has uncovered clustering in PC-space.

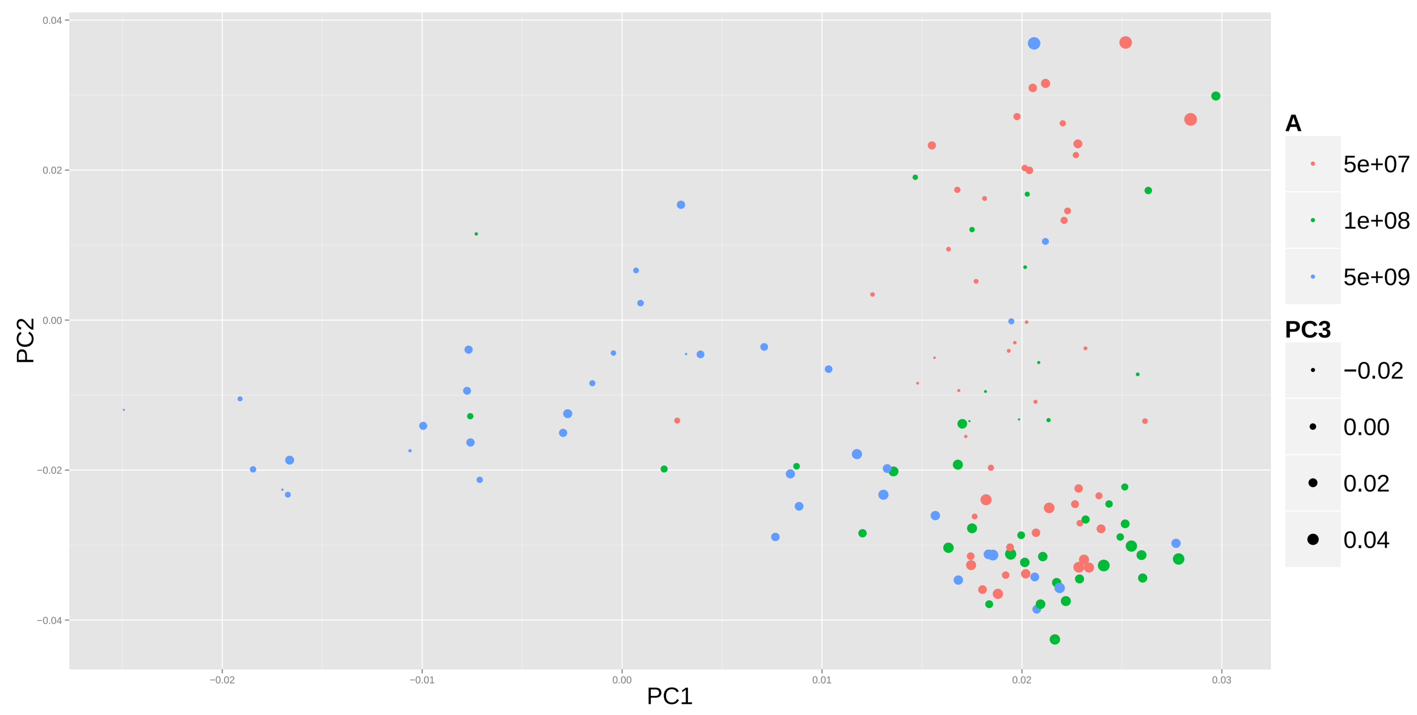


Figure 3: Clustering of differential rotation A in three dimensional PC-space. PC coefficients were estimated using step 1 above, with $k = 10$ PCs and SNR = 20.

Figure 3 shows a tendency for waveforms with almost uniform differential rotation ($A = 5 \times 10^9$ cm) to have smaller PC1 coefficients than more rapidly rotating models.

Waveforms in catalogue [1] have either Lattimer and Swesty (LS) or Shen EOS. To detect EOS, we apply a similar model comparison method to [5]. We use the deviance information criterion (DIC), a generalisation of the AIC for hierarchical models. DIC is a convenient method for MCMC samples and is preferable to Bayes factors as it is easily computed and relevant in the absence of informative priors. Bayes factors cannot be used under our flat prior specification. For all $i \in \{1, 2, \ldots, 128\}$:

1. remove waveform i, split the catalogue by EOS, and run PCA on each sub-catalogue;
2. fit the Bayesian PC regression model twice for waveform i (separately for LS PCs and Shen PCs);
3. calculate DIC for the two sets of PCs to determine the best fit and determine if the EOS is correctly identified (an absolute difference in DIC of ≤ 5 is *unidentified*).

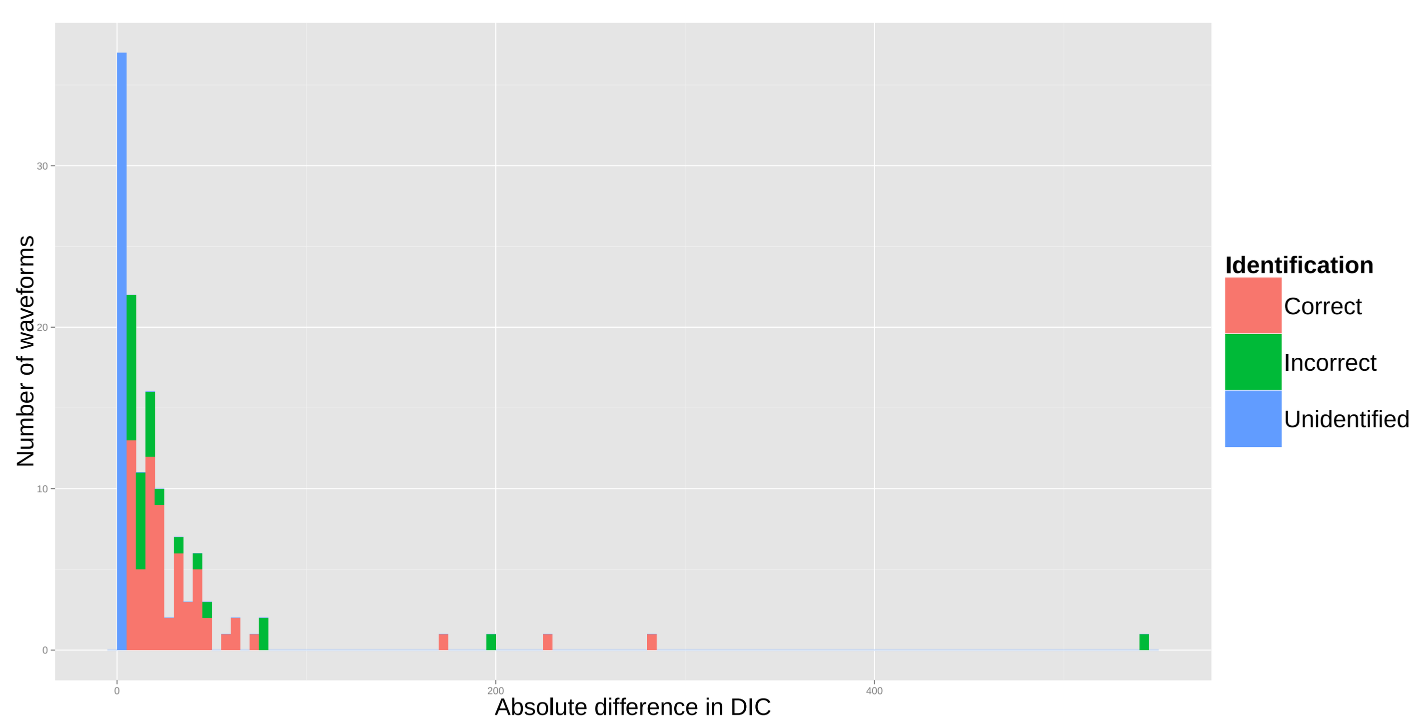


Figure 4: Identification of nuclear equation of state as a function of DIC, using $k = 10$ PCs and SNR = 20.

50% of the 128 waveforms were correctly identified, 21% incorrectly identified, and 29% unidentified.

Future directions

- a reversible jump MCMC algorithm for signal reconstruction and model selection;
- a Bayesian copula to model dependence between angular velocity and differential rotation;
- extend the analysis to other gravitational waveform catalogues;
- improve the *a priori* spectral density estimation.