LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Radiation Transfer at the Face of a LIGO III Test Mass

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Contents

1 Introduction

This is an attempt to model the transfer of heat due to radiation in a LIGO III system from the warm beam tube and the face of the silicon test mass over an arbitrary distance. This work is meant to expand on the work done by R. Weiss in document T1200093-v1.

2 References

1. LIGO-T1200093-v1
   Radiation cooldown simuation done by R. Weiss

2. LIGO-P990007-00-B
   This source provided us with some of the neccesary measurements for this scenario, most importantly the diameter of the beam tube.

3. "A Heat Transfer Textbook", 4th Edition, Lienhard, John H. IV and John H. V, `http://web.mit.edu/lienhard/www/downloadform1.html`
   This source describes the steps required in completing the neccesary calculations for this scenario.

4. "A Catalog of Radiation Configuration Factors", Howell, J.R., `http://www.engr.uky.edu/rtl/Catalog/intro.html`
   This source provides a list of common view factor equations, some of which are applicable to this scenario.

3 Assumptions

We assume that the beam tube is completely black with an emissivity of 1, while the reflective silicon test mass has an emissivity of 0.1 (at 120 K). We also assume that each surface radiates uniformly in regard to direction. We are focusing solely on the radiation between the beam tube and the face of the test mass with no regard for the other radiative elements of the system. If we assume that the system is working to specification, we know that the temperature of the test mass is 120 K and that the temperature of the beam tube is 295 K. Finally, we assume that the test mass has a radius of 0.28 meters and that the beam tube has a radius of 0.6225 m. We will be modeling this scenario as if the warm section of the beam tube is a 2-dimensional disk, and as if it is coaxial and parallel with the face of the test mass at alll times.

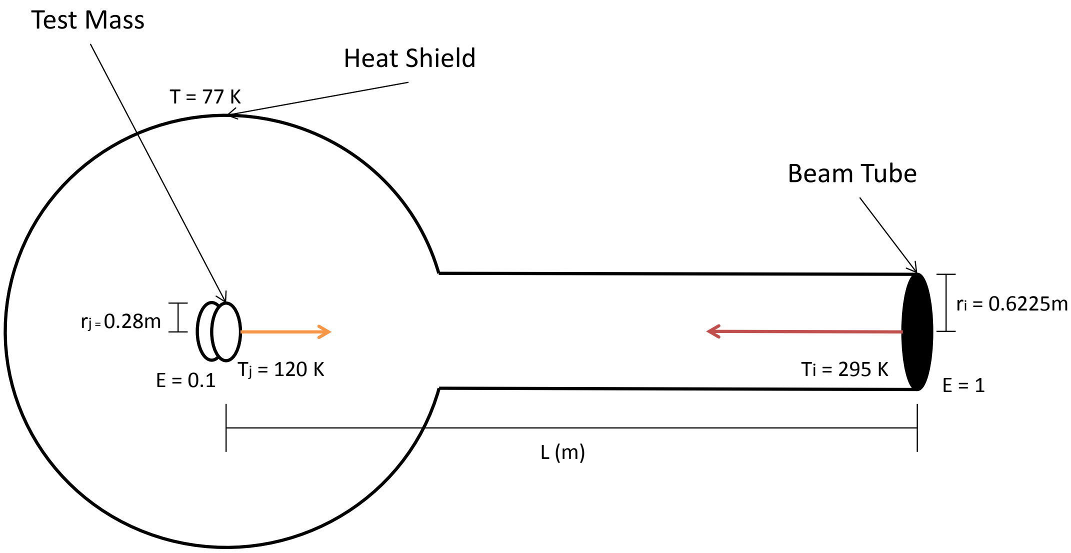

Figure 1: A diagram of the section of LIGO III discussed here

4 Process

From reference [4] we obtain equations (1-6) from which we calculate the view factor (V) between our two coaxial parallel disks. r is the radius, and L is the distance between the test mass and the warm section of the beam tube. i represents the beam tube disk, and j represents the test mass face. So r_i is the radius of the beam tube disk, and V_{ij} is the proportion of total emitted power from the beam tube that will reach the test mass.

$$R_i \equiv r_i/L \tag{1}$$

$$R_j \equiv r_j/L \tag{2}$$

$$S_{ij} \equiv 1 + \frac{1+R_j^2}{R_i^2} \tag{3}$$

$$S_{ji} \equiv 1 + \frac{1+R_i^2}{R_j^2} \tag{4}$$

$$V_{ij} = \frac{1}{2}\{S_{ij} - [S_{ij}^2 - 4(\frac{r_j}{r_i})^2]^{\frac{1}{2}}\} \tag{5}$$

$$V_{ji} = \frac{1}{2}\{S_{ji} - [S_{ji}^2 - 4(\frac{r_i}{r_j})^2]^{\frac{1}{2}}\} \tag{6}$$

In equations (7-12) we determine the net power transfer between the face of the test mass and the warm section of the beam tube, as well as the net power change within the test mass. E_i , T_i and A_i are the emissivity, temperature and face area (respectively). P_i and P_j are the total power being radiated from each of the objects, while I_{ij} is the total incident power transfer on the test mass from the beam tube disk face. P_{ij} is the one-way power transfer from the beam tube to the test mass. Q_{ij} is the net power transfer between the two bodies, and Q_j is the net heat gain/loss within the test mass itself. σ is the Stefan-Boltzmann constant ($5.6704 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$).

$$P_i = \sigma E_i T_i^4 A_i \tag{7}$$
$$P_j = \sigma E_j T_j^4 A_j \tag{8}$$
$$I_{ij} = P_i V_{ij} \tag{9}$$
$$P_{ij} = P_i V_{ij} E_j \tag{10}$$
$$P_{ji} = P_j V_{ji} E_i \tag{11}$$
$$Q_{ij} = P_{ij} - P_{ji} \tag{12}$$
$$Q_j = P_{ij} - P_j \tag{13}$$

5 Results

After completion of calculation, we arrive at these results:

The power emitted from the disk face that represents the beam tube (P_i) is **522.7937 W**

The power emitted from the face of the test mass (P_j) is **0.2896 W**

The rest of the results are dependent on the distance between the objects (L), and are graphed here by MATLAB 7.11.0 (R2010b) and labeled with their curves of best fit. We use a best fit curve despite having the original equations becuase writing out the entire equation without the substitution we used above would be unfeasible.

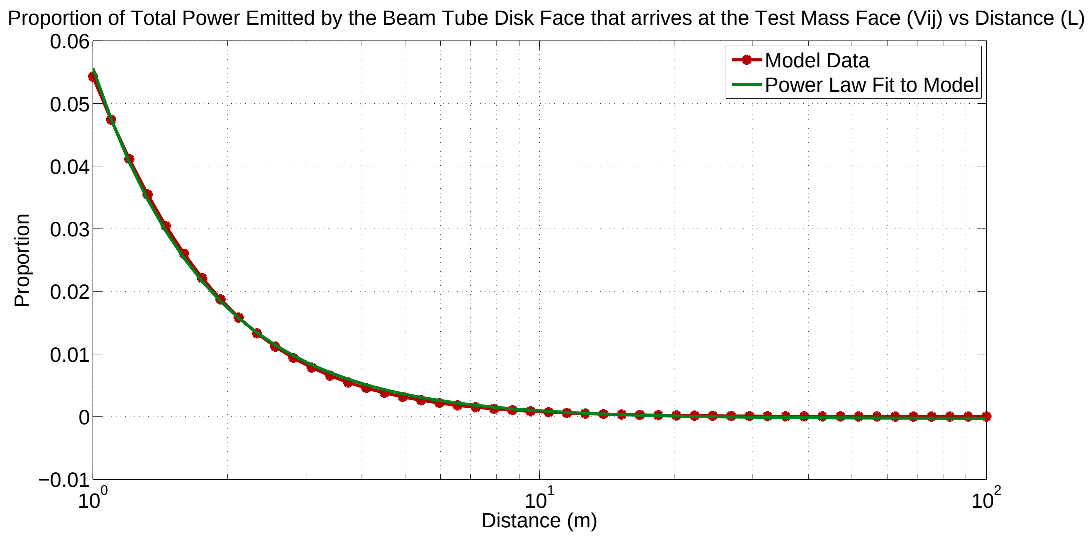

Figure 2: $V_{ij} \approx 0.05588L^{-1.663} - 0.000282$

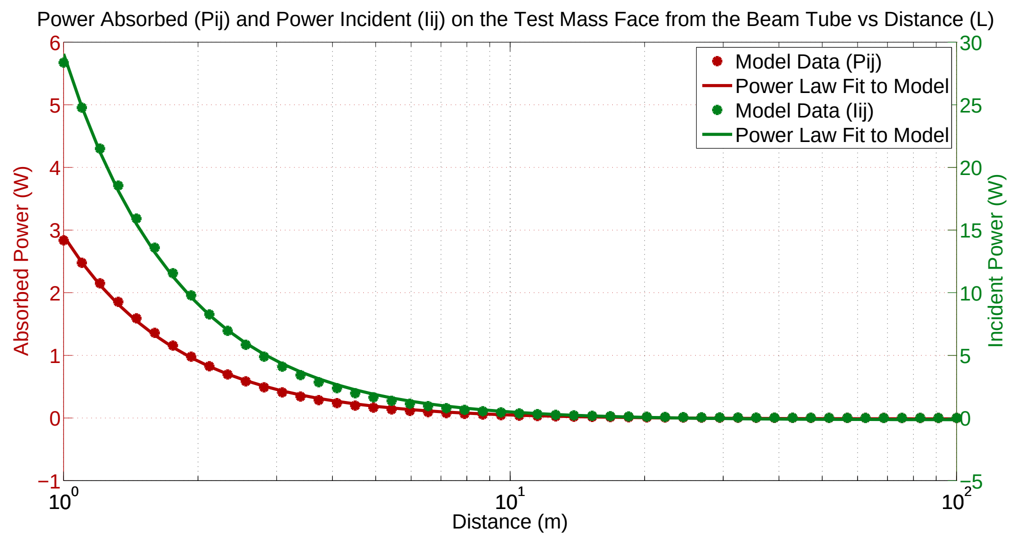

Figure 3: $P_{ij} \approx 2.921L^{-1.663} - 0.01484$; $I_{ij} \approx 29.21L^{-1.663} - 0.1483$

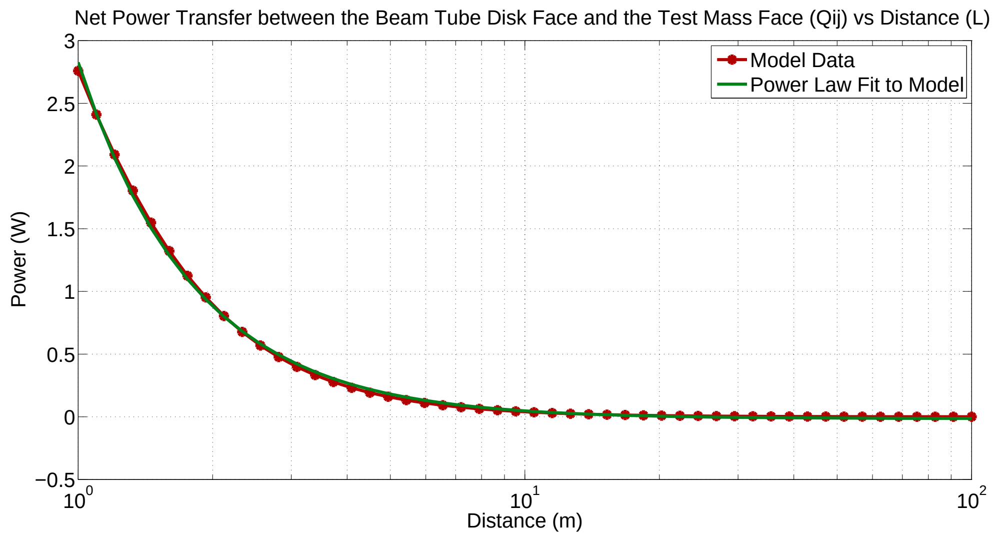

Figure 4: $Q_{ij} \approx 2.841L^{-1.663} - 0.01443$

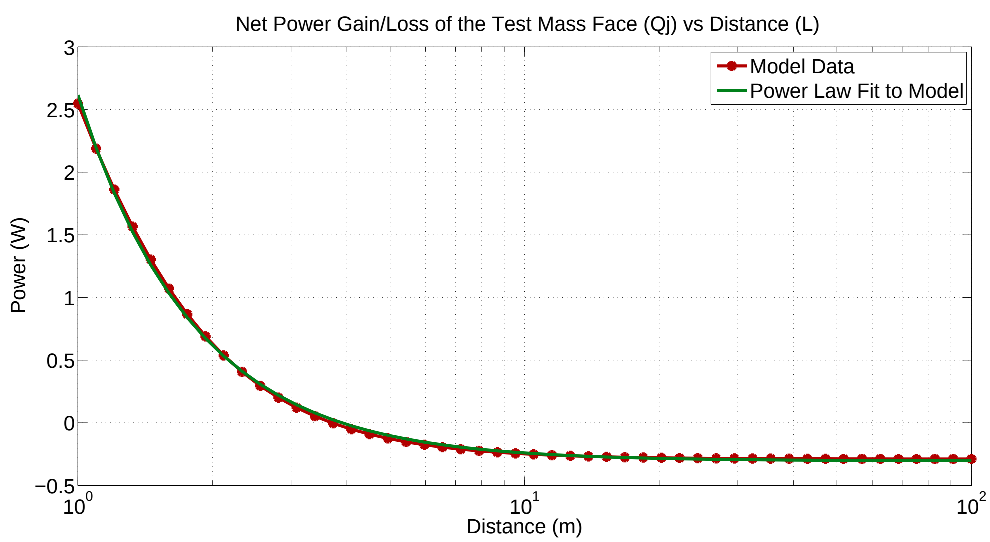

Figure 5: $Q_j \approx 2.921L^{-1.663} - 0.3044$

6 Conclusion

According to this model, the power emmitted from the beam tube and absorbed by the test mass is less than the total emmited power from the test mass itself provided that the tube and mass are greater than 3.8 meters from each other. Aditionally, after the 10 meter mark, the power transfer drops to negligible levels. Our recommendation is that in the final design, the distance between the two objects should be at least 4 meters, although it should be noted that increasing the distance if it is already past 10 meters in an attempt to lower heat transfer is unlikely to have a meaningful effect.