*LIGO Laboratory / LIGO Scientific Collaboration*

LIGO-T1300926-v2 *ADVANCED LIGO* 10 September 2014

HLTS Violin Mode Q

Mark Barton

Distribution of this document:

DCC

This is an internal working note

of the LIGO Laboratory.

|  |  |
| --- | --- |
| **California Institute of Technology****LIGO Project – MS 18-34****1200 E. California Blvd.****Pasadena, CA 91125**Phone (626) 395-2129Fax (626) 304-9834E-mail: info@ligo.caltech.edu | **Massachusetts Institute of Technology****LIGO Project – NW22-295****185 Albany St****Cambridge, MA 02139**Phone (617) 253-4824Fax (617) 253-7014E-mail: info@ligo.mit.edu |
| **LIGO Hanford Observatory****P.O. Box 1970****Mail Stop S9-02****Richland WA 99352**Phone 509-372-8106Fax 509-372-8137 | **LIGO Livingston Observatory****P.O. Box 940****Livingston, LA 70754**Phone 225-686-3100Fax 225-686-7189 |

http://www.ligo.caltech.edu/

**Table of Contents**

1 Introduction 3

1.1 Purpose and Scope 3

1.2 References 3

1.3 Version history 3

2 Measurement 3

3 Theory 3

3.1 Mode frequencies 3

3.2 Damping 5

4 Model parameter values 6

5 Results 7

6 Conclusion 8

# Introduction

## Purpose and Scope

This is the HLTS version of T1200418 (theory and measured violin mode Q’s of the HSTS, originally the MC2 suspension at LLO).

## References

LLO alog entries TBD

G. Cagnoli et al., Phys. Lett. A 255 (1999), p230

[T0900415](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=5084): Upper Limit to Suspension Thermal Noise from LIGO 1 and Implications for Wire Suspensions in Advanced LIGO

T070101: [Dissipation Dilution](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=27812)

T080096: [Wire Attachment Points and Flexure Corrections](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=10955)

D070447-v2: [HLTS Overall Assembly](https://dcc.ligo.org/LIGO-D070447)

Cumming et al., Design and development of the advanced LIGO monolithic fused silica suspension, Class. Quantum Grav. 29 (2012) 035003.

## Version history

11/7/13: -v1 with just theory.

9/10/14: -v2 with renumbering of equations and fix to Eq. 1.10 ( should have been ).

# Measurement

As of 11/7/13, Keiko Kokeyama has measured the fundamental violin modes of the four bottom wires of the HLTS suspension PR3 in LLO alog [9418](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=9418), plus one n=2 harmonic, with a similar technique to that used on the MC2 (LLO alog entry [5097](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5097)).

This data is not yet quite good enough or complete enough to do much with, but in -v1 of this document we present the relevant theory and a preliminary comparison.

# Theory

## Mode frequencies

In much the same way as for T1200418, the frequency and Q were calculated using the Mathematica model of the suspension, specifically case {"mark.barton", "20120120hltsPR3damp"} of the TripleLite2 model. This is based on 20120120hlts, which is equivalent to the Matlab parameter set ^/trunk/Common/MatlabTools/TripleModel\_Production/hltsopt\_metal.m revision 2034 and has given a good fit with measured TFs. It also includes modifications, used below, for optionally assigning a separate damping function on each of the four final wires, so as to allow net pendulum mode thermal noise to be calculated from fitted parameters on the respective wires. However since neither the Mathematica nor Matlab models includes violin modes explicitly, calculating these was a matter of using numerical values from the parameter sets in general formulae as described below.

Per Eq. 2.67 of Fletcher and Rossing, to second order in small quantities, the frequency of a violin mode is

(Their  has been renamed to avoid confusion with the thermodynamic material property used below.)

Here is the mode number, and

 ,

is the frequency of a wire without bending stiffness but the same length , tension and mass per length .

The dimensionless quantity (formerly) is



where is the radius of gyration of the wire, is the Young’s Modulus, and is the cross-sectional area, but it is closely related to the usual flexure length, defined (T080096) as



Here, is the second moment of area of the wire in the bending direction, equal to in any direction for a wire of circular cross-section. (The moments of area of the bottom wires in the longitudinal and transverse directions are called M31 and M32 in the model code.)

It is convenient and instructive to put the above formula in terms of :



This makes it obvious that to first order in  (≈ 0.00248 for the HSTS) the effect is simply to shorten the wire by one flexure length  at each end for all harmonics. This is consistent with the fact that a wire of non-zero bending stiffness does not bend sharply at the clamp point but along a curve that for most purposes gives the effect of a pivot away from the attachment point. In addition, there is also a tiny shortening second order in both and mode number . The plain term disappears because it turns out to be an artifact of doing the expansion in the numerator rather than the denominator, i.e.,



In a practical suspension with multiple wires which may not be exactly vertical, the tension is given by

 

where m is the net mass supported by a set of wires, g is local gravity (taken to be 9.81 m/s2),  is the number of wires sharing the load, and  is the angle of the wires to the vertical. The cross-sectional area and moment of area are

 

and

 

where r is the radius.

## Damping

The of the violin mode depends on the material damping factor and the dissipation dilution factor . The damping factor is modeled as a frequency-independent structural term  (Cagnoli et al. 1999; also T0900415) plus a thermoelastic term:



where (e.g., Cumming et al.)

 

is a time constant for heat diffusion across the wire ( is heat capacity is heat conductivity and  is diameter), and



is twice the thermoelastic damping at the peak frequency ( is temperature, is linear expansion, , and is stress). The magic number 0.0732 is a geometrical factor for wires of cylindrical shape, equal to where is the first zero of the derivative of the first Bessel function of the first kind:



Because the energy in a violin mode is stored in second-order stress changes of the elastic material, dissipation dilution is applicable (T070101) and the quality factor is not just for the material, but  where



Again there is a higher order term proportional to , which turns out to be significant.


# Model parameter values

The following table gives symbol names and values for key parameters from the “production” HLTS model as of 1/20/2012 through the date of this report, which aims to be a good approximation to a generic HLTS suspension and has given good fits to measured transfer functions. The model can be found in the SUS SVN at

 ^/trunk/Common/MathematicaModels/TripleLite2/mark.barton/20120120hlts

Table 1: Key parameter values from Mathematica model “20120120hlts”

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter (Theory) | Parameter (Mathematica) | Value (SI Units) | Note |
|  | m3 | 12.142 | optic mass (generic HLTS value) |
|  | l3 | 0.255 | wire length |
|  | Y3==Ysteel | 2.119\*10^11 | Young’s modulus |
|  | r3 | 0.00013462 | wire radius |
|  | flex3 | 0.00268734 | flexure length (generic HSTS value) |
|  | M31 | 2.57946\*10^-16 | wire second moment of area  |
|  | betasteel | -2.5\*10^-4 | logarithmic rate of change of Young’s modulus with temperature |
|  | alphasteel | 12\*10^-6 | thermal expansion coefficient |
|  | rhosteel | 7800 | density |
|  | Csteel | 486 | heat capacity |
|  | phisteel | 2\*10^-4 | structural component of phi |
|  | taufibre | 0.00041358 | thermoelastic time constant |
|   | deltafibre | 0.00258057 | thermoelastic half maximum phi |
|  (n=1) | D1 | 0.0109046 | dissipation dilution (n=1) |
|  (n=2) | D2 | 0.0117404 | dissipation dilution (n=2) |
|  (n=3) | D3 | 0.0131334 | dissipation dilution (n=3) |
|  (n=4) | D3 | 0.0150836 | dissipation dilution (n=4) |

It is interesting to note that the thermoelastic peak in the damping function is at a substantially lower frequency for HLTS due to the increased time for heat to flow across the thicker wires - see Figure 1.

Figure 1: Comparison of HLTS and HSTS thermoelastic phi



The predicted frequency and Q values are given in Table 2.

Table 2: Predicted violin mode frequency and Q values

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| f1(Hz) | Q1 | f2(Hz) | Q2 | f3(Hz) | Q3 | f4(Hz) | Q4 |
| 513.273 | 63747.6 |  1026.98 |  81281.3 | 1541.56 | 94421.3 | 2057.44 | 99492.5 |

# Results

The raw data from LLO alog [9418](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=9418) is given in Table 3.

Table 3: Raw data

|  |  |  |  |
| --- | --- | --- | --- |
| f1(Hz) | FittedQ | f2(Hz) | FittedQ |
| 513.219 | 82442 |   |   |
| 513.547 | 89783 | 1026.92 | 108367 |
| 516.562 | 82895 |   |   |
| 517.594 | 107637 |   |   |

# Conclusion

Three of the measured Q’s are around 85000, which is quite close to the predicted Q of 63748. The fourth Q is somewhat larger. Looking at the plots in the alog, it is apparent that this ringdown had a visibly lower initial excitation and a consequently noisier tail to the ringdown, which is not to produce spuriously good Q’s. (Some of this same effect may be present in the three apparently good ringdowns - it would be desirable to have the error estimates from the linear regression.) The single n=2 Q value is also a little higher than predicted, but in rough proportion.

Thus the preliminary conclusion is that the Q’s are very much in the right range and there is no rubbing or the like spoiling them.