# Gravitational Wave Data Analysis: a Mathematical and Statistical Challenge

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# Outline

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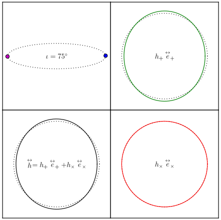
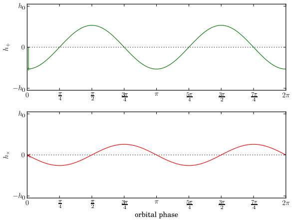
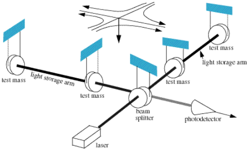
1. Dealing With the Data
2. Looking for the Signal
   - Modelled Signals
   - Unmodelled Signals
3. Interpreting the Results
   - Absence of Signal
   - Presence of Signal

# Gravitational Wave Basics





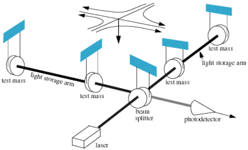



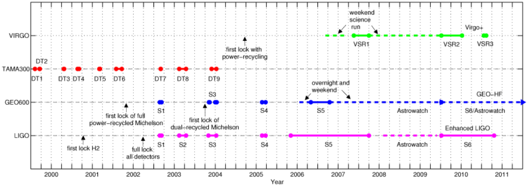

- Linearized gravity $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- TT gauge $h_{ij}\vec{e}_i\vec{e}_j = \overleftrightarrow{h} = h_+\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_+ + h_\times\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_\times$
- GW detector measures $h(t) = \overleftrightarrow{h} : \overleftrightarrow{d} = F_+ h_+(t) + F_\times h_\times(t)$ where $\overleftrightarrow{d} = \frac{\vec{u}\otimes\vec{u} - \vec{v}\otimes\vec{v}}{2}$ w/ $\vec{u}$ & $\vec{v}$ along arms

# Gravitational Wave Observations

- Limiting attention to ground-based interferometers
  Not considering space-based detectors, pulsar timing, etc

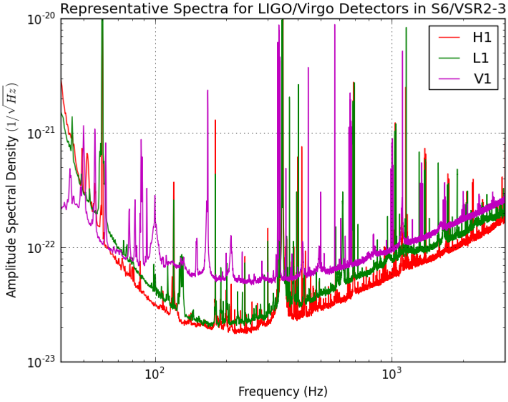
# Gravitational Wave Observations




- km-scale ifos in US & Europe; "initial detector era" 2002–11
- Currently upgrading for "advanced detector era" 2015+
- Data analysts finishing last of initial detector analyses & preparing improved analysis methods for ADE

# Characterization of Noise




Abadie et al (LSC/Virgo) arXiv:1203.2674

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# Characterization of Noise

- "Noise curve" is estimate of ASD $\sqrt{S_n(f)}$ where $S_n(f)$ is one-sided power spectral density
- For wide-sense stationary noise

$$E\left[n(t)n(t')\right] = \int_0^\infty \cos(2\pi f[t-t'])\, S_n(f)\, df$$

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$$E\left[\tilde{n}(f)\tilde{n}(f')\right] = \frac{1}{2}\delta(f - f')S_n(f)$$

- If noise is Gaussian, its probability distribution is

$$p(n) \propto \exp\left(-2\int_0^\infty \frac{|\tilde{n}(f)|^2}{S_n(f)}\,df\right)$$

- Real noise has non-Gaussian "glitches", non-stationarities, correlations between detectors, narrow "lines" etc
- Idealized model is a good starting point, but need to cope with complications

# Data Quality Vetos

Examining auxiliary data channels allows "bad" times to be flagged and/or vetoed according to categories, e.g.:

- **Cat 1** Do not include data in Fourier transforms
- **Cat 2** Okay to include in Fourier transform, but ignore any transient event at this time
- **Cat 3** Regard transient event at this time w/suspicion
- **Cat 4** Transient events somewhat more likely to be noise

Slutsky et al, *CQG* **27**, 165023 (2010), arXiv:1004.0998

# Is Vetoing Enough?

Class. Quantum Grav. **27** (2010) 194010 N Christensen

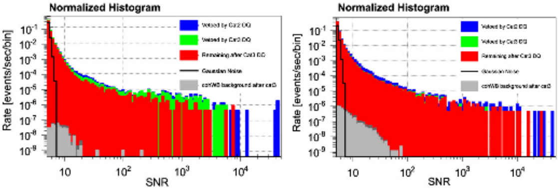


**Figure 1.** Single interferometer SNR plots from the coherent wave burst [25] pipeline overlaid with the single interferometer Omega [24] burst triggers; the Gaussian distribution is also given for comparison. Note that the SNR ~10 events are the problem for the coherent analysis; the single interferometer rate of SNR ~10 events is very large. The effect of the successive application of the DQ flag categories can be seen in the results for H1 (left) and L1 (right) from S6.

Vetoing bad times reduces noise "tails"
but data still not Gaussian

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# Classification of GW Signals

At freqs relevant to ground-based detectors (10s-1000s of Hz), natural division of sources:

| | modelled | unmodelled |
|---|---|---|
| long | Periodic Sources (aka Continuous Waves) (e.g., Rotating Neutron Star) | Stochastic Background (Cosmological or Astrophysical) |
| short | Binary Coalescence (Inspiral+Merger+Ringdown) (Black Holes, Neutron Stars) | Bursts (Supernova, BH Merger, etc.) |

# Outline

# Matched Filtering

Given noise and signal hypotheses
$\mathcal{H}_N: \quad x(t) = n(t) \qquad \mathcal{H}_S: \quad x(t) = n(t) + s(t)$
odds ratio is

$$\frac{p(\mathcal{H}_S|x)}{p(\mathcal{H}_N|x)} = \frac{p(x|\mathcal{H}_S)}{p(x|\mathcal{H}_N)} \frac{p(\mathcal{H}_S)}{p(\mathcal{H}_N)}$$

For Gaussian noise, Bayes factor $\equiv$ likelihood ratio is

$$\frac{p(x|\mathcal{H}_S)}{p(x|\mathcal{H}_N)} = \exp\left(-2\int_0^\infty \frac{|\tilde{x}(f) - \tilde{s}(f)|^2}{S_n(f)}\,df\right) \Big/ \exp\left(-2\int_0^\infty \frac{|\tilde{x}(f)|^2}{S_n(f)}\,df\right)$$

so log-likelihood ratio is

$$\ln \Lambda = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{s}(f)^* \tilde{x}(f)}{S_n(f)}\,df - 2\int_0^\infty \frac{|\tilde{s}(f)|^2}{S_n(f)}\,df$$

# Idealized Matched Filter & Complications

Define $(x|y) = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{x}(f)^*\tilde{y}(f)}{S_n(f)}\,df$

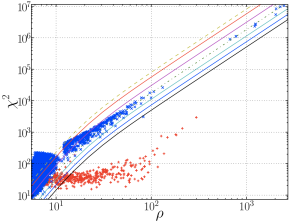
- Likelihood is $p(x|\mathcal{H}_N) \propto e^{-(x|x)/2}$
- Likelihood ratio is $\ln\frac{p(x|\mathcal{H}_S)}{p(x|\mathcal{H}_N)} = (s|x) - \frac{(s|s)}{2}$
- Could use $\rho = \frac{(s|x)}{\sqrt{(s|s)}}$ as detection statistic; $p(\rho|\mathcal{H}_N) \propto e^{-\rho^2/2}$

Complications:

- Data not Gaussian; true $p(\rho|\mathcal{H}_N)$ has large outliers
- $\mathcal{H}_S(\lambda)$ is composite hypothesis;
  $s(t;\lambda)$ depends on unknown signal parameters
- Data taken by detectors w/different location & orientation

# Suppressing Noise Outliers (Transient Search)

- Large values of matched filter SNR $\rho = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{s}(f)^*\tilde{x}(f)}{S_n(f)}\,df$ for signals, or for non-Gaussian glitches
- $\chi^2$ method (Allen *PRD* **71**, 062001 (2005); gr-qc/0405045):
  - Split into $p$ frequency intervals $\rho_i = 4\,\mathrm{Re}\int_{f_i}^{f_{i+1}} \frac{\tilde{s}(f)^*\tilde{x}(f)}{S_n(f)}\,df$ into which signal SNR would be evenly distributed.
  - Construct $\chi^2 = p\sum_{i=1}^{p}(\rho_i - \rho/p)^2$
- Empirically determine contours in $(\rho, \chi^2)$ which separate simulated signals from background events (Babak et al arXiv:1208.3491.)

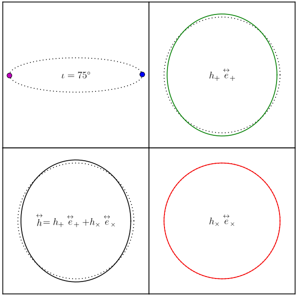


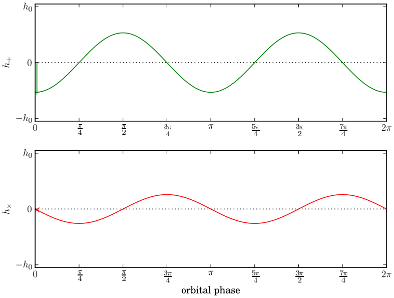
# Parameter Space

Signal expected in a detector depends on unknown params:

- distance, arrival time/phase, sky position $(\alpha, \delta)$, inclination $\iota$, polarization angle $\psi$
- For binary coalescence: masses, spins
- For periodic: NS spin, spindown, ellipticity, orbit if in binary

$$s(t) = A(t)\left(\frac{1+\cos^2\iota}{2}F_+\cos\phi(t) + \cos\iota\, F_\times \sin\phi(t)\right)$$



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$$s(t) = A(t)\left(\frac{1+\cos^2\iota}{2} F_+ \cos\phi(t) + \cos\iota\, F_\times \sin\phi(t)\right)$$

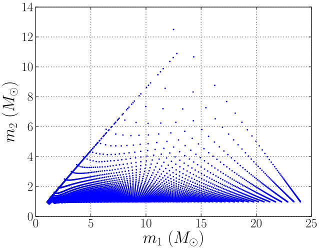
- CBC: shape of transient signal in a detector depends on masses & spins; other params just change amplitude $A(t)$
- CW: signal factors into $s(t) = \sum_{\mu=1}^{4} \mathcal{A}^\mu h_\mu(t)$ where $\{\mathcal{A}^\mu\}$ depend on amplitude params $\{h_0, \iota, \psi, \phi_0\}$; template shape depends on phase params $f_0$, $\frac{df}{dt}$, $\alpha$, $\delta$, etc. Jaranowski et al *PRD* **58**, 063001 (1998); gr-qc/9804014

# Template Banks

- Need to search for signal w/unknown parameters
- $(s_1|s_2) = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{s}_1(f)^*\tilde{s}_2(f)}{S_n(f)}\,df$
  lets us define "distance" btwn nearby param space pts
  (Owen *PRD* **53**, 6749 (1996); gr-qc/9511032)

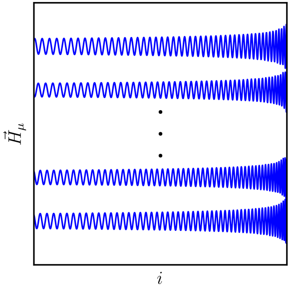
$$(s(\boldsymbol{\lambda})|s(\boldsymbol{\lambda}+\boldsymbol{d\lambda})) = 1 - \sum_{ij} g_{ij}\,d\lambda^i\,d\lambda^j$$

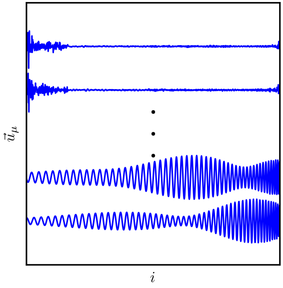
- "metric" $g_{ij}$ used to determine spacing btwn templates
  (figure from Babak et al arXiv:1208.3491.)



# Singular Value Decomposition for Template Banks

- Standard method specifies 97% overlap between signal & nearest template; neighboring templates overlap by $\gtrsim 94\%$
- Can speed up for low-latency analysis by using SVD to resolve templates in orthonormal basis & dropping least significant basis vectors
- Cannon et al *PRD* **82**, 044025 (2010); arXiv:1005.0012



# Coherent Search for Periodic Signals

- For continuous waves, divide signal parameters into
  - amplitude params: $\{h_0, \iota, \psi, \phi_0\}$
  - phase params: $\lambda \equiv \{\alpha, \delta, f_0, f_1, \ldots\}$
- Jaranowski et al *PRD* **58**, 063001 (1998); gr-qc/9804014 showed signal linear in $\{\mathcal{A}^\mu\}$, fcns of amplitude params

$$s(t) = \mathcal{A}^\mu h_\mu(t) \qquad (\text{assume } \textstyle\sum_{\mu=1}^{4})$$

  template waveforms $h_\mu(t)$ depend on phase params
- Log-likelihood ratio quadratic in $\{\mathcal{A}^\mu\}$:

$$\ln\Lambda(\mathcal{A},\lambda) = (s|x) - \frac{(s|s)}{2} = 2\mathcal{A}^\mu x_\mu(\lambda) - \mathcal{A}^\mu \mathcal{M}_{\mu\nu}(\lambda)\mathcal{A}^\nu$$

- $\mathcal{F}$-stat method uses best-fit amp params $\widehat{\mathcal{A}}^\mu = \mathcal{M}^{\mu\nu}(\lambda)x_\nu(\lambda)$ ($\mathcal{M}^{\mu\nu}$ is inv of $\mathcal{M}_{\mu\nu}$); detection statistic is max log-likelihood

$$\mathcal{F} = \ln\Lambda(\widehat{\mathcal{A}},\lambda) = \frac{1}{2}x_\mu(\lambda)\mathcal{M}^{\mu\nu}(\lambda)x_\nu(\lambda)$$

# Bayesian Interpretation ($\mathcal{B}$-statistic)

- Assume $\lambda$ known; likelihood $p(x|\mathcal{A}) \propto e^{-\chi^2(\mathcal{A})/2}$
- Bayes's theorem says $p(\mathcal{H}|x) = \frac{p(x|\mathcal{H})p(\mathcal{H})}{p(x)}$
- Odds ratio $\frac{p(\mathcal{H}_S|x)}{p(\mathcal{H}_N|x)} = \frac{p(x|\mathcal{H}_S)}{p(x|\mathcal{H}_N)} \frac{p(\mathcal{H}_S)}{p(\mathcal{H}_N)}$; Bayes Factor $\mathcal{B}_{10} = \frac{p(x|\mathcal{H}_S)}{p(x|\mathcal{H}_N)}$
- $\mathcal{H}_S \equiv$ noise + signal w/some $\mathcal{A}$; $\mathcal{H}_N \equiv$ noise only
- $\mathcal{F}$-stat is maximized log-likelihood: $\max_{\mathcal{A}} \frac{p(x|\mathcal{A})}{p(x|0)} = e^{\mathcal{F}}$
- But $\mathcal{H}_S$ is composite hypoth. $p(x|\mathcal{H}_S) = \int p(x|\mathcal{A})p(\mathcal{A}|\mathcal{H}_S)d^4\mathcal{A}$
- Don't maximize; marginalize! $\mathcal{B}$-statistic (Prix): $\mathcal{B} = \int \frac{p(x|\mathcal{A})}{p(x|0)} p(\mathcal{A}|\mathcal{H}_S)d^4\mathcal{A} = \int e^{-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu}\mathcal{A}^\nu + \mathcal{A}^\mu x_\mu} p(\mathcal{A}|\mathcal{H}_S)d^4\mathcal{A}$
- Prix & Krishnan *CQG* **26**, 204013 (2009): If $p(\mathcal{A}|\mathcal{H}_S)$ uniform in $\{\mathcal{A}^\mu\}$, $\mathcal{B} = e^{\mathcal{F}}$
  Unphysical; implies $p(h_0, \cos\iota, \psi, \phi_0|\mathcal{H}_S) \propto h_0^3(1-\cos^2\iota)^3$
- Prix & JTW working on approximations for evaluating $\mathcal{B}$-stat integral w/physical priors

# Computational Costs for CW Searches

- If $\lambda \equiv$ {freq, sky pos etc} known, can do most sensitive fully coherent search (correlate all data)
- If some params unknown, have to search over them
- Long coherent observation $\rightarrow$ fine resolution in freq etc $\rightarrow$ need too many templates $\rightarrow$ computationally impossible

$$\text{e.g.} \qquad N_{\text{tmplts}} \sim \frac{1}{\Delta f}\frac{1}{\Delta \dot{f}}\frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches semi-coherent: deliberately limit coherent integration time & param space resolution to keep number of templates manageable

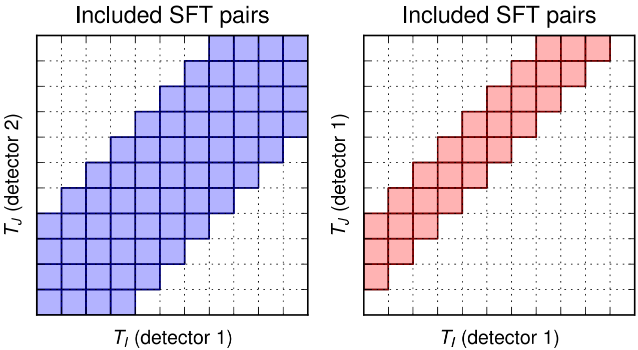
# One Semicoherent Method: Cross-Correlation

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)
Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)
(Currently being applied by JTW, Larson, Krishnan, et al)

- Divide data into segments of length $T_{\text{sft}}$ & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$
- Label SFTs by $I$, $J$, ... and pairs by $\alpha$, $\beta$, ...
  - ☞ $I$ & $J$ can be same or different times or detectors
- Construct cross-correlation $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{\tilde{k}_I})\tilde{x}_J(f_{\tilde{k}_J})}{(T_{\text{sft}})^2}$
  - ☞ $f_{\tilde{k}_I} \approx$ signal freq @ time $T_I$ Doppler shifted for detector $I$
- Use CW signal model to determine expected cross-correlation btwn SFTs & combine pairs into optimal statistic $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$

# Tuning the Cross-Correlation Search

- Computational considerations limit coherent integration time
- Can make tunable semi-coherent search by restricting which SFT pairs $\alpha$ are included in $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$
- E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \leq T_{\max}$



# Outline

# Stochastic Background Search Method

- Noisy data from GW Detector:
  $x(t) = n(t) + s(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Look for correlations between detectors

$$E\left[x_1 x_2\right] = \overbrace{\cancel{E\left[n_1 n_2\right]}}^{\text{avgto0}} + \overbrace{\cancel{E\left[n_1 s_2\right]}}^{\text{avgto0}} + \overbrace{\cancel{E\left[s_1 n_2\right]}}^{\text{avgto0}} + E\left[s_1 s_2\right]$$

- Expected cross-correlation (frequency domain)

$$E\left[\tilde{x}_1^*(f)\tilde{x}_2(f')\right] = E\left[\tilde{s}_1^*(f)\tilde{s}_2(f')\right] = \overleftrightarrow{d}_1 : E\left[\overleftrightarrow{\tilde{h}}{}_1^*(f) \otimes \overleftrightarrow{\tilde{h}}_2(f')\right] : \overleftrightarrow{d}_2$$

- For stochastic backgrounds

$$E\left[\tilde{s}_1^*(f)\tilde{s}_2(f')\right] = \delta(f-f')\gamma_{12}(f)\frac{S_{\text{gw}}(f)}{2}$$

$S_{\text{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

# Stochastic Background Detection Statistic

- Expected cross-correlation (frequency domain)

$$E\left[\tilde{x}_1^*(f)\tilde{x}_2(f')\right] = E\left[\tilde{s}_1^*(f)\tilde{s}_2(f')\right] = \delta(f-f')\gamma_{12}(f)\frac{S_{\text{gw}}(f)}{2}$$

- Optimally filtered cross-correlation statistic

$$Y = \int df\, \tilde{x}_1^*(f)\, Q(f)\, \tilde{x}_2(f)$$

- Filter encodes expected spectrum & spatial distribution (isotropic, pointlike, spherical harmonics . . . )

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{\text{gw}}^{\text{exp}}(f)}{S_{n1}(f) S_{n2}(f)}$$

# Burst Search Methods

- Robust method for finding transients:
  look for coincident signals in multiple detectors
- For bursts, no template, so look for e.g., excess power
- Can also combine detector data coherently

# One Coherent Burst Search Method

Sutton et al, *NJP* **12**, 053034 (2010); arXiv:0908.3665

- Vector $\mathbf{x}$ of $D$ detector outputs is

$$\mathbf{x} = \mathbf{F}\mathbf{h} + \mathbf{n} = \begin{pmatrix} x_1 \\ \vdots \\ x_D \end{pmatrix} = \begin{pmatrix} F_{1+} & F_{1\times} \\ \vdots & \vdots \\ F_{D+} & F_{D\times} \end{pmatrix} \begin{pmatrix} h_+ \\ h_\times \end{pmatrix} + \begin{pmatrix} n_1 \\ \vdots \\ n_D \end{pmatrix}$$

- Log-likelihood ratio is $\ln\Lambda = (\mathbf{F}\mathbf{h}|\mathbf{x}) - \frac{(\mathbf{F}\mathbf{h}|\mathbf{F}\mathbf{h})}{2}$
- Maximize by taking $\mathbf{h} = \widehat{\mathbf{h}} = (\mathbf{F}^\dagger\mathbf{F})^{-1}\mathbf{F}^\dagger\mathbf{h}$; detection stat is

$$E_{\text{gw}} = 2\ln\widehat{\Lambda} = (\mathbf{F}\widehat{\mathbf{h}}|\mathbf{F}\widehat{\mathbf{h}}) = (\mathbf{P}^{\text{gw}}\mathbf{x}|\mathbf{P}^{\text{gw}}\mathbf{x})$$

- $\mathbf{P}^{\text{gw}} = \mathbf{F}(\mathbf{F}^\dagger\mathbf{F})^{-1}\mathbf{F}^\dagger$ projects onto 2-dim space of GW signals
- $\mathbf{P}^{\text{null}} = \mathbf{1}_{D\times D} - \mathbf{P}^{\text{gw}}$ projects onto $(D-2)$-dim null space
- $E_{\text{null}} = (\mathbf{P}^{\text{null}}\mathbf{x}|\mathbf{P}^{\text{null}}\mathbf{x})$ used to veto noise transients

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# Setting Upper Limits

- If no significant signal seen, can set upper limit on event rate, known pulsar ellipticity, GW background strength, etc.
- E.g., inspiral event rate set using loudest event statistic
  Biswas et al *CQG* **26**, 175009 (2009); arXiv:0710.0465
- Typically use data $x$ to set limit on physical quantity $\mu$ by constructing posterior pdf

$$p(\mu|x, I) = \frac{p(x|\mu, I)p(\mu|I)}{p(x|I)}$$

  & integrating to find upper limit $\mu^{\text{ul}}$: $\int_0^{\mu^{\text{ul}}_{95\%}} p(\mu|x, I)\, d\mu = 0.95$
- Don't generally include much non-GW prior info in $p(\mu|I)$; in initial detector era, would often find $p(\mu|x, I) \approx p(\mu|I)$ if we did!
- Do use this method to combine independent experiments

$$p(\mu|x_2, x_1, I) = \frac{p(x_2, x_1|\mu, I)p(\mu|I)}{p(x_2, x_1|I)} = \frac{p(x_2|\mu, I)p(x_1|\mu, I)p(\mu|I)}{p(x_2|x_1, I)p(x_1|I)}$$

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# Blind Injection Challenge

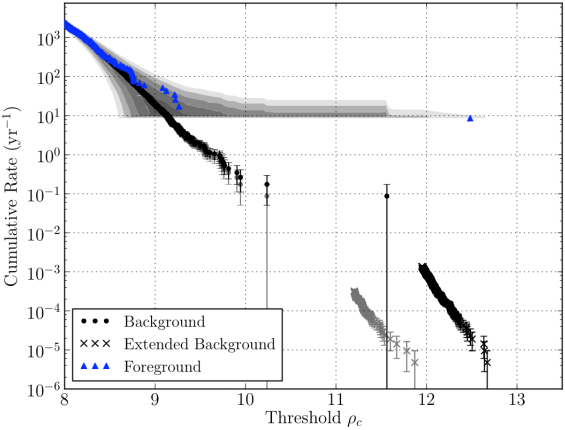
- No direct detections of GW so far; detection & parameter estimation methods tested by Blind Injection Challenge: http://www.ligo.org/science/GW100916/
- LIGO & Virgo routinely perform "hardware injections"; simulated signals added to data via control loop.
- For blind injection, time & parameters were concealed until analysis was complete
- Reported as part of S6/VSR2/VSR3 inspiral search
  Abadie et al (LSC/Virgo) *PRD* **85**, 082002 (2012); arXiv:1111.7314

# Detection Confidence

- Non-Gaussian data ☞ can't trust false alarm rate from $p(\rho|\mathcal{H}_N)$
- Can't "turn off" GW to get background (unless using EM triggers)
- Seek events found in coincidence in different detectors; estimate background by "time-sliding" data relative to each other. (Low thresholds $\rightarrow \exists$ many triggers in each detector.)
- Routinely do 100 time-slides to estimate significance of events. Only allows false alarm probability $\gtrsim 1\%$
- For BIC, used trigger lists to synthesize slides for all of S6
- Only louder "background" events were signal trigger in one detector + glitch in other.
  Different FAR estimates if you exclude "signal" trigger or not!

# Detection Confidence for Rare Events

Different FAR estimates if you exclude “signal” trigger or not!




Abadie et al (LSC/Virgo) *PRD* **85**, 082002 (2012); arXiv:1111.7314

# Parameter Estimation

- Matched-filter searches return best-fit pt in param space; not generally best estimate of true signal parameters:
  - Single-detector triggers independent of some params
  - Other parameter degeneracies
  - Coarse template banks
- Follow up detections with dedicated parameter estimation using Markov-Chain Monte Carlo, nested sampling, etc
- Produce posterior PDFs for signal parameters
- LSC/Virgo parameter estimation paper forthcoming . . .

# Summary

- GW data analysis involves not just model + experiment also statistical and mathematical signal processing
- Looking for modelled/unmodelled signal in non-ideal noise
- Matched filtering, but also coherent/semicoherent analyses, template banks, sensitivity vs computational cost
- Statistical inference used to
  - Set upper limits in the absence of a detection (now)
  - Assign confidence to a potential detection (soon!)
  - Determine parameters of detected systems (later)