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HLTS Violin Mode Q

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1 Introduction

1.1 Purpose and Scope

This is the HLTS version of T1200418 (theory and measured violin mode Q's of the HSTS, originally the MC2 suspension at LLO).

1.2 References

LLO alog entries TBD

G. Cagnoli et al., Phys. Lett. A 255 (1999), p230

T0900415: Upper Limit to Suspension Thermal Noise from LIGO 1 and Implications for Wire Suspensions in Advanced LIGO

T070101: Dissipation Dilution

T080096: Wire Attachment Points and Flexure Corrections

D070447-v2: HLTS Overall Assembly

Cumming et al., Design and development of the advanced LIGO monolithic fused silica suspension, Class. Quantum Grav. 29 (2012) 035003.

1.3 Version history

11/7/13: -v1 with just theory.

9/10/14: -v2 with renumbering of equations and fix to Eq. 1.10 (r should have been d).

2 Measurement

As of 11/7/13, Keiko Kokeyama has measured the fundamental violin modes of the four bottom wires of the HLTS suspension PR3 in LLO alog 9418, plus one n=2 harmonic, with a similar technique to that used on the MC2 (LLO alog entry 5097).

This data is not yet quite good enough or complete enough to do much with, but in -v1 of this document we present the relevant theory and a preliminary comparison.

3 Theory

3.1 Mode frequencies

In much the same way as for T1200418, the frequency and Q were calculated using the Mathematica model of the suspension, specifically case `{"mark.barton", "20120120hltsPR3damp"}` of the `TripleLite2` model. This is based on `20120120hlts`, which is equivalent to the Matlab parameter set `^/trunk/Common/MatlabTools/TripleModel_Production/hltsopt_metal.m` revision 2034 and has given a good fit with measured TFs. It also includes modifications, used below, for optionally assigning a separate damping function on each of the four final wires, so as to allow net pendulum mode thermal noise to be calculated from fitted parameters on the respective wires. However since neither the Mathematica nor Matlab models includes violin modes explicitly, calculating these was a matter of using numerical values from the parameter sets in general formulae as described below.

Per Eq. 2.67 of Fletcher and Rossing, to second order in small quantities, the frequency of a violin mode is

$$f_n = nf_1^0\left(1+b+b^2+\frac{n^2\pi^2}{8}b^2\right) \tag{1.1}$$

(Their β has been renamed b to avoid confusion with the thermodynamic material property β used below.)

Here $n=1,2,3...$ is the mode number, and

$$f_1^0=\frac{1}{2L}\sqrt{\frac{T}{\rho_L}}\ , \tag{1.2}$$

is the frequency of a wire without bending stiffness but the same length L, tension T and mass per length ρ_L.

The dimensionless quantity b (formerly β) is

$$b=\frac{2K}{L}\sqrt{\frac{YA}{T}} \tag{1.3}$$

where K is the radius of gyration of the wire, Y is the Young's Modulus, and A is the cross-sectional area, but it is closely related to the usual flexure length, defined (T080096) as

$$a=\sqrt{\frac{YI}{T}}=\frac{bL}{2} \tag{1.4}$$

Here, I is the second moment of area of the wire in the bending direction, equal to $\pi r^4/4$ in any direction for a wire of circular cross-section. (The moments of area of the bottom wires in the longitudinal and transverse directions are called `M31` and `M32` in the model code.)

It is convenient and instructive to put the above formula in terms of a:

$$f_n=\frac{n}{2L\left(1-\frac{2a}{L}-\frac{n^2\pi^2a^2}{2L^2}\right)}\sqrt{\frac{T}{\rho_L}} \tag{1.5}$$

This makes it obvious that to first order in $a/L = b/2$ ($\approx$ 0.00248 for the HSTS) the effect is simply to shorten the wire by one flexure length a at each end for all harmonics. This is consistent with the fact that a wire of non-zero bending stiffness does not bend sharply at the clamp point but along a curve that for most purposes gives the effect of a pivot a away from the attachment point. In addition, there is also a tiny shortening $n^2\pi^2a^2/2L$ second order in both a/L and mode number n. The plain b^2 term disappears because it turns out to be an artifact of doing the expansion in the numerator rather than the denominator, i.e.,

$$1/(1-b) = 1 + b + b^2 + O\left(b^3\right) \tag{1.6}$$

In a practical suspension with multiple wires which may not be exactly vertical, the tension is given by

$$T = \frac{mg}{n_W \cos\theta}$$

where m is the net mass supported by a set of wires, g is local gravity (taken to be 9.81 m/s^2), n_W is the number of wires sharing the load, and θ is the angle of the wires to the vertical. The cross-sectional area and moment of area are

$$A = \pi r^2 \tag{1.7}$$

and

$$I = \frac{\pi r^2}{4} \tag{1.8}$$

where r is the radius.

3.2 Damping

The Q of the violin mode depends on the material damping factor ϕ and the dissipation dilution factor D. The damping factor is modeled as a frequency-independent structural term $\phi_{struct} = 2\times10^{-4}$ (Cagnoli et al. 1999; also T0900415) plus a thermoelastic term:

$$\phi(f) = \phi_{struct} + \phi_{thermo} = \phi_{struct} + \frac{2\pi f \tau \Delta}{1+\left(2\pi f \tau\right)^2} \tag{1.9}$$

where (e.g., Cumming et al.)

$$\tau = 0.0732 C d^2 \rho_V / \kappa \tag{1.10}$$

is a time constant for heat diffusion across the wire (C is heat capacity κ is heat conductivity and $d = 2r$ is diameter), and

$$\Delta = \frac{Y T_W}{\rho_V C}\left(\alpha - \frac{\sigma\beta}{Y}\right)^2 \tag{1.11}$$

is twice the thermoelastic damping at the peak frequency $1/2\pi\tau$ (T_W is temperature, α is linear expansion, $\beta = \frac{1}{Y}\frac{dY}{dT_W}$, and $\sigma = T/A$ is stress). The magic number 0.0732 is a geometrical factor for wires of cylindrical shape, equal to $1/4\xi^2$ where ξ is the first zero of the derivative of the first Bessel function of the first kind:

$$\frac{dJ_1(x)}{dx}\bigg|_{x=\xi} = \tfrac{1}{2}\left(J_0(\xi) - J_2(\xi)\right) = 0 \tag{1.12}$$

Because the energy in a violin mode is stored in second-order stress changes of the elastic material, dissipation dilution is applicable (T070101) and the quality factor Q is not just $1/\phi$ for the material, but D/ϕ where

$$D = \frac{2a}{L}\left(1 + \frac{n^2\pi^2 a}{2L}\right) \tag{1.13}$$

Again there is a higher order term proportional to n^2, which turns out to be significant.

4 Model parameter values

The following table gives symbol names and values for key parameters from the "production" HLTS model as of 1/20/2012 through the date of this report, which aims to be a good approximation to a generic HLTS suspension and has given good fits to measured transfer functions. The model can be found in the SUS SVN at

`^/trunk/Common/MathematicaModels/TripleLite2/mark.barton/20120120hlts`

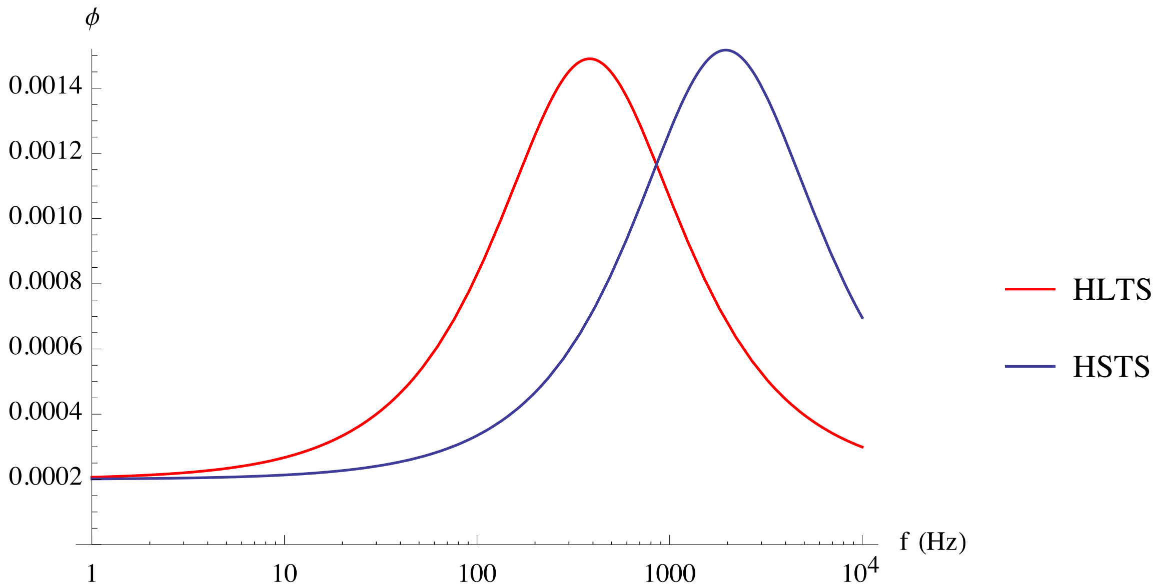
Table 1: Key parameter values from Mathematica model "`20120120hlts`"

Parameter (Theory)	Parameter (Mathematica)	Value (SI Units)	Note
m	`m3`	12.142	optic mass (generic HLTS value)
L	`l3`	0.255	wire length
Y	`Y3==Ysteel`	2.119*10^11	Young's modulus
r	`r3`	0.00013462	wire radius
a	`flex3`	0.00268734	flexure length a (generic HSTS value)
I	`M31`	2.57946*10^-16	wire second moment of area I
β	`betasteel`	-2.5*10^-4	logarithmic rate of change of Young's modulus with temperature
α	`alphasteel`	12*10^-6	thermal expansion coefficient
ρ_V	`rhosteel`	7800	density
C	`Csteel`	486	heat capacity
ϕ_{struct}	`phisteel`	2*10^-4	structural component of phi
τ	`taufibre`	0.00041358	thermoelastic time constant
Δ	`deltafibre`	0.00258057	thermoelastic half maximum phi
D (n=1)	`D1`	0.0109046	dissipation dilution (n=1)
D (n=2)	`D2`	0.0117404	dissipation dilution (n=2)

D (n=3)	`D3`	0.0131334	dissipation dilution (n=3)
D (n=4)	`D3`	0.0150836	dissipation dilution (n=4)

It is interesting to note that the thermoelastic peak in the damping function is at a substantially lower frequency for HLTS due to the increased time for heat to flow across the thicker wires - see Figure 1.

Figure 1: Comparison of HLTS and HSTS thermoelastic phi



The predicted frequency and Q values are given in Table 2.

Table 2: Predicted violin mode frequency and Q values

f1 (Hz)	Q1	f2 (Hz)	Q2	f3 (Hz)	Q3	f4 (Hz)	Q4
513.273	63747.6	1026.98	81281.3	1541.56	94421.3	2057.44	99492.5

5 Results

The raw data from LLO alog 9418 is given in Table 3.

Table 3: Raw data

f1 (Hz)	Fitted Q	f2 (Hz)	Fitted Q
513.219	82442		
513.547	89783	1026.92	108367
516.562	82895		
517.594	107637		

6 Conclusion

Three of the measured Q's are around 85000, which is quite close to the predicted Q of 63748. The fourth Q is somewhat larger. Looking at the plots in the alog, it is apparent that this ringdown had a visibly lower initial excitation and a consequently noisier tail to the ringdown, which is not to produce spuriously good Q's. (Some of this same effect may be present in the three apparently good ringdowns - it would be desirable to have the error estimates from the linear regression.) The single n=2 Q value is also a little higher than predicted, but in rough proportion.

Thus the preliminary conclusion is that the Q's are very much in the right range and there is no rubbing or the like spoiling them.