Searching for Gravitational Waves

John T. Whelan
john.whelan@astro.rit.edu

Center for Computational Relativity & Gravitation
& School of Mathematical Sciences
Rochester Institute of Technology

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Outline

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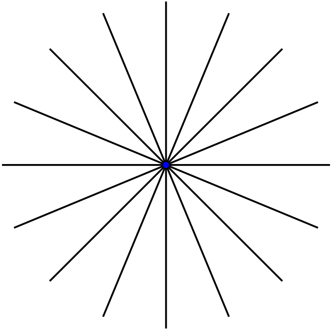
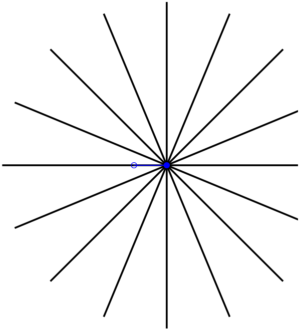
Action at a Distance

- Newtonian gravity: mass generates gravitational field
- Lines of force point towards object



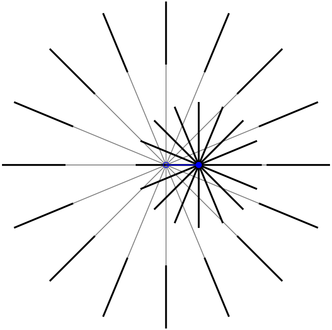
Issues with Causality

- Move object; Newton says: lines point to new location
- Relativity says: can't communicate faster than light to avoid paradoxes
- You could send me supraluminal messages via grav field



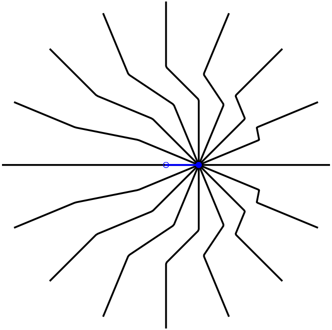
Gravitational Speed Limit

- If I'm 10 light years away,
  I can't know you moved
  the object 6 years ago
- Far away, gravitational field lines
  have to point to
  old location of the object



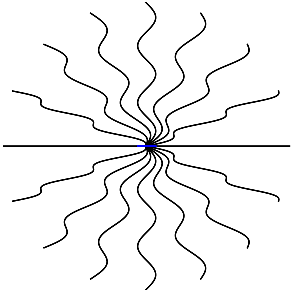
Gravitational Shock Wave

- Sudden motion (acceleration) of object generates gravitational shock wave expanding at speed of light

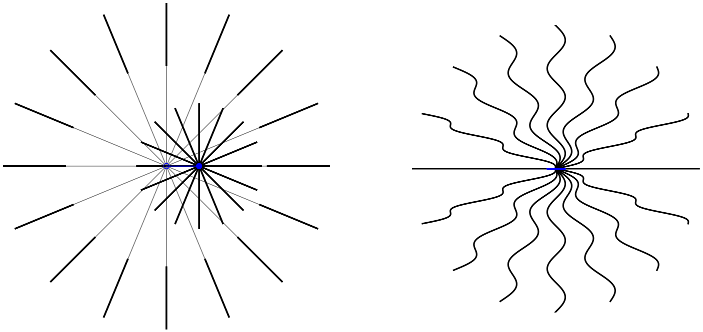


Ripples in the Gravitational Field

- Move object back & forth
  $\longrightarrow$ gravitational wave
- Same argument applies to electricity:
  - can derive magnetism as relativistic effect
  - accelerating charges generate electromagnetic waves propagating @ speed of light



Gravity + Causality = Gravitational Waves



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime (Special Relativity):
  Invariant spacetime interval (all inertial observers agree):

$$ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

$$= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

  Metric tensor $\{g_{\mu\nu}(\{x^\lambda\})\}$ determined by masses via Einstein's equations. (10 non-linear PDEs!)

Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

  ($h_{\mu\nu}$ "small" in weak-field regime, e.g. for GW detection)
- $h_{\mu\nu}$ is like electromagnetic potentials φ, $\vec{A}$; small coordinate changes like gauge transformations
- Convenient choice of gauge is transverse-traceless: In this gauge:
  - Test particles w/constant coords are freely falling
  - Vacuum Einstein eqns $\Longrightarrow$ wave equation for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{ij} = 0$$

Gravitational Wave Polarization States

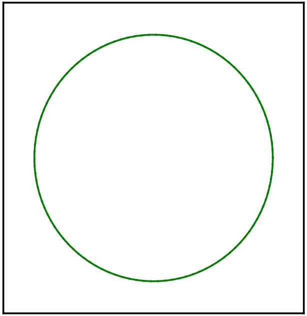
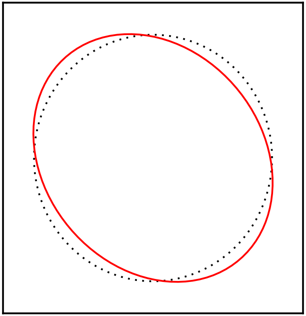
Far from source, GW looks like plane wave prop along $\vec{k}$
TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $h_+\left(t - \frac{x^3}{c}\right)$ and $h_\times\left(t - \frac{x^3}{c}\right)$ are components in "plus" and "cross" polarization states

Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization	Cross (×) Polarization
	

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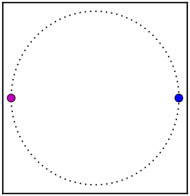
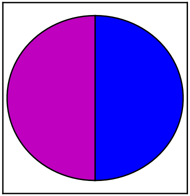
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
   - Gravitational-Wave Observations & Detectors
2. Upper Limit Results from Initial Detectors
   - Gamma-Ray Bursts
   - Known Pulsars
   - Gravitational-Wave Backgrounds
3. Prospects for Detections with Advanced Detectors
   - Compact Binaries
   - Unknown Neutron Stars
   - Accreting Neutron Stars

Generation of Gravitational Waves

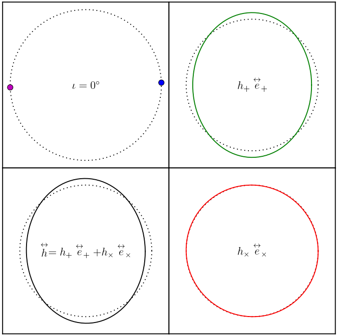
- EM waves generated by moving/oscillating charges
- GW generated by moving/oscillating masses
- Lowest multipole is quadrupole
- Different types of signals:
  - Burst (transient, unmodelled)
  - Stochastic (long-lived, unmodelled)
  - Binary coalescence (transient, modelled)
  - Periodic (long-lived, modelled)

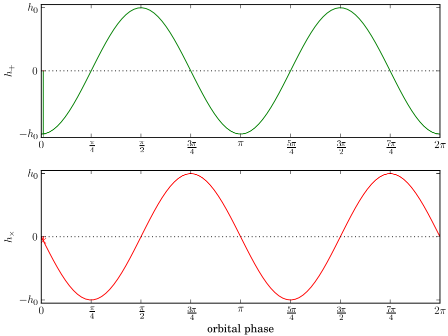




Gravitational Waves from Binary Orbit

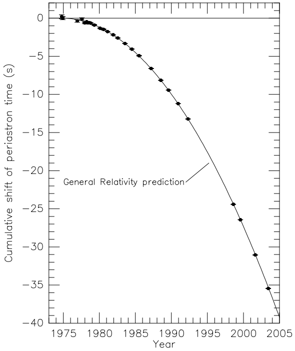
- Orbital motion $\rightarrow$ oscillating quadrupole moment $\rightarrow$ GWs





Gravitational Waves from Binary Orbit

- Orbital motion → oscillating quadrupole moment → GWs
- GW emission removes energy → orbit gets tighter → amplitude & freq increase in "chirp"
- Hulse & Taylor saw this evolution in binary pulsar 1993 Nobel Prize



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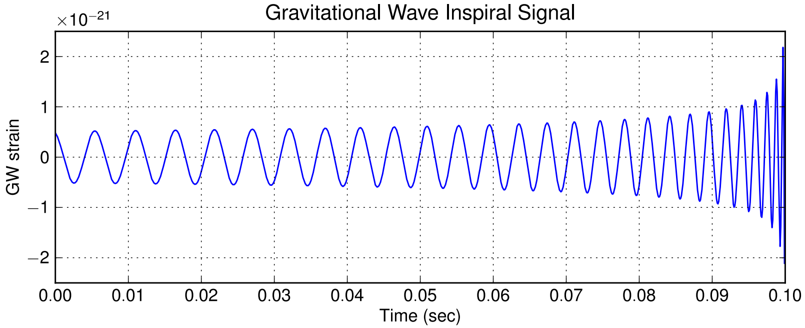

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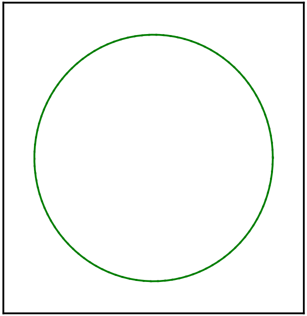
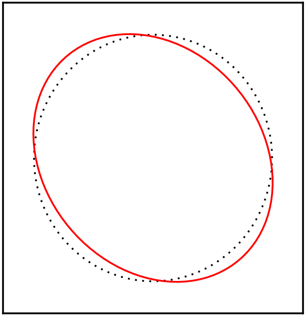
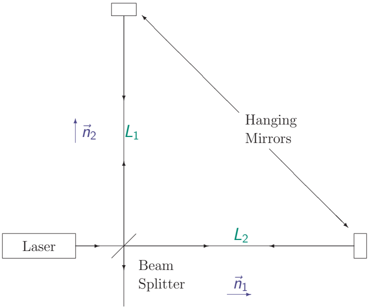
	

Measuring GWs w/Laser Interferometry

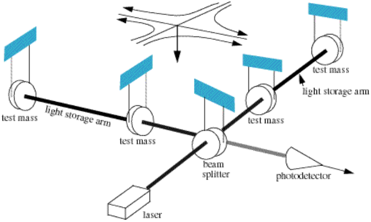
Interferometry: Measure GW-induced distance changes



- Measure small change in
  $$L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$
- Plausible signals: $h \lesssim 10^{-20}$
  $\rightarrow$ need L_0 very big!
- For LIGO, $L_0 = 4\,\mathrm{km} = 2.5\,\mathrm{mi}$

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in $L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$
- Plausible signals: $h \lesssim 10^{-20}$ $\rightarrow$ need L_0 very big!
- For LIGO, $L_0 = 4\,\text{km} = 2.5\,\text{mi}$

Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

LIGO Livingston (La.)

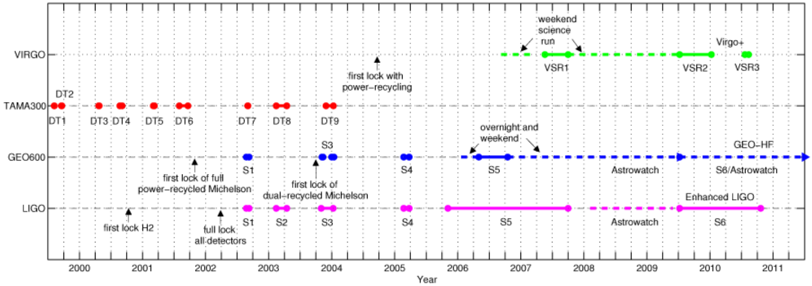
GEO-600 (Germany)

Virgo (Italy)

Initial Gravitational Wave Detector Network

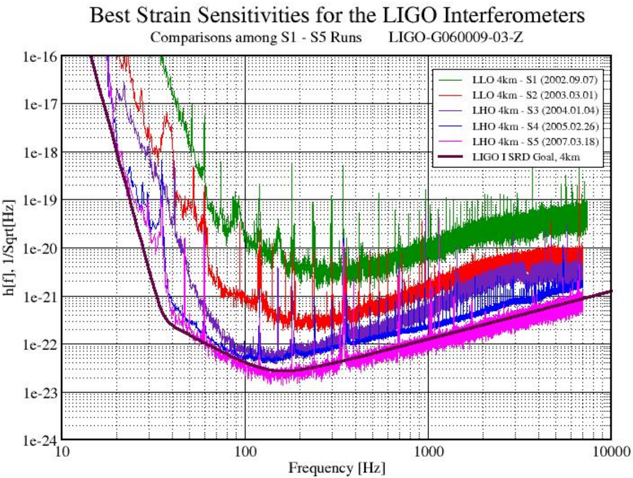
- “1st generation” ground-based interferometertic GW detectors (kilometer scale):
  - TAMA 300 (Tokyo, Japan) first online, late 90s; now offline
  - LSC (LIGO Scientific Collaboration) detectors conducting science runs since 2002
    - LIGO Hanford (4km H1 & 2km H2)
    - LIGO Livingston (4km L1)
    - GEO-600 (600m G1)
  - Virgo (3km V1) started science runs in 2007
  - LSC-Virgo long joint runs @ design sensitivity 2005-2010
- LIGO and Virgo being upgraded to 2nd generation “advanced” detectors ($10\times$ improvement in sensitivity)
- GEO-600 remains operational in “astrowatch” mode in case there’s a nearby supernova

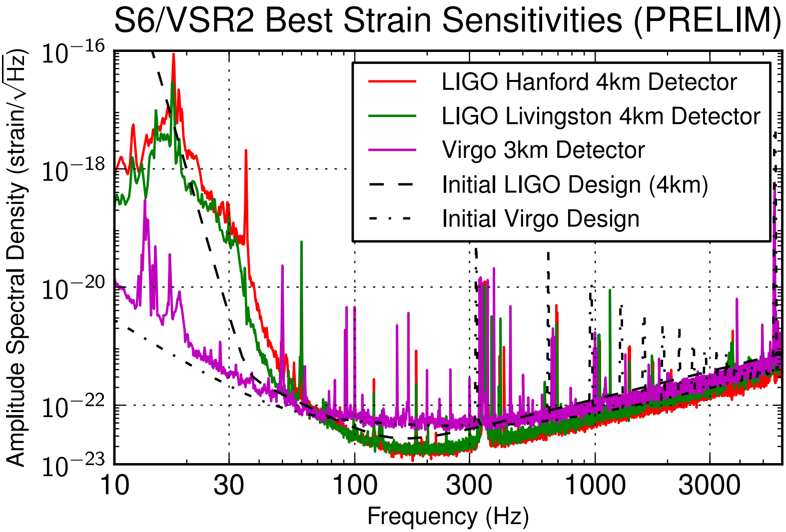
Initial Gravitational Wave Detector Network



Living Reviews in Relativity **14**, 5 (2011)

Evolution of LIGO Sensitivity S1-S5




S6/VSR2 Best Strain Sensitivities (PRELIM)
Amplitude Spectral Density (strain/√Hz)
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design
10^−16
10^−18
10^−20
10^−22
10^−23
10
30
100
300
1000
3000

Advanced Gravitational Wave Detector Network

- "2nd generation" ground-based interferometric GW detectors:
  - Adv LIGO expected to take science data from 2015
    4km detectors in Livingston, La. & Hanford, Wa.
  - Advanced Virgo should be on comparable timescale
  - KAGRA (cryogenic detector in Kamioka mine, Japan)
    uses 2.5-generation technology
  - Third advanced LIGO detector (4km)
    may be installed in India, taking data c.2019+
    Big payoff for sky localization via triangulation
- Planning for 3rd generation already underway:
  - Einstein Telescope in Europe
  - USA 3G plans still under development

Outline

Results of Initial Detector Observations

- 70+ Observational papers from initial LIGO/Virgo/GEO: https://www.lsc-group.phys.uwm.edu/ppcomm/Papers.html
- No detections (although some analyses still trickling out)
- Assortment of null results and upper limits
- As sensitivity improves, some of these results give new information to complement astronomical observations: "Multi-Messenger Astronomy"
- Some highlights . . .

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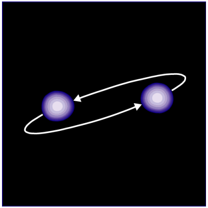
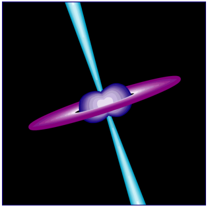
Gravitational Waves from Gamma-Ray Bursts

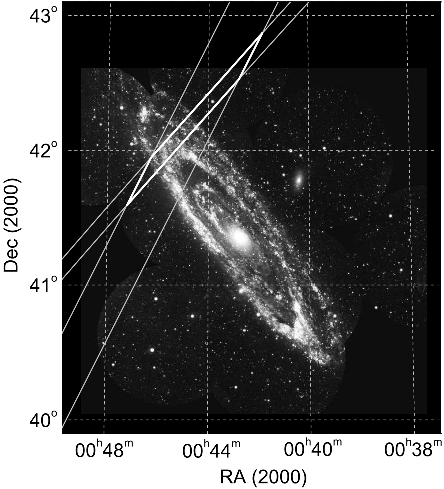




- GRBs are bursts of high energy photons observed by orbiting satellites like Swift and Fermi
- One possible source is the merger of a neutron star w/another neutron star or a black hole
- Search for the GWs emitted by the neutron star as it inspirals; search is “triggered” by the GRB, so can compare data at GRB time to data at other times

GRB070201

- 2007 Feb 1: short GRB whose error box overlapped spiral arm of M31 (770 kpc* away)
- LHO 4 km & 2 km detectors operating & sensitive to inspiral out to 35.7& 15.3 Mpc
- No GW seen; rule out binary progenitor in M31 w/> 99% conf
- *ApJ* **681**, 1419 (2008)



Similar result for GRB051103 & M81; *ApJ* **755**, 2 (2012)

*1 parsec (pc) = 3.26 light years

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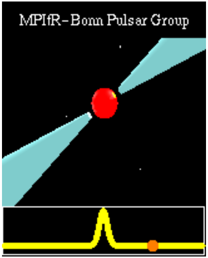
Searching for Known Pulsars



- Pulsar=rapidly rotating neutron star emitting radio or X-ray "pulses" as it spins (pulse comes when magnetic pole points at Earth)
- Pulsars spin down mostly due to drag of magnetic field through nebula
- If pulsar has small bump, will emit GWs
- Can search for periodic GW signal modulated by Doppler effect as Earth rotates & orbits Sun
- Parameters like freq, sky position, etc known from pulsar
- Spindown produces indirect upper limit
  - GW emission above limit $\longrightarrow$ more spindown than seen
  - Pulsars w/rapid spindown have "more room" for GW
  - LIGO/Virgo have surpassed spindown limit for Crab & Vela

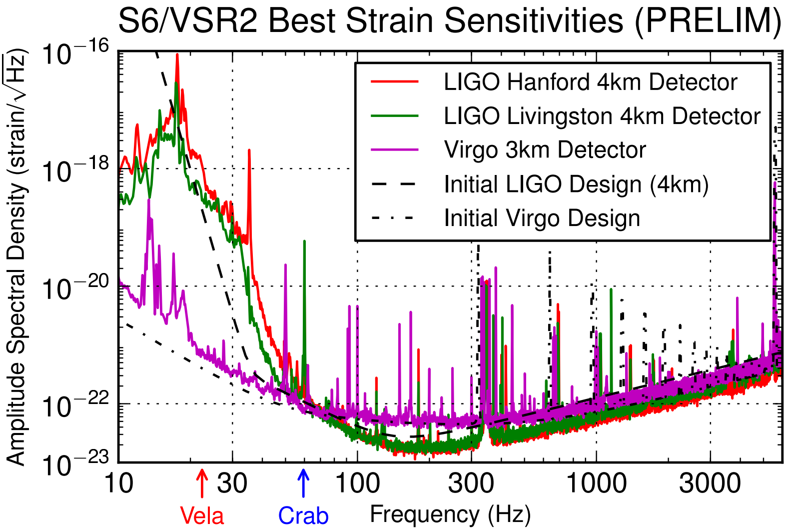
Crab Pulsar Upper Limit

- Pulsar in Crab Nebula
- Created by SN 1054
- $\sim 2\,\mathrm{kpc}$ away
- $f_{\mathrm{rot}} = 29.7\,\mathrm{Hz}$
- $f_{\mathrm{gw}} = 59.4\,\mathrm{Hz}$

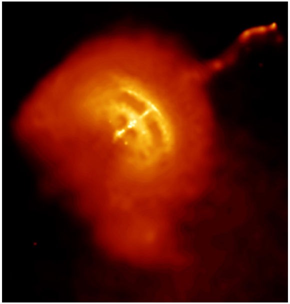
Image credit: Hubble/Chandra

- Initial LIGO (S5) upper limit beats spindown limit
- Abbott et al (LSC) *ApJL* **683**, L45 (2008)
- Abbott et al (LSC & Virgo) + Bégin et al *ApJ* **713**, 671 (2010)
- No more than 2% of spindown energy loss can be in GW

Initial Virgo Targets the Vela Pulsar



Vela Pulsar Upper Limit



- Pulsar in Vela SN remnant
- Created $\sim$ 12,000 years ago
- $\sim$ 300 pc away
- $f_{\text{rot}} = 11.2\,\text{Hz}$
- $f_{\text{gw}} = 22.4\,\text{Hz}$

Image credit: Chandra

- GW frequency below initial LIGO “seismic wall”
- Virgo has better low-frequency sensitivity
- VSR2 upper limit beats spindown limit
- No more than 10% of spindown energy loss can be in GW

Abadie et al (LSC & Virgo) + Buchner et al *ApJ* **737**, 93 (2011)

Outline

Searching for a Stochastic Background

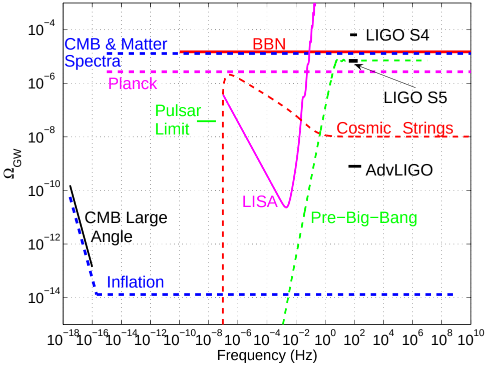
- Expect a random (stochastic) background of GWs left over from Big Bang (like the cosmic microwave background radiation) or from confusions of many faint sources
- Need to find a random signal in random noise!
- Noisy data from GW Detector:
  $x(t) = n(t) + h(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Look for correlations between detectors

$$\langle x_1 x_2 \rangle = \overbrace{\cancel{\langle n_1 n_2 \rangle}}^{\text{avgto0}} + \overbrace{\cancel{\langle n_1 h_2 \rangle}}^{\text{avgto0}} + \overbrace{\cancel{\langle h_1 n_2 \rangle}}^{\text{avgto0}} + \langle h_1 h_2 \rangle$$

- Details of expected correlation will depend on sky distribution of background

Allen & Romano *PRD* **59**, 102001 (1999)

Isotropic Stochastic Background Limit



S5 limit $\Omega_{gw}(f) < 6.9 \times 10^{-6} \left(\frac{72\,\text{km/s/Mpc}}{H_0}\right)^2$
[Abbott et al (LSC & Virgo) *Nature* **460**, 990 (2009)]
surpasses indirect limit from Big-Bang Nucleosynthesis

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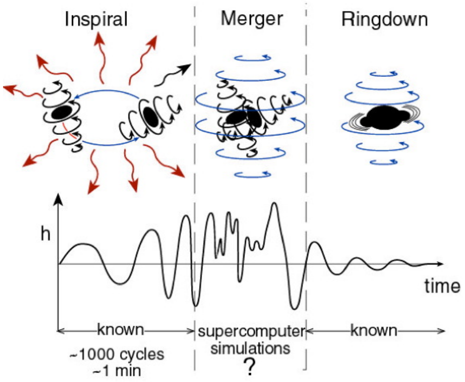
Template Waveforms for Binary Coalescence

- Inspiralling binaries produce well-modelled GW signals; Search with pattern-match filter
- Compact object binary coalescence consists of inspiral / plunge / merger / ringdown



Cartoon by Kip Thorne

Template Waveforms for Binary Coalescence

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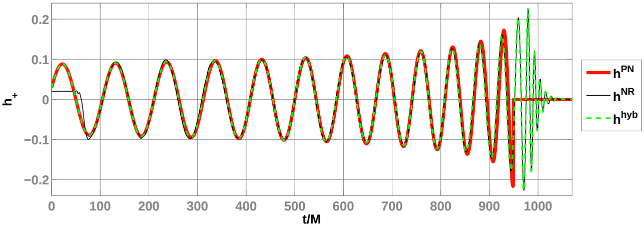


Ajith et al, *CQG* **24**, S689 (2007)

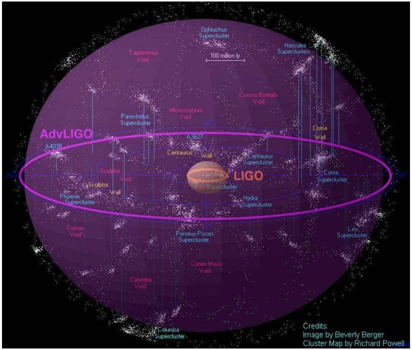
Template Waveforms for Binary Coalescence

- Compact object binary coalescence consists of inspiral / plunge / merger / ringdown
- For first part of inspiral, orbits not too relativistic can expand in powers of $\frac{v}{c} \longrightarrow$ post-Newtonian methods
  Can estimate orb vel from Kepler's 3rd law: $v \approx (\pi G M f)^{1/3}$
  - Low Mass $\longrightarrow$ plunge @ high freq
    $1.4M_\odot/1.4M_\odot$ NS/NS binary has $v \approx 0.3c$ @ 800 Hz;
    PN OK in LIGO band
  - High Mass $\longrightarrow$ plunge @ low freq
    $10M_\odot/10M_\odot$ BH/BH binary has $v \approx 0.4c$ @ 200 Hz;
    merges in LIGO band
- Different template families used for different mass ranges

Expected Event Rates w/Advanced Detectors

CQG **27**, 173001 (2010)

- Advanced detectors should see NS binary inspiral up to 400 Mpc & BH binary coalescence up to 2 Gpc away
- $\Longrightarrow$ Expect between a few and hundreds of events/year



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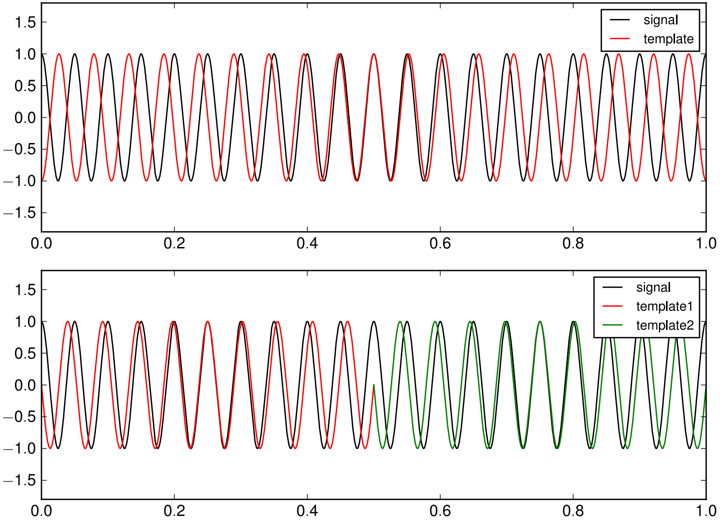
Searching for Unknown Neutron Stars

- Look for GWs from NSs not seen as pulsars
- Since freq, spindown, sky position, etc unknown, need to try different guesses in matched filter "template bank"
- Need to make bank dense enough so that true signal close to some template
- The longer you observe, the finer the needed resolution in frequency, sky position, etc
  E.g, for all-sky search with one spindown,

$$N_{\text{tmplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta \dot{f}} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2 \propto T^5$$

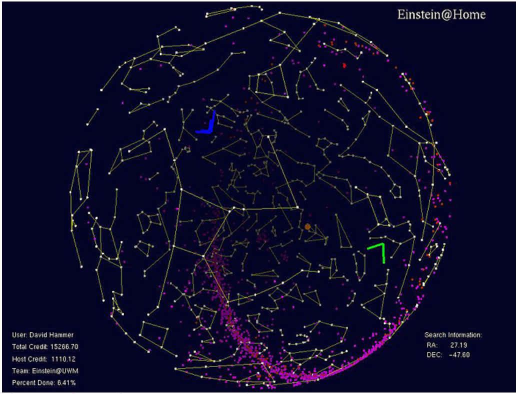
- Need to combine shorter coherent searches semicoherently

Coherent vs Semicoherent Searches



Searching for Unknown NSs: Einstein@Home

Semicoherent methods needed to handle phase param space;
Increase computing resources by enlisting volunteers
Distributed using BOINC & run as screensaver



http://www.einsteinathome.org/

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Gravitational Waves from Low-Mass X-Ray Binaries

- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides “hot spot”; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* **501**, L89 (1998) suggested GW spindown may balance accretion spinup; GW strength can be estimated from X-ray flux
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit

Brightest LMXB: Scorpius X-1

- Scorpius X-1
  - $1.4M_\odot$ NS w/$0.4M_\odot$ companion
  - unknown params are f_0, $a \sin i$, orbital phase
- LSC/Virgo searches for Sco X-1:
  - Coherent $\mathcal{F}$-stat search w/6 hr of S2 data
    Abbott et al (LSC) *PRD* **76**, 082001 (2007)
  - Directed stochastic ("radiometer") search (unmodelled)
    Abbott et al (LSC) *PRD* **76**, 082003 (2007)
    Abbott et al (LSC) arXiv:1109.1809
- Proposed directed search methods:
  - Look for comb of lines produced by orbital modulation
    Messenger & Woan, *CQG* **24**, 469 (2007)
  - Cross-correlation specialized to periodic signal
    Dhurandhar et al *PRD* **77**, 082001 (2008)
- Promising source for Advanced Detectors

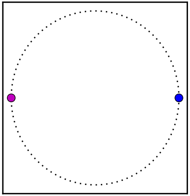
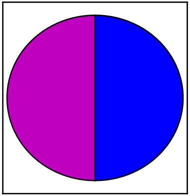
Resources for Further Investigation

- LIGO Science Pages:
  http://www.ligo.org/science/overview.php
- List of LSC and Virgo papers:
  https://www.lsc-group.phys.uwm.edu/ppcomm/Papers.html
  Includes links to free versions of papers on arXiv.org
- Summaries of recent LIGO science publications:
  http://www.ligo.org/science/outreach.php
- LIGO data releases:
  http://www.ligo.org/science/data-releases.php

EXTRA SLIDES

Multipole Expansion for Gravitational Radiation

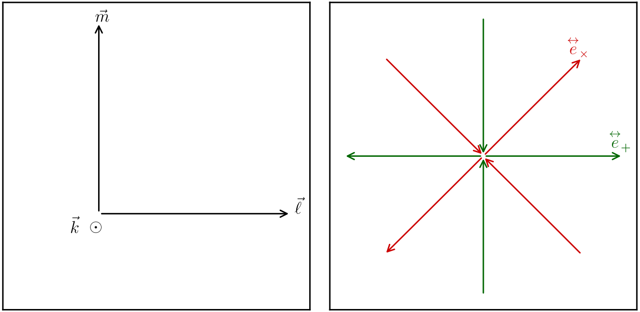
- **"Electric Dipole"?**
  No, "dipole moment" $\int \vec{r}\, dm \propto$ ctr of mass
  COM can't oscillate (also no negative "charge" in GR)
- **"Magnetic Dipole"?** No, "mag moment"
  $\frac{1}{2}\int \vec{r} \times \vec{v}\, dm \propto$ spin, another conserved quantity
- **"Electric Quadrupole"?** Yes! E.g., orbiting/rotating system w/ang vel Ω has GW frequency $f_{\text{gw}} = 2\frac{\Omega}{2\pi}$

The Polarization Basis

- wave propagating along $\vec{k}$;
  construct $\overleftrightarrow{e}_{+,\times}$ from $\perp$ unit vectors $\vec{\ell}$ & $\vec{m}$:

$$\overleftrightarrow{e}_{+} = \vec{\ell}\otimes\vec{\ell} - \vec{m}\otimes\vec{m} \qquad \overleftrightarrow{e}_{\times} = \vec{\ell}\otimes\vec{m} + \vec{m}\otimes\vec{\ell}$$



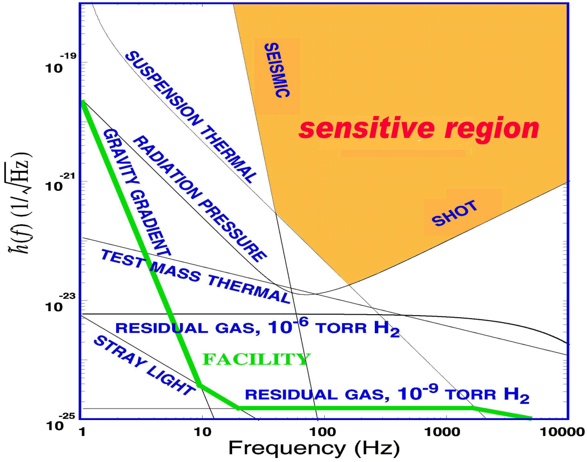
Some Sources of Gravitational Waves

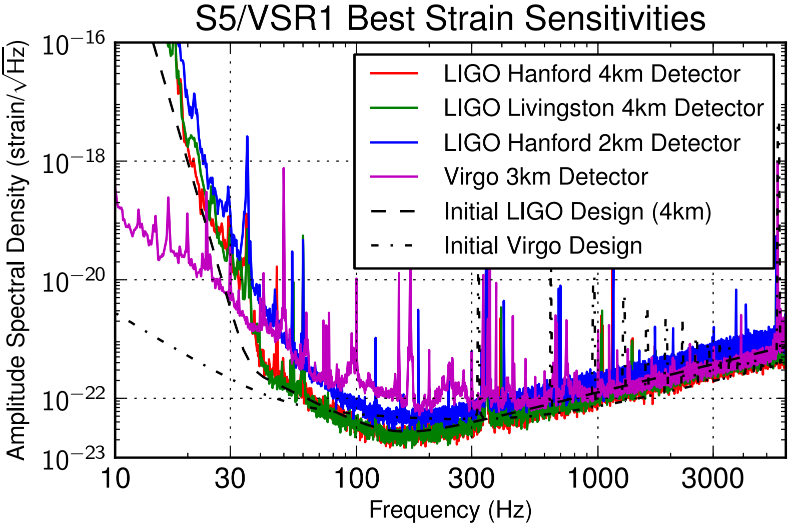
Useful to divide up by frequency band:

- Very Low Freq ($10^{-9}\,\mathrm{Hz} \lesssim f_{\mathrm{gw}} \lesssim 10^{-7}\,\mathrm{Hz}$)
- Low Freq ($10^{-3}\,\mathrm{Hz} \lesssim f_{\mathrm{gw}} \lesssim 10^{-1}\,\mathrm{Hz}$)
- High Freq ($10^{1}\,\mathrm{Hz} \lesssim f_{\mathrm{gw}} \lesssim 10^{3}\,\mathrm{Hz}$)

- Binary coalescence (inspiral+merger+ringdown):
  - Supermassive black hole binary
  - extreme mass ratio (stellar mass + SMBH)
  - Stellar mass BH and/or neutron star
- Galactic white dwarf binary orbit (continuous source)
- Rotating neutron star (pulsar, LMXB, etc)
- Supernova, Soft Gamma Repeater
- Cosmological background (primordial, phase transitions, cosmic superstrings, etc)
- SMBH flyby

LIGO's Sensitive Frequency Band




S5/VSR1 Best Strain Sensitivities
Amplitude Spectral Density (strain/√Hz)
10−16
10−18
10−20
10−22
10−23
10
30
100
300
1000
3000
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
LIGO Hanford 2km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design