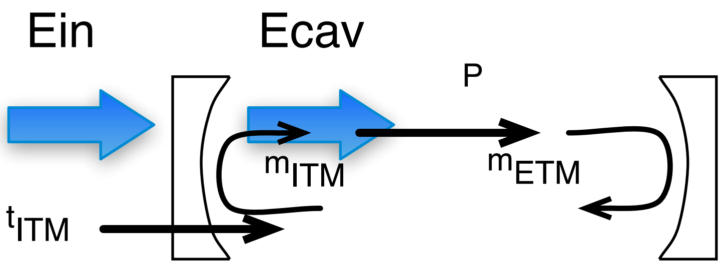
HankelIFO

simulations based on the Hankel transform

- J-Y Viney and P. Hello, J. Mod. Opt **40** 1981-1993 (1993)
- 2D FFT → 1D FFT assuming axi-symmetry
  - » 2D FFT : field DOF = N^2, operation matrix = $N^4 \sim 10^8$
  - » 1D FFT : field DOF = N/2, operation matrix = $N^2 / 4 \sim 10^4$
- Limitation
  - » only axi-symmetric case, no tilt, no astigmatism, no oval cross section of BS
- Merits
  - » Fast and accurate
    - – Explicit calculation of field by inversion of matrix
    - – No iteration or convergence problem
  - » Axi-symmetric boundary conditions, can be explicitly implemented
    - – Finite aperture, curvature mismatch

Simple example



$$E_{cav} = \frac{1}{1 - e^{i\phi} C} \cdot t_{ITM} \cdot E_{in}$$

$$C = m_{ITM} \cdot P \cdot m_{ETM} \cdot P$$

$r_k = a\, \zeta_k / \zeta_N$ $\rho_k = \zeta_k / a$ ζ_k : zero of J1 bessel

Diagonal operator

$$m_{mirror} = -r \cdot \exp(i 2k \frac{r_\alpha^2}{2R} + 2ik\delta(r_\alpha))$$

$$t_{mirror} = t(r_\alpha) \cdot \exp(i\phi(r_\alpha)) = t(r_\alpha) \cdot \exp(-ik(\frac{(n-1)}{2R} - \frac{1}{2f}) r_\alpha^2)$$

Non Diagonal operator

$$P_{\alpha\beta}(z) = \sum_{\sigma=0}^{N} H_{\alpha\sigma}^{(-)} \widetilde{G}_\sigma(z) H_{\sigma\beta}^{(+)}$$

$$\widetilde{G}_\alpha(z) = \exp(i \frac{z}{2k} \rho_\alpha^2)$$

$$\widetilde{P}(z,\gamma) = C_2 \cdot P(z/\gamma) \cdot C_1$$

Matlab code

- T1000254 : Hankel Cavity simulation package
  - With matlab code and examples
  - Filter cavity loss calculation from short to long : Patric Kwee

```
[fromITM, toETM, fromETM, toITM, roundLoss, Ein, rList] =
   HankelFP( N, a, T1, T2, ROC1, ROC2, del1, del2, invF, R, Lcav, phi )
```

- Martin Regehr
  - Mathematica based field and transfer function calculator
    - Scalar field
    - Construct IFO using building blocks
- Directly expandable to use Hankel elements
  - Under way