T1300538 BS_wide-angle_scatter_ZEMAX
6/10/13

Displacement noise requirement @ 100 Hz, m/rt Hz	$D_{req} := 1 \cdot 10^{-21}$
Motion of manifold @ 100 Hz, m/rt Hz	$x_{manifold} := 8 \cdot 10^{-11}$
ITM Elliptical Baffle displacement @ 100 Hz, m/rtHz	$x_{itmellbaf} := 1 \cdot 10^{-12}$
ACB displacement @ 100 Hz, m/rtHz	$x_{ACB} := 1 \cdot 10^{-12}$
ISI optical table displacement @ 100 Hz, m/rtHz	$x_{ISI} := 3 \cdot 10^{-14}$
Transfer function @ 100 Hz, ITM AR	$TF_{itmar} := 3.16 \cdot 10^{-11}$
Transfer function @ 100 Hz, BS from SR	$TF_{srbs} := 4.46 \cdot 10^{-11}$
BRDF of BD, sr^-1	$BRDF_{bd} := 0.030$
BRDF of oxidized un-polished steel, sr^-1	$BRDF_{oxiunpolish} := 0.03$
BRDF of chamber wall, sr^-1	$BRDF_{wall} := 0.1$
BRDF of ITM SUS frame @ normal incidence, sr^-1	$BRDF_{itmsus} := 1$
laser wavelength, m	$\lambda := 1.064 \cdot 10^{-6}$
wave number, m^-1	$k := 2 \cdot \frac{\pi}{\lambda}$
ITM beam radius, m	$w_{itm} := 0.053168$

see T070303 with arm cavity gain = 13000

Ref. T070247

Transmissivity of ITM HR $T_{itmhr} := 0.0140$

input laser power, W $P_{psl} := 125$

arm cavity gain $G_{ac} := 13000$

arm cavity power, W $P_a := \frac{P_{psl}}{2} \cdot G_{ac}$

Ref. Hiro e-mail 8/29/11

power in power recycling cavity arm, W $P_{rca} := \frac{P_a \cdot T_{itmhr}}{4}$

$P_{rca} = 2.844 \times 10^{3}$

radius of ITM, m $r_{itm} := 0.170$

hemispherical scattering loss fraction TM wide angle(ref: T070089) $\alpha_{TM} := 10 \times 10^{-6}$

see T070303 with arm cavity gain = 13000

The following data comes from ZEMAX Lambertian scatter sensor data; see /ALIGO/SLC/BS scatter power summary.xlsx

SCATTER PATH LENGTH

distance from BSAR to ITM ELL BAF PLATE1, m $L_{itmellp1} := 0.887$

distance from BSAR to ITM ELL BAF PLATE2, m $L_{itmellp2} := 0.912$

distance from BSAR to ITM ELL BAF PLATE3, m — $L_{itmellp3} := 0.912$

distance from BSAR to BSAR ELL BAF PLATE, m — $L_{bsell} := 0.14$

distance from BSAR to BSC spool +X, m — $L_{bscspoolx} := 1.699$

distance from BSAR to BSC flange +X, m — $L_{bscsflngx} := 1.802$

distance from BSAR to spool BSC2-BSC3, m — $L_{bscspoolbsc2\_3} := 2.508$

distance from BSAR to BSC spool -Y, m — $L_{bscspooly} := 1.403$

distance from BSAR to BSC flange -Y, m — $L_{bscsflngy} := 1.423$

distance from BSAR to BSC2 cylinder, m — $L_{bsccyl} := 1.282$

distance from BSAR to BSC2 top, m — $L_{bsctop} := 1.717$

distance from BSAR to BSC2 bottom, m — $L_{bscbot} := 1.497$

distance from BSAR to HAM4/MC tube, m — $L_{ham4mc} := 1.905$

distance from BSAR to ITM middle frame, m — $L_{itmframe} := 4.714$

distance from BSAR to ITM SUS ring, m — $L_{itmsusring} := 4.739$

distance from BSAR to ACB Plate6, m — $L_{acbp6} := 5.87$

distance from BSAR to WIDE ANGLE BAF BOTTOM LEDGE ITMX, m — $L_{widbot} := 5.89$

distance from BSAR to WIDE ANGLE BAF side ITMX, m — $L_{widside} := 5.9$

BS INCIDENT ANGLE

incident angle from BSAR to ITM ELL BAF PLATE1, rad	$\theta_{itmellp1} := .79$
incident angle from BSAR to ITM ELL BAF PLATE2, rad	$\theta_{itmellp2} := 1.02$
incident angle from BSAR to ITM ELL BAF PLATE3, rad	$\theta_{itmellp3} := 1.02$
incident angle from BSAR to BSAR ELL BAF PLATE, rad	$\theta_{bsell} := 1.33$
incident angle from BSAR to BSC spool +X, rad	$\theta_{bscspoolx} := -0.17$
incident angle from BSAR to BSC flange +X, rad	$\theta_{bscsflngx} := -0.26$
incident angle from BSAR to spool BSC2-BSC3, rad	$\theta_{bscspoolbsc2\_3} := -0.45$
incident angle from BSAR to BSC spool -Y, rad	$\theta_{bscspooly} := 0.09$
incident angle from BSAR to BSC flange -Y, rad	$\theta_{bscsflngy} := 0.24$
incident angle from BSAR to BSC2 cylinder, rad	$\theta_{bsccyl} := -0.05$
incident angle from BSAR to BSC2 top, rad	$\theta_{bsctop} := 0.88$
incident angle from BSAR to BSC2 bottom, rad	$\theta_{bscbot} := 0.7$
incident angle from BSAR to HAM4/MC tube, rad	$\theta_{ham4mc} := 0.24$
incident angle from BSAR to ITM middle frame, rad	$\theta_{itmframe} := -0.78$
incident angle from BSAR to ITM SUS ring, rad	$\theta_{itmsusring} := -0.74$

incident angle from BSAR to ACB Plate6, rad $\theta_{acbp6} := -0.7$

incident angle from BSAR to WIDE ANGLE BAF BOTTOM LEDGE ITMX, rad $\theta_{widbot} := -0.68$

incident angle from BSAR to WIDE ANGLE BAF side ITMX, rad $\theta_{widside} := -0.68$

The calculation is done for BSAR surface scatter in the X-direction; assume that the same scatter occurs from the BSHR surface in the Y-direction

FRACTION of LAMBERTIAN SCATTER FROM COC HITTING SURFACE

fractional power from BSAR to ITM ELL BAF PLATE1, rad $PF_{itmellp1} := 0.0002$

ifractional power from BSAR to ITM ELL BAF PLATE2, rad $PF_{itmellp2} := 0.0003$

fractional power from BSAR to ITM ELL BAF PLATE3, rad $PF_{itmellp3} := 0.023$

fractional power from BSAR to BSAR ELL BAF PLATE, rad $PF_{bsell} := 0.0005$

fractional power from BSAR to BSC spool +X, rad $PF_{bscspoolx} := 0.0404$

fractional power from BSAR to BSC flange +X, rad $PF_{bscsflngx} := 0.0255$

fractional power from BSAR to spool BSC2-BSC3, rad $PF_{bscspoolbsc2\_3} := 0.0292$

fractional power from BSAR to BSC spool -Y, rad $PF_{bscspooly} := 0.0972$

ifractional power from BSAR to BSC flange -Y, rad $PF_{bscsflngy} := 0.0579$

fractional power from BSAR to BSC2 cylinder, rad $PF_{bsccyl} := 0.3237$

fractional power from BSAR to BSC2 top, rad $PF_{bsctop} := 0.0571$

fractional power from BSAR to BSC2 bottom, rad $PF_{bscbot} := 0.1608$

fractional power from BSAR to HAM4/MC tube, rad $PF_{ham4mc} := 0.1521$

fractional power from BSAR to ITM middle frame, rad $PF_{itmframe} := 0.0041$

fractional power from BSAR to ITM SUS ring, rad $PF_{itmsusring} := 0.0006$

fractional power from BSAR to ACB Plate6, rad $PF_{acbp6} := 0.0011$

ifractional power from BSAR to WIDE ANGLE BAF BOTTOM LEDGE ITMX, rad $PF_{widbot} := 0.0001$

fractional power from BSAR to WIDE ANGLE BAF side ITMX, rad $PF_{widside} := 0.0002$

INCIDENT POWER

ITM ELL BAF PLATE1, W

$$P_{itmellp1} := P_{rca} \cdot PF_{itmellp1} \cdot \alpha_{TM} = 5.688 \times 10^{-6}$$

ITM ELL BAF PLATE2, W

$$P_{itmellp2} := P_{rca} \cdot PF_{itmellp2} \cdot \alpha_{TM} = 8.531 \times 10^{-6}$$

ITM ELL BAF PLATE3, W

$$P_{itmellp3} := P_{rca} \cdot PF_{itmellp3} \cdot \alpha_{TM} = 6.541 \times 10^{-4}$$

BSAR ELL BAF PLATE, W

$$P_{bsell} := P_{rca} \cdot PF_{bsell} \cdot \alpha_{TM} = 1.422 \times 10^{-5}$$

BSC spool +X, W

$$P_{bscspoolx} := P_{rca} \cdot PF_{bscspoolx} \cdot \alpha_{TM} = 1.149 \times 10^{-3}$$

BSC flange +X, W

$$P_{bscsflngx} := P_{rca} \cdot PF_{bscsflngx} \cdot \alpha_{TM} = 7.252 \times 10^{-4}$$

spool BSC2-BSC3, W

$$P_{bscspoolbsc2\_3} := P_{rca} \cdot PF_{bscspoolbsc2\_3} \cdot \alpha_{TM} = 8.304 \times 10^{-4}$$

BSC spool -Y, W

$$P_{bscspooly} := P_{rca} \cdot PF_{bscspooly} \cdot \alpha_{TM} = 2.764 \times 10^{-3}$$

BSC flange -Y, W

$$P_{bscsflngy} := P_{rca} \cdot PF_{bscsflngy} \cdot \alpha_{TM} = 1.647 \times 10^{-3}$$

BSC2 cylinder, W

$$P_{bsccyl} := P_{rca} \cdot PF_{bsccyl} \cdot \alpha_{TM} = 9.205 \times 10^{-3}$$

BSC2 top, W

$$P_{bsctop} := P_{rca} \cdot PF_{bsctop} \cdot \alpha_{TM} = 1.624 \times 10^{-3}$$

BSC2 bottom, W

$$P_{bscbot} := P_{rca} \cdot PF_{bscbot} \cdot \alpha_{TM} = 4.573 \times 10^{-3}$$

HAM4/MC tube, W

$$P_{ham4mc} := P_{rca} \cdot PF_{ham4mc} \cdot \alpha_{TM} = 4.325 \times 10^{-3}$$

ITM middle frame, W

$$P_{itmframe} := P_{rca} \cdot PF_{itmframe} \cdot \alpha_{TM} = 1.166 \times 10^{-4}$$

ITM SUS ring, W

$$P_{itmsusring} := P_{rca} \cdot PF_{itmsusring} \cdot \alpha_{TM} = 1.706 \times 10^{-5}$$

ACB Plate6, W

$$P_{acbp6} := P_{rca} \cdot PF_{acbp6} \cdot \alpha_{TM} = 3.128 \times 10^{-5}$$

WIDE ANGLE BAF BOTTOM
LEDGE ITMX, W

$$P_{widbot} := P_{rca} \cdot PF_{widbot} \cdot \alpha_{TM} = 2.844 \times 10^{-6}$$

WIDE ANGLE BAF side ITMX, W

$$P_{widside} := P_{rca} \cdot PF_{widside} \cdot \alpha_{TM} = 5.688 \times 10^{-6}$$

POWER SCATTERED INTO IFO MODE

$PF_s := 1$

$P_{inc} := 1$

$w_{ifo} := 1$

$BRDF_s := 1$

$\theta_{inc} := 1$

$L_s := 1$

wide angle scattered power from TM hitting the adjacent
surface

$$P_{inc} := P_{rca} \cdot \alpha_{TM} \cdot PF_s$$

power re-scattered by TM into IFO
mode

$$P_{sifo} := P_{inc} \cdot BRDF_s \cdot \frac{\lambda^2}{L_s^2} \cdot \alpha_{TM} \cdot \frac{\cos(\theta_{inc})}{\pi}$$

combining these equations, we get

$$P_{sTMifo} := P_{inc} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_s^{\,2}} \cdot BRDF_s \cdot \cos\left(\theta_{inc}\right) \right)$$

ITM ELL BAF PLATE1, W

$$P_{itmellp1ifo} := P_{itmellp1} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{itmellp1}^{\,2}} \cdot BRDF_{oxiunpolish} \cdot \cos\left(\theta_{itmellp1}\right) \right)$$

$$P_{itmellp1ifo} = 5.501 \times 10^{-25}$$

ITM ELL BAF PLATE2, W

$$P_{itmellp2ifo} := P_{itmellp2} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{itmellp2}^{\,2}} \cdot BRDF_{oxiunpolish} \cdot \cos\left(\theta_{itmellp2}\right) \right)$$

$$P_{itmellp2ifo} = 5.803 \times 10^{-25}$$

ITM ELL BAF PLATE3, W

$$P_{itmellp3ifo} := P_{itmellp3} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{itmellp3}^{\,2}} \cdot BRDF_{oxiunpolish} \cdot \cos\left(\theta_{itmellp3}\right) \right)$$

$$P_{itmellp3ifo} = 4.449 \times 10^{-23}$$

TOTAL ITM ELL BAF, W
note: power adds cohherently

$$P_{itmellbafifo} := \left(P_{itmellp1ifo} + P_{itmellp2ifo} + P_{itmellp3ifo}\right)$$

$$P_{itmellbafifo} = 4.562 \times 10^{-23}$$

BSAR ELL BAF PLATE, W

$$P_{bsellifo} := P_{bsell} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bsell}^2} \cdot BRDF_{oxiunpolish} \cdot \cos\left(\theta_{bsell}\right) \right)$$

$$P_{bsellifo} = 1.87 \times 10^{-23}$$

BSC spool +X, W

$$P_{bscspoolxifo} := P_{bscspoolx} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bscspoolx}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bscspoolx}\right) \right)$$

$$P_{bscspoolxifo} = 1.414 \times 10^{-22}$$

BSC flange +X, W

$$P_{bscsflngxifo} := P_{bscsflngx} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bscsflngx}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bscsflngx}\right) \right)$$

$$P_{bscsflngxifo} = 7.777 \times 10^{-23}$$

BSC spool -Y, W

$$P_{bscspoolyifo} := P_{bscspooly} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bscspooly}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bscspooly}\right) \right)$$

$$P_{bscspoolyifo} = 5.04 \times 10^{-22}$$

BSC flange -Y, W

$$P_{bscsflngyifo} := P_{bscsflngy} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bscsflngy}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bscsflngy}\right) \right)$$

$$P_{bscsflngyifo} = 2.846 \times 10^{-22}$$

BSC2 cylinder, W

$$P_{bsccylifo} := P_{bsccyl} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bsccyl}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bsccyl}\right) \right)$$

$$P_{bsccylifo} = 2.016 \times 10^{-21}$$

BSC2 top, W

$$P_{bsctopifo} := P_{bsctop} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bsctop}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bsctop}\right) \right)$$

$$P_{bsctopifo} = 1.265 \times 10^{-22}$$

BSC2 bottom, W

$$P_{bscbotifo} := P_{bscbot} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{bscbot}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{bscbot}\right) \right)$$

$$P_{bscbotifo} = 5.624 \times 10^{-22}$$

TOTAL BSC2, W
note: power adds incohherently

$$P_{bsc2ifo} := \left(P_{bscspoolxifo}^2 + P_{bscsflngxifo}^2 + P_{bscspoolyifo}^2 + P_{bscsflngyifo}^2 + P_{bsccylifo}^2 + P_{bsctopif}$$

$$P_{bsc2ifo} = 2.181 \times 10^{-21}$$

HAM4/MC tube, W

$$P_{ham4mcifo} := P_{ham4mc} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{ham4mc}^2} \cdot BRDF_{wall} \cdot \cos\left(\theta_{ham4mc}\right) \right)$$

$$P_{ham4mcifo} = 4.172 \times 10^{-22}$$

ITM middle frame, W

$$P_{itmframeifo} := P_{itmframe} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{itmframe}^2} \cdot BRDF_{itmsus} \cdot \cos\left(\theta_{itmframe}\right) \right)$$

$$P_{itmframeifo} = 1.344 \times 10^{-23}$$

ITM SUS ring, W

$$P_{itmsusringifo} := P_{itmsusring} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{itmsusring}^2} \cdot BRDF_{itmsus} \cdot \cos\left(\theta_{itmsusring}\right) \right)$$

$$P_{itmsusringifo} = 2.022 \times 10^{-24}$$

TOTAL ITM SUS FRAME, W
note: power adds cohherently

$$P_{itmsusifo} := \left(P_{itmframeifo} + P_{itmsusringifo}\right)$$

$$P_{itmsusifo} = 1.546 \times 10^{-23}$$

ACB Plate6, W

$$P_{acbp6ifo} := P_{acbp6} \cdot \left(\alpha_{TM} \cdot \frac{\lambda^2}{\pi \cdot L_{acbp6}^2} \cdot BRDF_{oxiunpolish} \cdot \cos\left(\theta_{acbp6}\right) \right)$$

$$P_{acbp6ifo} = 7.506 \times 10^{-26}$$

WIDE ANGLE BAF BOTTOM LEDGE ITMX, W

$$P_{widbotifo} := P_{widbot}\cdot\left(\alpha_{TM}\cdot\frac{\lambda^2}{\pi\cdot L_{widbot}^{\ 2}}\cdot BRDF_{oxiunpolish}\cdot\cos\left(\theta_{widbot}\right)\right)$$

$$P_{widbotifo} = 6.891\times 10^{-27}$$

WIDE ANGLE BAF side ITMX, W

$$P_{widsideifo} := P_{widside}\cdot\left(\alpha_{TM}\cdot\frac{\lambda^2}{\pi\cdot L_{widside}^{\ 2}}\cdot BRDF_{oxiunpolish}\cdot\cos\left(\theta_{widside}\right)\right)$$

$$P_{widsideifo} = 1.373\times 10^{-26}$$

TOTAL WIDE ANGLE ITM ACB, W
note: power adds cohherently

$$P_{acbifo} := \left(P_{acbp6ifo} + P_{widbotifo} + P_{widsideifo}\right)$$

$$P_{acbifo} = 9.569\times 10^{-26}$$

DISPLACEMENT NOISE @ 100 Hz, m/rtHz
Note: the following calculation is the total noise from the BSAR and BSHR wide angle scattering, which will add incoherently

$$\theta_t := 0 \qquad x_s := 1$$

Displacement Noise Requirement @ 100 Hz, m/rt Hz

$$D_{req} = 1\times 10^{-21}$$

$$DN_s := \sqrt{2}\cdot TF_{itmar}\cdot\left(\frac{P_{sTMifo}}{P_{psl}}\right)^{0.5}\cdot\frac{2\cdot k\cdot x_s}{\sqrt{2}}$$

TOTAL ITM ELL BAF

$$DN_{itmellbaf} := \sqrt{2}\cdot TF_{itmar}\cdot\left(\frac{P_{itmellbafifo}}{P_{psl}}\right)^{0.5}\cdot x_{itmellbaf}\cdot\frac{2}{\sqrt{2}}\cdot l$$

$$DN_{itmellbaf} = 2.255\times 10^{-28}$$

BSAR ELL BAF PLATE

$$DN_{bsell} := \sqrt{2}\cdot TF_{itmar}\cdot\left(\frac{P_{bsellifo}}{P_{psl}}\right)^{0.5}\cdot x_{ISI}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsell} = 4.331\times 10^{-30}$$

TOTAL BSC2

$$DN_{bsc2} := \sqrt{2}\cdot TF_{itmar}\cdot\left(\frac{P_{bsc2ifo}}{P_{psl}}\right)^{0.5}\cdot x_{manifold}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsc2} = 1.247\times 10^{-25}$$

HAM4/MC TUBE

$$DN_{ham4mc} := \sqrt{2}\cdot TF_{srbs}\cdot\left(\frac{P_{ham4mcifo}}{P_{psl}}\right)^{0.5}\cdot x_{manifold}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{ham4mc} = 7.698\times 10^{-26}$$

TOTAL ITM SUS FRAME

$$DN_{itmsus} := \sqrt{2}\cdot TF_{itmar}\cdot\left(\frac{P_{itmsusifo}}{P_{psl}}\right)^{0.5}\cdot x_{ISI}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{itmsus} = 3.938\times 10^{-30}$$

TOTAL WIDE ANGLE ITM ACB

$$DN_{acb} := \sqrt{2} \cdot TF_{itmar} \cdot \left(\frac{P_{acbifo}}{P_{psl}} \right)^{0.5} \cdot x_{ACB} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{acb} = 1.033 \times 10^{-29}$$

$$_{0}^{2} + P_{bscbotifo}^{2}\Big)^{0.5}$$

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k