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- LIGO -  
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<b>Technical Note</b>	<b>LIGO-T1300556-v1</b>	Date: 6/17/2013
<b>Radiation Transfer at the Face of a LIGO III Test Mass</b>		
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## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>References</b>	<b>2</b>
<b>3</b>	<b>Analysis</b>	<b>2</b>
<b>4</b>	<b>Process</b>	<b>3</b>
<b>5</b>	<b>Results</b>	<b>4</b>
<b>6</b>	<b>Conclusion</b>	<b>4</b>

## 1 Introduction

This is an attempt to model the transfer of heat due to radiation in a LIGO III system from the warm beam tube and the face of the silicon test mass. This work is meant to expand on the work done by R. Weiss in document T1200093-v1.

## 2 References

1. T1200093-v1
2. A Heat Transfer Textbook 4th Edition, John H. Lienhard IV and John H. Lienhard V, <http://web.mit.edu/lienhard/www/downloadform1.html>

## 3 Analysis

We assume that the beam tube is completely black with an emissivity of one, while the reflective silicon test mass has a emissivity of 0.1. We assume that the beam tube is completely coaxial and parallel with the face of the silicon test mass. We also assume that each surface radiates uniformly in regard to direction. We are focusing solely on the radiation between the beam tube and the face of the test mass with no regard for the other radiative elements of the system, chiefly the heat shield. If we assume that the system is working to specification, we know that the temperature of the test mass is 120 K and that the temperature of the beam tube is 295 K. Finally, we assume that the test mass has a radius of 0.28 meters, that the beam tube has a radius of 0.6225, and that the shortest distance between them is 20 meters.

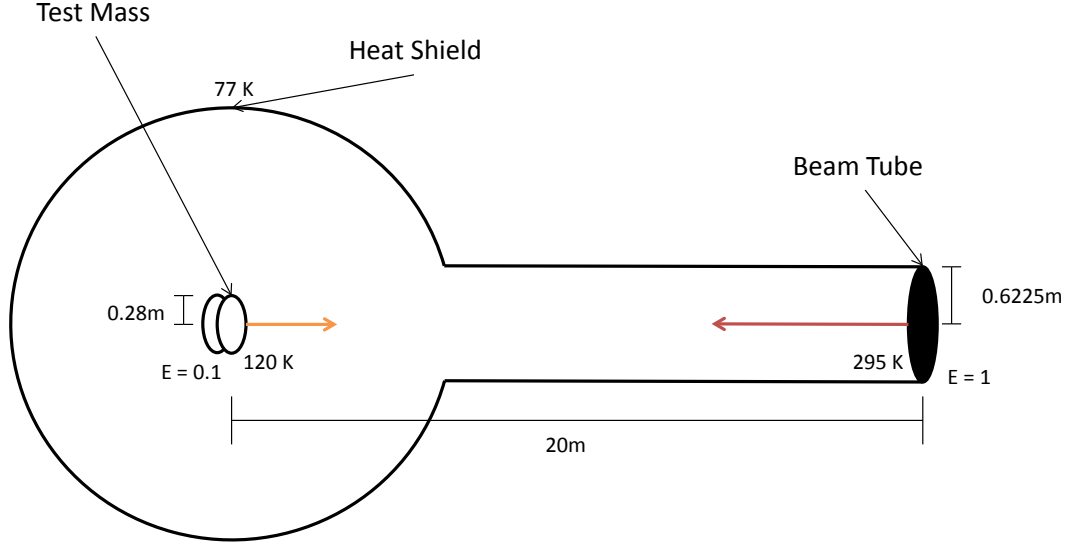


Figure 1: A diagram of the section of LIGO III discussed here

## 4 Process

$$R_i = \frac{r_i}{L} \quad (1)$$

$$R_j = \frac{r_j}{L} \quad (2)$$

$$S_{ij} = 1 + \frac{1 + R_j^2}{R_i^2} \quad (3)$$

$$S_{ji} = 1 + \frac{1 + R_i^2}{R_j^2} \quad (4)$$

$$V_{ij} = \frac{1}{2} \{ S_{ij} - [S_{ij}^2 - 4(\frac{r_j}{r_i})^2]^{\frac{1}{2}} \} \quad (5)$$

$$V_{ji} = \frac{1}{2} \{ S_{ji} - [S_{ji}^2 - 4(\frac{r_i}{r_j})^2]^{\frac{1}{2}} \} \quad (6)$$

We first use these equations to calculate the view factor between our two coaxial parallel disks.  $r$  is the radius, and  $L$  is the distance between the test mass and the beam tube.  $i$  represents the beam tube, and  $j$  represents the test mass. So  $r_i$  is the radius of the beam tube, and  $V_{ij}$  is proportion of total emitted power from the beam tube that will reach the test mass.

$$P_i = E_i T_i^4 A_i * 5.670410^{-8} \quad (7)$$

$$P_j = E_j T_j^4 A_j * 5.670410^{-8} \quad (8)$$

$$P_{ij} = P_i V_{ij} E_j \quad (9)$$

$$P_{ji} = P_j V_{ji} E_i \quad (10)$$

$$Q_{ij} = P_{ij} - P_{ji} \quad (11)$$

$$Q_j = P_{ij} - P_j \quad (12)$$

Here we do our final calculations to determine the Net Heat Transfer between the two objects of interest.  $E_i$  and  $T_i$  are the emissivity and temperature (respectively).  $P_i$  and  $P_j$  are the total power being radiated from each of the objects, while  $P_{ij}$  is the one-way power transfer from the beam tube to the test mass.  $Q_{ij}$  is the net heat transfer between the two bodies, and  $Q_j$  is the net heat gain/loss within the test mass itself.  $5.670410^{-8}$  is the Stefan-Boltzmann Constant.

## 5 Results

After completion of calculation, we arrive at these results (All results in terms of Watts):

The total power emitted from the Beam Tube ( $P_i$ ) is 522.7937

The total power emitted from the Test Mass ( $P_j$ ) is 0.2896

The power emitted by the Beam Tube and absorbed into the Test Mass ( $P_{ij}$ ) is 0.0102

The power emitted by the Test Mass and absorbed into the Beam Tube ( $P_{ji}$ ) is  $2.8023 * 10^{-4}$

The net power transfer between the Beam Tube and the Test Mass ( $Q_{ij}$ ) is 0.0100

The net power gain/loss within the Test Mass ( $Q_j$ ) is -0.2794

## 6 Conclusion

According to this model, the power emitted from the beam tube and absorbed by the test mass is small, so small that it is insignificant compared to the total power emission from the test mass itself.