Report on the Leak Localization on the Y arm at LLO

M. Meyer, H. Overmeir, K. Thorne, R. Weiss, M. Zucker October 10 – November 30, 2012

Abstract: A leak of close to the estimated size has been found at 2258 meters from the LVEA port on the yarm. The best estimate for the location of the leak was at 2067 +- 39 meters . The 10 inch ports in the mid building are at 1966 and 1972 meters with the next port toward the Yend being at 2284 meters. Based on the initial estimate, we mounted the helium leak detector for the leak hunt in the mid building. The leak behaves as though it is in a single location and is constant to 5% at 2.495+-0.03 x 10⁻⁴ torr liters/sec from August 15 to October 15, 2012. The report gives the results of the leak localization and its limitations. It presents the analysis methods used as well as the results of numerical modeling based on the underlying diffusion process.

Introduction RGA's and discharge gauges were used at the LVEA and yend to measure the pressure of atmospheric gas species and the total pressure change in two methods with sensitivity to the location of the leak. The simplest technique was to close valves at the ion pumps at both ends and accumulate the gas – the dynamic method. The difference of the ratio of the pressure derivative with time to the pressure at the two ends allows for a localization. The more difficult but potentially more accurate method was to modulate the pumps with valve closure with a sequence that first closed the valve at the LVEA leaving the yend valve open and then, after the system had come to equilibrium, to open the LVEA valve and close the yend valve. The DC method uses the pressure differences to solve for the leak position. A key element in the localization strategy is to calibrate the sensors by recognizing that the pressure derivative with time, when the system is not being pumped, is independent of the sensor location and leak position after a stationary flow pattern has been established in the tube.

The errors in the leak location are not Gaussian. Typical random error for any several day measurement as determined by the shot noise in the ion currents both in the discharge gauges and the RGA's would give localization errors of 5 to 10 meters. The measurements are insensitive to parameters common to both ends (ie., uniform outgassing, common temperature changes...). The measurements are dominated by uncontrolled (and currently unmeasured) sources of systematic errors such as temperature gradients along the tube and differential non-linearities in the sensors. These cause the errors to be larger and non-Gaussian. There are outliers in the data at distances as large as 90 meters from the most likely position. Taking more data without understanding and controlling or measuring the systematic error sources in the system would have been unproductive. If we had not found the leak, the model of a single leak might have been wrong and more pressure sensors would have been needed at different places along the tube to solve for the leak locations. If there are n leaks in the tube it requires n+1 sensors to uniquely solve for their positions. In addition, further efforts at localization would have hopefully gotten closer to the random statistical errors which will require an array of thermometers along the beam tube. The almost 200 meter difference between the average localization position and the actual leak is not vet explained. We will know more once the leak is covered and the new leak rate into the tube is measured.

The measurements used the RGA at amu 2, 14, 28, 32 and 40. The initial localizations were made with amu 28 and 14, the molecular nitrogen peaks, until we noticed that the discharge gauges mounted next to the RGA gave similar results and that the noise in the localization was not dominated by the shot noise but was correlated between the discharge gauges and the RGA due to a systematic we were not controlling. The other consideration for using the RGA rather than the total pressure measurements of the discharge gauges was the molecular hydrogen, amu 2, outgassing which could have skewed the results if it was inhomogeneous. With a leak as large as the current one, the Hydrogen outgassing is a few percent of the signal and the inhomogeneity a factor of 10 to 100 smaller.

We noticed non-linearity in the RGA once the Faraday mode ion current exceeded 10^{-10} amperes or the total pressure was larger than 3×10^{-6} torr. The amu 40 peak, due to the argon entering through the leak, has a much longer time constant than the nitrogen both because of the reduced thermal velocity (reduced diffusion constant) and the storage time of Argon in the cryo pumps. The amu 32 peak from molecular oxygen follows the nitrogen closely but is not at the atmospheric ratio to nitrogen due to gettering by the tube surface.



Figure 1 Schematic of the yarm. The measurements were carried out with gate valves 3,4,mid,9 and 10 open and the liquid nitrogen traps cold. The ion pump at the left (LVEA pump labeled by 313 l/s) was measured to have a pumping speed of 215 liters/sec for nitrogen and the one on the right (Yend labeled 1291 l/s) was measured to have a pumping speed of 1640 liters/sec for nitrogen. The valves modulated were the smaller gate valves associated with these two pumps. During an accumulation the smaller gate valves were both shut. The RGAs and discharge gauges used to perform the measurements are mounted on 10 inch ports on the beamtube adjacent to the gate valve 4 (the LVEA pressure sensors) and gate valve 9 (the yend pressure sensors).

Calibration

A critical part of the leak localization is to establish the relative sensitivity of the discharge gauges and the RGAs to a few parts in a thousand. The absolute calibration is not critical except to estimate the size of the leak which is determined by accumulation of the leak into the known volume of the beamtube. The localization trades on pressure ratios and their differences. The relative sensitivity of the RGA's and discharge gauges is determined during accumulation measurements. Once the diffusion transients have damped out, the time derivative of the pressure in the tube due to the leak and outgassing is independent of where in the tube the pressure is measured (shown in **Appendix 1**). The fact that dp/dt is the same everywhere in the tube allows the pressure sensors to be normalized by simply dividing the sensor output by the derivative of the sensor output during the accumulation after the transients have died away.



Figure 2 The accumulation begins at 5250 minutes and is ended at about 6300 minutes as the yend and lvea valves are open in sequence. The steady pressure derivative begins at 5800 minutes and shows a 10% difference in the gauge sensitivities. 10 hours is typical for the transients to die off.



Figure 3 A complete leak localization sequence showing both the initial accumulation between 0 to 1000 minutes followed by a valve modulation. The data has been normalized to remove the differences in gauge sensitivity: the lvea discharge gauge output has been multiplied by 1.051 while the yend output has been multiplied by 0.9496. The values are determined by the transient free parts of the accumulations at the beginning and end of the sequence. In the time between 1000 to 2800 minutes, the yend and lvea valves are open. Between 2800 minutes to 5200 minutes, the yend valve is closed and the lvea valve is open. Between 5200 minutes, the yend valve is open and the lvea valve is closed. The pumping speed at the lvea is about ¼ of that at the yend which accounts for the difference in time to reach steady state for the two valve configurations. As shown in **Appendix 1**, the transient solutions to the diffusion are a sum of exponentials with different time constants with time origin at the moment when a valve is activated. The times to achieve steady state were not always attained in the data taking and we had to resort to exponential curve fitting at the ends of a valve state to determine the equilibrium pressure values in the valve modulation method. The difference in pressure between the lvea and yend during the steady state part of the accumulation carries the localization information.

Dynamic method The beamtube is isolated from the pumps and the pressure at the yend and lvea as a function time is recorded. In the steady state dp/dt is a constant independent of location in the tube and the pressure difference between the gauges after normalization provides the leak localization. As is shown in **Appendix 1**, the location of the leak is given by

$$\mathbf{x}_{lk} = \left(\frac{2av}{3L}\right) \left[\frac{\frac{P_{yend}(t)}{dP_{yend}} - \frac{P_{lvea}(t)}{\frac{dP_{lvea}}{dt}}}{\frac{dP_{lvea}}{dt}}\right] + \frac{L}{2} \quad t > \frac{2L^2}{av} \quad eq 1$$

L is the length of the beam tube between the ports at the LVEA and the yend, a is the radius of the beamtube and v the thermal velocity of the gas for which the pressure is being measured.



Figure 4 Model generated by the FORTRAN program btleakfind5 of the accumulation of the 2.5 x 10⁻⁴ torr liters/sec placed at 2089 meters from the LVEA 10 inch port on the beamtube. The pressure at the lvea and yend are plotted along with the leak location estimates of eq. 1. The transients in the accumulation are damped out well enough by 10 hours after the accumulation was started.

An accumulation is part of each localization sequence and reaches steady state more quickly than the valve modulation technique. More than 2/3 of the data we have taken used the dynamic method.

Model of dynamic accumulation method: pressure distribution 480 ő 9.475 pressure in units of 10^-6 torr 9.470 9**.**465 460 ő 500 2500 3500 1000 1500 2000 3000 position meters

Figure 5 The model pressure distribution in the tube after the transients have decayed for the accumulation of a 2.5×10^{-4} torr liter/sec leak at 2089 meters from the lvea 10" flange on the tube.

Valve modulation method (DC method) The technique is easier to understand and was the first proposed. The concept is a straightforward application of the linear relations between the pressure in the tube and the pumping impedence of the tube which varies linearly with the length (the baffles and bellows are a perturbation and discussed later under error sources). **Figure 6** shows the method. It is first order independent of the pumping speeds of the pumps.



Figure 6 The static pressure distribution of nitrogen in the beamtube after the transients have died off for a leak of 2.5 x 10⁻⁴ torr liters/sec located 2089 meters from the LVEA beamtube 10 inch pump out port. The red curve is with both LVEA and yend gate valves open, the green curve is with the lvea valve closed and yend open while the violet curve has the lvea gate open and the yend closed.



Figure 7 Model of the valve modulation after several cycles of the valve modulation. The equilibrium values are those in **Figure 6**. The cycle times are longer than those used in the actual data taking shown in **Figure 8**. A comparison of the model with the data shows that the pumping speed of the modulated pumps, data at the minimum pressures, have been assigned correctly but that the peak pressures, which involve the impedence of the beamtube, do not match the data; the tube impedence is about 25% larger than that used in the model. The model assumes a tube with radius 62cm, the discrepancy is most likely due to the baffles and bellows in the tube.



Figure 8 Data from the discharge gauges at the LVEA and yends. The LVEA valve is closed at 900 minutes while the yend valve is open. At 2900 minutes the LVEA valve is open and the yend valve is closed. The valve modulation times are too short to achieve equilibrium and an exponential end fit has been done to project to the equilibrium values. The exponential end fit to the LVEA pressure at 2900 minutes was carried out but is not shown in the figure. The pressure was normalized by dividing by the transient free values of dp/dt: the raw LVEA pressure was multiplied by 1.0516 and the yend pressure by 0.95323.

The position of the leak is determined from



where the pressures are the equilibrium values after the valve modulation has taken place and the pressure derivatives with time are the transient free values during the accumulations. The notation *lvea-on* is the valve state with the lvea valve open and the yend closed while *yend-on* is the case for the yend valve open and the lvea valve closed. The equation is derived in **Appendix 1**.

Sensitivity and Errors Table 1 lists the sensitivity of the localization to variations in the measured variables. The units in the table are in meters of leak position for a 1% change in the measured variable.

Table 1 Sensitivity of leak position to 170 change in measured variable						
measured variable	change in leak	measured variable	change in leak			
	position meters		position meters			
dp _{lvea} /dt	13.7	P _{lvea} (lvea-on)	-3.5			
dp _{yend} /dt	-13.7	P _{lvea} (yend-on)	-10.1			
$dp_{yend}/dt + dp_{lvea}/dt$	0.08	Pyend(lvea-on)	13.1			
sum of all p	0.38	Pyend(yend-on)	0.38			

Table 1 Sensitivity of leak position to 1% change in measured variable

Table 2 shows the average pressure values derived from the valve modulation technique, the statistical errors of the pressures are all about 5×10^{-9} torr.

Table 2 Average equilibrium pressure for the unterent valve states						
location and state	pressure 10 ⁻⁶ torr	location and state	pressure 10 ⁻⁶ torr			
lvea (yend-on)	2.991	yend (yend-on)	0.1522			
lvea (lvea-on)	1.161	yend (lvea-on)	4.401			

 Table 2
 Average equilibrium pressure for the different valve states

Uniform temperature changes do not effect the localization, however, temperature gradients along the tube are significant. The temperature gradients cause several different errors. The gas pressure is linear with the temperature, a 1K temperature difference between the LVEA and the yend will cause a 3 meter error in the localization if not corrected in the analysis. Another contribution comes from the change in the diffusion constant in the tube with temperature which amounts to about 1 meter for one 1K

difference between the tube sections between the lvea and the leak and the yend and the leak. The pumping speed of ion pumps at the LVEA and yend change with temperature by close to 1% for 1K which corresponds approximately to a 10 meter change in the localization. The unmeasured temperature gradients in the beamtube could account for between 30 to 60 meters of noise in the localization. If there is further work needed to localize the leak or other leaks form, it would be valuable to monitor thermometers every 250 meters along the beamtube.

The baffles were not included in the modeling which assumes a tube with constant radius of 62 cm. The baffles extend 8 cm into the tube. In the middle of the arm at LLO there is close to 1 baffle per 20 meters of tube length. This reduces the pumping speed of each 20 meter section by 10% and upto 30% at the ends where there are multiple baffles per 20 meter section. The leak is close to the middle of the tube and the reduction in pumping speed is symmetric about the leak causing little error (less than 1 meter) in the leak localization. If the leak had been near one end of the tube at LLO or should there be a leak at either end of the tube modules at LHO, the corrections would be significant and the model would need to be modified. The reduction in the pumping speed due to the baffles can be seen in the discrepancy between the modeled and measured pressure in the valve modulation method where the measured pressures are higher than the modeled ones in those cases when the pump is separated from the leak by a long section of tube.

Appendix 1 : One dimensional diffusion in a tube

The molecular flow in a tube when the pressure is low enough so that the transverse dimensions of the tube are smaller than the mean free path is described by the diffusion equation for the molecular density $\rho(x,t)$

$$D\frac{\delta^2\rho(x,t)}{\delta x^2} = \frac{\delta\rho(x,t)}{\delta t}$$

where $D=\frac{2}{3}av_{th}$ is the diffusion constant, a the radius of the tube, $v_{th} = \sqrt{\frac{8kT}{\pi m}}$ the most probable thermal speed. k is Boltzmann's constant, T the absolute temperature and m the mass of the molecule. The transient equation is solved by separation of variables $\rho(x,t)=X(x)T(t)$ which yields the ordinary differential equations

$$\frac{d^2 X(x)}{dx^2} + \frac{\lambda^2}{D} X(x) = 0 \rightarrow X(x) = A \sin(\frac{n\pi x}{L}) + B \cos(\frac{n\pi x}{L})$$
$$\frac{dT(t)}{dt} + \lambda^2 T(t) = 0 \rightarrow T(t) = \sum_{n=1}^{\infty} \alpha_n e^{-\frac{(n\pi)^2 D t}{4L^2}}$$

The transients die off exponentially with the longest time constants, n =1, associated with the largest spatial dimensions , $\tau = \frac{6 L^2}{\pi^2 a v_{th}}$ about 8 hours for nitrogen at 296K.

The special solutions associated with the transient free accumulations and steady state pressure distributions, are determined from the boundary conditions imposed by the leak and the pumping. A useful quantity is the diffusive particle flux determined by the equations above coupled to the continuity equation for conservation of the molecules

$$J=D\frac{d\rho}{dx}$$

In the valve modulation method the only flow is from the leak at x_{lk} through the beamtube to the pump at x = 0 or x = L depending on which valve is open. J is constant in the beamtube (as long as the radius a of the tube is fixed) and therefore the density or pressure gradient is constant. The flow between the leak to the open valve is the same for both valve modulations while the pressure difference between the two ends of the tube is proportional to the tube length – the distance between the leak and the pump. The assumption made is that the quantity of gas leaking into the tube is independent of the pressure in the tube, valid as long as the pumping speed of the leak is tiny compared to that of the beamtube or pumps. The flow in the tube is then proportional to

Flow
$$\propto \frac{(P_2 - P_1)_{1\text{open}}}{x_{1k}} = \frac{(P_1 - P_2)_{2\text{open}}}{L - x_{1k}}$$

which gives equation 1 when the individual pressure readings are normalized for the gauge constants using the transient free accumulation rates.

The transient free accumulation is most easily treated by noting that J decreases linearly with distance from the leak to the ends where it becomes equal to zero. J is not the same at the leak into the two regions x_{lk} to x = 0 and x_{lk} to L if the leak is not exactly in the middle. At the leak, J into the direction with the longer tube is larger than J into the direction with the smaller tube length in proportion to the length (the volume that needs to be filled). When the transients have died off, the second derivative of the pressure with distance in the tube becomes constant and the first derivative of the pressure with time in the tube (as indicated by the diffusion equation) at any location becomes the same. Combining these relations with the need to have the total J into the two sections of tube be given by the leak size Q_{lk} gives

$$J_1 + J_2 = \frac{Q_{lk}}{\pi a^2}$$
 at the leak and everywhere $\frac{dP}{dt} = \frac{Q_{lk}}{\pi a^2 L}$

The relations discussed in the paragraph above and the two equations lead to equation 2.

Both equations 1 and 2 were checked in detail by using a finite element diffusion model for the tube incorporated into the FORTRAN program btleakfind4 and btleakfind5. The programs determine the pressure profiles for different pumping speeds at the ends and the leak with adjustable size and position. If it should ever become necessary to fit to the full solution for the evolution of the pressure in space and time, the FORTRAN programs are the best way to determine the solutions.

Appendix 2: The data

The table on the next and following page show the reduced data for both methods as well as some tests to determine the effect of constant outgassing and the solutions for other than amu 14 and 28. The third page gives a graphical display of the solutions. The data is clearly not Gaussian with a distribution of points much larger than simply due to random errors which in principle have an rms of 10 meters. Table 1 gives direct estimates of the localization errors as determined by running the FORTRAN programs with a range of parameters. The actual errors are about three times larger than one would determine from the variation of the experiment parameters, especially, estimated temperature gradients. It is very hard to believe that temperature gradients of 10K occur along the north and south sections of the tube. The discrepancy between the found leak position and the various localization solutions has not yet been explained. The tube dimensions come from the CB&I table in Appendix 3. There is some suspicion that these numbers are not the as built values and we will check them with GPS in the near future.

type	method	directory/files pressure or current	directory/files dp/dt, di/dt	calibration	lk position meters from LVEA RGA all +- 5 to 10 m	notes
accumulation	discharge	$\frac{1015_17}{624_10_16_15}$ $\frac{1015_17}{723_10_16_15}$		dp/dt(lvea) 5.98994e-11+-3.0e-13 dp/dt(yend) 6.7250e-11+-3.0e-13	2070.07	no traps
accumulation	amu28	1015_17/amu28_10_16_yend 1015_17/amu28_10_16_lvea		di/dt(lvea) 1.46493e-15+-3.0e-18 di/dt(yend) 1.10594e-15+-2.0e-18	2068.25	no traps
accumulation	amu14	1015_17/amu14_10_16_lvea 1015_17/amu14_10_16_yend		di/dt(lvea) 1.62750e-16+-2.0e-19 di/dt(yend) 1.50098e-16+-4.0e-18	2089.01	no traps
accumulation	amu32	1015_17/amu32_10_16_lvea 1015_17/amu32_10_16_yend		di/dt(lvea) 2.432995e-16+-6.0e-19 di/dt(yend) 1.84592e-16+-4.0e-19	2111.18	no traps
accumulation	amu40	1015_17/amu40_10_16_lvea 1015_17/amu40_10_16_yend		di/dt(lvea) 1.82476e-17+-2.0e-19 di/dt(yend) 1.53863e-17+-2.0e-19	2008.21	no traps
accumulation	amu2	1015_17/amu2_10_16_lvea 1015_17/amu2_10_16_yend		di/dt(lvea) 2.34403e-17+-2.0e-19 di/dt(yend) 1.66064e-17+-2.0e-19	1620 f(temp)	test of concept no agree, no traps
valve mod	discharge	1011_14/624_10_09_08_07_06_05 1011_14/723_10_09_08_07_06_05	$1011 - 14/624_10_14_13_\dots \\ 1011_14/723_10_14_13_\dots$	dp/dt(lvea) 5.63517e-11+-1.0e-13 dp/dt(yend) 6.29781e-11+-1.0e-13	2074.91	
valve mod	amu28	1011_14/amu28_10_14_lvea 1011_14/amu28_10_14 yend	1011_14/amu28_10_14_yend 1011_14/amu28_10_14_lvea	di/dt(lvea) 1.412302e-15+-2.0e-18 di/dt(yend) 1.0460e-15+-3.0e-18	2028.21	
valve mod	amu14	1011_14/amu14_10_14_lvea 1011_14/amu14_10_14 yend	1011_14/amu14_10_14_yend 1011_14/amu14_10_14_lvea	di/dt(lvea) 1.553415e-16+-4.0e-19 di/dt(yend) 1.39821e-16+-6.0e-19	2060.25	
valve mod	amu2	1011_14/amu2_10_14_lvea 1011_14/amu2_10_14_yend	1011_14/amu2_10_14_yend 1011_14/amu2_10_14_lvea	di/dt(lvea) 4.09979e-17+-3.0e-19 di/dt(yend) 2.36028e-17+-2.0e-19	2732 f(temp)	test of concept not to agree
accumulation	discharge	1011_14/624_10_14_13_12_11_10 1011_14/723_10_14_13_12_11_10		dp/dt(lvea) 5.63517e-11+-1.0e-13 dp/dt(yend) 6.29781e-11+-1.0e-13	2136.7	
accumulation	amu28	1011_14/amu28_10_14_yend 1011_14/amu28_10_14_lvea		di/dt(lvea) 1.393565e-15+-2.0e-18 di/dt(yend) 1.0460e-15+-3.0e-18	2088.8	
accumulation	amu14	1011_14/amu14_10_14_lvea 1011_14/amu14_10_14_yend		di/dt(lvea) 1.55341e-16+-4.0e-19 di/dt(yend) 1.39821e-16+-6.0e-18	2190.3	
accumulation	amu32	1011_14/amu32_10_14_lvea 1011_14/amu32_10_14_yend		di/dt(lvea) 2.28186e-16+-6.0e-19 di/dt(yend) 1.73542e-16+-5.0e-19	2109.7	
accumulation	amu40	1011_14/amu40_10_14_lvea 1011_14/amu40_10_14_yend		di/dt(lvea) 1.762180e-17+-4.0e-19 di/dt(yend) 1.45388e-17+-2.0e-19	2071.6	
accumulation	amu2	1011_14/amu2_10_14_lvea 1011_14/amu2_10_14_yend		di/dt(lvea) 4.09979e-17+-3.0e-19 di/dt(yend) 2.36028e-17+-2.0e-19	2804.6 f(temp)	test of concept not to agree
valve mod	discharge	$1006_09/624_10_09_08_07_06_05$ $1006_09/723_10_09_08_07_06_05$	$1003_05/624_10_05_04_03$ $1003_05/723_10_05_04_03$	dp/dt(lvea) 4.9620e-11+-1.0e-13 dp/dt(yend) 5.7894e-11+-1.0e-13	2082.9	
valve mod	amu28	1006_09/amu28_10_9_lvea 1006_09/amu28_10_09 yend	1003_05/amu28_10_05_yend 1003_05/amu28_10_05_lvea	di/dt(lvea) 1.18803e-15+-1.0e-18 di/dt(yend) 9.50980e-16+-1.0e-18	1974.8	

									same as above new calibration				12pts	11pts	10 pts	9pts	
2069.6	2058	2017	2183	2191	2084.3	2046.8	2102.2	2109.7	2104.7	2145.5	2077.7		2110+-58	2067+-39	2113+-51	2073+-24	
di/dt(lvea) 1.3740e-16+-1.0e-18 di/dt(yend) 1.2850e-16+-1.0e-18	dp/dt(lvea) 4.9620e-11+-1.0e-13 dp/dt(yend) 5.7894e-11+-1.0e-13	di/dt(lvea) 1.18803e-15+-1.0e-18 di/dt(yend) 9.50980e-16+-1.0e-18	di/dt(lvea) 1.3740e-16+-1.0e-18 di/dt(yend) 1.2850e-16+-1.0e-18	dp/dt(lvea) 5.1380e-11+-1.0e-13 dp/dt(yend) 5.6660e-11+-1.0e-13	di/dt(lvea) 1.37656e-16+-1.0e-18 di/dt(yend) 1.4939e-16+-1.0e-18	dp/dt(lvea) 4.7251e-11+-1.0e-13 dp/dt(yend) 5.2127e-11+-1.0e-13	dp/dt(lvea) 4.7251e-11+-1.0e-13 dp/dt(yend) 5.2127e-11+-1.0e-13	dp/dt(lvea) 4.7251e-11+-1.0e-13 dp/dt(yend) 5.2127e-11+-1.0e-13	dp/dt(lvea) 5.3846e-11+-2.0e-13 dp/dt(yend) 5.7868e-11+-2.0e-13	dp/dt(lvea) 4.8205e-11+-2.0e-13 dp/dt(yend) 5.2394e-11+-2.0e-13	dp/dt(lvea) 5.3846e-11+-2.0e-13 dp/dt(yend) 5.7868e-11+-2.0e-13	Do not use amu: 2,32,40	accumulation method	valve modulation method	mid pt avg accumulation	mid pt avg valve mod	
1003_05/amu14_10_05_yend 1003_05/amu14_10_05_lyea					0924_25/amu14_10_05_yend 0924_25/amu14_10_05_lvea	0924_25/624_09_26_25_24 0924_25/723_09_26_25_24	0924_25/624_09_26_25_24 0924_25/723_09_26_25_24	0924_25/624_09_26_25_24 0924_25/723_09_26_25_24	0910_15/624_09_13_12_11 0910_15/723_09_13-12_11								
1006_09/amu14_10_9_lvea 1006_09/amu14_10_09 yend	$1003_05/624_10_05_04_03$ $1003_05/723_10_05_04_03$	1003_05/amu28_10_05_lvea 1003_05/amu28_10_05_yend	1003_05/amu14_10_05_yend 1003_05/amu14_10_05_lyea	$1003_05/624_10_03_02_01$ $1003_05/723_10_03_02_01$	0920_23/amu14_09_25_lvea 0920_03/amu14_09_25 yend	0920_23/624_09_23_22_21_20 0920_23/723_09_23_22_21_20	0915_18/624_09_17_16_15_14 0915_18/723_09_17_16_15_14	0920_23/624_09_20_19_18 0920_23/723_09_20_20_18	0920_23/624_09_20_19_18 0920_23/723_09_20_19_18	0924_25/624_09_26_25_24 0924_25/723_09_26_25_24	0910_15/624_09_13_12_11 0910_25/723_09_13_12_11						
amu14	discharge	amu28	amu14	discharge	amu14	discharge	discharge	discharge	discharge	discharge	discharge						
valve mod	accumulation	accumulation	accumulation	accumulation	valve mod	valve mod	valve mod	valve mod	valve mod	accumulation	accumulation						



summary of leak localization solutions

first port toward yend

mid bldg port toward y end Appendix 3: The CB&I port positions



· · · · · · · · · · · · · · · · · · ·				CED BEAM TUBE LIGO BEAM TUBE HANFORD, WA & LIVINGSTON, LA SUB-ASSEMBLY END LOCATIONS FOR LIVINGSTON MODULES (X) & (Y) ARD END MODULES (X) & (Y) ARD END ASSIGNED DAGE TO DAGE
[END-B]	595669 575669 57566945 57666945 5766045 616421 496609 456985 437173 397549 377549 377549 377549 377549 338113 3381123	318301 278677 298489 2588657 239853 239855 239855 239855 239865 239865 199429 159885 159885 159885 129993 129993 120181 1		→ 5 → BEARIONZ BEWYBKZ OVIE C-35% COVIE C-35% COVIE C-35% COVIE C-35% COVIE C-35% COVIE C-35% COVIE C-30515 COVIE COVIE
[END-A] 635293	605046 5756681 5756681 5566845 5566845 476797 476797 41736 337756 3377736 41736 3377736 3377736	338113 338113 318301 298489 298489 239053 199429 159805 159805 139993 139993 120181 120181 120181 1201801 1201801 1201801 139959		
LENGTH	19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812	19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812		WED 6-29-9 DATE DATE DATE DATE
TYPE		× × ×	A MODULE LY1 A MODULE LY1	APPRO M. Lease
TUBE NO.	2012 2012	2000 2000 2000 2000 2000 2000 2000 200		DOICATES CAUNCE FROM PR
· · ·				
[END-B]	1999071 19825097 19825097 1982697 1923073 1923073 1923073 1923073 1923073 1923073 1923073 1923073 1803257 1863637 1864577 1764557 17645577	1/24303 1/24303 1/2655129 166555129 166655129 1666981 1566457 1566457 1566457 1566457 1487209 1487209 1487209 1487209 14475855 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 1447558555 144758555 144758555 144758555 144758555 144758555 144758555 144758555 1447585555 14475855555 1447585555555555555555555555555555555555	$\begin{array}{c} 1467961\\ 1467961\\ 1368337\\ 1368337\\ 1368337\\ 1368713\\ 1368713\\ 1289089\\ 1229553\\ 1229653\\ 1220666\\ 1226965\\ 1226953\\ 12269523\\ 1226953\\ 1226953\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 12269523\\ 122695252\\ 122695252\\ 12269523\\ 12269523\\ 12269523\\ 1226952525\\ 1226952525\\ 12269523\\ 12269523\\ 12269523\\ 1226952525252\\ 12269523\\ 12269523\\ 12269525252525\\ 12269523\\ 122695252525252\\ 12269523\\ 12269525252525252\\ 122695252525252525252522\\ 122695252525252525252525252525252220222222202020202000000$	972097 952285 952285 912661 873037 8532255 813649 7339189 7339189 7339189 7339189 7339189 714541 7154541 715555555555
[END-A] [END-B]	2016432 1999071 1992509 19825097 19825097 19825097 19825097 19825097 19825097 1942885 1942885 1942885 1942885 1942885 1942885 1942885 1942885 1942885 1942885 1942885 1963637 18833449 18833449 18833449 18833457 1843825 1843825 1844201 1804201 1784389 1764577 1744765	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
LENGTH [END-A] [END-B]	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	19812 $1/44/65$ $1/24953$ $1/24953$ 19812 1724953 1705141 19812 1724953 1705141 19812 1785141 16655329 19812 1665537 1665517 19812 1665593 1665517 19812 1665693 1665517 19812 1666681 1685329 19812 16258933 16666831 19812 1586269 1566457 19812 15862691 15862691 19812 15862691 1566457 19812 1566457 15268333 19812 15268333 1507021 19812 1507021 1487209 19812 1507021 1447585 19812 1487209 1447585	19812 14773 142773 19812 142773 1427961 19812 142795 1388149 19812 1388149 1388337 19812 1388149 1388337 19812 1388149 1388149 19812 1388149 1388149 19812 1388149 13884981 19812 1388981 1328713 19812 1388981 1289277 19812 12898989 1269277 19812 1289889 122896841 19812 1289889 12299653 19812 12898841 1198029 19812 1289841 1198029 19812 1289841 1198029 19812 1198029 1170217 19812 1198029 1170217 19812 1198029 111090993 19812 1198029 111090993 19812 1198029 111090993 19812 10910769 1071157 19812 1091	1 19812 991909 972097 1 19812 972097 952285 1 19812 972097 952285 1 19812 972097 952285 1 19812 972097 952285 1 19812 912661 892849 1 19812 912661 892849 1 19812 873037 8532255 1 19812 873037 8532255 1 19812 873037 8532255 1 19812 813601 793789 1 19812 73373 813661 1 19812 734353 714541 1 19812 734353 714541 1 19812 734353 714541 1 19812 734353 714541 1 19812 734353 714541 1 19812 734353 714541 1 19812 734353 657493 1 19
TYPE LENGTH [END-A] [END-B] [END-B]	N*, P** 17361 2016432 1999071 M 16562 1999071 19825097 S 19812 19825097 1962697 A 19812 19825097 1962697 A 19812 19825097 1942885 A 19812 1942885 19426895 A 19812 1923073 1962697 B 19812 1923073 1962667 A 19812 1923617 1963637 B 19812 1923617 1963637 A 19812 1883449 18833449 B 19812 18833469 1863637 A 19812 1883449 1863637 B 19812 1863637 1843825 A 19812 1864201 1784389 B 19812 1824013 1804201 B 19812 1804201 1784389 A 19812 1784389 1744765 A 19812 1764577 1744765	B 19812 1/44/65 1/24953 1/265517 D 19812 1/24953 1/05141 1/05141 B 19812 1/665517 1665517 1665517 B 19812 1665517 1645795 B 19812 1665517 1645795 B 19812 1665517 1645795 B 19812 1625893 1665517 B 19812 1625893 1665615 B 19812 1586269 1566457 B 19812 1586269 1566457 B 19812 1586269 1566457 A 19812 1586269 1566457 B 19812 1526833 1507021 B 19812 1507021 1477209 A 19812 1467397 1467397	H 1 9812 1 427733 1 427733 B 1 9812 1 427733 1 427733 A 1 9812 1 388149 1 368337 A 1 9812 1 388149 1 368337 A 1 9812 1 388149 1 368337 B 1 9812 1 388149 1 388337 A 1 9812 1 388337 1 388941 B 1 9812 1 388337 1 388941 B 1 9812 1 388989 1 289989 A 1 9812 1 289989 1 289989 B 1 9812 1 289653 1 2899891 B 1 9812 1 289687 1 1 308941 B 1 9812 1 306923 1 1 308969 B 1 9812 1 1 30693 1 1 30693 B 1 9812 1 1 30693 1 1 30693 B 1 9812 1 1 306978 1 1 306969 <t< td=""><td>$\begin{array}{c c c c c c c c c c c c c c c c c c c$</td></t<>	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $
TUBE NO. TYPE LENGTH [END-A] [END-B]	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	30 30 4 17912 $1477/503$ $1477/503$ 31 32 8 19812 14079613 $1477/503$ 32 34 6 19812 1368337 138537 138537 35 8 19812 1368337 13485255 1328713 $1477/57$ 35 8 19812 1388149 1328713 1328713 $1477/57$ 37 8 19812 1388225 1328713 1328713 1328337 37 8 19812 1388267 1328713 1328713 37 8 19812 1269277 1229653 1229653 37 41 0 19812 1269277 1279653 47 47 8 19812 1269277 1229653 1229653 44 47 19812 119812 128918465 1170212 1289889441 47 48 19812 119812 1198029 11170212 1109029	53 0 19812 991909 972097 54 A 19812 972097 952285 55 56 A 19812 972097 952285 57 B 19812 972037 912661 972037 57 B 19812 912661 8923413 912861 57 B 19812 912661 8973037 912861 59 B 19812 873037 853225 833413 61 B 19812 873037 8532255 833413 61 B 19812 873037 8532255 833413 63 A 19812 873037 8532255 833413 65 A 19812 813601 793789 773977 65 A 19812 734353 714541 734353 66 A 19812 734355 714541 694729 65 A 19812 714541 694729 655105 69 A 19812 714541 694729 674917 694729 655105 674917 655105
TUBE NO. TYPE LENGTH [END-A] [END-B]	1 N* P** 17361 2016432 1999071 2 N* F* 16562 1999071 19825097 3 S 19812 19825097 1942885 5 B 19812 19825697 1942885 7 B 19812 19825697 1942885 7 B 19812 1982561 1962697 8 19812 1983261 19836537 1942885 9 B 19812 1983261 19626677 8 19812 1983261 19823617 9 19812 1983261 19626577 11 B 19812 18834491 18834491 12 B 19812 18834491 18636377 13 B 19812 18636377 1844301 11 B 19812 1864201 18636377 13 B 19812 1864201 1764507 14 19812 1844201 17844865 1864201 13 19812 18842401 1864201 17844865 13 19812 188122 1864201 17844657 13 19812 1864201 178446577 </td <td>15 B 19812 $1/44/65$ $1/24953$ $1/65517$ 17 0 19812 1724953 1705141 1705141 19 19 19812 1705141 1705141 1705141 19 19 19812 1665517 1665517 1665517 1665517 20 B 19812 1687329 16665517 1665517 1665517 21 B 19812 16812 1665517 1665517 1665517 22 B 19812 16812 1665833 1666981 168633 23 B 19812 1686681 15862697 1566457 1566457 26 A 19812 15862697 1566457 1566457 1566457 1566457 27 B 19812 15862697 1566457 1566457 1566457 28 B 19812 1507021 1507021 1487209 29 B 19812 1507021 14677397 1447585<td>$\begin{array}{ c c c c c c c c c c c c c c c c c c c$</td><td>$\begin{array}{ c c c c c c c c c c c c c c c c c c c$</td></td>	15 B 19812 $1/44/65$ $1/24953$ $1/65517$ 17 0 19812 1724953 1705141 1705141 19 19 19812 1705141 1705141 1705141 19 19 19812 1665517 1665517 1665517 1665517 20 B 19812 1687329 16665517 1665517 1665517 21 B 19812 16812 1665517 1665517 1665517 22 B 19812 16812 1665833 1666981 168633 23 B 19812 1686681 15862697 1566457 1566457 26 A 19812 15862697 1566457 1566457 1566457 1566457 27 B 19812 15862697 1566457 1566457 1566457 28 B 19812 1507021 1507021 1487209 29 B 19812 1507021 14677397 1447585 <td>$\begin{array}{ c c c c c c c c c c c c c c c c c c c$</td> <td>$\begin{array}{ c c c c c c c c c c c c c c c c c c c$</td>	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$

.

•		· · ·	•	· · · ·		
						LIGO BEAM TUBE HANFORD, WA & LIVINGSTON, LA SUB-ASSEMBLY END LOCATIONS FOR LIVINGSTON MODULES (X2) & (Y2) ER 3 MO CONTRACT MO CONTRACT MO CONTRACT MO CONTRACT MO CONTRACT MO CONTRACT MO CONTRACT MO CONTRACT MO CONTRACT MO SUB-ASSEMED MOLLES (X2) & (Y2) ER 333771 M. L. TELLALIAN SUT 4 4 4
[END-B]	3419019 3458643 3458643 3458643 3458643 3498267 3537891 3537891 3557793 35577615 3557703 35577615 3597327 3617139	3636951 3656763 3676575 3676575 3696387 3716199 3736011 3755823 3775675 37756775 37756775 37756775 37756775 37756775 37756775 37756775 377567575 37756775 3775775 3775775 37757775 377577777777	3934131 3953943 3978584 3988588			⇒ 3 5 BEARIZIONZ BENYBKZ OVIE <->>A-SK 25000 KCE OVIE <->>A-SK 25000 KCE BA VVIE <->>A-SK OVIE <->>A-SK 25000 KCE BA VVIE DYLE <->>A-SK 25000 KCE NOIL IO BE DYLE <->>A-SK DYLE <->>> DYLE
[END-A]	3399207 3419019 3419019 3458643 3458643 3478455 3478455 3478455 3478455 3478257 3517991 3557793 35577615 3597327	3617139 3656763 3656763 3656763 3696387 3756975 3756911 3756823 3755823 3755823 3755823 3755823 38755823 38755823 38755823 38755823 38755823 38755823 38755823 38755823 38755823 38755823 38755823 38755823 3875597 38755597 38755597 387755597 387755597 3877555977 3877555977 3877555977 38775559777 38775559777 38775559777 3877555977777777777777777777777777777777	3914319 39344131 39705044 3970504	•		
LENGTH	19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812	19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812	19812			APPROVED
ТҮРЕ				R MODULE LY2 R MODULE LY2		M. Leve CBI CALTECH CALTECH
TUBE NO.				× × TYPE FO	· .	NDICATES CRANCE FROM P
		· · · · · · · · · · · · · · · · · · ·				
[END-B]	2035429 2051991 2071803 2071803 2111427 2111427 2131239 2131239 2130653 2151051 2151051 2150655 2130675 22190675	2258131 22690111 22690353 22897353 23897353 23897353 2349171 23887955 2468607 2448231 2448231 2448231 2488607 2488607 2488607 2488607 2488667 2488667 2488667 2487855	2547291 2567103 2567103 2606727 2626539 2646351 2666163 2685975 2685975 2705787	2725599 2745411 2765223 2864285035 28844471 28844471 28844283 28844283 28844283 28844283 2884495 28844283 28844095	2923719 2943531 2943531 2963343 3002967 3022779 3022779 3022779 3062403 310227 31027 30027 30000000000	$\begin{array}{c} 3141651\\ 3161463\\ 3181275\\ 3281275\\ 3280399\\ 32208999\\ 32208999\\ 322080335\\ 322080335\\ 333971\\ 3339771\\ 33399583\\ 33399583\\ 3399587\\ 33399587\\ 33399587\\ 3399587\\ 3339587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 33357587\\ 3335757267\\ 333575757\\ 333575757\\ 333575757\\ 333575757\\ 333575757\\ 3337575757\\ 3337575757\\ 333757575757\\ 3337575757575757\\ 333757575757575757\\ 33375757575757575757\\ 33375757575$
[END-A]	2017433 2035429 2035429 2051991 2071803 2071803 2071803 2071803 2071803 2131239 2131239 2131239 2131239 2131239 2131651 2131239 2130675	221048/ 2230299 225302999 2269923 2269923 2309547 2309547 23887959 2388795 2388795 2488607 2488607 2488607 2488607 2488607 2488231 2488231 2488231 2488231 2488231 24887855	2527479 25577479 2567701 2567103 2586915 2686727 2686727 2666163 2666163 2685975	2705787 2725599 2765999 2765223 2765223 2765223 2765223 2765035 28644471 28644471 28644471 28644471 28644471 28644471	2903907 2903907 2903719 2903719 2903343 2903343 3002967 3002967 3002967 30022779 3002967 30022779 30022403 30622403 30622403	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
LENGTH	17996 16562 19812 19812 19812 19812 19812 19812 19812 19812	19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812	19812 19812 19812 19812 19812 19812 19812 19812 19812	19812 19812 19812 19812 19812 19812 19812 19812	19812 19812 19812 19812 19812 19812 19812 19812 19812 19812 19812	1 9812 1 981 1 9812 1 9812
ТҮРЕ						
TUBE NO.				0,000,00-4,4,4,4,4,4		
				•		
				1		NOO. 50401722

.

Appendix 4 The FORTRAN program to estimate the pressure in the tube vs position and time. The program requires the pumping speed at the tube ends, the size and position of the leak, the amu and temperature of the molecule being measured.

```
c ******btleakfind4.for ***** August 30, 2012
c try to fix problem with incorrect results due to number of steps
c Include writing file of the pressure across the tube at a fixed time
c iterative and computational intense variables in the Runge-Kutta
croutines are double
c precision. Runge - Kutta now has fixed step size
c Program shows the pressure at the 4km tube ends as one goes through
c the process of finding a single leak in the module at position x. The
technique is to
c close off first the pump at one and and then the pump at the other end
while observing
c the pressure at both ends.
С
c The program uses the Runge Kutta techniques from Numerical Recipes
c The calculation is made in cgs units with the pressure in torr
С
  The outgassing is described by:
С
С
       aj = fixed outgassing rate
С
С
       a = tube radius
С
С
С
       al = length of module/number of sections = section length
С
С
       fp = pumping speed of the pumps
С
       v = thermal velocity of molecule at 300K
С
С
       alk = leak at position x along the tube
С
c Difference equation to solve
С
c Section 1: dp/dt = ((2*v*a)/(3*al**2))*(p(2)-p(1)) + (2/a)*aj
                    + (aqlk(1))-fp(1)*p(1))/(pi*al*a**2)
c end pt
С
c Section n: dp/dt = ((2*v*a)/(3*al**2))*2*(p(n-1)-p(n))
c sym mid pt
                        + (2/a)*(aj)
                        + (aqlk(n)))/(pi*al*a**2)
С
С
c Section k : dp/dt = ((2*v*a)/(3*a1**2))*(p(k-1)-2*p(k) +p(k+1))
                      + (2/a)*(aj)
С
                      + (aqlk(k))/(pi*al*a**2)
С
С
c section m : dp/dt = ((2*v*a)/(3*al**2))*(p(m-1)-p(m)) +b(2/a)*aj
С
                      +(aqlk(m)-f(m)*p(m))/(pi*al*a**2)
С
С
        use winteracter
        character fileout*100
        real*8 ystart,dydx
        dimension p(1000,8000),ystart(1000),time(8000)
        dimension fp(1000),alk(1000),dydx(1000)
```

```
common /deriv/ pstart,aa,bb,cc,dd,ajt,nsec,tbon,tboff,tpon
        write(unit=*,fmt=601)
601
        format(' enter # of sections :' $)
         read(unit=*,fmt=602)nsec
602
        format(i6)
c set pump speeds and leaks in all sections to 0
         do 620 \text{ k}=1, \text{nsec}
          fp(k)=0.0
          alk(k)=0.0
620
         continue
          write(unit=*,fmt=1)
1
        format(' enter pump speed lit/sec at x=0: ' $)
          read(unit=*,fmt=2)fpp1
          fpp1 = fpp1*1000.0
2
         format(e15.6)
           write(unit=*,fmt=1111)
1111
       format(' enter pump speed lit/sec at x = L:' $)
        read(unit=*,fmt=2)fpp2
        fpp2=fpp2*1000.0
        write(unit=*,fmt=605)
605
         format(' enter amu of gas : ' $)
         read(unit=*,fmt=4)amu
         write(unit=*,fmt=3)
         format(' enter J, torr liters/sec/cm<sup>2</sup> : ' $)
3
          read(unit=*,fmt=4)ajt
          ajt=ajt*1000.0
        format(e15.6)
4
c enter the fixed parameters
c beam tube radius cm
       a = 61.91
c thermal speed of gas in cm/sec
       v = (5.901e4) * sqrt(18.0/amu)
c module length cm
       al0 = 3.94108e5
c number of sections
        an = real(nsec)
c section length cm
       al = al0/an
       write(unit=*,fmt=9)
9
         format(' enter starting pressure in torr : '$)
        read(unit=*,fmt=4)pstart
         write(unit=*,fmt=1202)
1202
       format(' enter period in hrs to turn pumps at end on/off :' $)
          read(unit=*,fmt=4)tau
         tau=tau*3600.0
          write(unit=*,fmt=1301)
1301
        format(' enter leak size (torr*lit/sec), sec# for leak ' $)
        read(unit=*,fmt=411)allk,nlksec
411
          format(e15.6,i6)
        alk(nlksec)=allk*1000.0
        aa = (2.0*v*a)/(3.0*al**2)
        bb = 2.0/a
        dd = 1.0/(3.14159*al*a**2)
         do 1000 kt = 1, nsec
         p(kt,1)=pstart
```

```
1000
         continue
9876
         write(unit=*,fmt=7)
7
         format(' time(hrs)/step, #steps total, #calsteps/step : '$)
           read(unit=*,fmt=8)tstep1,mstep,intstep
8
        format(e15.6,2i6)
c convert time to seconds
         tstep = tstep1*3600.0
         x2=0.0
         nvar = nsec
         do 100 k=1,mstep
         x1 = x2
         x2 = x1 + tstep
         phase = 2.0*3.14159*x1/tau
         phase = sin(phase)
         if (phase.le.0.0) tbon = -1.0
         if(phase.gt.0.0)tbon = 1.0
         if(tbon.gt.0.5) then
         fp(1)=0.0
         fp(nsec)=fpp2
          end if
          if(tbon.lt.-0.5)then
          fp(1) = fpp1
          fp(nsec) = 0.0
          end if
           if(k.eq.1)then
           do 1002 n=1,nsec
            ystart(n)= pstart
1002
         continue
         time(k) = x1/3600.0
         p(nsec+1,k)=pstart
         end if
         call rkdumb(ystart,x1,x2,dydx,intstep,fp,alk)
          do 1003 n=1,nsec
           p(n,k+1)=ystart(n)
1003
        continue
     time(k+1)=x2/3600.0
c calculate the average pressure
          sum = 0.0
          do 400 n=1, nsec
           sum = sum + p(n, k+1)/an
400
        continue
        p(nsec+1, k+1) = sum
        write(unit=*,fmt=11)time(k),p(nsec+1,k)
11
        format(' time (hours) = '1pe15.6, 'avg p (torr) = '1pe15.6)
100
         continue
        write(unit=*,fmt=77)
2000
77
        format('enter fileout: '$)
          read(unit=*,fmt=78)fileout
78
        format(a100)
           open(unit=2,file=fileout)
           write(unit=*,fmt=4001)
       format(' enter 1 for p vs x, 2 for p vs t : '$)
4001
            read(unit=*,fmt=4002)ich
4002
       format(i3)
```

```
if(ich.eq.2)then
           write(unit=*,fmt=2001)nsec+1
2001
        format(' enter #segment ('i4,' = avg pressure) : '$)
           read(unit=*,fmt=2002)ks
2002
        format(i6)
           write(unit=2,fmt=79)mstep+1
79
        format(i5)
          do 200 k=1,mstep+1
           write(unit=2,fmt=80)time(k),p(ks,k)
80
        format(1pe15.6,1pe15.6)
200
        continue
           close (2)
           end if
           if(ich.eq.1)then
           write(unit=2,fmt=4011)nsec
4011
       format(i5)
          m=mstep+1
           do 4020 j=1,nsec
c set to center of section
           xs = al*(real(j-1)+0.5)/100.0
          write(unit=2,fmt=4021)xs,p(j,m)
4021
       format(1pe15.6,1pe15.6)
4020
       continue
          end if
           write(unit=*,fmt=2003)
2003
        format(' enter 1 to write another file: '$)
            read(unit=*,fmt=2004)igo
2004
        format(i3)
            if(igo.eq.1)go to 2000
3000
        continue
            end
        subroutine derivs(y,dydx,fp,alk)
                   real*8 y,dydx
                  dimension y(1000), dydx(1000), fp(1000), alk(1000)
        common /deriv/ pstart,aa,bb,cc,dd,ajt,nsec,tbon,tboff,tpon
            do 100 n=2,nsec-1
            xy = aa*(y(n-1)+y(n+1)-2.0*y(n))+bb*(ajt)
            dydx(n) = xy + dd*(alk(n)-fp(n)*y(n))
100
        continue
            dydx(1) = aa*(y(2)-y(1))+bb*(ajt)+dd*(alk(1)-fp(1)*y(1))
            xy = aa*(y(nsec-1)-y(nsec))+bb*(ajt)
            dydx(nsec) = xy+dd*(alk(nsec)-fp(nsec)*y(nsec))
            return
             end
        SUBROUTINE rk4(v,dydx,x,h,yout,fp,alk)
        INTEGER nsec, i
        REAL*8 dydx,yout,ystart,v,dv
        REAL*8 dym,dyt,yt
        real*4 x,h6,hh,xh,h
        dimension fp(1000),alk(1000),dydx(1000)
        dimension yout(1000),v(1000),dv(1000)
        dimension dym(1000), dyt(1000), yt(1000)
        common /deriv/ pstart,aa,bb,cc,dd,ajt,nsec,tbon,tboff,tpon
```

	hh=h*0.5
	h6=h/6.
	xh=x+hh
	do 11 $i=1$, nsec
	$v_{t}(i) = v(i) + bh + dv dv(i)$
11	gontinuo
ΤT	conclude call derive(ut dut fn alk)
	da 12 i-1 maan
	do $12 1=1$, nsec
1.0	yt(1)=v(1)+nn*dyt(1)
12	continue
	call derivs(yt,dym,fp,alk)
	do 13 1=1,nsec
	yt(i)=v(i)+h*dym(i)
	dym(i)=dyt(i)+dym(i)
13	continue
	call derivs(yt,dyt,fp,alk)
	do 14 i=1,nsec
	yout(i)=v(i)+h6*(dydx(i)+dyt(i)+2.*dym(i))
14	continue
	return
	END
С	(C) Copr. 1986-92 Numerical Recipes Software 7%W3.
	SUBROUTINE rkdumb(ystart,x1,x2,dydx,intstep,fp,alk)
	INTEGER nsec, intstep
	REAL*4 x1,x2,x,h
	real*8 ystart,v,dydx,dv,yout
	dimension fp(1000),alk(1000),ystart(1000),dydx(1000)
	dimension yout(1000),v(1000),dv(1000)
CU	USES rk4
	INTEGER i,k
	<pre>common /deriv/ pstart,aa,bb,cc,dd,ajt,nsec,tbon,tboff,tpon</pre>
	do 11 i=1,nsec
	v(i)=ystart(i)
11	continue
	x=x1
	h=(x2-x1)/real(intstep)
	do 13 k=1,intstep
	call derivs(v,dv,fp,alk)
	<pre>call rk4(v,dv,x,h,yout,fp,alk)</pre>
	<pre>if(x+h.eq.x)pause 'stepsize not significant in rkdumb'</pre>
	x=x+h
	do 12 i=1,nsec
	v(i) = yout(i)
12	continue
13	continue
	do 14 i=1,nsec
	<pre>ystart(i)=yout(i)</pre>
14	continue
	return
	END
С	(C) Copr. 1986-92 Numerical Recipes Software 7%W3.