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ITM Elliptical Baffle--Black Glass Hybrid Design Scattering Analysis

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CHANGE LOG

Date, version	Summary of Changes
6/27/13 V1	• New document
8/15/13 V2	• Increased BRDF black glass @ 57 deg to 4E-5 sr^-1 • Updated scattering solid angle • Increased rough cut hole edge area

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1 Introduction

A hybrid design for the ITM Elliptical Baffle is proposed, in which a plate of black glass is attached to the existing center skin to cover the stainless steel (SS) surface; the only modification to the SS skin is to add several clearance holes for attachment bolts.
The cut edge of the beam hole in the black glass is a major scattering element and the edge must be fire-polished so that this is not a dominant scattering surface.
In the following analysis, the scattered light displacement noise is calculated for two cases: 1) all oxidized polished stainless steel surfaces, 2) the hybrid structure as described above. Greater than a factor 70 times reduction in scattered light displacement noise could be achieved with the Hybrid baffle.

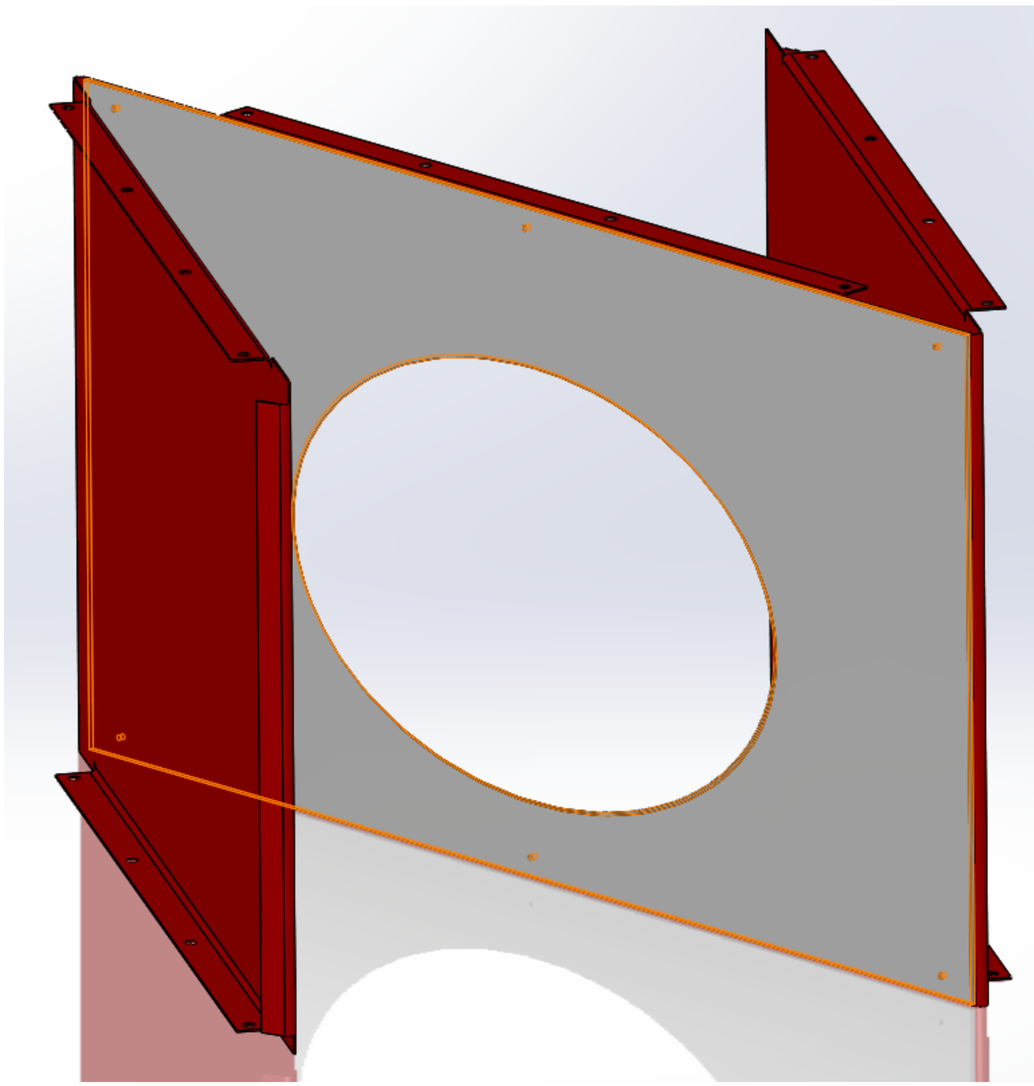

Figure 1: ITM Elliptical Baffle with Black Glass Plate Insert

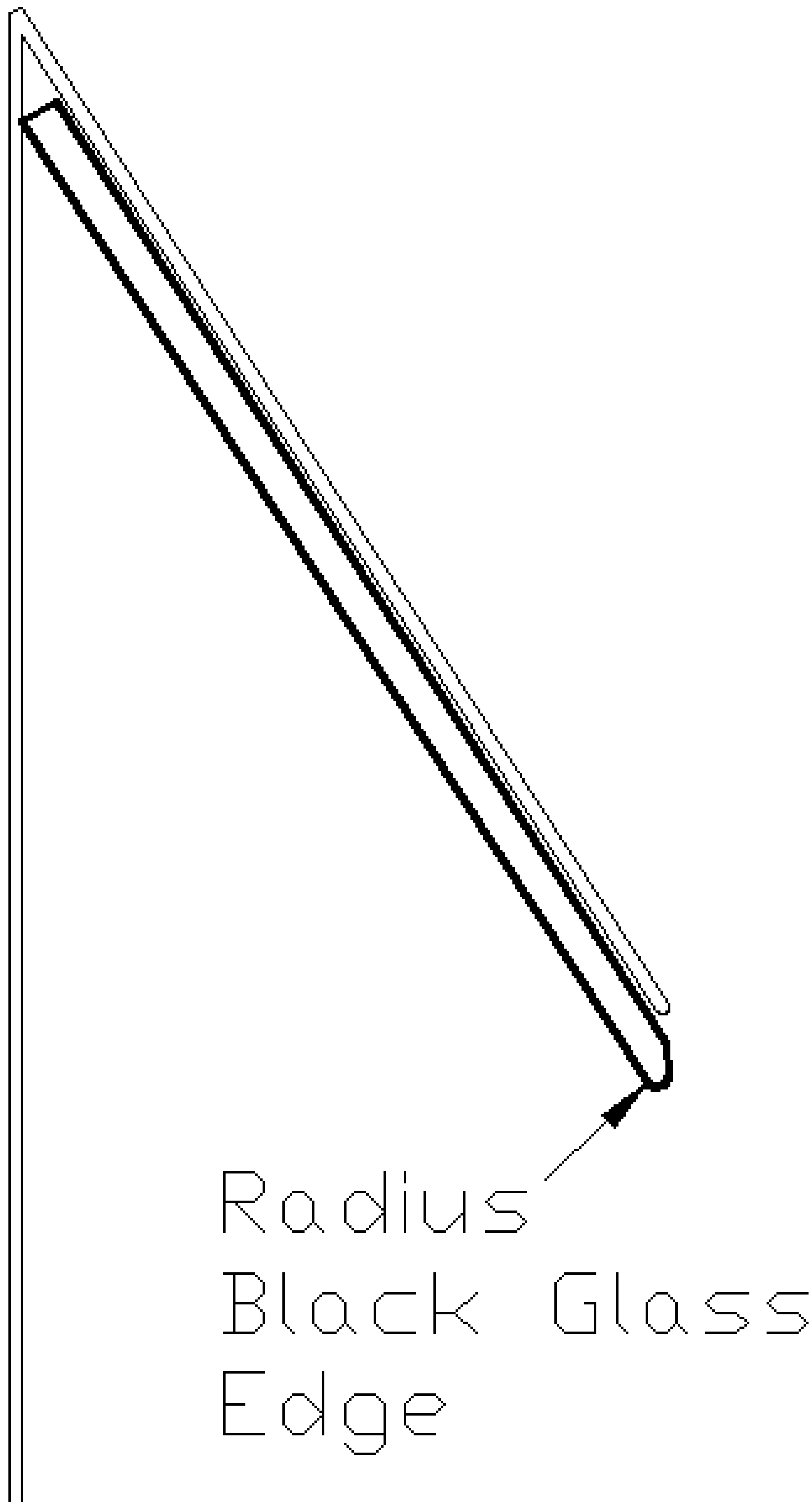

Figure 2: Cross Section of Baffle, Showing Beveled Edge of Black Glass Hole

2 Scattering Analysis

The power recycling cavity beam passes from the ITM through the hole in the ITM Elliptical Baffle, and the wings of the Gaussian beam get clipped by the baffle hole.

arm cavity power, W	$P_a = 8.125\times 10^5$	
Transmissivity of ITM HR	$T_{itmhr} := 0.014$	
power exiting from ITM toward elliptical baffle, W	$P_{itm} := P_a \cdot T_{itmhr}$	$P_{itm} = 1.1375\times 10^4$
power hitting the ITM elliptical baffle from ITM side, W	$P_{itmellbaf}(0,0) = 0.5342$	

2.1 Oxidized Stainless Steel Baffle

2.1.1 BRDF of Oxidized Stainless Steel

The incident angle of the light hitting the surface of the ITM Elliptical Baffle is 57 deg, and the incident angle hitting the edge of the central plate of the baffle is centered at 0 deg.

The BRDF was measured for oxidized polished stainless steel at incidence angles of 57 deg and 3 deg as a function of the angle away from the specular directions, as shown below. The BRDF data is approximately symmetrical for plus and minus angles about the specular direction.

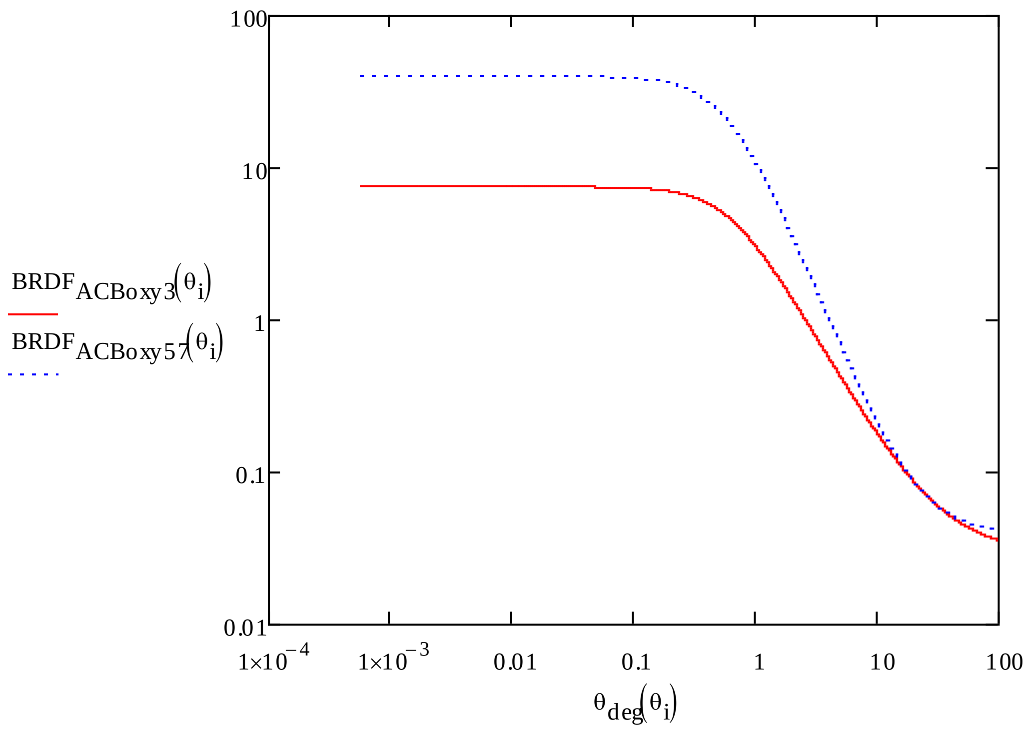

Figure 3: BRDF of Oxidized Stainless Steel

2.1.2 Scatter by Elliptical Baffle Portion

Power passing through the elliptical Baffle hole, W

$$P_{itmellbaftran}(\delta x,\delta y) := \left(\int_{\delta y-b}^{\delta y+b} \int_{\delta x-a\cdot\sqrt{1-\frac{y^2}{b^2}}}^{\delta x+a\cdot\sqrt{1-\frac{y^2}{b^2}}} I_{itm}(x,y)\,dx\,dy \right)$$

$$P_{itmellbaftran}(0,0) = 1.1374\times 10^4$$

power hitting the ITM elliptical baffle from ITM side, W

$$P_{itmellbaf}(\delta x,\delta y) := P_{itm} - P_{itmellbaftran}(\delta x,\delta y)$$

$$P_{itmellbaf}(0,0) = 0.5342$$

Power scattered into IFO mode
from both arms, W

$$P_{itmellbafsss} := \sqrt{2} \cdot P_{itmellbaf}(0,0) \cdot BRDF_{ellbafss} \cdot \Delta_{itma}$$

$$P_{itmellbafsss} = 1.1735 \times 10^{-10}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmellbafss} := TF_{itmar} \cdot \left(\frac{P_{itmellbafsss}}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmellbafss} = 7.671 \times 10^{-24}$$

2.1.3 Scatter by SS Baffle Hole Edge

horizontal edge, m

$$x := a$$

vertical edge, m

$$y := 0$$

exitance function from ITM at edge, W/m^2

$$I_{itm}(x,y) := 2 \cdot \frac{4 \cdot P_{0rc}}{\pi \cdot w_{itm}^{2}} \cdot e^{-2 \cdot \left(\frac{x^2+y^2}{w_{itm}^{2}}\right)}$$

$$I_{itm}(x,y) = 358.2562$$

maximum width of exposed edge, m

$$w_{itmbafe} := 2 \cdot r_{edgess}$$

Radius of baffle hole, m

$$R_{itmbaf} := a$$

exposed area of baffle hole edge, m^2

$$A_{itmbafess} := \int_{-R_{itmbaf}}^{0} 2\cdot\sqrt{R_{itmbaf}^2 - x^2}\,dx - \int_{-R_{itmbaf}+w_{itmbafe}}^{0} 2\cdot\sqrt{R_{itmbaf}^2 - (x - w_{itmbafe})^2}\,dx$$

$$A_{itmbafess} = 1.1379\times 10^{-5}$$

power incident on ITM Baf hole edge, W

$$P_{itmbafedgess} := I_{itm}(a,0)\cdot A_{itmbafess}$$

$$P_{itmbafedgess} = 4.077\times 10^{-3}$$

power scattered from two ITM Ellip Baf hole edge toward ITM, W

$$P_{itmbafedgesss} := \sqrt{2}\cdot P_{itmbafedgess}\cdot BRDF_{edgess}\cdot \Delta_{itma}$$

$$P_{itmbafedgesss} = 2.9855\times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafedgess} := TF_{itmar}\cdot\left(\frac{P_{itmbafedgesss}}{P_{psl}}\right)^{0.5}\cdot x_{baf}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{itmbafedgess} = 1.2235\times 10^{-24}$$

ratio of edge scatter to baf scatter

$$\frac{DN_{itmbafedgess}}{DN_{itmellbafss}} = 0.1595$$

2.1.4 Total Scatter by SS ACB

Total SS baf scatter power, W

$$P_{itmbafssts} := P_{itmellbafsss} + P_{itmbafedgesss}$$

$$P_{itmbafssts} = 1.2034\times 10^{-10}$$

total SS baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafsst} := TF_{itmar} \cdot \left(\frac{P_{itmbafssts}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafsst} = 7.768 \times 10^{-24}$$

2.2 Hybrid Stainless Steel Baffle with Black Glass Panel

2.2.1 BRDF of Black Glass

The BRDF was measured for black glass at incidence angles of 57 deg and 5 deg, as a function of the angle away from the specular direction.
As will be shown in the following section, the cut edges of the beveled black glass hole of the baffle are the dominant scattering surfaces, and it will be necessary to fire-polish the cut edge of the glass. An estimated BRDF of the fire-polished edge at 0 deg was constructed, based on the BRDF data at 5 deg; the two free parameters BRDFbgfpθ2 and BRDF0 can be adjusted to represent the degree of roughness of the fire-polished cut edge.

BRDF Black Glass fire polish (empirical estimate)

break-over angle, rad $\theta_1 := 0.5 \frac{\pi}{180} = 8.7266 \times 10^{-3}$

micro-roughness angle, rad $\theta_2 := 5 \cdot \frac{\pi}{180} = 0.0873$

max BRDF, sr^-1 $BRDF_0 := 0.1$

final slope modifier $\beta := 0.8$

micro-roughness constant

$$C_{mr} := \frac{2^{\frac{1}{(\beta)}} - 1}{\theta_1^{\,2}}$$

$$C_{mr} = 1.6548 \times 10^{4}$$

large angle BRDF, fire polish, sr^-1 $BRDF_{bgfp\,\theta 2} := 4 \cdot 10^{-3}$

BRDF function, sr^-1 $BRDF_{bgfp}(\theta_i) := \frac{BRDF_0}{\left(1 + C_{mr} \cdot \theta_i^{\,2}\right)^{\beta}} + BRDF_{bgfp\,\theta 2}$

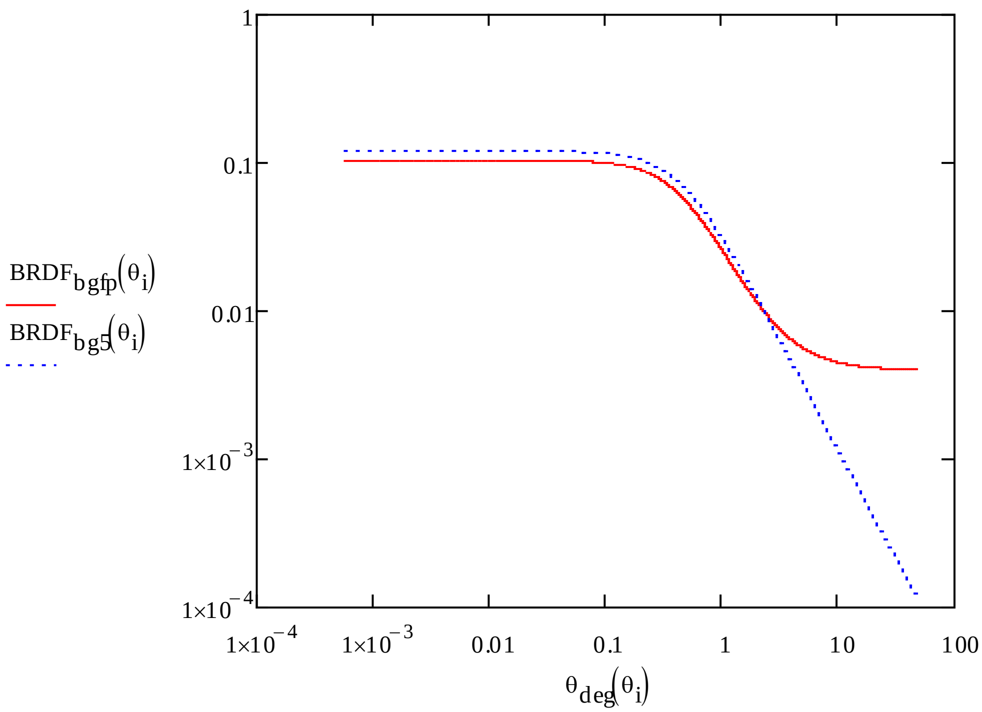

Figure 4: Measured BRDF @ 5 deg and Estimated BRDF of Fire-polished Black Glass @ 0 Deg Incidence

2.2.2 Scatter from Black Glass Elliptical Baffle Surface

Power scattered into IFO mode
from both arms, W

$$P_{itmellbafbgs} := \sqrt{2} \cdot P_{itmellbaf}(0,0) \cdot BRDF_{ellbafbg} \cdot \Delta_{itma}$$

$$P_{itmellbafbgs} = 1.5647 \times 10^{-13}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmellbafbg} := TF_{itmar} \cdot \left(\frac{P_{itmellbafbgs}}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmellbafbg} = 2.8011 \times 10^{-25}$$

2.2.3 Scatter from Black Glass Baffle Hole Edge

maximum width of exposed edge, m

$$w_{itmbafe} := 2 \cdot r_{edgebgr} \qquad r_{edgebgr} = 2.54 \times 10^{-4}$$

Radius of baffle hole, m

$$R_{itmbaf} = 0.112$$

exposed area of baffle hole edge, m^2

$$A_{itmbafebg} := \int_{-R_{itmbaf}}^{0} 2 \cdot \sqrt{R_{itmbaf}^{2} - x^{2}}\, dx - \int_{-R_{itmbaf}+w_{itmbafe}}^{0} 2 \cdot \sqrt{R_{itmbaf}^{2} - \left(x - w_{itmbafe}\right)^{2}}\, dx$$

$$A_{itmbafebg} = 1.1379 \times 10^{-4}$$

power incident on ITM Baf hole edge, W

$$P_{itmbafedgebg} := I_{itm}(a,0) \cdot A_{itmbafebg}$$

$$P_{itmbafedgebg} = 0.041$$

power scattered from two BG ITM Ellip Baf hole edge toward ITM, W

$$P_{itmbafedgebgrs} := \sqrt{2} \cdot P_{itmbafedgebg} \cdot BRDF_{edgebgr} \cdot \Delta_{itmar}$$

$$P_{itmbafedgebgrs} = 2.9855 \times 10^{-11}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafedgebgr} := TF_{itmar} \cdot \left(\frac{P_{itmbafedgebgrs}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafedgebgr} = 3.8691 \times 10^{-24}$$

ratio of rough cut edge scatter to baf scatter

$$\frac{DN_{itmbafedgebgr}}{DN_{itmellbafbg}} = 13.8131$$

2.2.4 Scatter from Black Glass Baffle Fire-polished Hole Edge

$$\theta_t := 0$$

$$\theta_i(\theta_t, \theta_{xy}) := \mathrm{acos}\left(\cos(\theta_{xy})\cdot\cos(\theta_t)\right)$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) := \int_0^{\theta_{xymaxedge}} \left[\int_{2\cdot\theta_i(\theta_t,\theta_{xy}) - \frac{w_{itmar0}}{l_{itmar0}}}^{2\cdot\theta_i(\theta_t,\theta_{xy}) + \frac{w_{itmar0}}{l_{itmar0}}} BRDF_{bgfp}\left(\theta_s + 2\cdot\theta_i(\theta_t,\theta_{xy})\right)\cdot\sqrt{w_{itmar0}^{\;2} - \left[l_{itmar0}\left(\theta_s - 2\cdot\theta_i(\theta_t,\theta_{xy})\right)\right]^2}\cdot\frac{l_{itmar0}}{l_{itmar0}^{\;2}}\, d\theta_s\right]\cdot\cos(\theta_{xy})\, d\theta_{xy}$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) = 2.883\times 10^{-13}$$

.

$$P_{itmbafedgebgfps} := I_{itm}(a,0)\cdot A_{itmbafebg}\cdot S_{edgebg}(\theta_t, BRDF_{bgfp})$$

$$P_{itmbafedgebgfps} = 1.1753\times 10^{-14}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafedgebgfp} := TF_{itmar} \cdot \left(\frac{P_{itmbafedgebgfps}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafedgebgfp} = 7.6767 \times 10^{-26}$$

ratio of edge scatter to baf scatter $\frac{DN_{itmbafedgebgfp}}{DN_{itmellbafbg}} = 0.2741$

2.2.5 Total Scatter by Hybrid ACB

total black glass baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafbgfpt} := TF_{itmar} \cdot \left(\frac{P_{itmbafbgfpts}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafbgfpt} = 2.9044 \times 10^{-25}$$

2.2.6 Comparison of Hybrid Baffle with Scatter from aLIGO Oxidized SS Baffle

Therefore, adding a black glass panel to the ITM Elliptical Baffle will decrease the scattered light displacement noise by > 26.

ratio of SS to BG baf scatter $\frac{DN_{itmellbafss}}{DN_{itmbafbgfpt}} = 26.4122$