LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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SURF Progress Report 2 -Modeling the Calibrated Response of the Advanced LIGO Detectors

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Abstract

The goal of LIGO is to detect and study gravitational waves, which are as yet undetected fundamental prediction of general relativity. Specifically the work focused on calibrating the gravitational wave strain data channel on the new Advanced LIGO detectors. This project is to develop a model focusing on the optical response part of the sensing chain element. These models are created with high attention to detail from real world disturbances. LIGO deals with gravitational waves that produce movement several orders of magnitude smaller than an atomic nucleus in the detectors. For precision of these measurements to be possible, understanding of the effect of external sources is of paramount importance. The end result of the project was a thorough examination of how real world changes to the detectors will affect the calibration in an error propagation study. This will improve the performance of the detector when brought into operation as well as ensure that real world changes are measured well enough to achieve accurate calibration.

1 Introduction

Gravitational waves are fundamental predictions of general relativity. They consist of waves of space-time curvature created by accelerating mass quadrupole moments. They are transverse waves that arent readily scattered or absorbed due to the relative weakness of the gravitational force. Any time mass accelerates it creates gravitational waves, even if its something as small and slow as a bird flapping its wings. However, these gravitational waves are too small to detect. The sources that LIGO hopes to detect are astrophysical sources capable of producing much stronger waves with more energy. These include, but are not necessarily limited to, collisions of binary neutron stars, supernova, spinning neutron stars, and the cosmic gravitational wave background left over from moments after the big bang. These waves are thought to propagate at the speed of light. If it turns out that they do not it would imply that the fundamental force particle of gravity, the graviton, has mass. Further studies of the gravitational waves could reveal an entire scope of science determining their causes and characteristics as well as use in gravitational wave astronomy.

aLIGO uses 4km dual recycled Fabry-Perot Michelson interferometers to detect these waves. With a 200W laser and the resonance of the cavities it is possible to bottle up about 800 kilowatts within the arms. This makes the arms incredibly sensitive to the smallest change in the position of the end test mass which would put the cavity off resonance. When this happens the total destructive interference at the dark port is interrupted and some light leaks through. This light is measured to provide the actuation functions of the arms the data they need to move the mirrors back into resonance. This also provides the signal of the passing gravitational wave.

The differences between aLIGO and the initial LIGO include, but are not limited to the following. The sensing system uses different electronics and DC readout. The actuator for Advanced LIGO contains a complex quadruple pendulum and the optical plant is now a more complex dual-recycled interferometer. Additionally, calibration done in the time domain will have to be much more detailed than initial LIGO. It also uses non-static FIR filters to capture changes in the detector. These changes and more will be elements to understand and study in the progression of the project and development of new filter designs. Also, a greater understanding of the physics of general relativity and gravitational waves will be helpful in pursuit of these goals.

2 Objectives

The goal of this project is to improve the model for the gravitational wave detector in Advanced LIGO. The focus will be on modeling the sensing function and the digital filters, which describes how the interferometer responds to differential changes in arm lengths and how that response is digitized. The first step consisted of learning how to work in the modeling environment as well as understanding the model itself. This was achieved through various online tutorials for Matlab/Simulink provided by the Mathworks website as well as direct experimentation with the model developed by Jeff Kissel. The next step was to learn as much as possible about the principles of LIGO and gravitational waves. This is an ongoing process throughout the summer as it is important to not only access the output of the model but also to understand what it means. Now that enough information on the subject has been learned production of plots from the model and available equations has begun. Ideally, enough will be learned to intuitively interpret changes in the data as a result to model perturbations as this will lead to more effective use of the model and other resources.

Moving forward, the goal will be to construct a subsystem within the model which reconstructs the gravitational wave strain data by inverting the response function of the detector. Upon the completion of this subsystem it will be tested for errors and perturbations arising from unaccounted changes to the front end systems. This will provide estimation of error for offline systems after the aLIGO begins observations.

3 Approach

The first task given was to justify the simplification of the dual-recycled Fabry-Perot Michelson into a single Fabry-Perot cavity. Then the reflectivity and transmissivity of the signal mirror were calculated using the equations:

$$r_{cm} = r_{ITM} - \frac{t_{ITM}^2 r_{sm} e^{-i\phi}}{1 - r_{ITM} r_{sm} e^{-i\phi}} \tag{1}$$

$$\phi = 2kl_s = \frac{4\pi l_s (f_{carr} + f_{sig})}{c} \tag{2}$$

$$R + T + L = 1 \tag{3}$$

$$[R, T] = [r^2, t^2] (\text{For power and energy respectively}) \tag{4}$$

$$\tag{5}$$

The reflectivity of the signal mirror was found to be .65. Figure 1 was generated also using these equations.

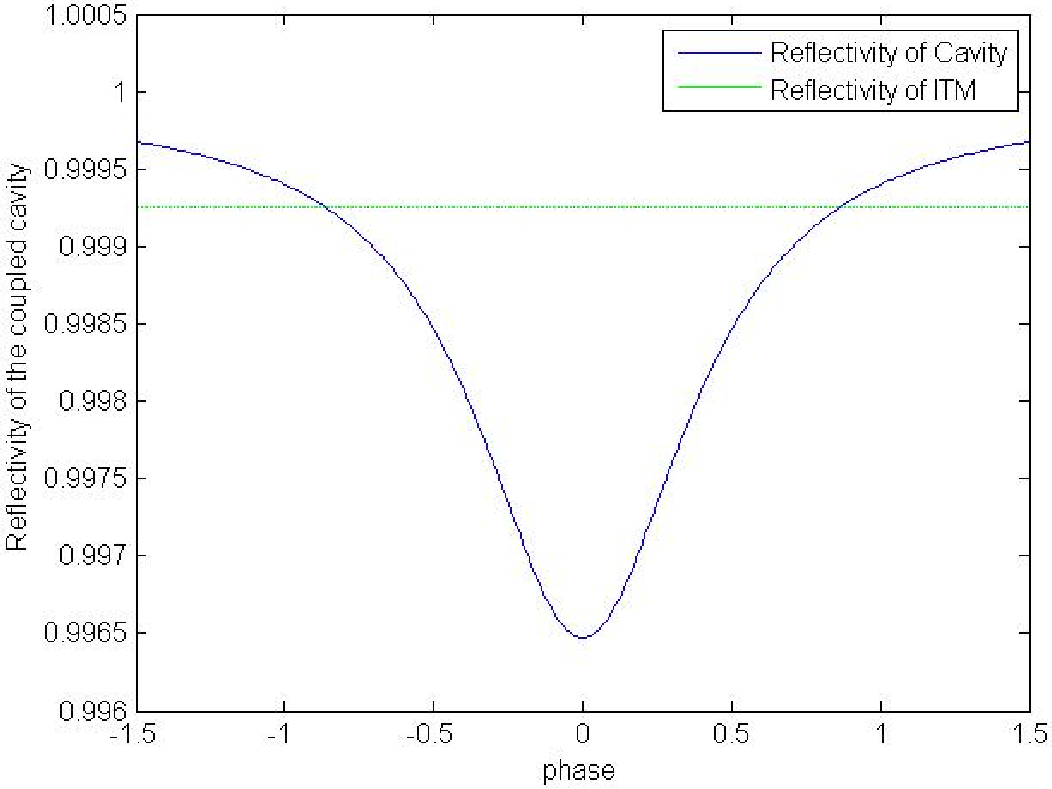

Figure 1: Reflectivity of the coupled cavity plotted against the phase.

Figure 3 shows the existing DARM model loop as well as the inverse response function block being developed [1]. This function is derived as such:

$$\begin{aligned}
&R = (1+G)/C \\
&R^{-1} = C/(1+G) \\
&G = CAD \\
&e = h(1/(1+G))C \\
&s = (G/(1+G))(1/A)h \\
&h = e/((1/(1+G))C) && (6) \\
&h = ((1+G)/G)As && (7)
\end{aligned}$$

At high freqencies the open loop gain (G) is small, such that 1 is much greater than G. This allows the extraction of h from e/C by Eqn 5. At low freqencies G is much larger such that G/(1+G) = 1. In this case h is extracted from sA by Eqn 6. By superposition:

$$h = (e/C) + (sA) \tag{8}$$

Substituting Eqn. 5 and Eqn. 6 into Eqn. 7 shows that this is rigourously true. This is demonstrated graphicallyl in Figure 2, where e is the error, s the control, and h the strain. Therefore, the strain is reconstructed by dividing the error signal by the sensing function and adding the product of the control signal and actuation function. This is done with no dependance on the digital filters. The error and control signals, as well as the reconstructed strain, are shown as functions of frequency in Figure 2 [6].

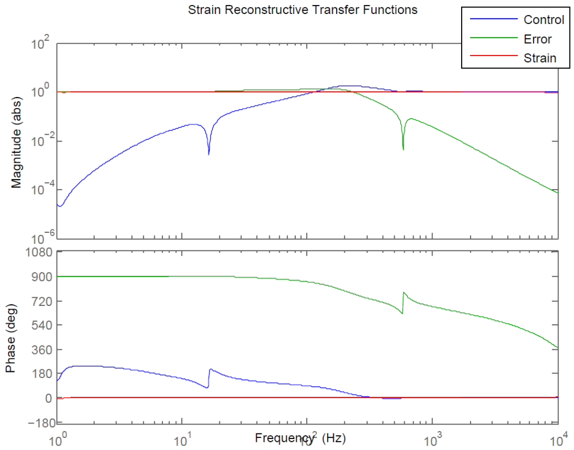

Figure 2: Strain Reconstructive Functions

Along with these equations, a new DARM model was adopted, created by Rana Adhikari. This was intended to be a simpler version of the Kissel model, and therefore easier to work with for the purposes of this project.

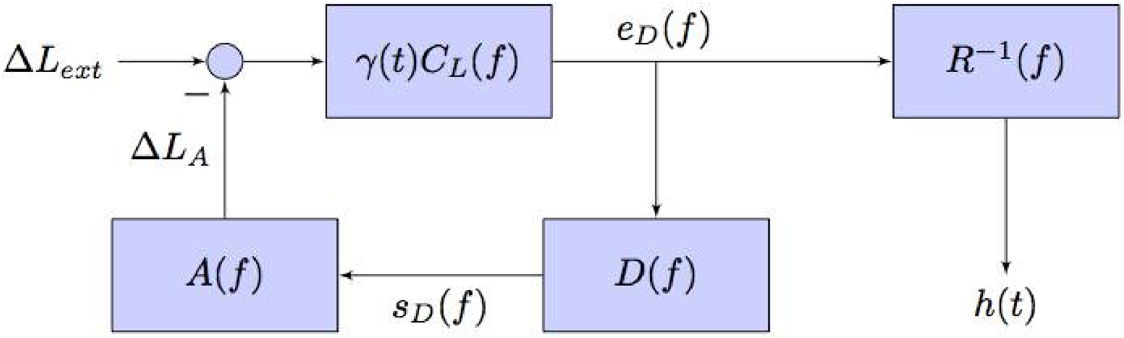

Figure 3: The DARM model loop with the inverse response block included (R^{-1})

The construction of this block is still in progress. Obstacles involved attempts to invert the sensing function. Early attempts met with exponentially increasing signals going to infinity

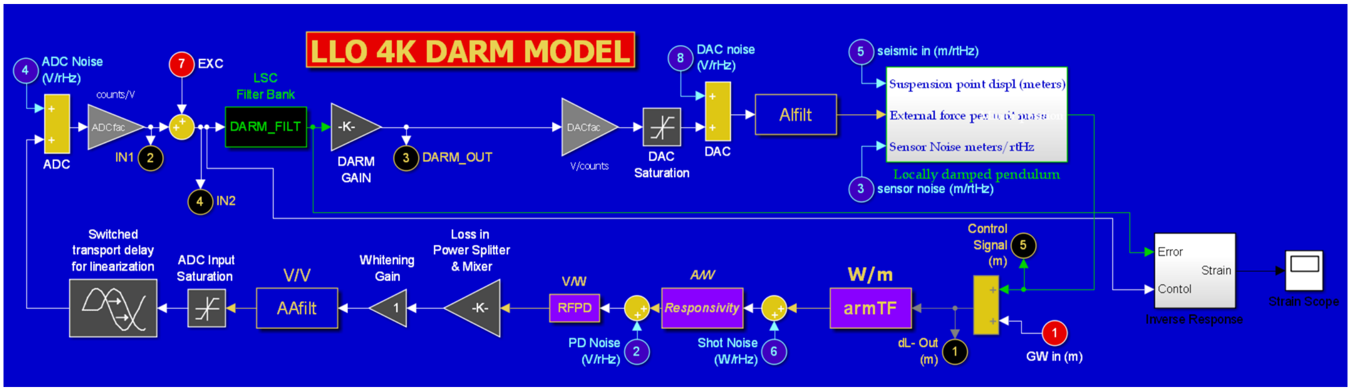

Figure 4: DARM Model developed by Rana Adhikari

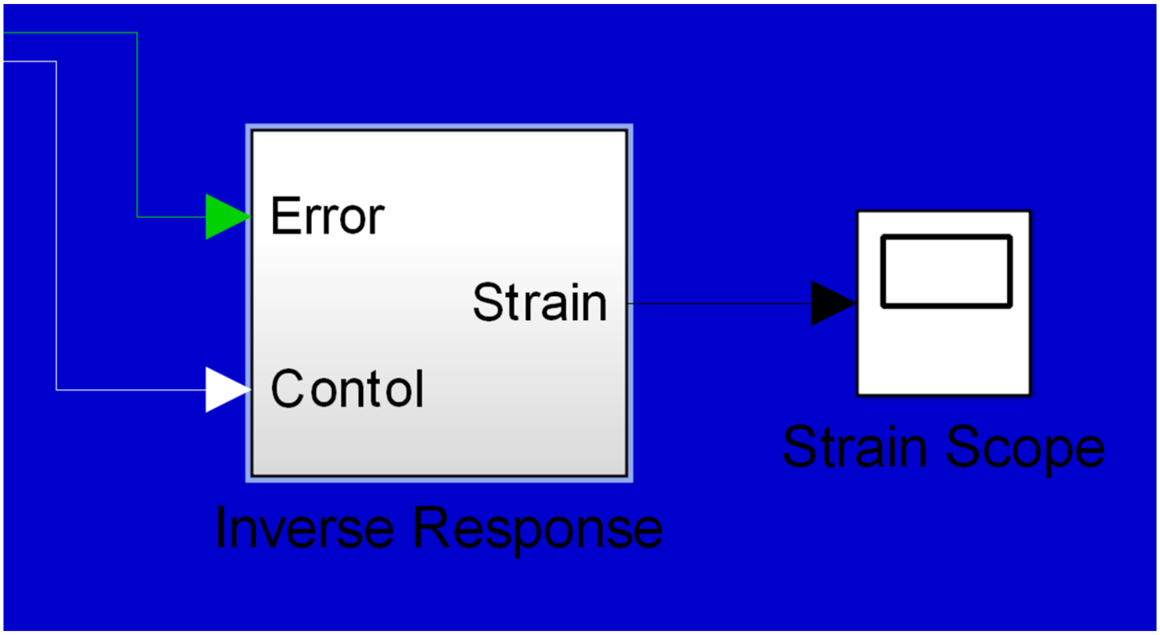

Figure 5: Inverse Response Block

with any input except zero. This was solved by substituting a simplified version of the sensing function, in the form of a zero-pole-gain function, into the model. This function was designed to be invertible and replicate the behavior of the sensing function within the LIGO frequency band. This resolved the exponential increase problem.

However, the output signal was still not satisfactory. The model is designed as a linear representation of a complex system, so a sine wave input should generate a sine wave output. This output may be shifted in phase or magnitude, but should not be fundamentally different if the inverse response function is properly constructed. In this case, the sine wave input generated a chirp-signal like strain reading with alternating positive and negative steps between chirps.

It was recently found that the analog to digital converter saturation block was involved in the creation of the chirp signal. This conclusion arose from an observation that the magnitude of the steps within the chirp signal was proportional to the saturation points set in the block. This problem is solved by inputting a signal of sufficiently small amplitude so the saturation point is not reached. This causes an output sine wave of shifted magnitude, with transient non-sinusoidal behavior in the first milli-second of simulations in the time domain. This transient behavior is currently suspected of resulting from the transport delay, and studies are ongoing as to whether the effect is negligible or negatable.

4 Project Schedule

1. June 19th- begin work on learning and understanding existing model
2. July 9th- Turn in first progress report
3. Characterize the calibrated detector response of the model
4. Construct Inverse Response function block
5. August 2nd- Turn in second progress report
6. Preform error analysis of offline strain reconstruction function
7. August 22nd- present final project report
8. August 23rd- end of project

References

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[3] Jeff Kissel, *CALIBRATING AND IMPROVING THE SENSITIVITY OF THE LIGO DETECTORS.* (2010).

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[6] Rana Adhikari