T1300692-v1 BS Elliptical Baffle black glass hybrid
8/15/12

BRDF of ellip baf SS, sr^-1 $BRDF_{ellbafss} := 0.030$

BRDF of black glass @ 45 deg, sr^-1
(ref: T1300691_ACB upgrade SS with
black glass) $BRDF_{ellbafbg} := 9 \cdot 10^{-5}$

BRDF of SS edge, sr^-1 $BRDF_{edgess} := 0.1$

BRDF of rough cut black glass edge, sr^-1 $BRDF_{edgebgr} := 0.1$

BRDF of fire-polish black glass edge, sr^-1 $BRDF_{edgebgfp} := 0.0001$

Motion of BS frame @ 100 Hz, m/rtHz $x_{sus} := 3.1 \cdot 10^{-14}$

laser wavelength, m $\lambda := 1.064 \cdot 10^{-6}$

wave number, m^-1 $k := 2 \cdot \frac{\pi}{\lambda}$ $k = 5.9052 \times 10^{6}$

Transfer function @ 100 Hz, ITM AR $TF_{itmar} := 3.16 \cdot 10^{-11}$

ITM beam radius, m $w_{itm} := 0.053168$

IFO waist size, m $w_{ifo} := 0.0120$

transformed beam waist after ITM AR surface
(see H1 Signal Recycling Cavity beam
size_8-12-13) $w_{itmar0} := 0.008342$

distance from ITM AR to virtual beam waist, m $l_{itmar0} := 1.293 \times 10^{3}$

solid angle of ITM AR beam waist, sr $\Delta_{itmar} := \pi \cdot \left(\frac{\lambda}{\pi \cdot w_{itmar0}} \right)^2$

$\Delta_{itmar} = 5.1784 \times 10^{-9}$

see BS Ellip Baf scatter overlap integral.xmcd

effective scattering solid angle
note: this work is not completed

$\Delta\omega_{effbsellipbaf4pt} := 3.507 \times 10^{-10}$

ITM elliptical baffle minor semi-axis, m

$a := \frac{0.21 + 0.014}{2}$ $a = 0.112$

ITM elliptical baffle major semi-axis, m

$b := \frac{0.260 + 0.014}{2}$ $b = 0.137$

vertical aperture in BS ellip baf, m

$r_{bsellipy} := 0.13$

horizontal aperture in BS ellip baf, m

$r_{bsellipx} := 0.105$

radius of SS edge, m

$r_{edgess} := 0.001 \cdot 0.0254$

$r_{edgess} = 2.54 \times 10^{-5}$

radius of BG baffle rough-cut edge, m

$r_{edgebgr} := 0.01 \cdot 0.0254$

$r_{edgebgr} = 2.54 \times 10^{-4}$

Ref. T070247

Transmissivity of ITM HR

$T_{itmhr} := 0.0140$

input laser power, W

$P_{psl} := 125$

arm cavity gain

$G_{ac} := 13000$

arm cavity power, W

$P_a := \frac{P_{psl}}{2} \cdot G_{ac}$ $P_a = 8.125 \times 10^{5}$

Ref. Hiro e-mail 8/29/11

power in power recycling cavity arm, W

$$P_{rca} := \frac{P_a \cdot T_{itmhr}}{4}$$

$$P_{rca} = 2.8438 \times 10^3$$

radius of ITM, m

$$r_{itm} := 0.170$$

exitance function from ITM, W/m^2

$$I_{itm}(x,y) := 2 \cdot \frac{4 \cdot P_{rca}}{\pi \cdot w_{itm}^2} \cdot e^{-2 \cdot \left(\frac{x^2+y^2}{w_{itm}^2} \right)}$$

power exiting from ITM toward elliptical baffle, W

$$P_{itm} := 4 \cdot \int_0^{r_{itm}} \int_0^{r_{itm} \cdot \sqrt{1-\frac{y^2}{r_{itm}^2}}} I_{itm}(x,y)\, dx\, dy$$

$$P_{itm} = 1.1375 \times 10^4$$

BS ELLIPTICAL BAFFLE INCIDENT POWER

Power passing through the ITM elliptical baffle, W

$$P_{itmellbaftran}(\delta x, \delta y) := \left(\int_{\delta y - b}^{\delta y + b} \int_{\delta x - a \cdot \sqrt{1-\frac{y^2}{b^2}}}^{\delta x + a \cdot \sqrt{1-\frac{y^2}{b^2}}} I_{itm}(x,y)\, dx\, dy \right)$$

$$P_{itmellbaftran}(0,0) = 1.1374 \times 10^4$$

arm power exiting from ITMAR passing through BS elliptical baffle, W

$$P_{itmarbsellbaf} := 4\cdot\int_0^{r_{bsellipy}}\int_0^{r_{bsellipx}\cdot\sqrt{1-\frac{y^2}{r_{bsellipy}^2}}} I_{itm}(x,y)\,dx\,dy$$

$$P_{itmarbsellbaf} = 1.1373\times 10^{4}$$

Power hitting BS baffle, W

$$P_{bsbaf} := P_{itmellbaftran}(0,0) - P_{itmarbsellbaf}$$

$$P_{bsbaf} = 1.0931$$

Scatter efficiency of BS Elliptical Baffle

$$\eta_{bsellbaf} := \frac{P_{bsbaf}}{P_{itm}}$$

$$\eta_{bsellbaf} = 9.6097\times 10^{-5}$$

SCATTER FROM OXIDIZED SS BS ELLIP BAFFLE SURFACE

Power scattered into IFO mode
from both arms, W

$$P_{bsellbafsss} := \sqrt{2}\cdot P_{bsbaf}\cdot BRDF_{ellbafss}\cdot\Delta_{itmar}$$

$$P_{bsellbafsss} = 2.4015\times 10^{-10}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{bsellbafss} := TF_{itmar}\cdot\left(\frac{P_{bsellbafsss}}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsellbafss} = 1.1339\times 10^{-23}$$

SCATTER FROM SS BS ELLIP BAFFLE HOLE EDGE

horizontal edge, m $x := r_{bsellipx}$

vertical edge, m $y := 0$

exitance function from ITM at edge, W/m^2 $I_{itm}(x,y) := 2\cdot\frac{4\cdot P_{rca}}{\pi\cdot w_{itm}^{2}}\cdot e^{-2\cdot\left(\frac{x^2+y^2}{w_{itm}^{2}}\right)}$

$I_{itm}(x,y) = 1.0494\times 10^{3}$

maximum width of exposed edge, m $w_{bsbafe} := 2\cdot r_{edgess}$

Radius of baffle hole, m $R_{bsbaf} := r_{bsellipx}$

exposed area of baffle hole edge, m^2

$$A_{bsbafess} := \int_{-R_{bsbaf}}^{0} 2\cdot\sqrt{R_{bsbaf}^{2}-x^{2}}\,dx - \int_{-R_{bsbaf}+w_{bsbafe}}^{0} 2\cdot\sqrt{R_{bsbaf}^{2}-\left(x-w_{bsbafe}\right)^{2}}\,dx$$

$A_{bsbafess} = 1.0668\times 10^{-5}$

power incident on BS Baf hole edge, W

$P_{bsbafedgess} := I_{itm}\left(r_{bsellipx},0\right)\cdot A_{bsbafess}$

$P_{bsbafedgess} = 0.011$

intercepton efficiency for ITM power $\eta_{itmellbafedgess} := \frac{P_{bsbafedgess}}{P_{itm}}$

$\eta_{itmellbafedgess} = 9.8415\times 10^{-7}$

power scattered from two BS Ellip Baf hole edge toward BS W

$$P_{bsbafedgesss} := \sqrt{2}\cdot P_{bsbafedgess}\cdot BRDF_{edgess}\cdot \Delta_{itmar}$$

$$P_{bsbafedgesss} = 8.1983 \times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafedgess} := TF_{itmar}\cdot\left(\frac{P_{bsbafedgesss}}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsbafedgess} = 2.0951 \times 10^{-24}$$

ratio of edge scatter to baf scatter

$$\frac{DN_{bsbafedgess}}{DN_{bsellbafss}} = 0.1848$$

Total SS baf scatter power, W

$$P_{bsbafssts} := P_{bsellbafsss} + P_{bsbafedgesss}$$

$$P_{bsbafssts} = 2.4835 \times 10^{-10}$$

total SS baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafsst} := TF_{itmar}\cdot\left(\frac{P_{bsbafssts}}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsbafsst} = 1.1531 \times 10^{-23}$$

SCATTER FROM BLACK GLASS BS ELLIP BAFFLE SURFACE

Power scattered into IFO mode from both arms, W

$$P_{bsellbafbgs} := \sqrt{2}\cdot P_{bsbaf}\cdot BRDF_{ellbafbg}\cdot \Delta_{itmar}$$

$$P_{bsellbafbgs} = 7.2046 \times 10^{-13}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsellbafbg} := TF_{itmar}\cdot\left(\frac{P_{bsellbafbgs}}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsellbafbg} = 6.2109\times 10^{-25}$$

SCATTER FROM ROUGH CUT BG ITM ELLIP BAFFLE HOLE EDGE

maximum width of exposed edge, m $w_{bsbafe} := 2\cdot r_{edgebgr}$

Radius of baffle hole, m $R_{bsbaf} = 0.105$

exposed area of baffle hole edge, m^2

$$A_{bsbafebg} := \int_{-R_{bsbaf}}^{0} 2\cdot\sqrt{R_{bsbaf}^{2} - x^{2}}\,dx - \int_{-R_{bsbaf}+w_{bsbafe}}^{0} 2\cdot\sqrt{R_{bsbaf}^{2} - \left(x - w_{bsbafe}\right)^{2}}\,dx$$

$$A_{bsbafebg} = 1.0668\times 10^{-4}$$

power incident on BS Baf hole edge, W

$$P_{bsbafedgebg} := I_{itm}\left(r_{bsellipx}, 0\right)\cdot A_{bsbafebg}$$

$$P_{bsbafedgebg} = 0.112$$

power scattered from two BG ITM Ellip Baf hole edge toward ITM, W

$$P_{bsbafedgebgrs} := \sqrt{2}\cdot P_{bsbafedgebg}\cdot BRDF_{edgebgr}\cdot\Delta_{itmar}$$

$$P_{bsbafedgebgrs} = 8.1982\times 10^{-11}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafedgebgr} := TF_{itmar}\cdot\left(\frac{P_{bsbafedgebgrs}}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsbafedgebgr} = 6.6253 \times 10^{-24}$$

ratio of rough cut edge scatter to baf scatter

$$\frac{DN_{bsbafedgebgr}}{DN_{bsellbafbg}} = 10.6673$$

SCATTER FROM FIRE-POLISHED BG BS ELLIP BAFFLE HOLE EDGE

max input angle, rad $\theta_{xymaxedge} := \frac{\pi}{2} = 1.5708$

max input angle, deg $\theta_{xymaxpdeg} := \theta_{xymaxedge} \cdot \frac{180}{\pi} = 90$

BRDF Black Glass BRDF @ 5 deg incidence

$$BRDF_0 := 0.12$$

$$\beta := .75$$

$$C_{mr} := 1.655 \times 10^4$$

$$BRDF_{\theta 2} := 1 \cdot 10^{-5}$$

BRDF function, sr^-1 $BRDF_{bg5}(\theta_i) := \frac{BRDF_0}{\left(1 + C_{mr} \cdot \theta_i^{\,2}\right)^{\beta}} + BRDF_{\theta 2}$

BRDF Black Glass fire polish (empirical estimate)

break-over angle, rad $\theta_1 := 0.5 \cdot \frac{\pi}{180} = 8.7266 \times 10^{-3}$

micro-roughness angle, rad $\theta_2 := 5 \cdot \frac{\pi}{180} = 0.0873$

max BRDF, sr^-1 $BRDF_0 := 0.1$

final slope modifier $\beta := 0.85$

micro-roughness constant $C_{mr} := \frac{2^{\left(\frac{1}{\beta}\right)} - 1}{\theta_1^{\,2}}$

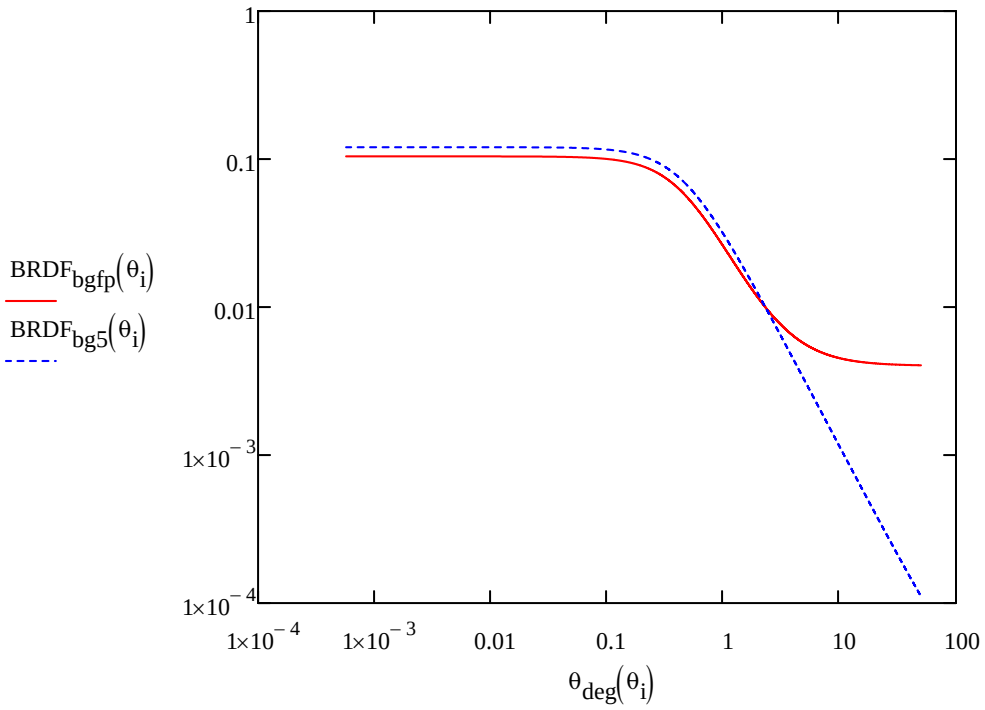
$C_{mr} = 1.6548 \times 10^4$

large angle BRDF, fire polish, sr^-1 $BRDF_{bgfp\theta 2} := 4 \cdot 10^{-3}$

BRDF function, sr^-1 $BRDF_{bgfp}(\theta_i) := \frac{BRDF_0}{\left(1 + C_{mr} \cdot \theta_i^{\,2}\right)^{\beta}} + BRDF_{bgfp\theta 2}$

$\theta_{deg}(\theta_i) := \theta_i \cdot \frac{180}{\pi}$

$\theta_i := 0, 0.00001 .. 10 \cdot \theta_2$



$$\theta_t := 0$$

$$\theta_i(\theta_t, \theta_{xy}) := \operatorname{acos}(\cos(\theta_{xy}) \cdot \cos(\theta_t))$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) := \int_0^{\theta_{xymaxedge}} \left[\int_{2\cdot\theta_i(\theta_t,\theta_{xy})-\frac{w_{itmar0}}{l_{itmar0}}}^{2\cdot\theta_i(\theta_t,\theta_{xy})+\frac{w_{itmar0}}{l_{itmar0}}} BRDF_{bgfp}(\theta_s + 2\cdot\theta_i(\theta_t,\theta_{xy})) \cdot \sqrt{w_{itmar}} \right]$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) = 2.883 \times 10^{-13}$$

$$P_{bsbafedgebgfps} := I_{itm}(r_{bsellipx}, 0) \cdot A_{bsbafebg} \cdot S_{edgebg}(\theta_t, BRDF_{bgfp})$$

$$P_{bsbafedgebgfps} = 3.2273 \times 10^{-14}$$

intercepton efficiency for ITM power

$$\eta_{bsellbafedgebg} := \frac{P_{bsbafedgebgfps}}{P_{itm}}$$

$$\eta_{bsellbafedgebg} = 2.8372 \times 10^{-18}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafedgebgfp} := TF_{itmar} \cdot \left(\frac{P_{bsbafedgebgfps}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsbafedgebgfp} = 1.3145 \times 10^{-25}$$

ratio of edge scatter to baf scatter

$$\frac{DN_{bsbafedgebgfp}}{DN_{bsellbafbg}} = 0.2116$$

Total BG baf scatter power, W

$$P_{bsbafbgfpts} := P_{bsellbafbgs} + P_{bsbafedgebgfps}$$

$$P_{bsbafbgfpts} = 7.5273 \times 10^{-13}$$

total black glass baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafbgfpt} := TF_{itmar} \cdot \left(\frac{P_{bsbafbgfpts}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsbafbgfpt} = 6.3485 \times 10^{-25}$$

ratio of SS to BG baf scatter

$$\frac{DN_{bsellbafss}}{DN_{bsbafbgfpt}} = 17.8617$$

=

$$\sqrt{0^2 - \left[l_{itmar0}\cdot\left(\theta_s - 2\cdot\theta_i\left(\theta_t, \theta_{xy}\right)\right)\right]^2}\cdot\frac{l_{itmar0}}{l_{itmar0}^2}\,d\theta_s\Bigg]\cdot\cos\left(\theta_{xy}\right)\,d\theta_{xy}$$