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To: Thomas W. Eagar, Sc.D.  
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**Subject: Review of LIGO tube structural stability**

Dr. Eagar,

At your request, I reviewed mechanical and material data regarding the subject tubes. I understand that small external cracks have recently been discovered near the fillet welds that are attaching circumferential stiffeners to the main tubing wall. I calculated the stability of a long crack in the system using fracture mechanics. The results show that significant leakage will occur well before these cracks become an issue for structural stability. This report summarizes my procedure and findings.

### **Tube support system**

I reviewed two documents describing the design of the tubes. One document is "beamtube090194.txt" by R. Weiss (Sept 1994) and the other document is "LIGO Beam Tube Component and Module Leak Testing" in American Vacuum Society Paper No. 1708 by W.A. Carpenter et al. (1999). Each (of two) beam tube modules is 2 km long and constructed from 20 meter long tube sections. The system was installed with anchors at each end and expansion joints (bellows) every 40 meters along the beam tube. There is a fixed support (mounting ring) in between each expansion joint (also every 40 meters).

### **Tube axial loads in service**

This system is loaded in compression during the bake-out. I estimated an extension of approximately 2 in. for the end of every tube section (Calculation 1). After cooling to ambient and completing the vacuum, there the contraction of the tubing from external pressure will be resisted at the ends by the fixed supports, resulting in a longitudinal tension in the tube wall. The longitudinal tensile stress in the tube wall is low – at most approximately 1.5 ksi (Calculation 2) assuming that the expansion joints only work in compression and even less if the joints do expand.

### Existing cracking

The cracking found recently is at fillet welds running around the circumference of the tube. The fillet welds attach external stiffeners to the main tubing wall. When the vacuum is applied, the exterior surface of the wall is in axial tension at that location because the tube wall contracts more in the unsupported areas than where the stiffeners are. This is only a concern when the system is exposed to an environment sufficiently corrosive to initiate stress corrosion cracks (SCC) in welded stainless steel 304L. For calculation purposes, I assumed that the crack reaches a length of one foot before it is detected and repaired. Field observations may help determine how conservative this assumption is.

### Crack stability

Stainless steel 304L maintains its toughness even if SCC cracks are initiated and grow through the thickness. Because of the limited wall thickness (3 mm), the material response to a long circumferential crack will be ductile (plane stress condition). For a one foot long crack, I estimated that the stress intensity factor would be only 13 ksi $\sqrt{\text{in}}$  when using 3 ksi as the nominal stress (Calculation 3), which is double the longitudinal stress calculated above. Although this could be confirmed by material testing, I suspect that the fracture toughness for this thickness is at least 40 ksi $\sqrt{\text{in}}$ . This gives a total safety margin of 6 between the existing stress and the expected minimum fracture toughness. For this analysis to be valid, I verified that the loading is in the region of K-dominance using a yield strength of 30 ksi for the stainless steel sheet (Calculation 4). With this data, the foot long crack falls in the region of K-dominance per ASTM E399.

### Tube stability

The calculations show that a crack of a large size (one foot long) would not cause a separation of the tube even if the nominal stress in the tube wall was 6 times the amount expected from tensioning the tube without expansion joints. Therefore a stability issue is not predicted. However, the calculation is simplified and excludes the effect of bending of the wall near the stiffeners, but the margin for safety is large. I have not been able to review the behavior of the expansion joints in the system due to insufficient information. These joints could add to the tension if they provide considerable friction between the contracted state (bake-out) and the service conditions. I would be happy to perform more analysis and provide more background information if desired. Shall you or others have any questions, please let me know.

Simon C. Bellemare, Ph.D., P.E.  
Principal, Bellemare LLC

(Calculations and references attached)

**CALCULATIONS****(1) Extension of tube ends [by linear expansion theory]**

$$\begin{aligned}
 &= (\text{thermal expansion coefficient}) \times (\Delta T) \times (\text{half length}) \\
 &= (17 \times 10^{-6} / ^\circ\text{C}) \times (140^\circ\text{C}) \times (20\text{m}) = 48 \text{ mm} \\
 &\cong 2 \text{ in.}
 \end{aligned}$$

**(2) Tension stress from vacuum [without expansion joints, thin wall pressure vessel]**

$$\begin{aligned}
 &= (\text{diameter}) \times (\text{pressure}) / (4 \times \text{thickness}) \\
 &= (48 \text{ in}) \times (14.7\text{psi}) / (4 \times 3 \text{ mm} / (25.4 \text{ mm} / \text{in.})) = 1490 \text{ psi} \\
 &\cong 1.5 \text{ ksi.}
 \end{aligned}$$

**(3) Stress intensity factor [1 ft long crack, infinite plate solution]**

$$\begin{aligned}
 &= (\text{nominal stress}) \times [\pi \times (\text{half crack length})]^{1/2} \\
 &= (3\text{ksi}) \times (\pi \times 6 \text{ in.})^{1/2} \\
 &\cong 13 \text{ ksi}\sqrt{\text{in.}} \text{ (need approximately } 6 \times 1.5 \text{ ksi to reach } 39 \text{ ksi}\sqrt{\text{in.}})
 \end{aligned}$$

**(4) K-Dominance verification [verified to approximately 6 x 1.5 ksi stress level]**

$$\begin{aligned}
 &= 2.5 \times (\text{stress intensity} / \text{yield stress})^2 \\
 &= 2.5 \times (40 \text{ ksi}\sqrt{\text{in}} / 30 \text{ ksi})^2 \\
 &\cong 4.4 \text{ in. (which is smaller than the half crack length of 6 in., therefore Ok)}
 \end{aligned}$$

(References attached on stress intensity and K-dominance.)