Manifold-Cryopump scatter
3/19/13

Arm cavity power, W	$P_a := 8.125\times 10^5$
PSL laser power, W	$P_{psl} := 125$
arm cavity gain	$G_{ac} := 13000$
arm cavity power, W	$P_a := \frac{P_{psl}}{2} \cdot G_{ac}$
	$P_a = 8.125\times 10^5$
laser wavelength, m	$\lambda := 1.064\cdot 10^{-6}$
wave number, m^-1	$k := 2\cdot\frac{\pi}{\lambda}$
IFO waist size, m	$w_{ifo} := 0.012$
solid angle of IFO mode, sr	$\Delta\Omega_{ifo} := \frac{\lambda^2}{\pi\cdot w_{ifo}^2} = 2.502\times 10^{-9}$
Transfer function @ 100 Hz, ITM HR	$TF_{itmhr} := 1.1\cdot 10^{-9}$
IFO arm length, m	$L := 4000$
ETM total annular scatter loss fraction	$\alpha_{etmannulus} := 6\cdot 10^{-6}$
CP intercepted annular power fraction	$\eta_{cpannulus} := 0.29$
BT intercepted annular power fraction	$\eta_{btannulus} := 0.71$
radius of ACB aperture, m	$R_{acb} := 0.170 + 0.004$
radius of beam tube, m	$R_t := 0.913$
radius of beam tube baffle aperture, m	$R_{bt} := 0.531$

radius of Mani/Cryopump baffle aperture, m	$R_{cp} := 0.3845$
half-angle from centerline to Rcp, rad	$\theta_{cp} := \frac{R_{cp}}{L}$
BRDF, sr^-1; CSIRO, surface 2, S/N 2	$BRDF_1(\theta) := \frac{2755.12}{\left(1 + 8.50787 \cdot 10^8 \cdot \theta^2\right)^{1.23597}}$
motion of beam tube @ 100 Hz, m/rt Hz	$x_{bt} := 8 \cdot 10^{-11}$
motion of manifold/cryopump baffle @ 100 HZ, m/rt HZ	$x_{cp} := 1 \cdot 10^{-12}$
motion of ACB @ 100 Hz, m/rt Hz	$x_{ACB} := 1 \cdot 10^{-12}$
location of near BT baffle from ETM, m	$L_{btbnear} := 130$
location of far BT baffle from ETM, m	$L_{btbfar} := L - 130$
	$L_{btbfar} = 3.87 \times 10^3$
number of manifold/cryopump baffles	$N_{cp} := 4$
number of BT baffles	$N_{bt} := 4$
number of ACB	$N_{acb} := 4$
half-angle from centerline to Racb, rad	$\theta_{acb} := \frac{R_{acb}}{L} = 4.35 \times 10^{-5}$
half-angle from centerline to Rcp, rad	$\theta_{cp} := \frac{R_{cp}}{L} = 9.612 \times 10^{-5}$
half-angle from centerline to Rbt, rad	$\theta_{bt} := \frac{R_{bt}}{L} = 1.327 \times 10^{-4}$

total scattered power from ETM, W

$$P_{etmtotal} := P_a \cdot \int_0^{\frac{\pi}{2}} 2 \cdot \pi \cdot \sin(\theta) \cdot BRDF_1(\theta)\, d\theta = 9.997$$

$$P_{etmtotal} = 9.997$$

power incident on cryopump baffle, W

$$P_{cp\theta} := P_a \cdot \int_{\theta_{cp}}^{\theta_{bt}} 2 \cdot \pi \cdot \sin(\theta) \cdot BRDF_1(\theta)\, d\theta = 2.723$$

$$P_{cp\theta} = 2.723$$

OFFSET ETM CALCULATION

horizontal offset, m

$$x_0 := 0.2$$

vertical offset, m

$$y_0 := 0.08$$

reference coordinates

$$x := 0 \qquad y := 0$$

transformation to x,y coords

$$\theta(x, y, x_0, y_0) := \frac{\sqrt{(x - x_0)^2 + (y - y_0)^2}}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + L^2}}$$

$$\theta := \theta(x, y, x_0, y_0)$$

BRDF, sr^-1; CSIRO, surface 2, S/N 2

$$BRDF_1(\theta) := \frac{2755.12}{\left(1 + 8.50787 \cdot 10^8 \cdot \theta(x, y, x_0, y_0)^2\right)^{1.23597}}$$

BRDF, sr^-1; CSIRO, surface 2, S/N 2 in xy coords

$$BRDF_{xy}(x, y, x_0, y_0, L) := \frac{2755.12}{\left[1 + 8.50787 \cdot 10^8 \cdot \left[\frac{\sqrt{(x - x_0)^2 + (y - y_0)^2}}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + L^2}}\right]^2\right]^{1.23597}}$$

CP Baffle incident Power, W

power scattered through the last beam tube baffle, W

$$R := R_{bt}$$

$$P_{Rbtoff} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \frac{BRDF_{xy}(x,y,x_0,y_0,L)}{L^2} \, dx \, dy \right)$$

$$P_{Rbtoff} = 15.975$$

power scattered through the CP baffle hole, W

$$R := R_{cp}$$

$$P_{Rcpoff} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \frac{BRDF_{xy}(x,y,x_0,y_0,L)}{L^2} \, dx \, dy \right)$$

$$P_{Rcpoff} = 12.363$$

scattered power hitting the CP baffle, W

$$P_{cpoff} := P_{Rbtoff} - P_{Rcpoff}$$

$$P_{cpoff} = 3.611$$

power loss fraction from COC to cryopump baffle

$$\eta_{lcp} := \frac{P_{cpoff}}{P_a}$$

$$\eta_{lcp} = 4.445 \times 10^{-6}$$

check non-offset COC calculation

$x_0 := 0 \qquad y_0 := 0$

$R := R_{bt}$

$$P_{Rbt} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2 - y^2}}^{\sqrt{R^2 - y^2}} \frac{BRDF_{xy}(x, y, x_0, y_0, L)}{L^2} \, dx \, dy \right)$$

$P_{Rbt} = 16.818$

$R := R_{cp}$

$$P_{Rcp} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2 - y^2}}^{\sqrt{R^2 - y^2}} \frac{BRDF_{xy}(x, y, x_0, y_0, L)}{L^2} \, dx \, dy \right)$$

$P_{Rcp} = 14.096$

scattered power hitting the CP baffle--x,y integral, W

$P_{cp} := P_{Rbt} - P_{Rcp}$

$P_{cp} = 2.723$

scattered power hitting the CP baffle--θ integral, W

$P_{cp\theta} = 2.723$

ACB incident Power, W

offset ETM $\qquad x_0 := 0.2$

$y_0 := 0.08$

power scattered through the CP hole (ignoring the beam block in the CP, W

$R := R_{cp}$ $R_{cp} = 0.385$

$$P_{Rcp} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2 - y^2}}^{\sqrt{R^2 - y^2}} \frac{BRDF_{xy}(x, y, x_0, y_0, L)}{L^2} \, dx \, dy \right)$$

$P_{Rcp} = 12.363$

power scattered through the ACB hole, W

on-axis ETM $x_0 := 0$

$y_0 := 0$

$R := R_{acb}$ $R_{acb} = 0.174$

$$P_{Racb} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2 - y^2}}^{\sqrt{R^2 - y^2}} \frac{BRDF_{xy}(x, y, x_0, y_0, L)}{L^2} \, dx \, dy \right)$$

$P_{Racb} = 7.096$

net scattered power hitting the ACB, W

$P_{acb} := P_{Rcp} - P_{Racb}$

$P_{acb} = 5.267$

Total Power Scattered from ETM, W

offset ETM $x_0 := 0.2$

$y_0 := 0.08$

power scattered out to the beam tube radius at the near end, W

distance from ETM to near BT baffle, m

$L_{btbnear} = 130$

$L := L_{btbnear}$

$R := R_t \qquad R_t = 0.913$

$$BRDF_{xy}(x, y, x_0, y_0, L) := \frac{2755.12}{\left[1 + 8.50787 \cdot 10^8 \cdot \left[\frac{\sqrt{(x - x_0)^2 + (y - y_0)^2}}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + L^2}}\right]^2\right]^{1.23}}$$

$$P_{R.t.near} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2 - y^2}}^{\sqrt{R^2 - y^2}} \frac{BRDF_{xy}(x, y, x_0, y_0, L)}{L^2}\, dx\, dy\right)$$

$P_{R.t.near} = 23.629$

power scattered out to the beam tube baffle radius at the far end, W

distance from ETM to far BT baffle, m

$L_{btbfar} = 3.87 \times 10^3$

$L := L_{btbfar}$

$R := R_{bt} \qquad R_{bt} = 0.531$

$$BRDF_{xy}(x, y, x_0, y_0, L) := \frac{2755.12}{\left[1 + 8.50787 \cdot 10^8 \cdot \left[\frac{\sqrt{(x - x_0)^2 + (y - y_0)^2}}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + L^2}}\right]^2\right]^{1.23597}}$$

$$P_{R.t.far} := P_a \cdot \left(\int_{-R}^{R} \int_{-\sqrt{R^2 - y^2}}^{\sqrt{R^2 - y^2}} \frac{BRDF_{xy}(x, y, x_0, y_0, L)}{L^2} \, dx \, dy\right)$$

$$P_{R.t.far} = 16.244$$

power scattered onto all beam tube baffles, W

$$P_{bttotal} := P_{R.t.near} - P_{R.t.far}$$

$$P_{bttotal} = 7.385$$

CP Power Scattered from measured ETM annular scatter function 3/19/13 onto BT Baffle

inner radius of annular scatter cone from ETM, m $R_{etmmin} := 0.5$

outer radius of annular scatter cone from ETM, m $R_{etmmax} := 0.6$

radius of beam tube, m $R_t := 0.913$

radius of beam tube baffle aperture, m $R_{bt} = 0.531$

radius of Mani/Cryopump aperture, m $R_{cp} = 0.385$

horizontal offset of ETM, m $x_{off} := 0.2$

vertical offset of ETM, m $y_{off} := 0.08$

intensity of ETM annulus scatter, assuming uniform scatter, W/m^2

$$I_{etmannul.0} := \frac{\alpha_{etmannulus} \cdot P_a}{\pi \cdot \left(R_{etmmax}^2 - R_{etmmin}^2\right)}$$

$$I_{etmannul.0} = 14.107$$

aperture function

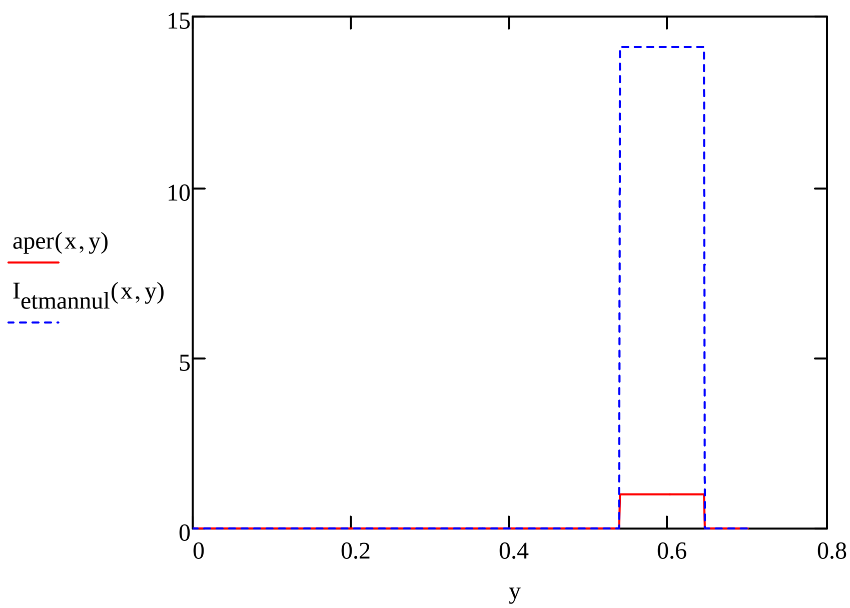
$$aper(x, y) := \Phi\left[\sqrt{\left(x - x_{off}\right)^2 + \left(y - y_{off}\right)^2} - 0.5\right] - \Phi\left[\sqrt{\left(x - x_{off}\right)^2 + \left(y - y_{off}\right)^2} - 0.6\right]$$

intensity distribution function near ITM, W/m^2

$$x := 0$$

$$I_{etmannul}(x, y) := I_{etmannul.0} \cdot aper(x, y)$$

$$y := 0, 0.001 .. 0.7$$



aper

Total annular scatter power, W

$$P_{etmannultot} := \left(\int_{-R_t}^{R_t} \int_{-\sqrt{R_t^2 - y^2}}^{\sqrt{R_t^2 - y^2}} I_{etmannul}(x, y)\, dx\, dy \right)$$

$$P_{etmannultot} = 4.875$$

$$\alpha_{etmannulus} \cdot P_a = 4.875$$

Power passing through the BT baffle aperture, W

$$P_{btaper} := \left(\int_{-R_{bt}}^{R_{bt}} \int_{-\sqrt{R_{bt}^2 - y^2}}^{\sqrt{R_{bt}^2 - y^2}} I_{etmannul}(x, y)\, dx\, dy \right)$$

$$P_{btaper} = 1.984$$

Power passing through the Manif/Cryo
baffle aperture, W

$$P_{mani.cryaper} := \int_{-R_{cp}}^{R_{cp}} \int_{-\sqrt{R_{cp}^2 - y^2}}^{\sqrt{R_{cp}^2 - y^2}} I_{etmannul}(x, y)\, dx\, dy$$

$$P_{mani.cryaper} = 0.805$$

Power hitting the BT baffle, W

$$P_{etmannul.btbaf} := P_{etmannultot} - P_{btaper}$$

$$P_{etmannul.btbaf} = 2.891$$

Power hitting the Mani/Cryo
baffle, W

$$P_{etmannul.cpbaf} := P_{btaper} - P_{mani.cryaper}$$

$$P_{etmannul.cpbaf} = 1.179$$

Power hitting the ACB, W

$$P_{etmannul.acb} := P_{mani.cryaper}$$

$$P_{etmannul.acb} = 0.805$$

BRDF porcelainized steel, #2, 57 deg inc.

large angle BRDF, sr^-1 $BRDF_{\theta 2} := 0.035$

CP Power Scattered into IFO from assumed pathfinder ETM BRDF

$$BRDF_1(\theta) := \frac{2755.12}{\left(1 + 8.50787\cdot 10^{8}\cdot\theta^{2}\right)^{1.23597}}$$

$$P_{cpetmbrdfifo} := \sqrt{N_{cp}}\cdot P_{cp}\cdot BRDF_{\theta 2}\cdot\frac{\pi\cdot w_{ifo}^{2}}{L^{2}}\cdot BRDF_1\left(30\cdot 10^{-6}\right)\cdot\Delta\Omega_{ifo}$$

$$P_{cpetmbrdfifo} = 1.966\times 10^{-17}$$

displacement noise @ 100 Hz from ETM BRDF scatter, m/rtHz

$$DN_{cpetmbrdf} := \frac{1}{\sqrt{2}}\cdot TF_{itmhr}\cdot\left(\frac{P_{cpetmbrdfifo}}{P_{psl}}\right)^{0.5}\cdot x_{cp}\cdot 2\cdot k$$

$$DN_{cpetmbrdf} = 3.643\times 10^{-24}$$

where the factor 1/rt2 was added to correct for the slow phase motion that is below the gravity wave band

CP Power Scattered into IFO from measured ETM annular scatter function 3/19/13

incident power onto CP baffle, W

$$P_{etmannul.cpbaf} = 1.179$$

scattered power into IFO from CP baffle, W

$$P_{cpetmannulifo} := \sqrt{N_{cp}}\cdot P_{etmannul.cpbaf}\cdot BRDF_{\theta 2}\cdot\frac{\pi\cdot w_{ifo}^{2}}{L^{2}}\cdot BRDF_1\left(30\cdot 10^{-6}\right)\cdot\Delta\Omega_{ifo}$$

$$P_{cpetmannulifo} = 8.5134\times 10^{-18}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{cpetmannul} := \frac{1}{\sqrt{2}} \cdot TF_{itmhr} \cdot \left(\frac{P_{cpetmannulifo}}{P_{psl}} \right)^{0.5} \cdot x_{cp} \cdot 2 \cdot k$$

$$DN_{cpetmannul} = 2.397 \times 10^{-24}$$

Beam Tube Baffle Power Scattered into IFO from measured ETM annular scatter function 3/19/13

incident power onto BT baffle, W

$$P_{etmannul.btbaf} = 2.891$$

scattered power into IFO from BT baffle, W

$$P_{btetmannulifo} := \sqrt{N_{bt}} \cdot P_{etmannul.btbaf} \cdot BRDF_{\theta 2} \cdot \frac{\pi \cdot w_{ifo}^{2}}{L_{btbfar}^{2}} \cdot BRDF_{1}\left(30 \cdot 10^{-6}\right) \cdot \Delta\Omega_{ifo}$$

$$P_{btetmannulifo} = 2.087 \times 10^{-17}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{btannulus} := \frac{1}{\sqrt{2}} \cdot TF_{itmhr} \cdot \left(\frac{P_{btetmannulifo}}{P_{psl}} \right)^{0.5} \cdot x_{bt} \cdot 2 \cdot k$$

$$DN_{btannulus} = 3.003 \times 10^{-22}$$

ACB Power Scattered into IFO from measured ETM annular scatter function 3/19/13

$$P_{etmannul.acb} = 0.805$$

scattered annular power into IFO from ACB, W

$$P_{acbetmannulifo} := \sqrt{N_{acb}} \cdot P_{etmannul.acb} \cdot BRDF_{\theta 2} \cdot \frac{\pi \cdot w_{ifo}^{2}}{L^{2}} \cdot BRDF_{1}\left(30 \cdot 10^{-6}\right) \cdot \Delta\Omega_{ifo}$$

$$P_{acbetmannulifo} = 5.809 \times 10^{-18}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{acbannulus} := \frac{1}{\sqrt{2}} \cdot TF_{itmhr} \cdot \left(\frac{P_{acbetmannulifo}}{P_{psl}}\right)^{0.5} \cdot x_{ACB} \cdot 2 \cdot k$$

$$DN_{acbannulus} = 1.98 \times 10^{-24}$$

scattered annular power into IFO from ACB, W

$$P_{acboxytsifo} := 7.229 \times 10^{-18}$$

displacement noise @ 100 Hz from
ETM BRDF scatter, m/rtHz

$$DN_{acboxyt} := 2.228 \times 10^{-24}$$

displacement noise from Manifold/cryo baf and ACB
with ETM BRDF scatter, m/rtHz

$$DN_{cp.acb.etmbrdf} := DN_{cpetmbrdf} + DN_{acboxyt}$$

$$DN_{cp.acb.etmbrdf} = 5.871 \times 10^{-24}$$

displacement noise from Manifold/cryo baf and ACB
with ETM annular scatter, m/rtHz

$$DN_{cp.acb.etmannul} := DN_{cpetmannul} + DN_{acbannulus}$$

$$DN_{cp.acb.etmannul} = 4.378 \times 10^{-24}$$

combined displacement noise of annular and BRDF scatter

$$DN_{cp.acb.etmcombine} := DN_{cp.acb.etmbrdf} + DN_{cp.acb.etmannul}$$

$$DN_{cp.acb.etmcombine} = 1.025 \times 10^{-23}$$

Fractional increase in displacement noise

$$F_{DNacb} := \frac{DN_{cp.acb.etmcombine}}{DN_{cp.acb.etmbrdf}}$$

$$F_{DNacb} = 1.746$$

-
7