



LIGO



Global longitudinal quad damping vs. local damping

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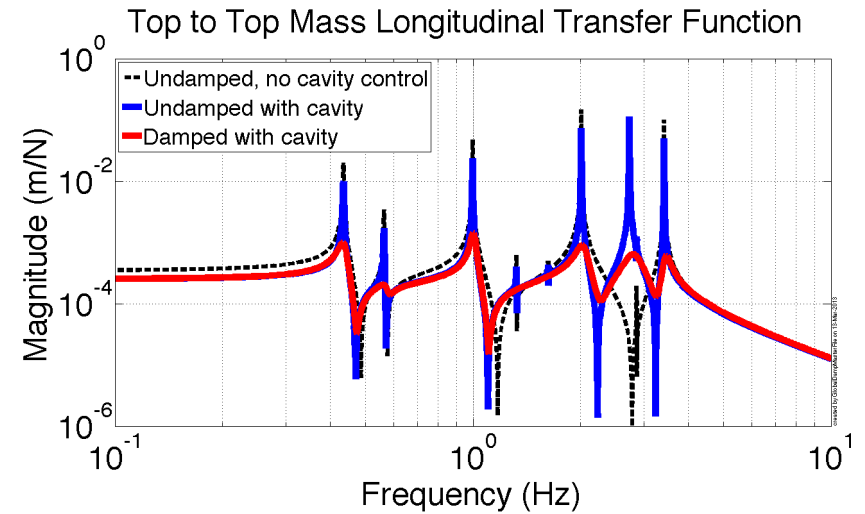
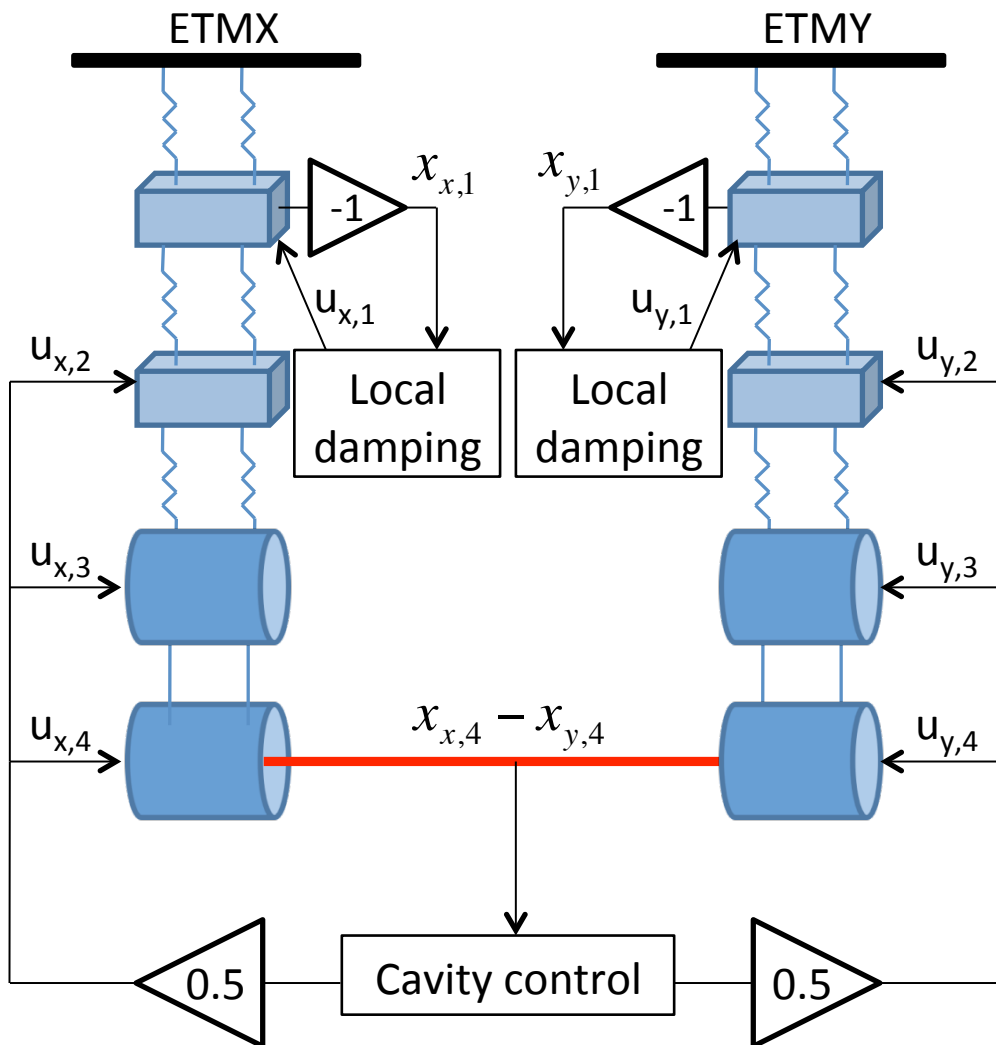
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Summary



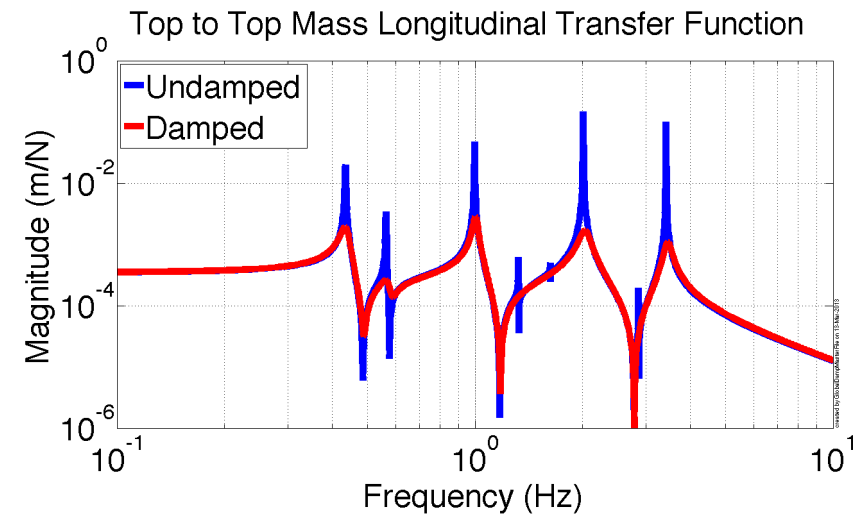
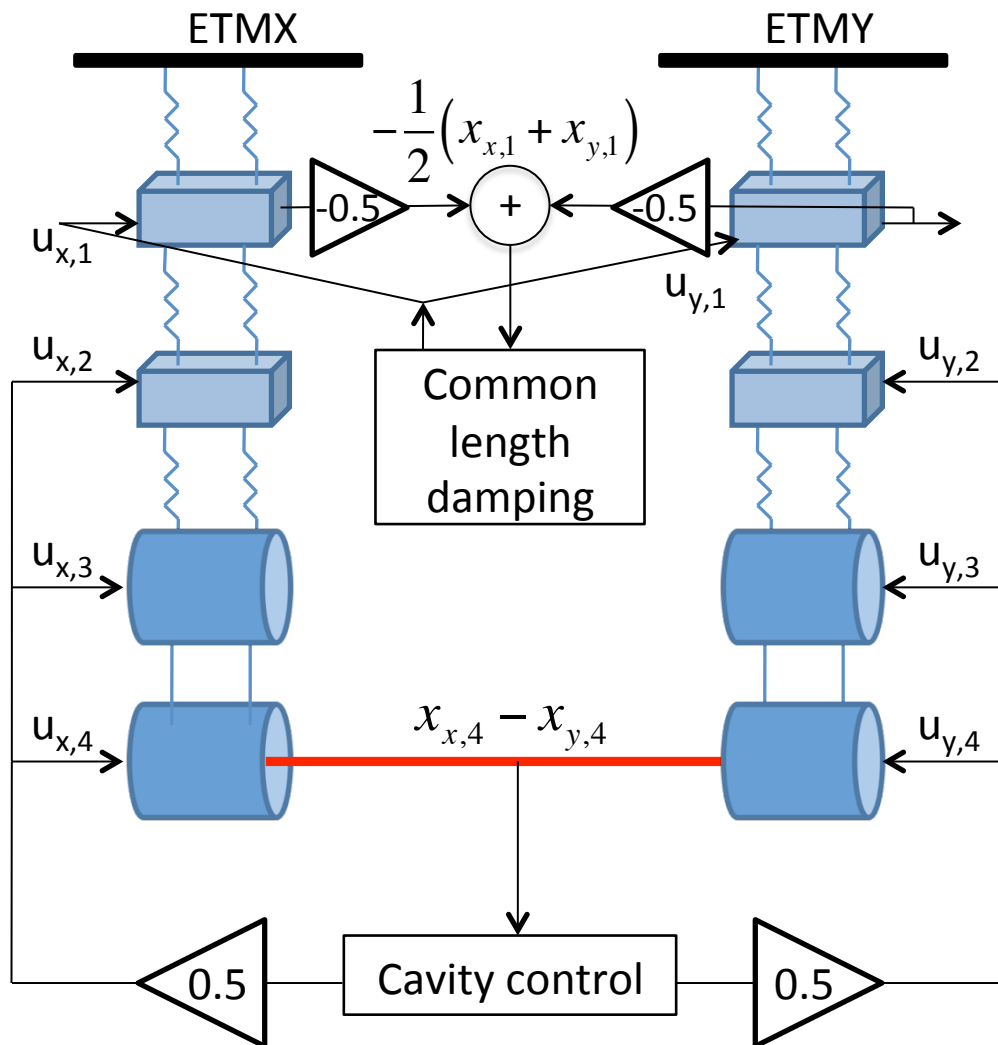
- Background: local vs. global damping
- Part I: global *common* length damping
 - Simulations
 - Measurements at 40 m lab
- Part II: global *differential* arm length damping without OSEMs
 - Simulations
 - Measurements at LIGO Hanford
- Conclusions

Usual Local Damping



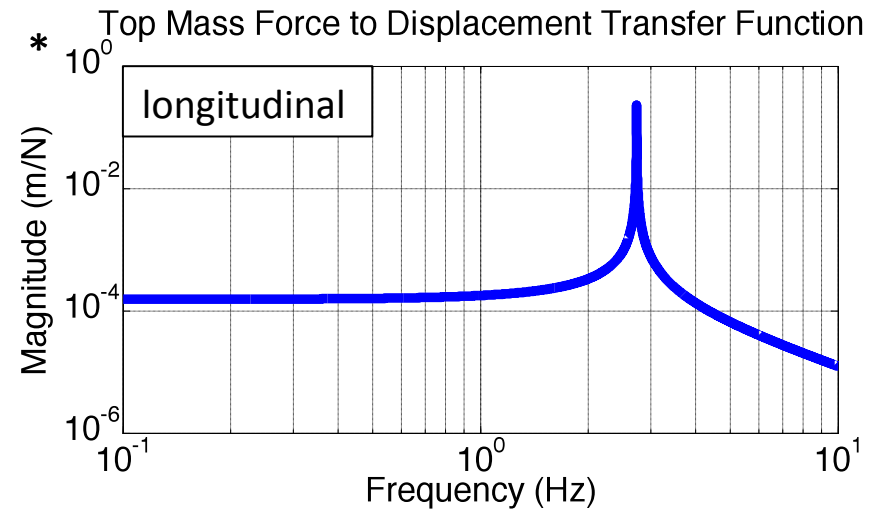
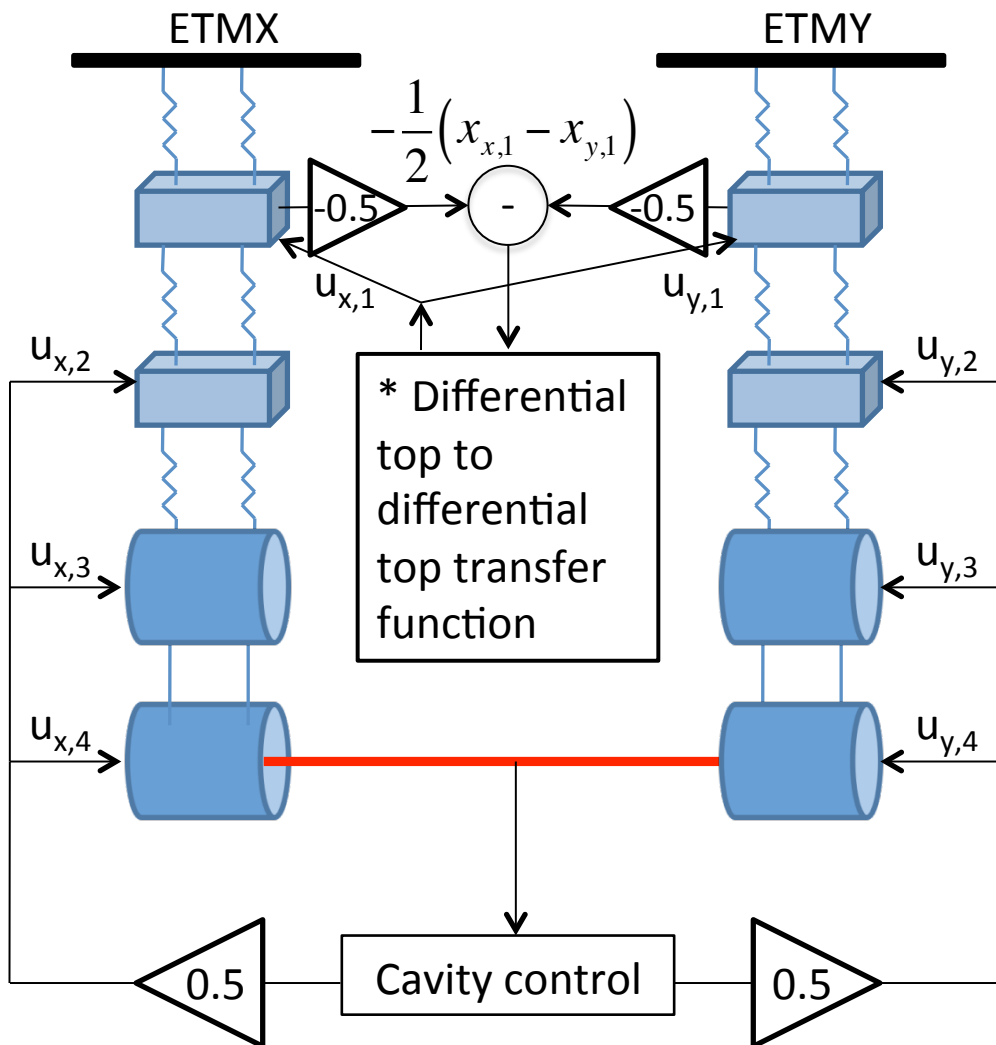
- The nominal way of damping
- OSEM sensor noise coupling to the cavity is non-negligible for these loops.
- The cavity control influences the top mass response.
- Damping suppresses all Qs

Common Arm Length Damping



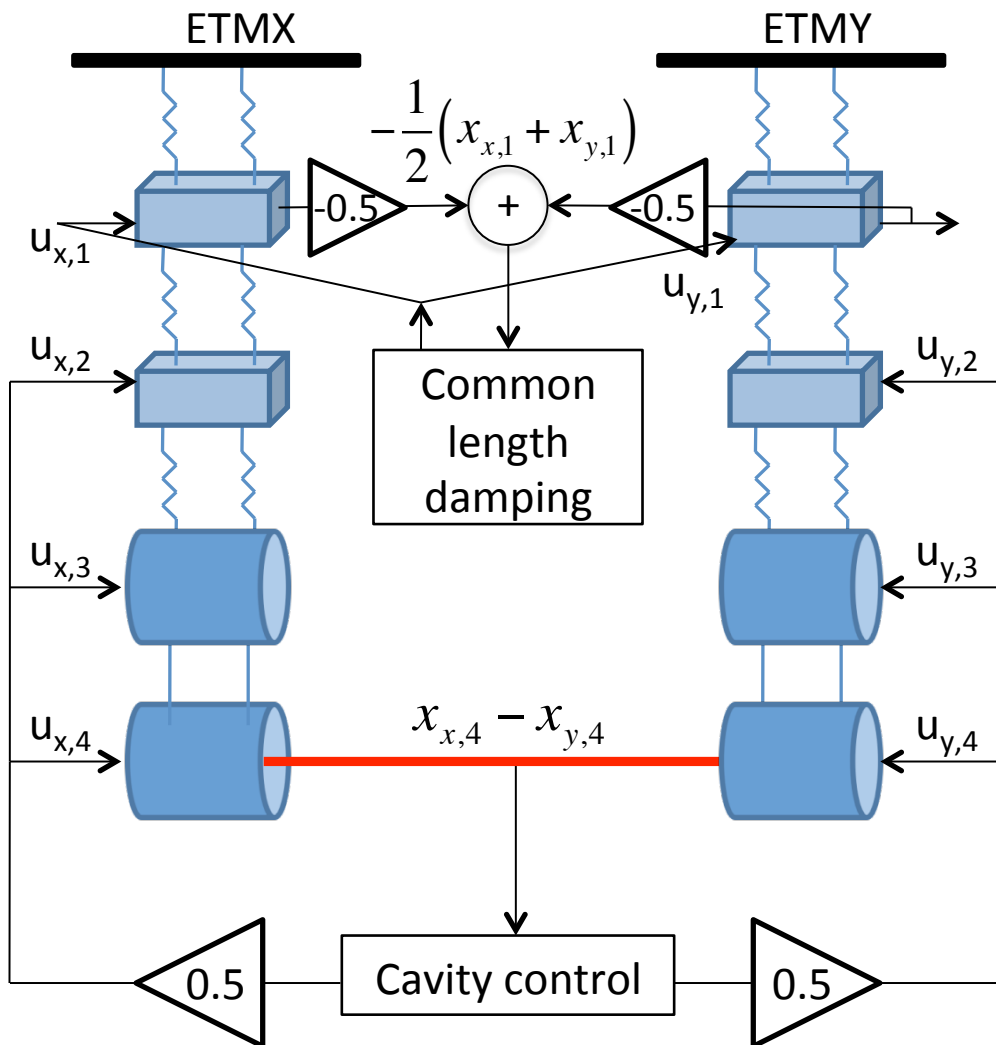
- Common length DOF independent from cavity control
- The global common length damping injects the same sensor noise into both pendulums
- Both pendulums are the same, so noise stays in common mode, i.e. no damping noise to cavity!

Differential Arm Length Trans. Func.



- The differential top mass longitudinal DOF behaves just like a cavity-controlled quad.
- Assumes identical quads (ours are pretty darn close).
- See 'Supporting Math' slides.

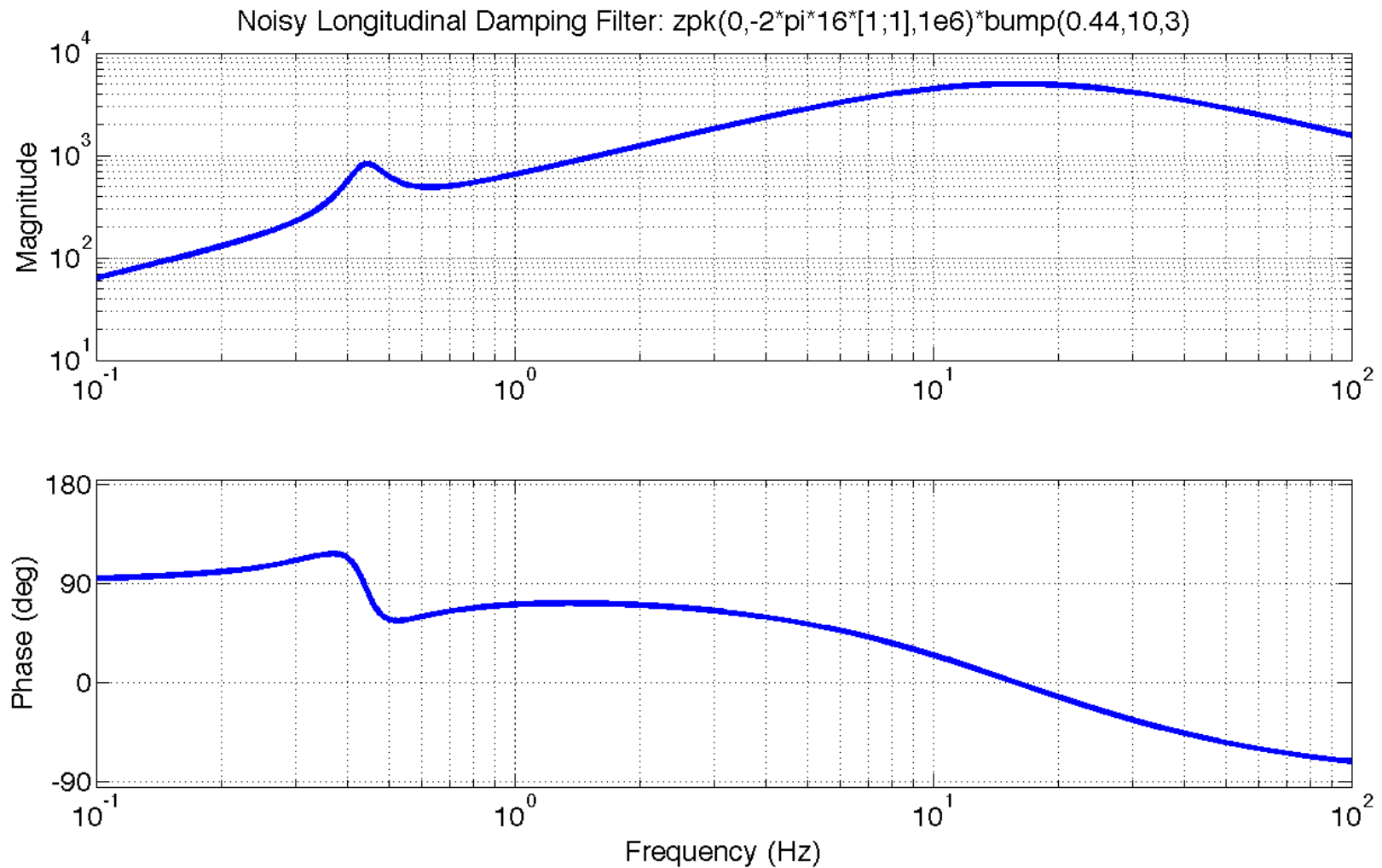
Simulated Common Length Damping



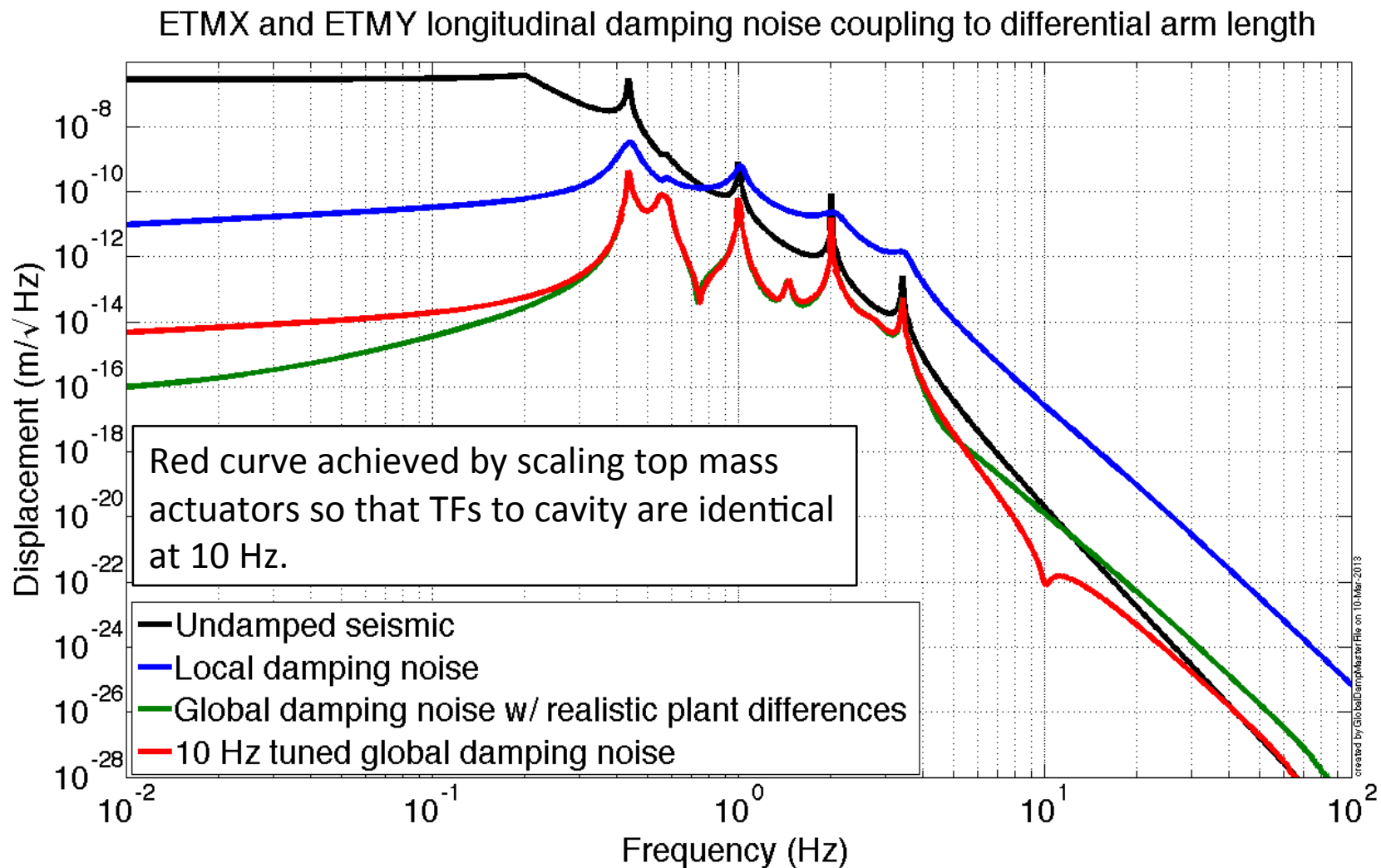
Realistic quads - errors on the simulated as-built parameters are:

- Masses: ± 20 grams
- d's (d_n, d_1, d_3, d_4): ± 1 mm
- Rotational inertia: $\pm 3\%$
- Wire lengths: ± 0.25 mm
- Vertical stiffness: $\pm 3\%$

Simulated Common Length Damping

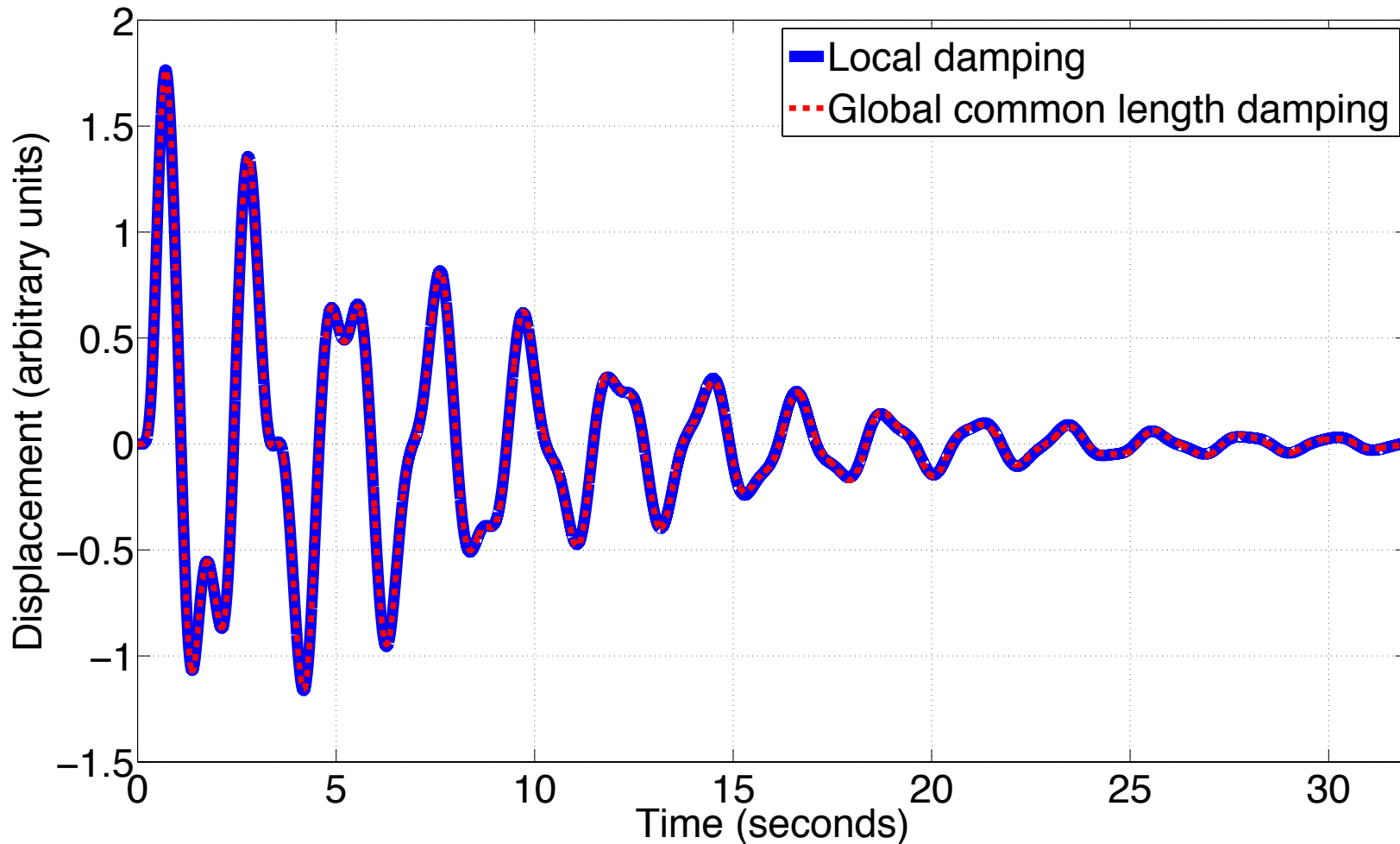


Simulated Damping Noise to Cavity



Simulated Damping Ringdown

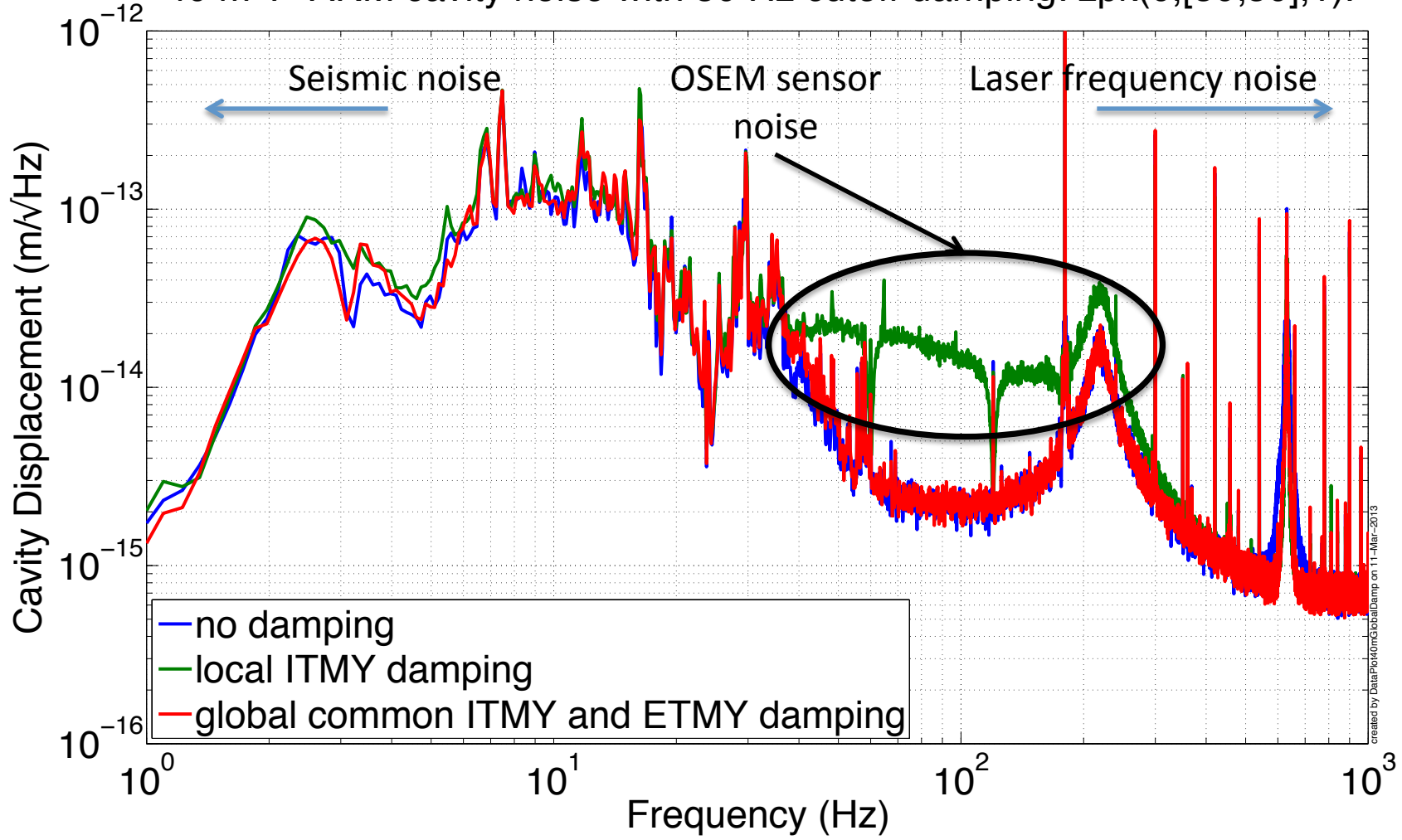
Test mass longitudinal damping from a top mass impulse



created by GlobalDampMasterFile on 11-Mar-2013

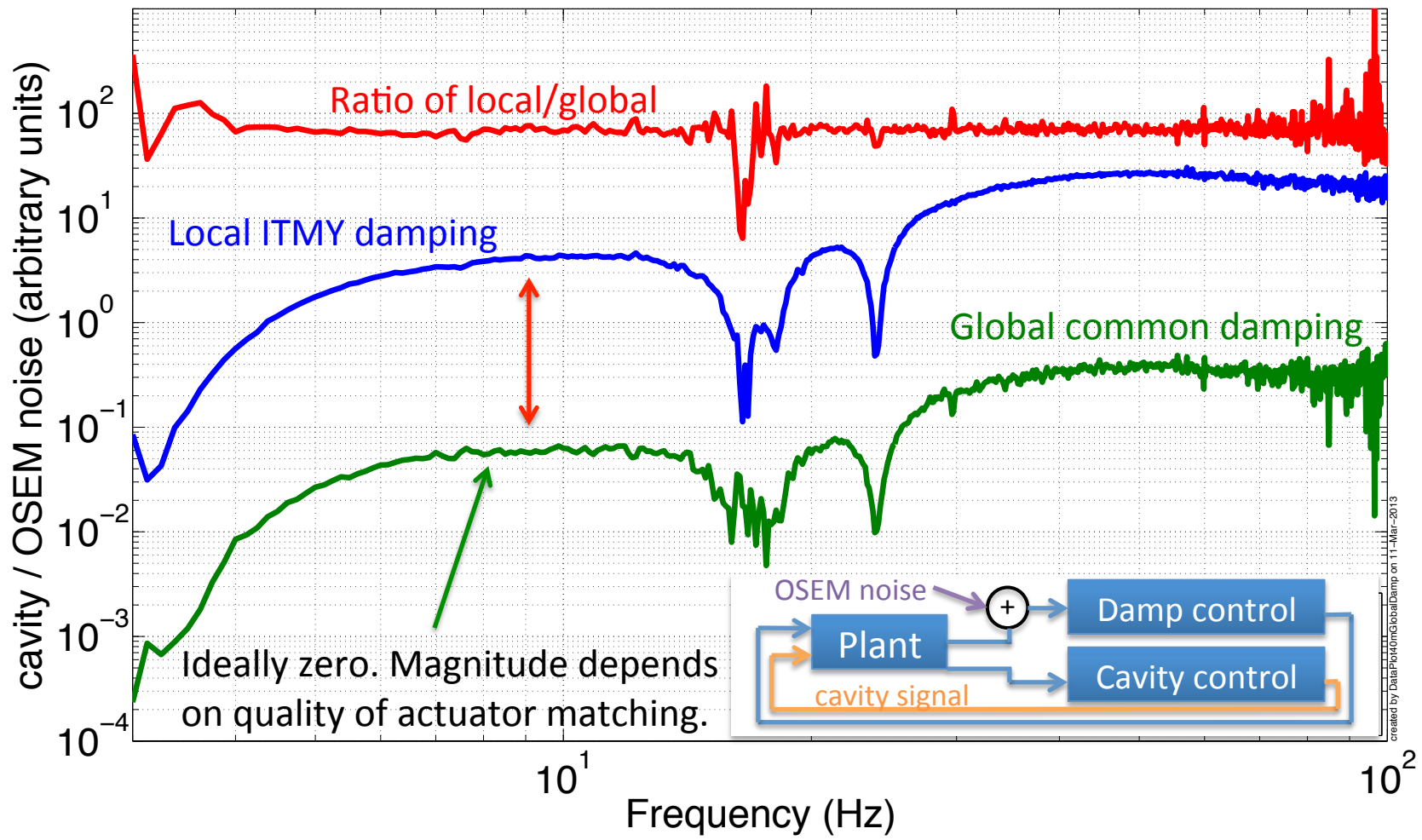
40 m Lab Noise Measurements

40 m Y-ARM cavity noise with 50 Hz cutoff damping: $\text{zpk}(0,[50;50],1)$.



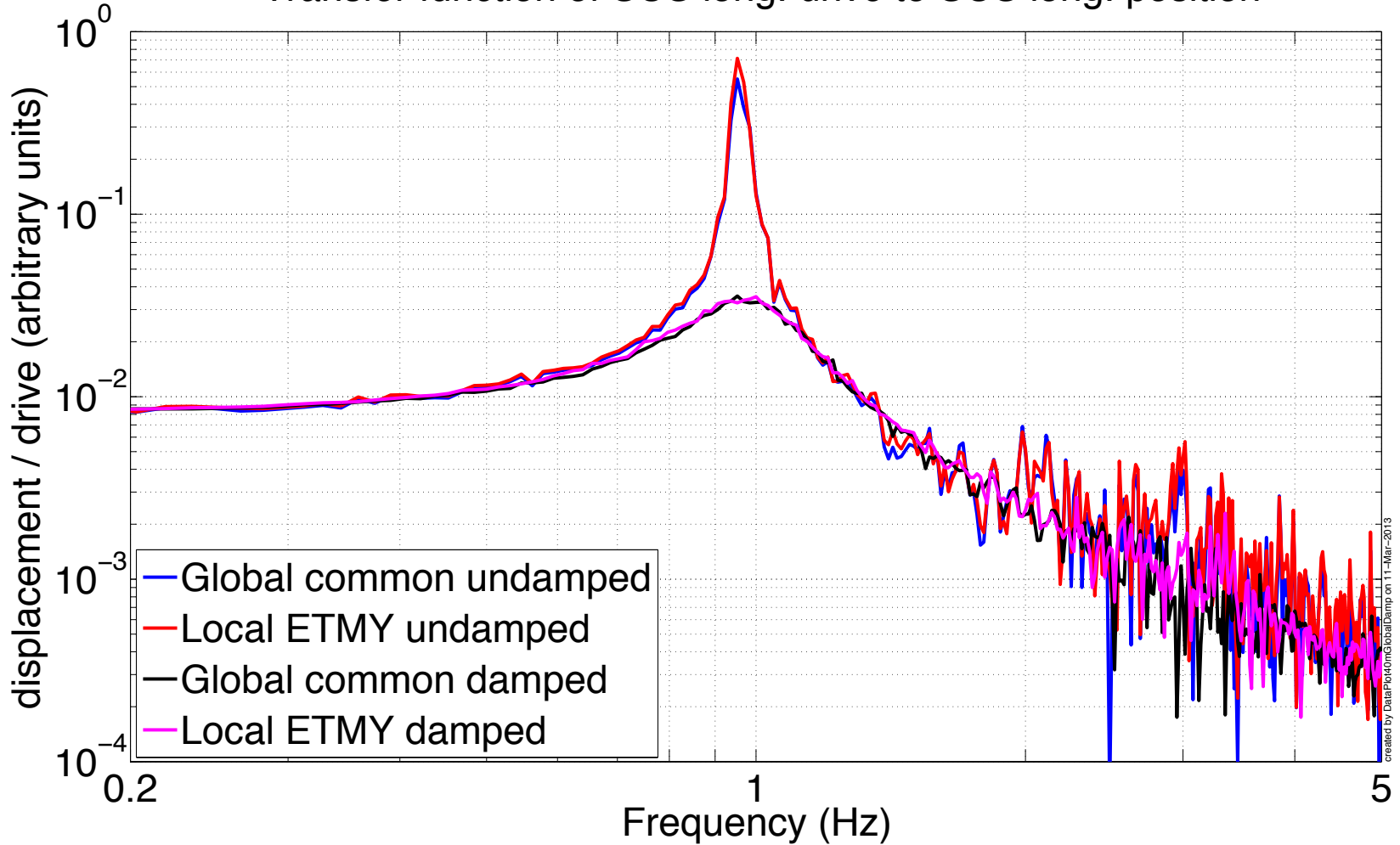
40 m Lab Noise Measurements

Transfer function of OSEM sensor noise to Y-arm cavity: (cavity)/(OSEM noise)



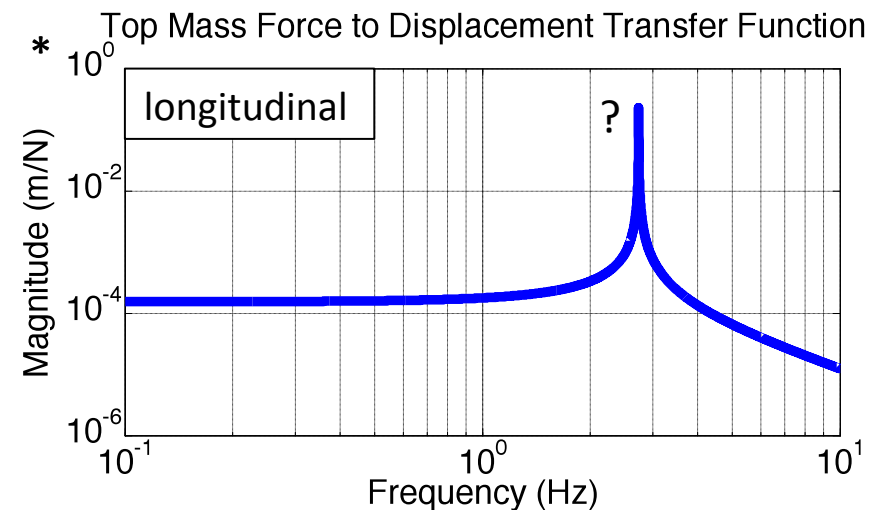
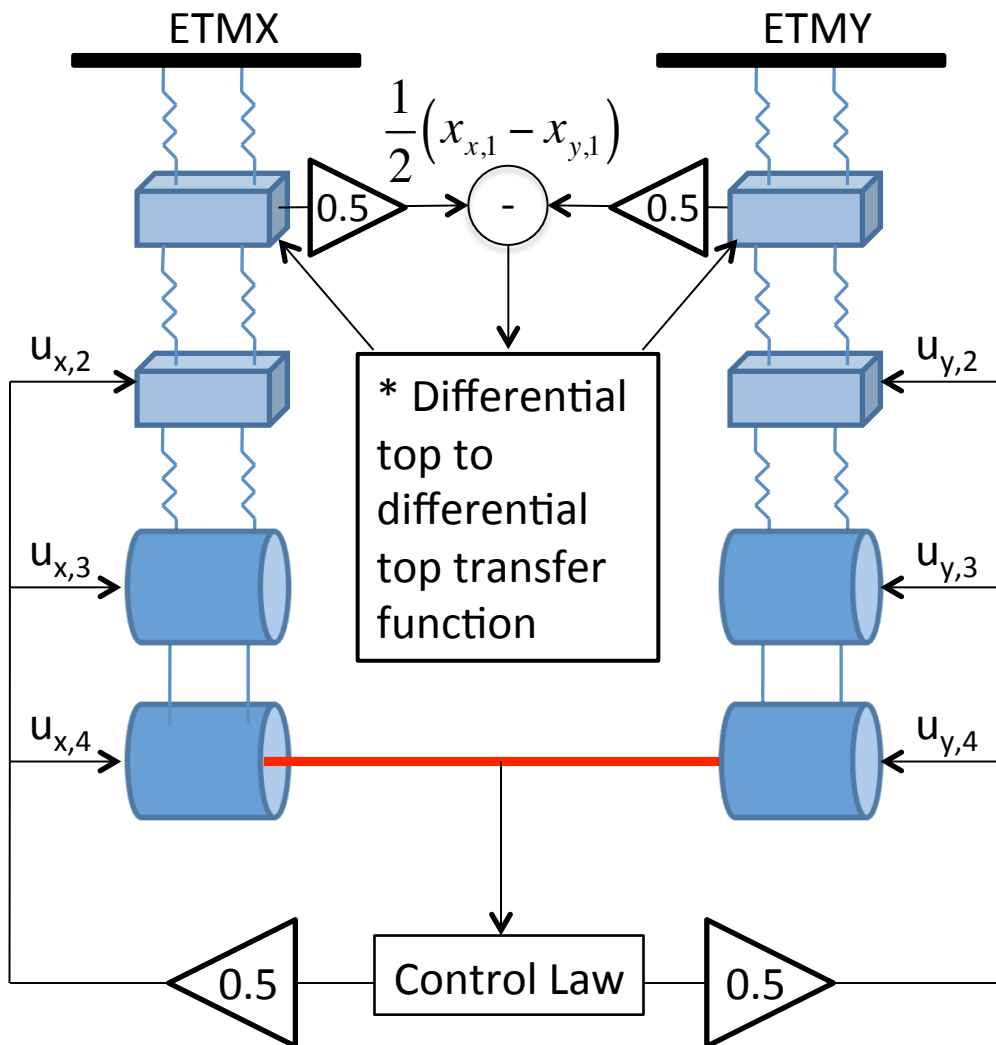
40 m Lab Damping Measurements

Transfer function of SUS long. drive to SUS long. position



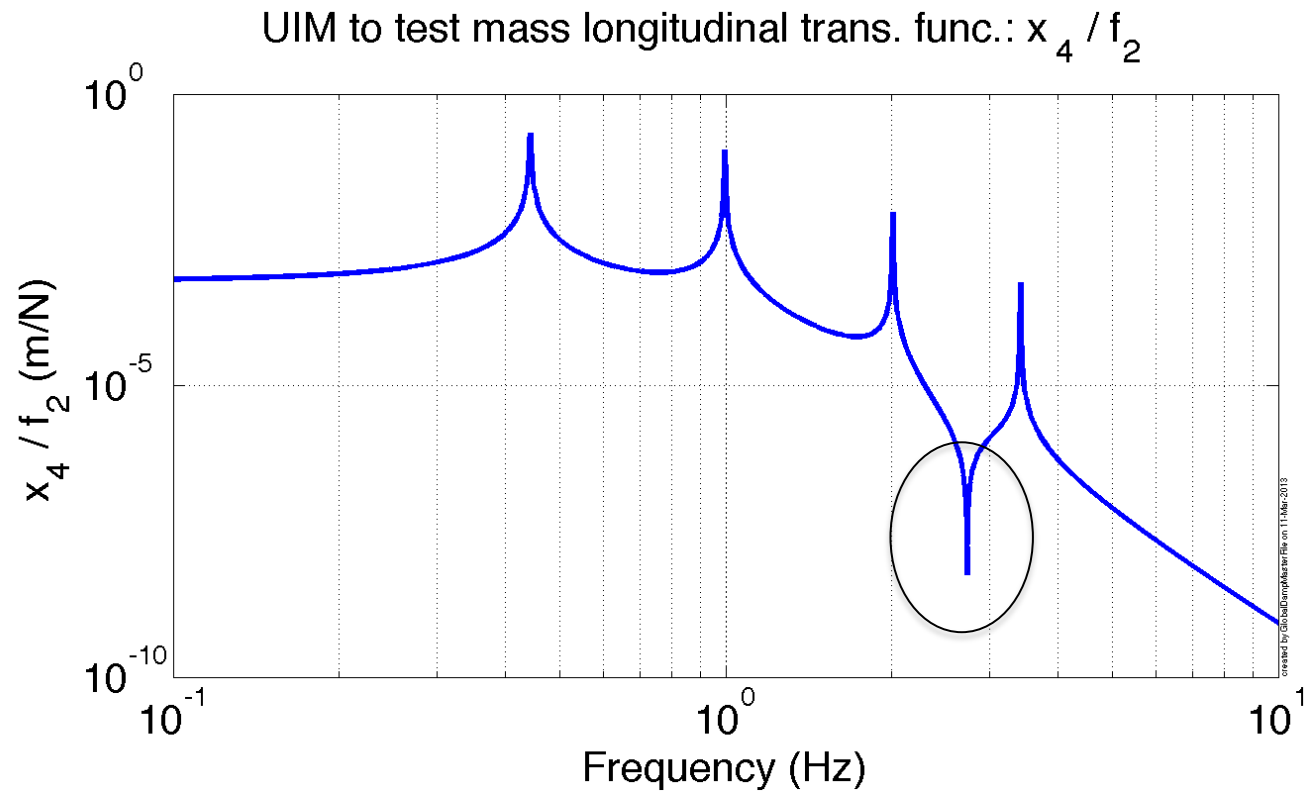
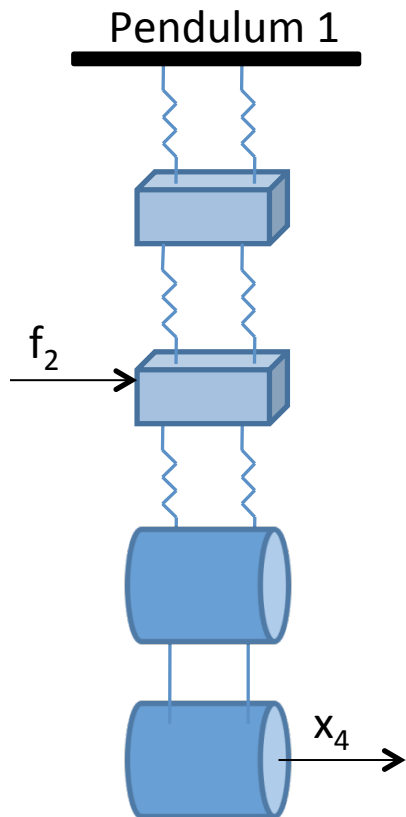
created by dataPlot0mGlobalDamp on 11-Mar-2013

Differential Arm Length Damping



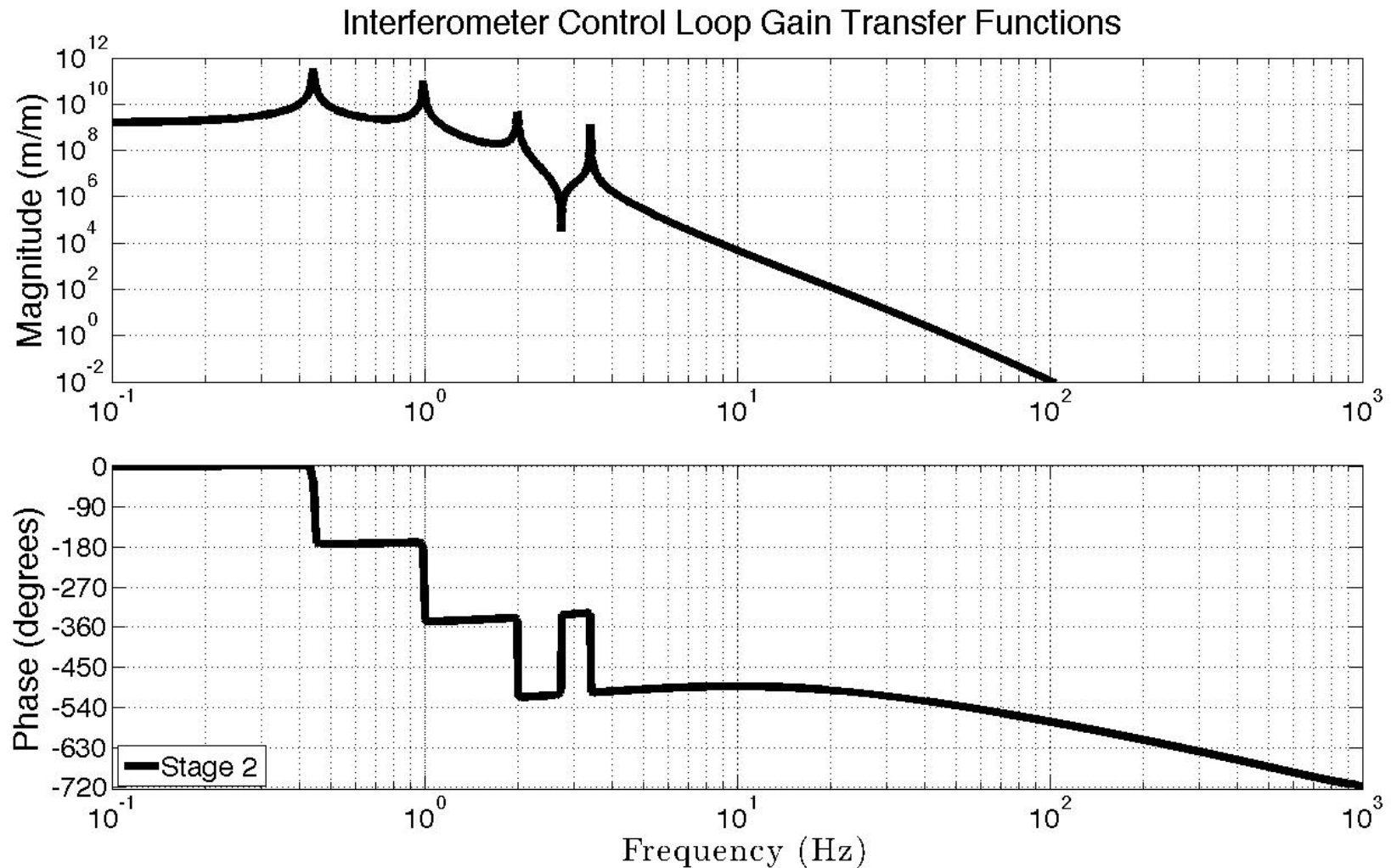
- If we understand how the cavity control produces this mode, can we design a controller that also damps it?
- If so, then we can turn off local damping altogether.

Differential Arm Length Damping

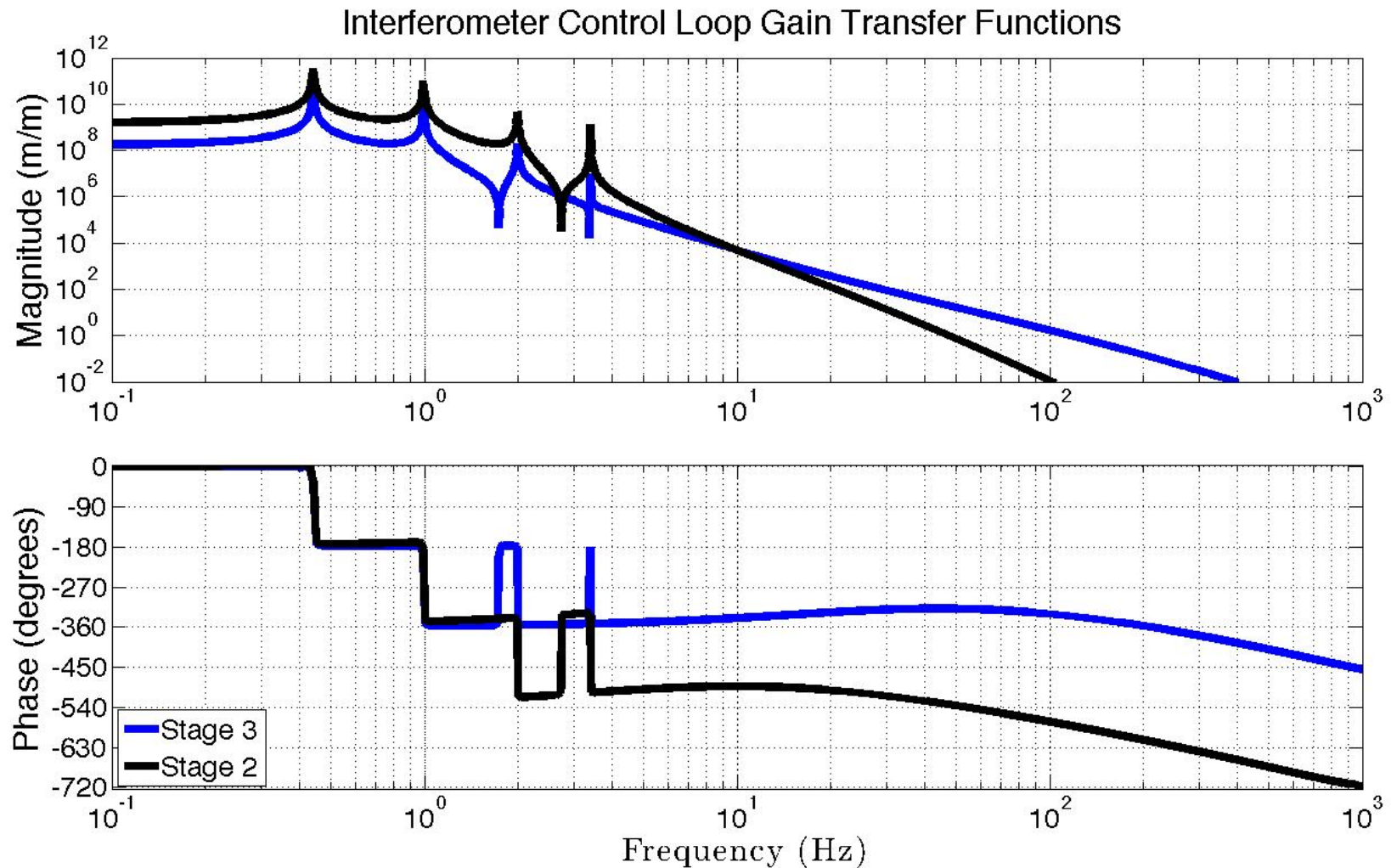


- The new top mass modes come from the zeros of the TF between the highest stage with large cavity UGF and the test mass. See more detailed discussion in the 'Supporting Math' section.
- This result can be generalized to the zeros in the cavity loop gain transfer functions (based on observations, no hard math yet).

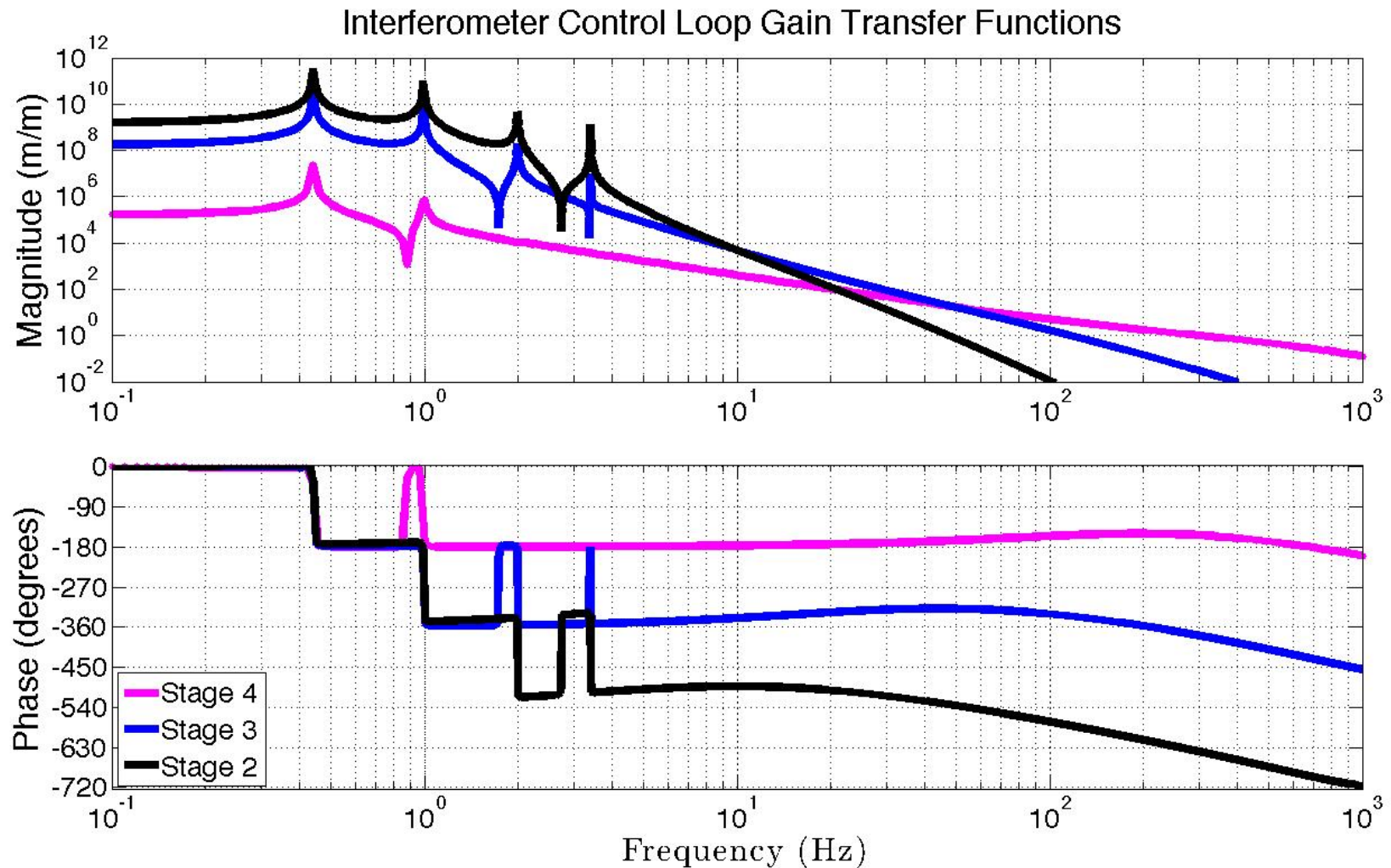
Differential Arm Length Damping



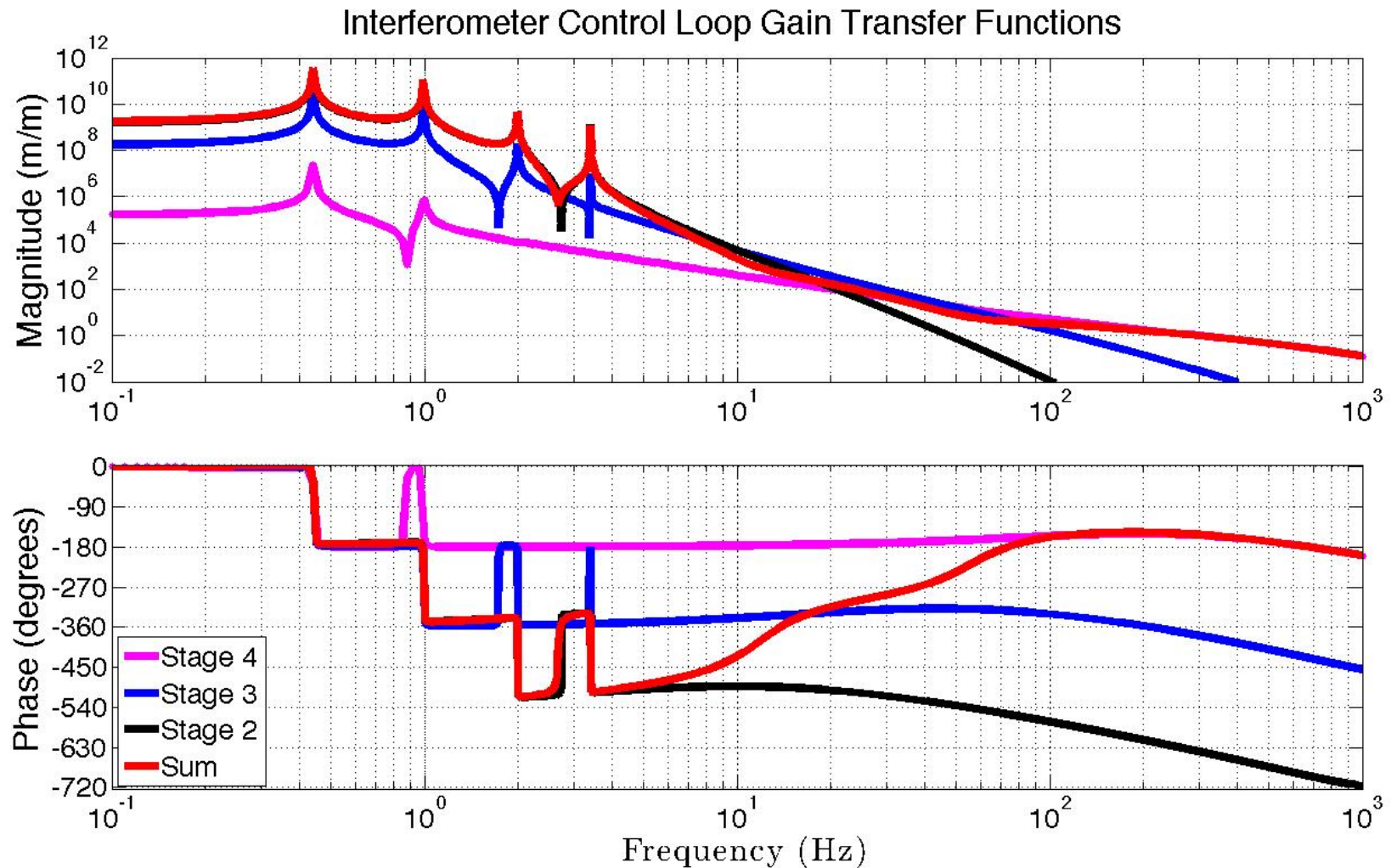
Differential Arm Length Damping



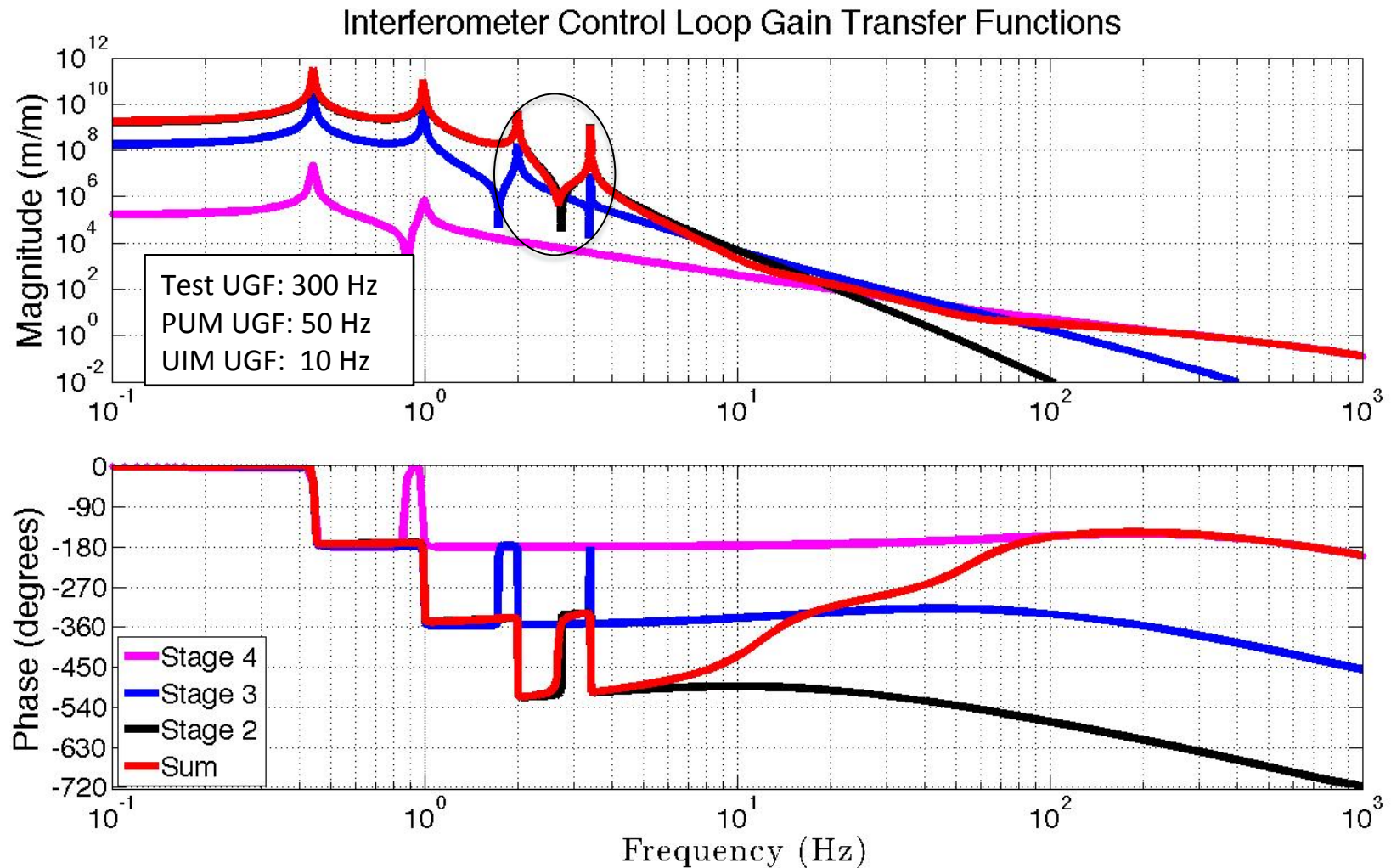
Differential Arm Length Damping



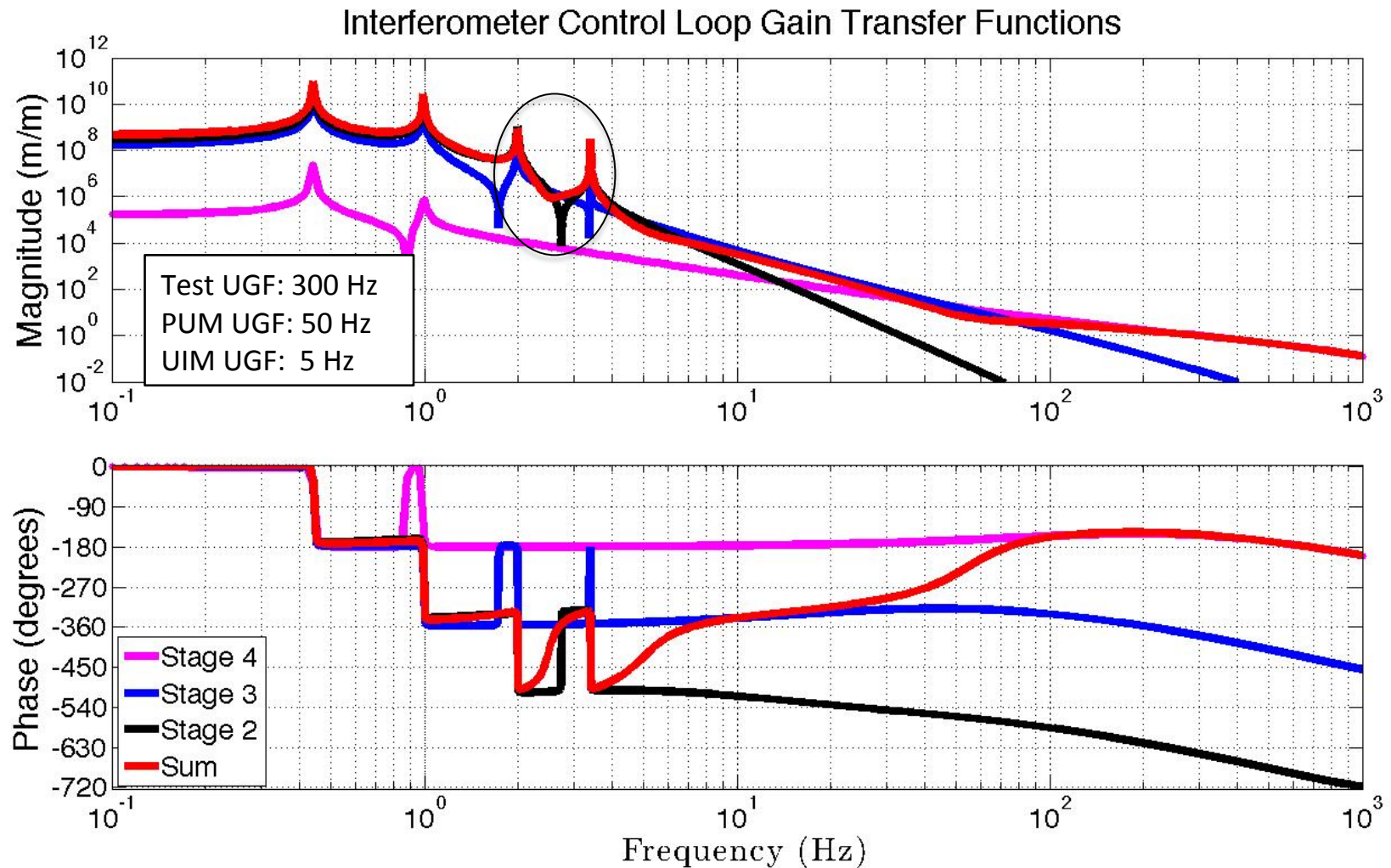
Differential Arm Length Damping



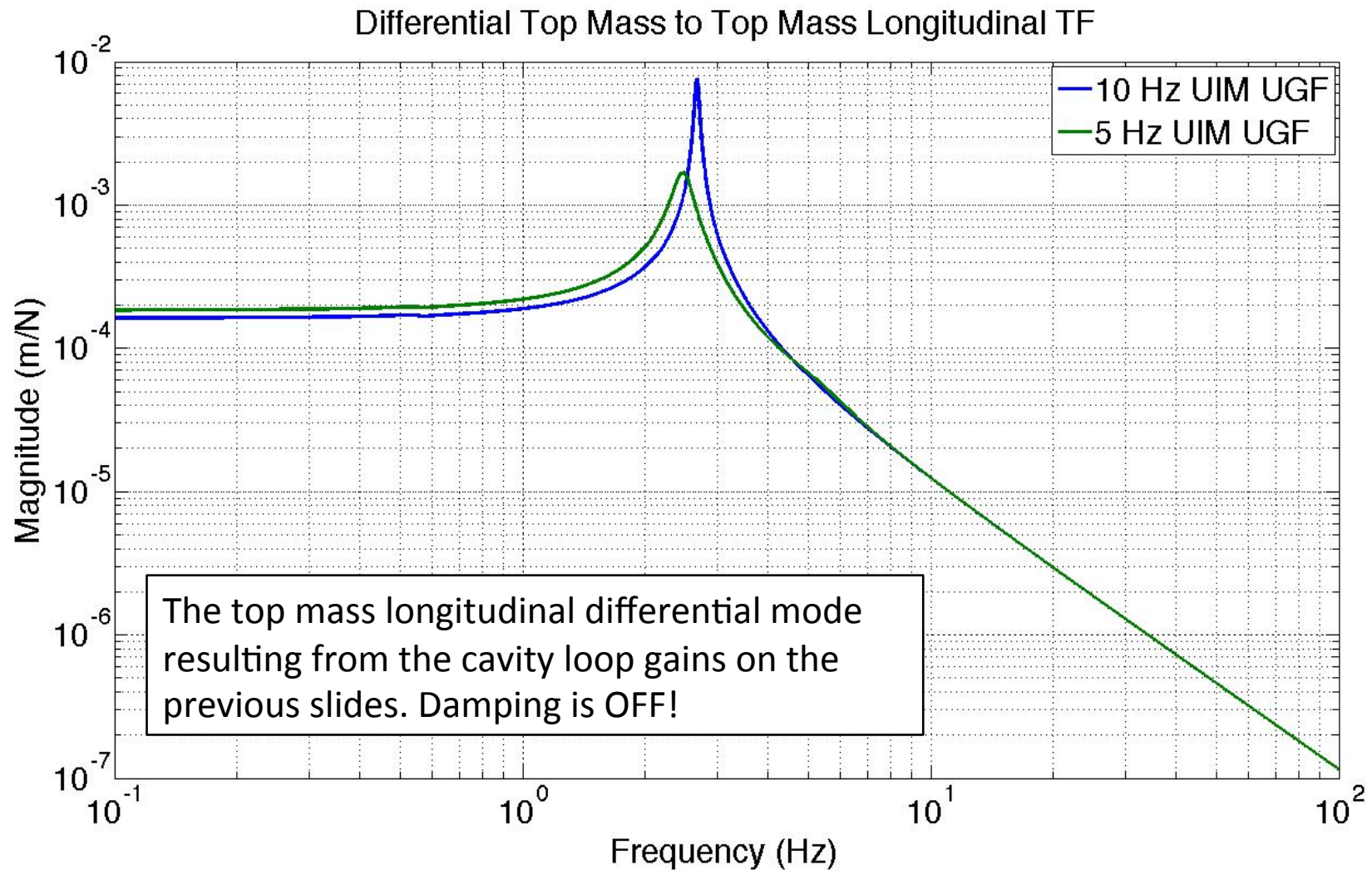
Differential Arm Length Damping



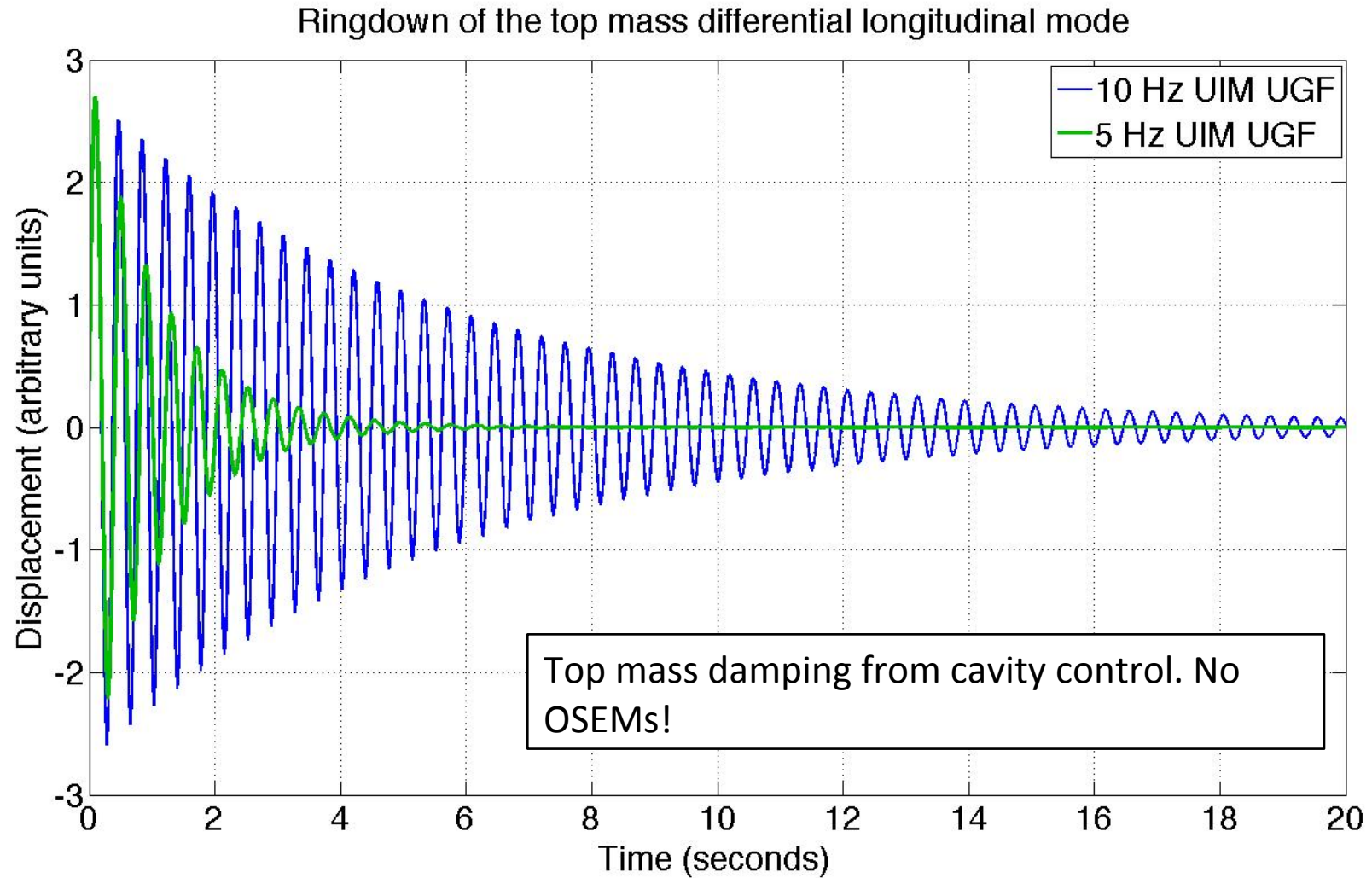
Differential Arm Length Damping



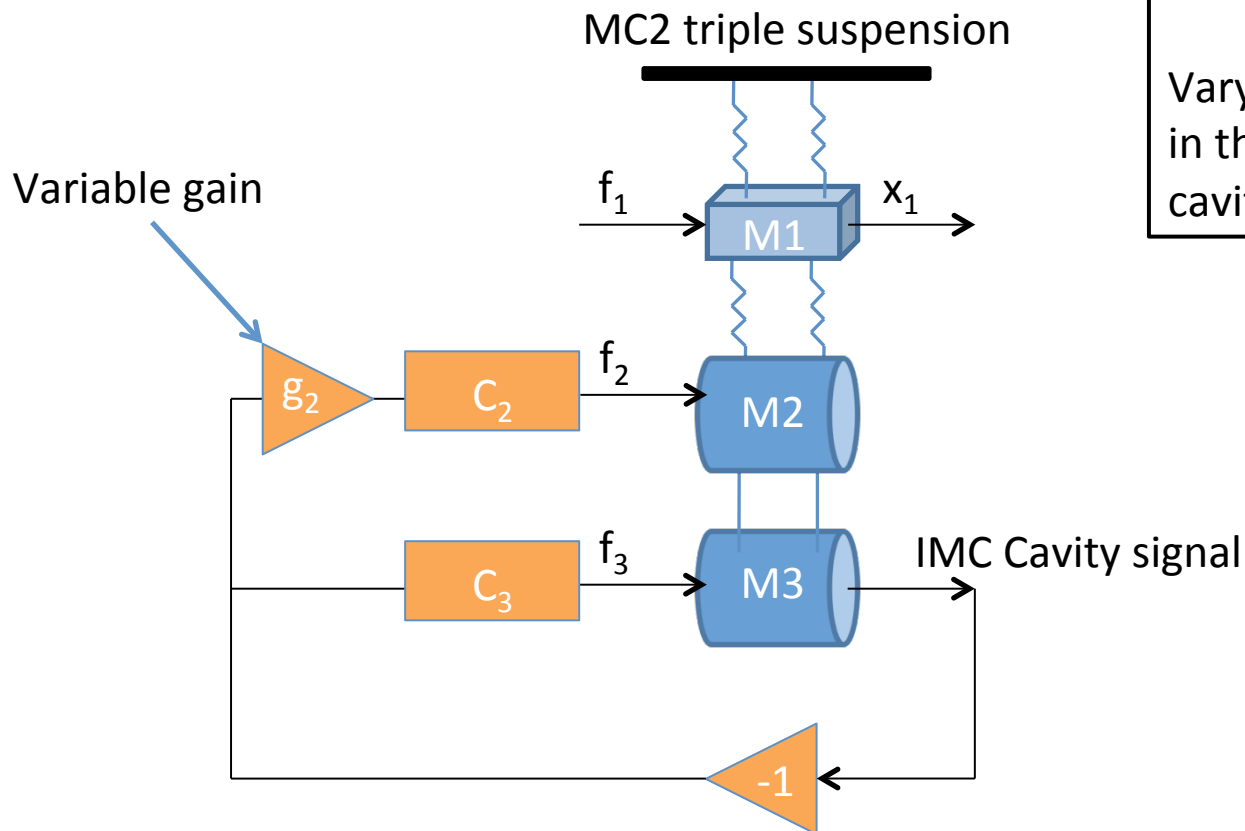
Differential Arm Length Damping



Differential Arm Length Damping



LHO Damping Measurements Setup

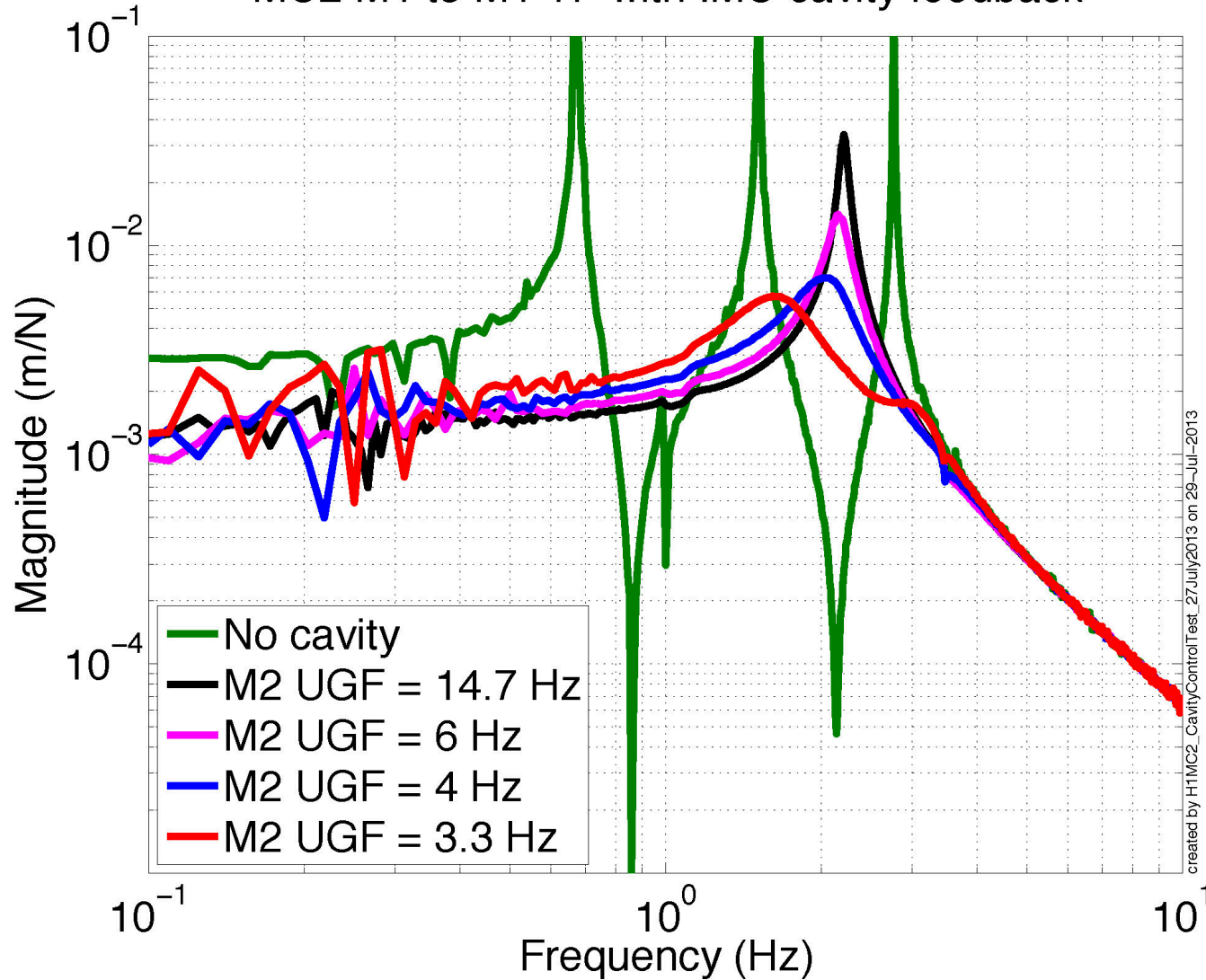


Test procedure
 Vary g_2 and observe the changes in the responses of x_1 and the cavity signal to f_1 .

Terminology Key
 IMC: input mode cleaner, the cavity that makes the laser beam nice and round
 M1: top mass
 M2: middle mass
 M3: bottom mass
 MC2: Mode cleaner triple suspension #2
 C_2 : M2 feedback filter
 C_3 : M3 feedback filter

LHO Damping Measurements

MC2 M1 to M1 TF with IMC cavity feedback

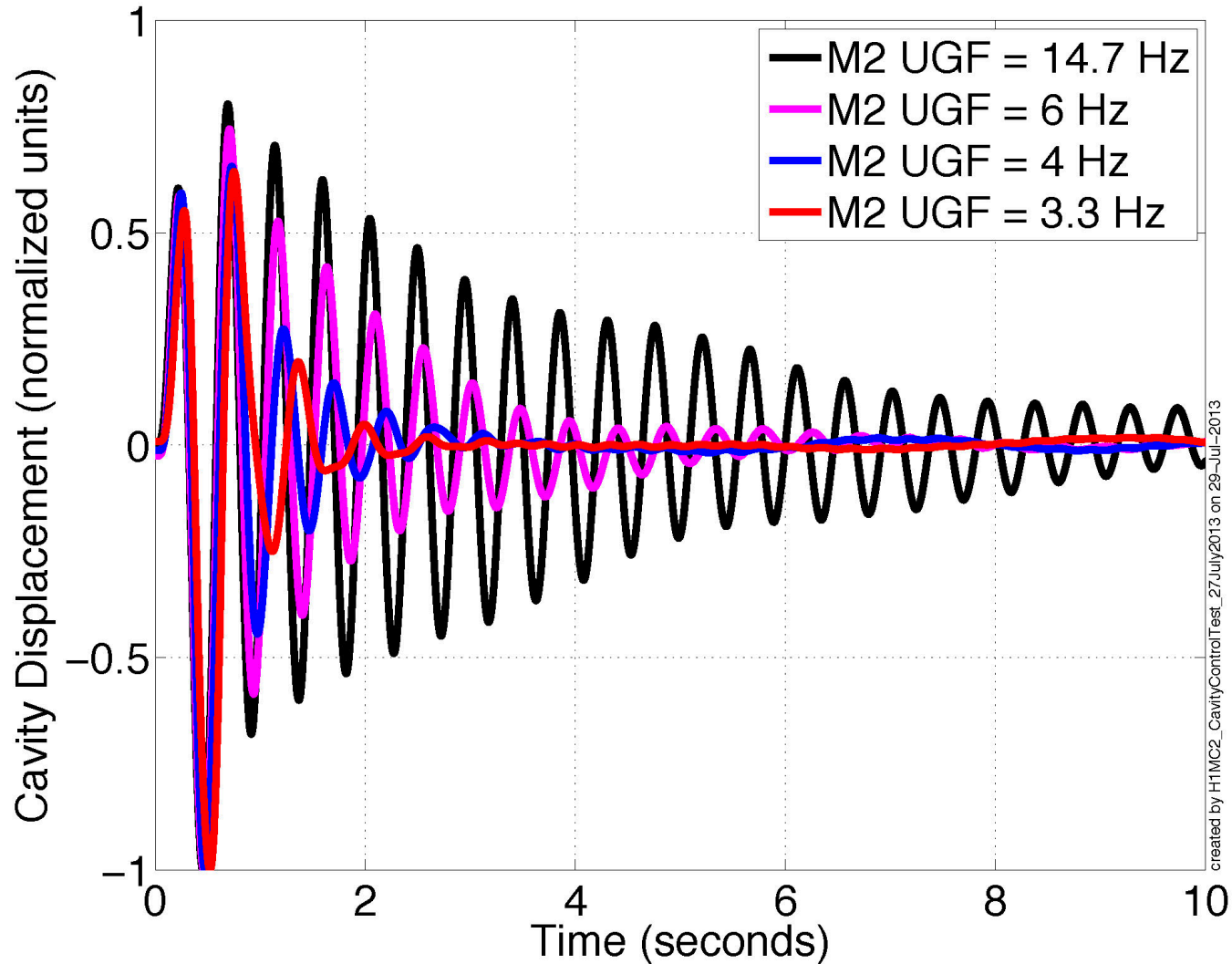


created by H1MC2_CavityControlTest_27 July 2013 on 29-Jul-2013

Terminology Key
M1: top mass
M2: middle mass
M3: bottom mass
MC2: triple suspension
UGF: unity gain
frequency or bandwidth

LHO Damping Measurements

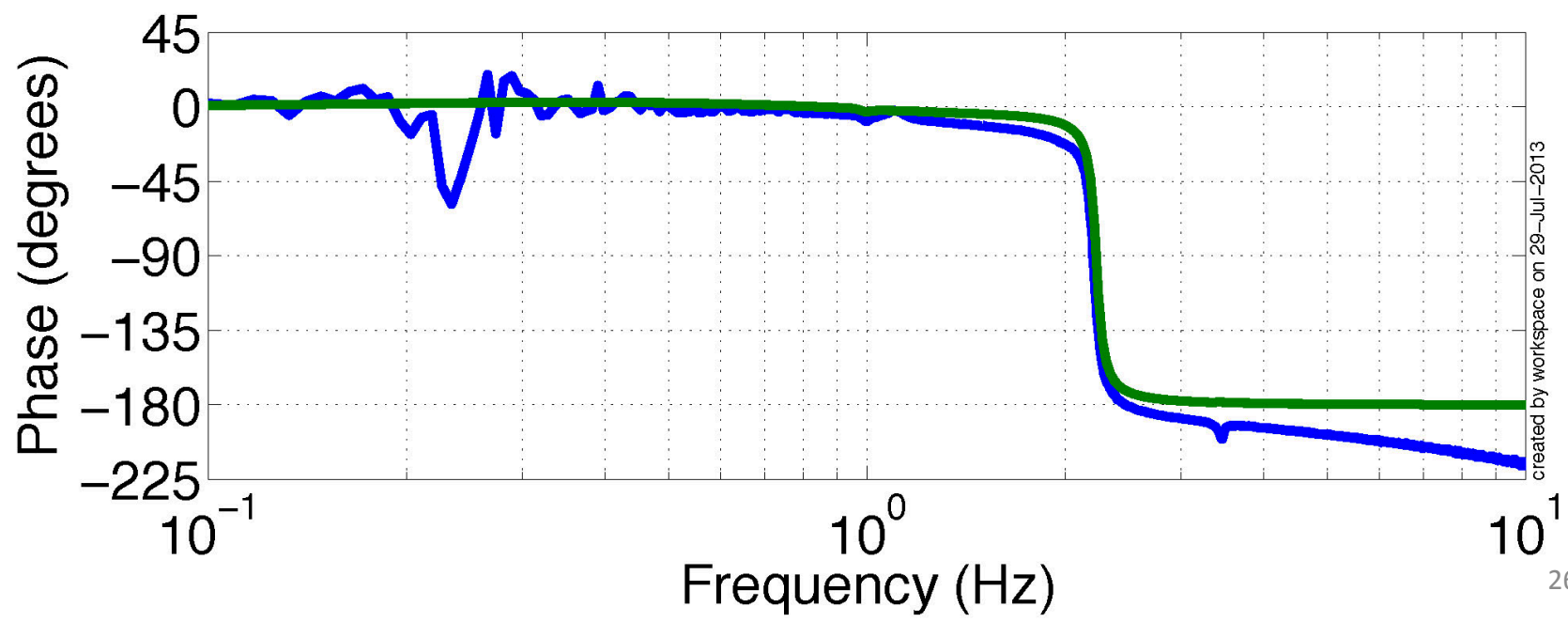
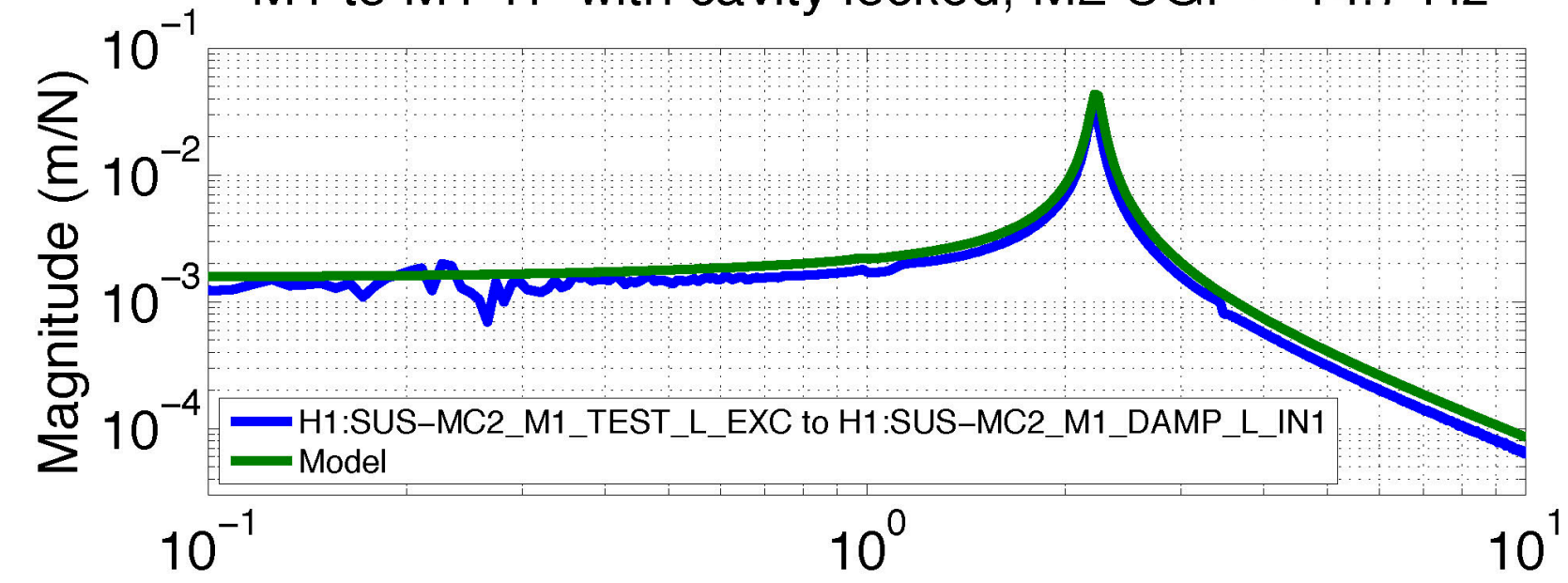
IMC cavity response to MC2 M1 impulse, feedback to M2 & M3



created by H1MC2_CavityControlTest_27.July2013 on 29-Jul-2013

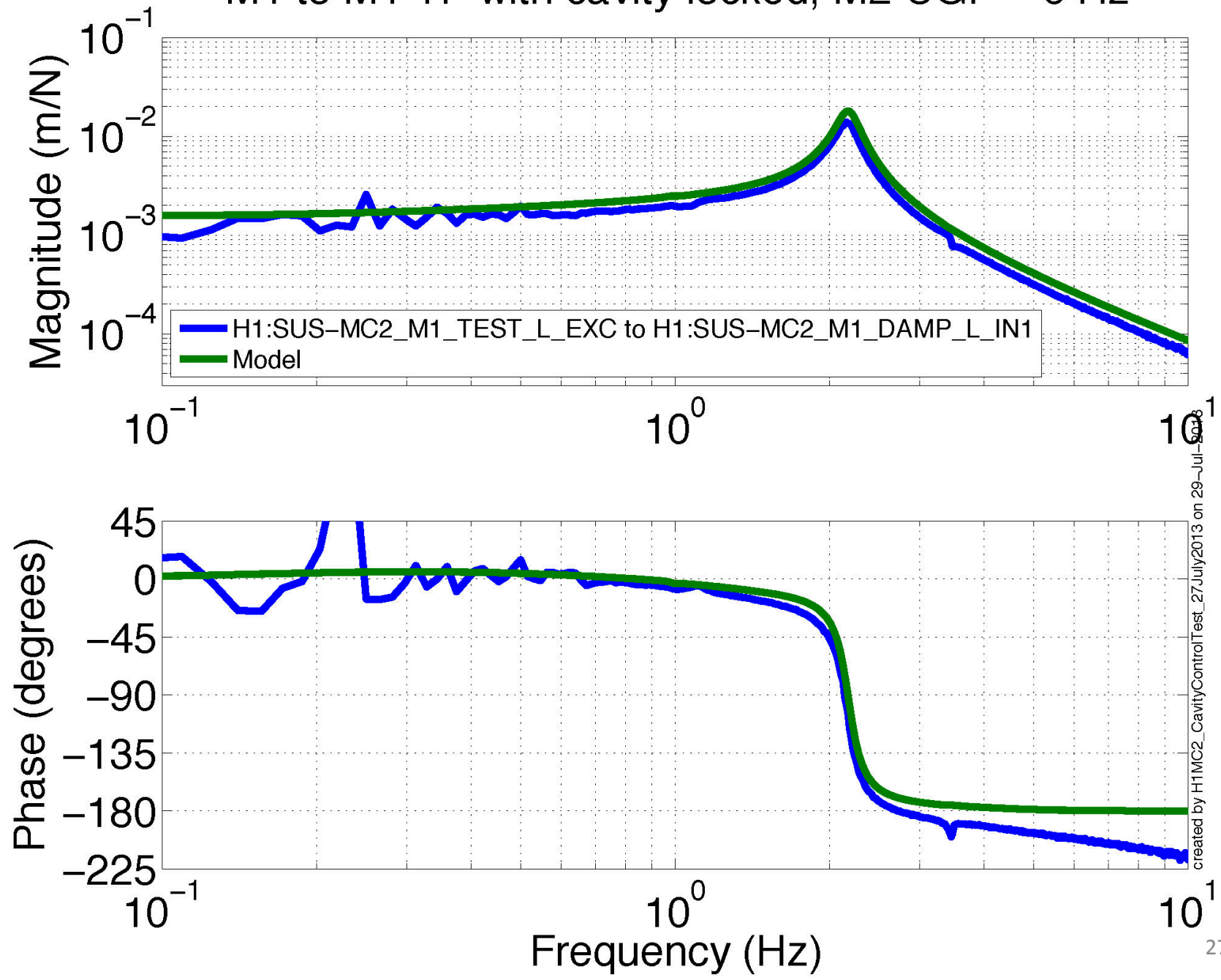
Terminology Key
IMC: cavity signal,
bottom mass sensor
M1: top mass
M2: middle mass
M3: bottom mass
MC2: triple suspension
UGF: unity gain
frequency or bandwidth

M1 to M1 TF with cavity locked, M2 UGF = 14.7 Hz



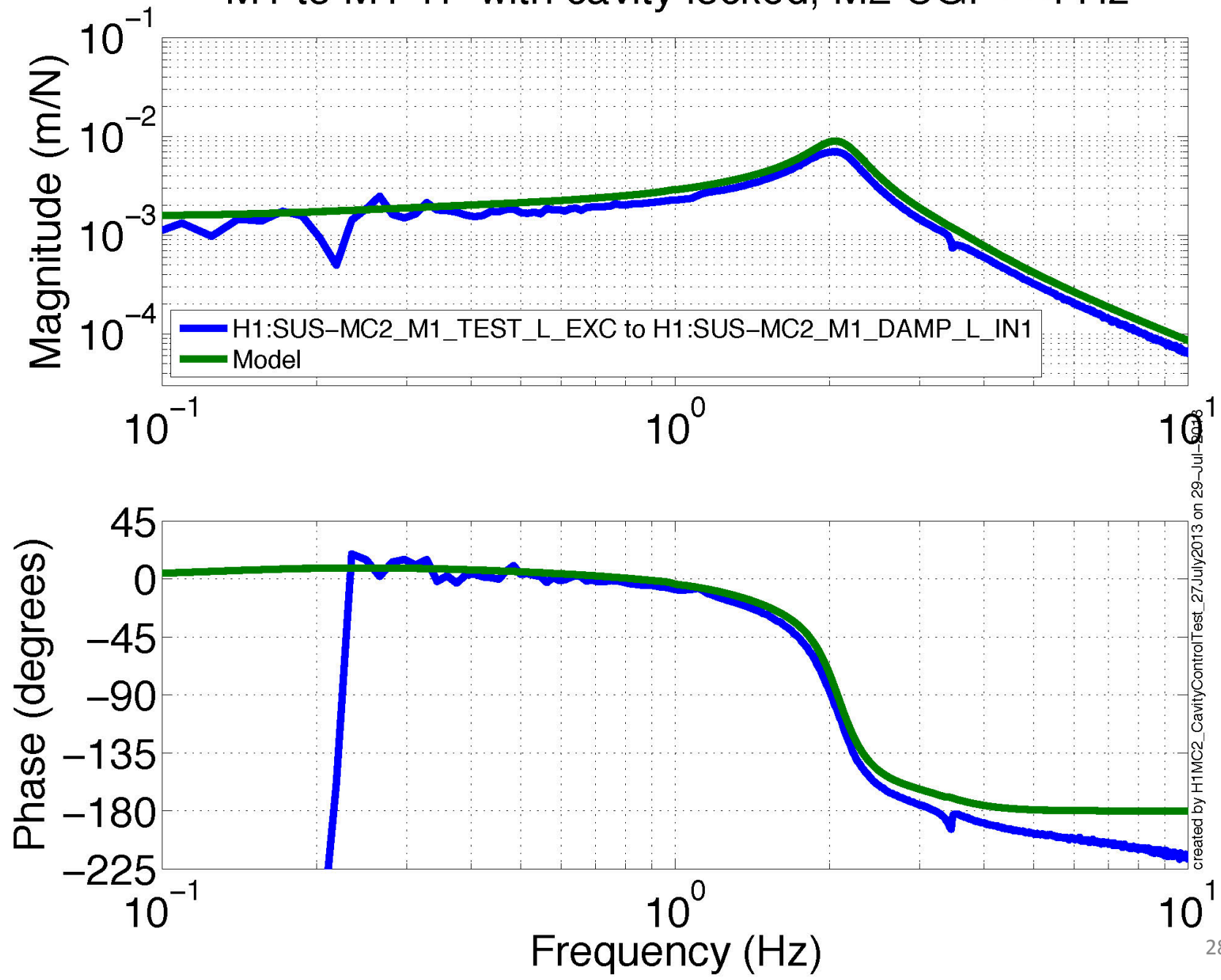
created by workspace on 29-Jul-2013

M1 to M1 TF with cavity locked, M2 UGF = 6 Hz



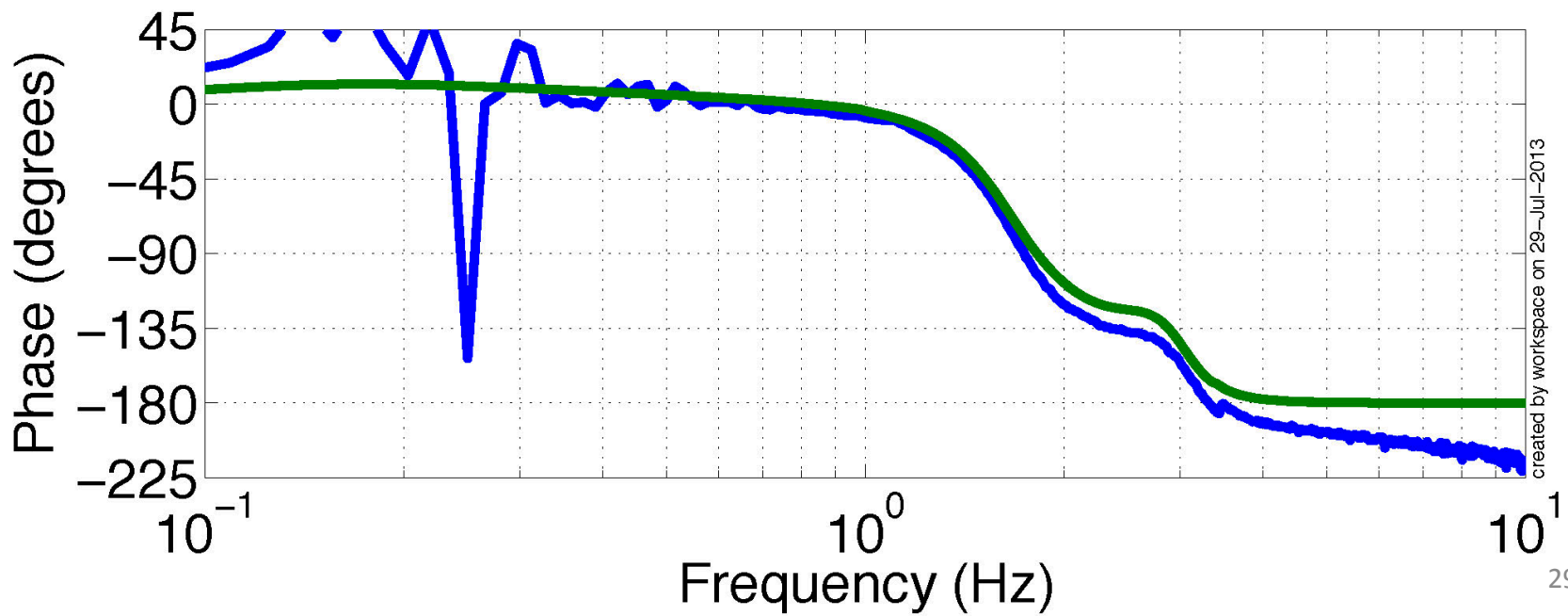
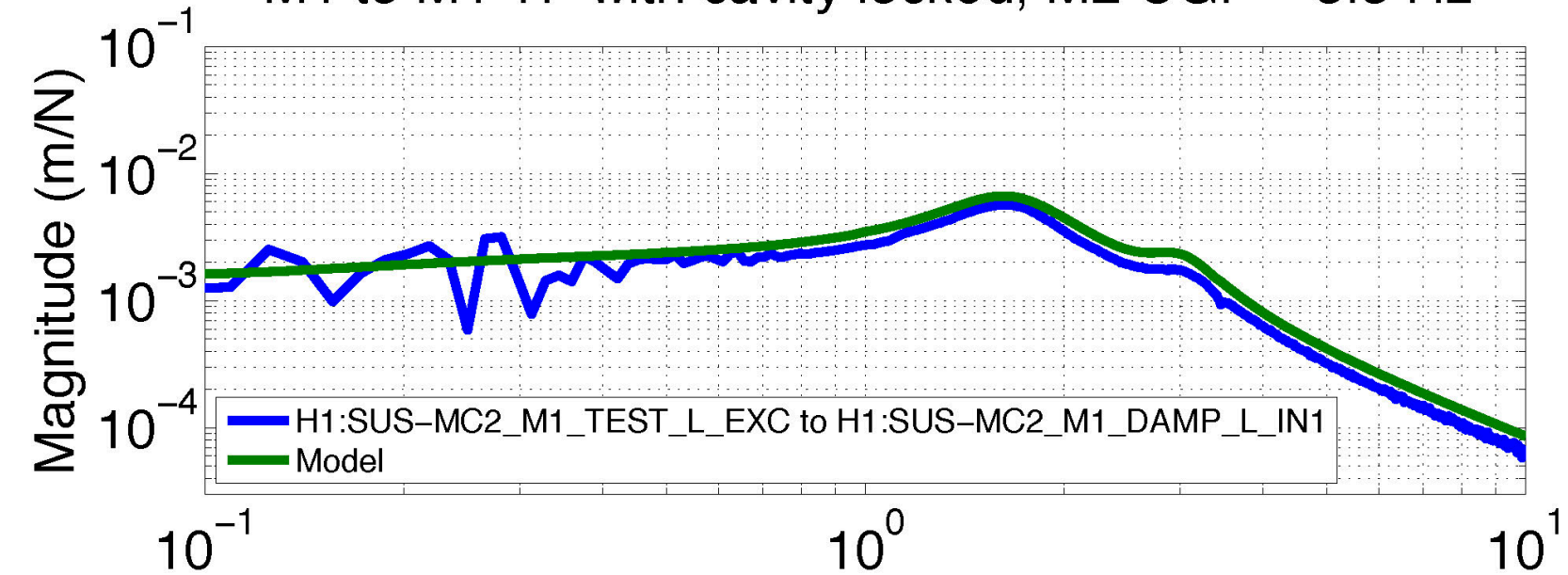
created by H1MC2_CavityControlTest_27July2013 on 29-Jul-2013

M1 to M1 TF with cavity locked, M2 UGF = 4 Hz



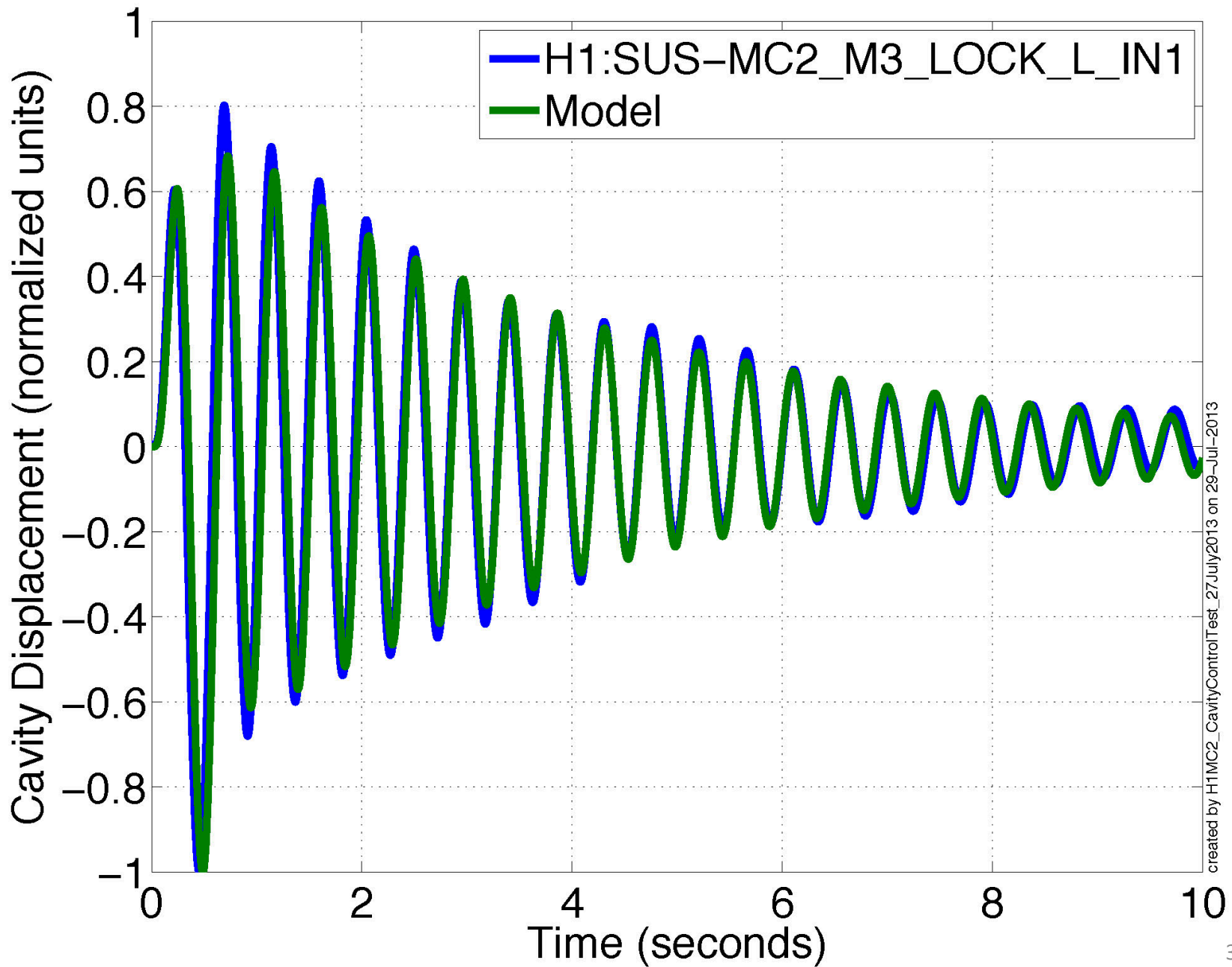
created by H1MC2_CavityControlTest_27July2013 on 29-Jul-2013

M1 to M1 TF with cavity locked, M2 UGF = 3.3 Hz



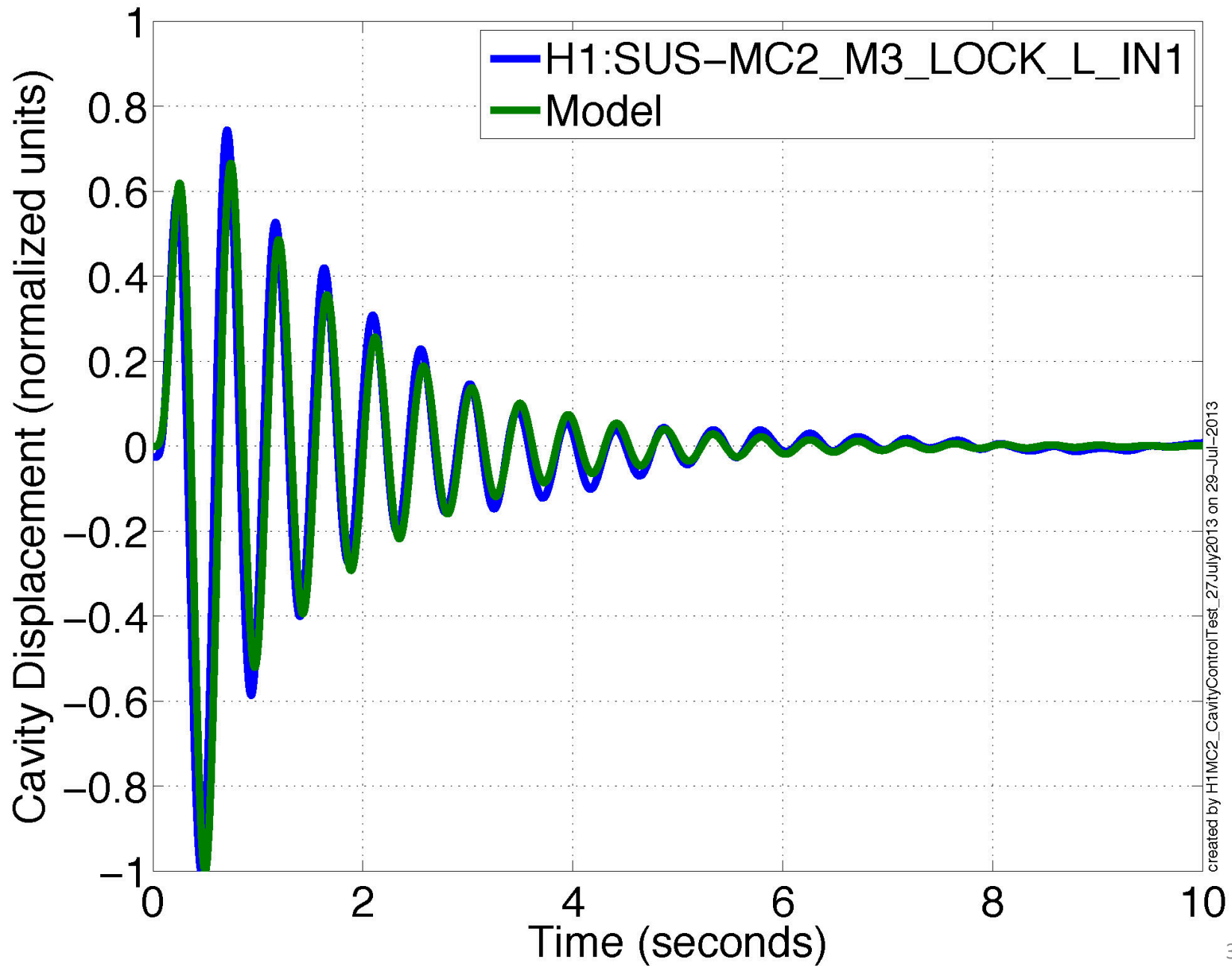
created by workspace on 29-Jul-2013

IMC cavity response to MC2 M1 impulse, M2 UGF = 14.7 Hz



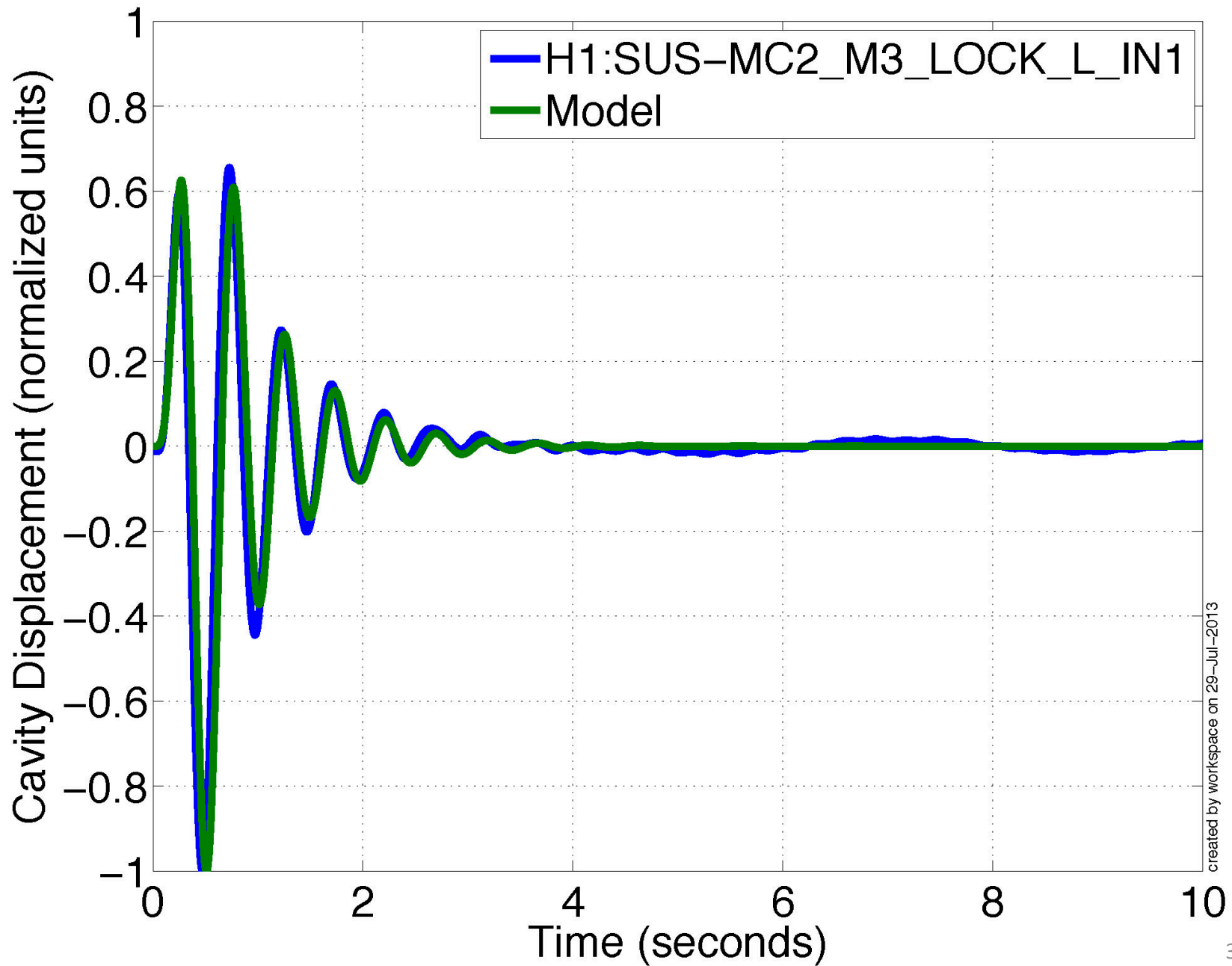
created by H1MC2_CavityControlTest_27July2013 on 29-Jul-2013

IMC cavity response to MC2 M1 impulse, M2 UGF = 6 Hz



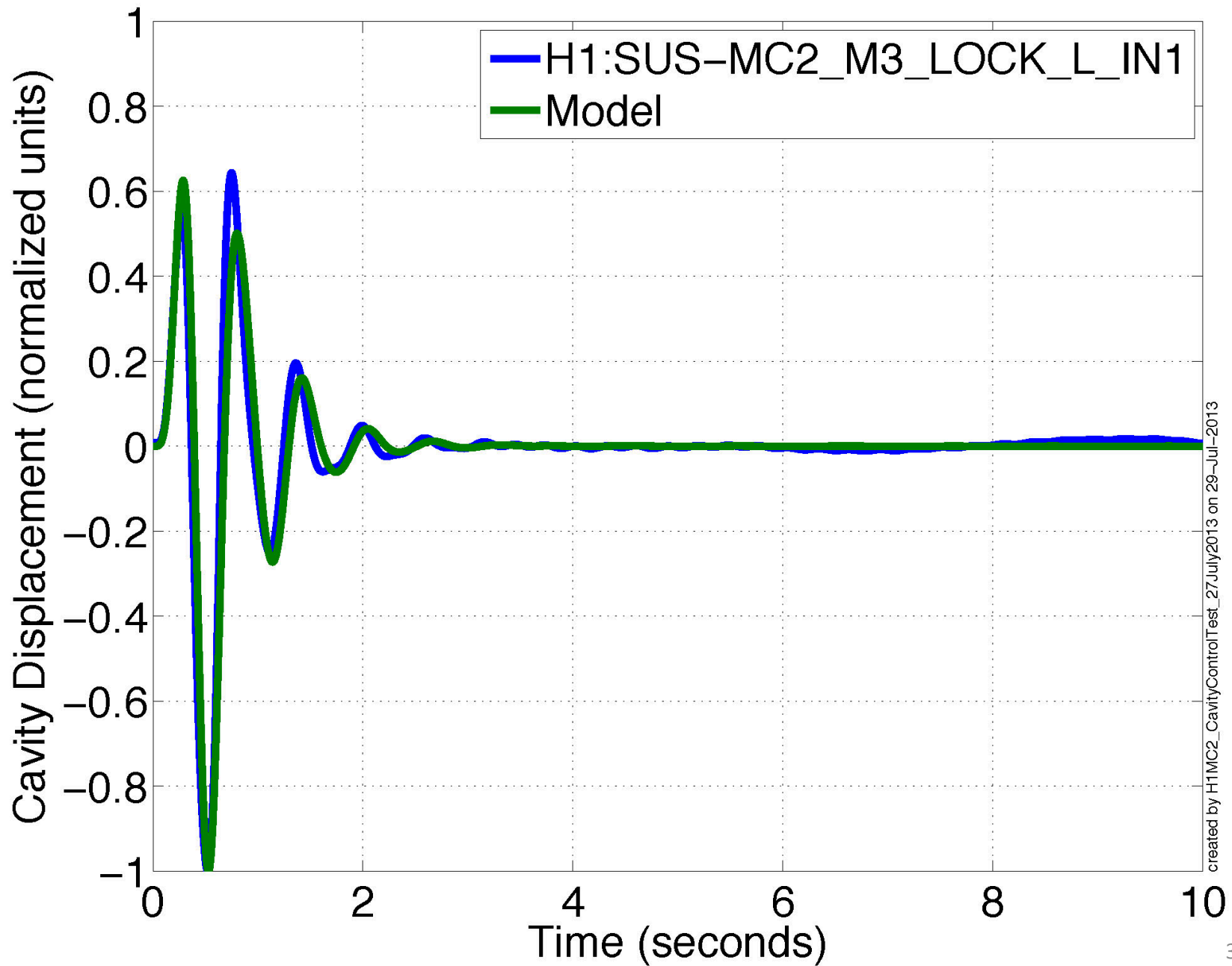
created by H1MC2_CavityControlTest_27July2013 on 29-Jul-2013

IMC cavity response to MC2 M1 impulse, M2 UGF = 4 Hz



created by workspace on 29-Jul-2013

IMC cavity response to MC2 M1 impulse, M2 UGF = 3.3 Hz



created by H1MC2_CavityControlTest_27July2013 on 29-Jul-2013



- Very simple implementation. A matrix transformation and a little bit of actuator tuning.
- Overall, global damping isolates OSEM sensor noise in two ways:
 - 1: **common length damping** -> damp global DOFs that couple weakly to the cavity
 - 2: **differential length damping** -> cavity control damps its own DOF
- Can isolate nearly all longitudinal damping noise.
- If all 4 quads are damped globally, the cavity control becomes independent of the damping design.

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Conclusions cont.



- Broadband noise reduction, both in band (>10 Hz) and out of band (<10 Hz).
- Applies to other cavities, e.g. IMC.
- Limitations:
 - Only works for longitudinal, maybe pitch and yaw
 - Adds an extra constraint to the cavity control, encouraging more drive on the lower stages

LIGO Acknowledgements



- Caltech: 40 m crew, Rana Adhikari, Jenne Driggers, Jamie Rollins
- LHO: commissioning crew
- MIT: Kamal Youcef-Toumi, Jeff Kissel.



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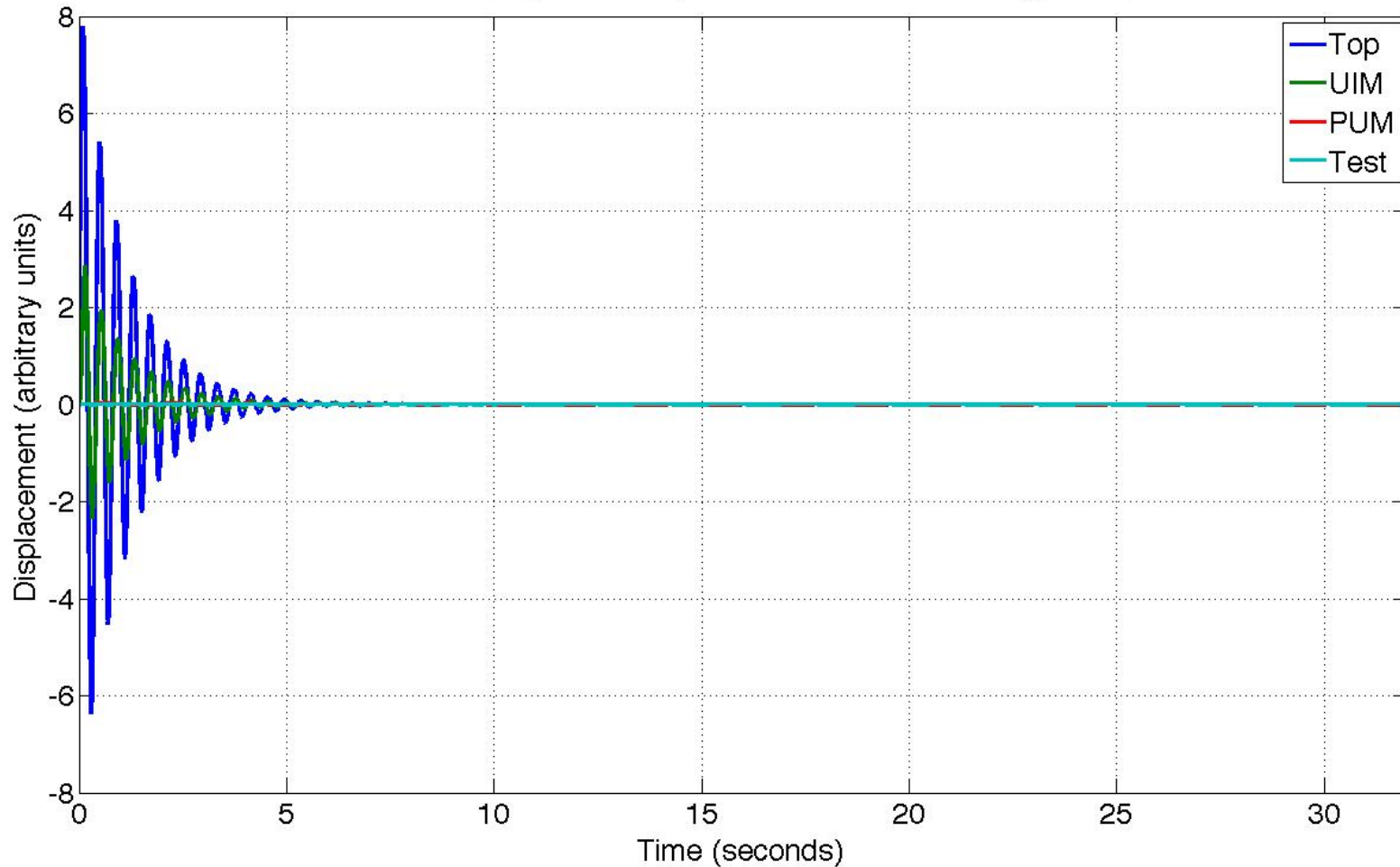
Backups



Differential Damping – all stages



Differential damping with cavity feedback from an ISI stage 2 impulse



Supporting Math

1. Dynamics of common and differential modes
 - a. Rotating the pendulum state space equations from local to global coordinates
 - b. Noise coupling from common damping to DARM
 - c. Double pendulum example
2. Change in top mass modes from cavity control – simple two mass system example.

DYNAMICS OF COMMON AND DIFFERENTIAL MODES

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

Local ETMX Longitudinal Plant

$$\dot{\mathbf{x}} = \mathbf{A}_x \mathbf{x} + \mathbf{B}_x \mathbf{u}_x$$

$$\mathbf{x}_m = \mathbf{R}\mathbf{x} + \mathbf{n}_x$$

Local ETMY Longitudinal Plant

$$\dot{\mathbf{y}} = \mathbf{A}_y \mathbf{y} + \mathbf{B}_y \mathbf{u}_y$$

$$\mathbf{y}_m = \mathbf{R}\mathbf{y} + \mathbf{n}_y$$

\mathbf{R} = sensing matrix

\mathbf{n} = sensor noise

Local to global transformations:

$\mathbf{d} = (\mathbf{x} - \mathbf{y})$ Differential displacement signals



$$\dot{\mathbf{d}} = \mathbf{A}_x \mathbf{x} - \mathbf{A}_y \mathbf{y} + \mathbf{B}_x \mathbf{u}_x - \mathbf{B}_y \mathbf{u}_y$$

$\mathbf{c} = (\mathbf{x} + \mathbf{y})/2$ Common displacement signals



$$\dot{\mathbf{c}} = [\mathbf{A}_x \mathbf{x} + \mathbf{A}_y \mathbf{y} + \mathbf{B}_x \mathbf{u}_x + \mathbf{B}_y \mathbf{u}_y]/2$$

$\mathbf{u}_d = \mathbf{u}_x - \mathbf{u}_y$ Differential control signals

$\mathbf{u}_c = (\mathbf{u}_x + \mathbf{u}_y)/2$ Common control signals

Ideal case: $\mathbf{A}_x = \mathbf{A}_y = \mathbf{A}$, $\mathbf{B}_x = \mathbf{B}_y = \mathbf{B}$

Combined Differential/Common system matrix

$\dot{\mathbf{d}} = \mathbf{A}\mathbf{d} + \mathbf{B}\mathbf{u}_d$ global differential plant

$\dot{\mathbf{c}} = \mathbf{A}\mathbf{c} + \mathbf{B}\mathbf{u}_c$ global common plant



$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

Real case: $\tilde{\mathbf{A}} = (\mathbf{A}_x - \mathbf{A}_y)/2$, $\tilde{\mathbf{B}} = (\mathbf{B}_x - \mathbf{B}_y)/2$, $\mathbf{A} = (\mathbf{A}_x + \mathbf{A}_y)/2$, $\mathbf{B} = (\mathbf{B}_x + \mathbf{B}_y)/2$

$\mathbf{A}_x = \mathbf{A} + \tilde{\mathbf{A}}$, $\mathbf{A}_y = \mathbf{A} - \tilde{\mathbf{A}}$

Combined Differential/Common system matrix

$\dot{\mathbf{d}} = \mathbf{A}\mathbf{d} + \mathbf{B}\mathbf{u}_d + 2\tilde{\mathbf{A}}\mathbf{c} + 2\tilde{\mathbf{B}}\mathbf{u}_c$

$\dot{\mathbf{c}} = \mathbf{A}\mathbf{c} + \mathbf{B}\mathbf{u}_c + 0.5\tilde{\mathbf{A}}\mathbf{d} + 0.5\tilde{\mathbf{B}}\mathbf{u}_d$



$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 2\tilde{\mathbf{A}} \\ 0.5\tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & 2\tilde{\mathbf{B}} \\ 0.5\tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

Determining the coupling of common mode damping to DARM

$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 2\tilde{\mathbf{A}} \\ 0.5\tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & 2\tilde{\mathbf{B}} \\ 0.5\tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

\mathbf{G}_{damp} = damping control
 $\mathbf{R}_{s,damp}$ = damping sensor matrix
 $\mathbf{R}_{a,damp}$ = damping actuation matrix
 $\mathbf{u}_d = 0$, ignoring the cavity control for now
 $\mathbf{u}_c = -\mathbf{R}_{a,damp} \mathbf{G}_{damp} (\mathbf{R}_{s,damp} \mathbf{c} + n_x / 2 + n_y / 2)$
 n_x = ETMX top mass long. sensor noise
 n_y = ETMY top mass long. sensor noise

- Now, substitute in the feedback and transform to Laplace space:

$$\begin{bmatrix} s\mathbf{I}\mathbf{d} \\ s\mathbf{I}\mathbf{c} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 2\tilde{\mathbf{A}} - 2\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \\ 0.5\tilde{\mathbf{A}} & \mathbf{A} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} -2\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \\ -\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \end{bmatrix} (n_x + n_y) / 2$$

- Grouping like terms:

$$\begin{bmatrix} s\mathbf{I} - \mathbf{A} & -2(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}) \\ -0.5\tilde{\mathbf{A}} & s\mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}) \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} = \begin{bmatrix} -2\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \\ -\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \end{bmatrix} \bar{n}$$

$$\bar{n} = (n_x + n_y) / 2$$

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

- Solving \mathbf{c} in terms of \mathbf{d} and $\bar{\mathbf{n}}$:

$$(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})\mathbf{c} = 0.5\tilde{\mathbf{A}}\mathbf{d} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\bar{\mathbf{n}}$$

$$\mathbf{c} = (s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})^{-1} (0.5\tilde{\mathbf{A}}\mathbf{d} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\bar{\mathbf{n}})$$

- Plugging \mathbf{c} in to \mathbf{d} equation:

$$(s\mathbf{I} - \mathbf{A})\mathbf{d} - 2(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})\mathbf{c} = -2\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\bar{\mathbf{n}}$$

$$(s\mathbf{I} - \mathbf{A})\mathbf{d} - 2(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})^{-1} (0.5\tilde{\mathbf{A}}\mathbf{d} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\bar{\mathbf{n}}) = -2\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\bar{\mathbf{n}}$$

- Defining intermediate variables to keep things tidy:

$$\mathbf{D} = s\mathbf{I} - \mathbf{A} - (\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})^{-1} \tilde{\mathbf{A}}$$

$$\mathbf{N} = 2\left[(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp})^{-1} \mathbf{B} + \tilde{\mathbf{B}} \right] \mathbf{R}_{a,damp}\mathbf{G}_{damp}$$

- Then \mathbf{d} can be written as a function of $\bar{\mathbf{n}}$:

$$\mathbf{D}\mathbf{d} = -\mathbf{N}\bar{\mathbf{n}}$$

$$\mathbf{d} = -\mathbf{D}^{-1}\mathbf{N}\bar{\mathbf{n}}$$

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

Then the transfer function from common mode sensor noise to DARM is:

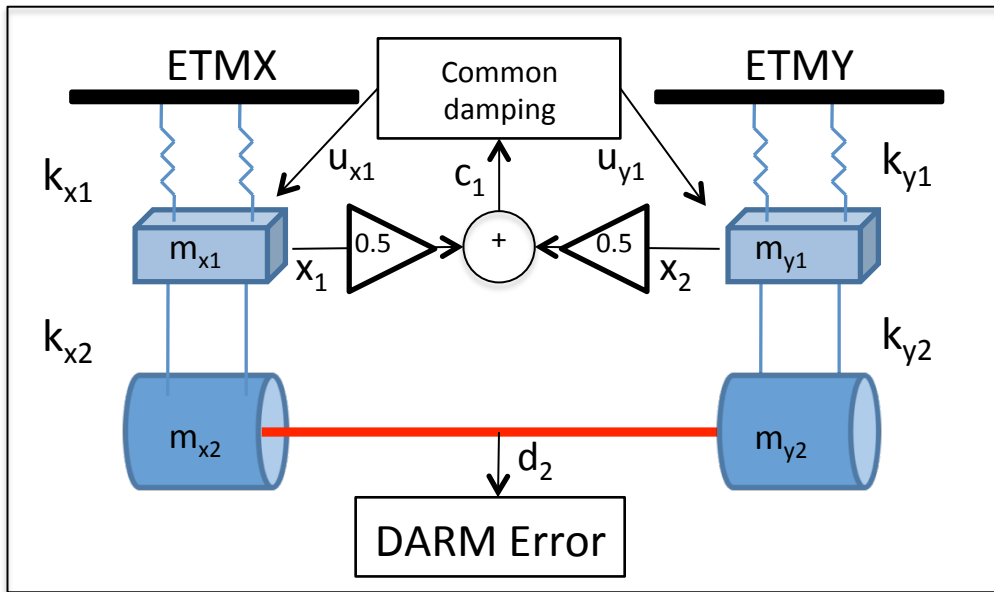
$$d_4 = \mathbf{R}_{S,cavity} \mathbf{d}, \text{ DARM cavity error}$$

$$\boxed{\frac{d_4}{\bar{n}} = -\mathbf{R}_{s,cavity} \mathbf{D}^{-1} \mathbf{N}}, \text{ TF between common mode top mass sensor noise and DARM error}$$

As the plant differences go to zero, \mathbf{N} goes to zero, and thus the coupling of common mode damping noise to DARM goes to zero.

Simple Common to Diff. Coupling Ex.

To show what the matrices on the previous slides look like.



System state space in diff-comm coordinates

$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 2\tilde{\mathbf{A}} \\ 0.5\tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & 2\tilde{\mathbf{B}} \\ 0.5\tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

$\mathbf{u}_d = 0$, ignoring the cavity control for now

$$\mathbf{u}_c = -\mathbf{R}_{a,damp} \mathbf{G}_{damp} (\mathbf{R}_{s,damp} \mathbf{c} + n_x / 2 + n_y / 2)$$

\mathbf{G}_{damp} = damping control filter

$\mathbf{R}_{s,damp}$ = damping sensor matrix

$\mathbf{R}_{a,damp}$ = damping actuation matrix

$\mathbf{R}_{s,cavity}$ = cavity sensor matrix

\mathbf{d} = differential DOFs

\mathbf{c} = common DOFs

n_x = ETMX top mass long. sensor noise

n_y = ETMY top mass long. sensor noise

$$\mathbf{c}_1 = \mathbf{R}_{s,damp} \mathbf{c} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \dot{c}_1 \\ \dot{c}_2 \end{bmatrix}$$

$$\mathbf{d}_2 = \mathbf{R}_{s,cavity} \mathbf{d} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \dot{d}_1 \\ \dot{d}_2 \end{bmatrix}$$

$$\mathbf{R}_{a,damp} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

ETMX A Matrix

$$\mathbf{A}_x = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1} + k_{x2})}{m_{x1}} & k_{x2} & 0 & 0 \\ k_{x2} & \frac{-k_{x2}}{m_{x2}} & 0 & 0 \end{bmatrix}$$

ETMY A Matrix

$$\mathbf{A}_y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{y1} + k_{y2})}{m_{y1}} & k_{y2} & 0 & 0 \\ k_{y2} & \frac{-k_{y2}}{m_{y2}} & 0 & 0 \end{bmatrix}$$

Common A Matrix

$$\mathbf{A} = (\mathbf{A}_x + \mathbf{A}_y) / 2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1} + k_{x2})m_{y1} - (k_{y1} + k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2} + k_{y2} & 0 & 0 \\ k_{x2} + k_{y2} & \frac{-k_{x2}m_{y2} - k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{bmatrix} / 2$$

Differential A Matrix

$$\tilde{\mathbf{A}} = (\mathbf{A}_x - \mathbf{A}_y) / 2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1} + k_{x2})m_{y1} + (k_{y1} + k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2} - k_{y2} & 0 & 0 \\ k_{x2} - k_{y2} & \frac{-k_{x2}m_{y2} + k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{bmatrix} / 2$$

Simple Common to Diff. Coupling Ex

ETMX B Matrix

$$\mathbf{B}_x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_{x1}} & 0 \\ 0 & \frac{1}{m_{x2}} \end{bmatrix}$$

Common B Matrix

$$\mathbf{B} = (\mathbf{B}_x + \mathbf{B}_y) / 2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{m_{x1} + m_{y1}}{m_{x1}m_{y1}} & 0 \\ 0 & \frac{m_{x2} + m_{y2}}{m_{x2}m_{y2}} \end{bmatrix} / 2$$

ETMY B Matrix

$$\mathbf{B}_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_{y1}} & 0 \\ 0 & \frac{1}{m_{y2}} \end{bmatrix}$$

Differential B Matrix

$$\mathbf{B} = (\mathbf{B}_x - \mathbf{B}_y) / 2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{m_{x1} - m_{y1}}{m_{x1}m_{y1}} & 0 \\ 0 & \frac{m_{x2} - m_{y2}}{m_{x2}m_{y2}} \end{bmatrix} / 2$$

Simple Common to Diff. Coupling Ex

$$\mathbf{D} = s\mathbf{I} - \mathbf{A} - \left(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{damp}^T \mathbf{G}_{damp} \mathbf{R}_{damp} \right) \left(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{damp}^T \mathbf{G}_{damp} \mathbf{R}_{damp} \right)^{-1} \tilde{\mathbf{A}}$$

$$\mathbf{N} = 2 \left[\left(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{damp}^T \mathbf{G}_{damp} \mathbf{R}_{damp} \right) \left(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{damp}^T \mathbf{G}_{damp} \mathbf{R}_{damp} \right)^{-1} \mathbf{B} + \tilde{\mathbf{B}} \right] \mathbf{R}_{damp}^T \mathbf{G}_{damp}$$

$d_4 = \mathbf{R}_{S,cavity} \mathbf{d}$, DARM cavity error

$\frac{d_4}{\bar{n}} = -\mathbf{R}_{S,cavity} \mathbf{D}^{-1} \mathbf{N}$, TF between common mode top mass sensor noise and DARM error

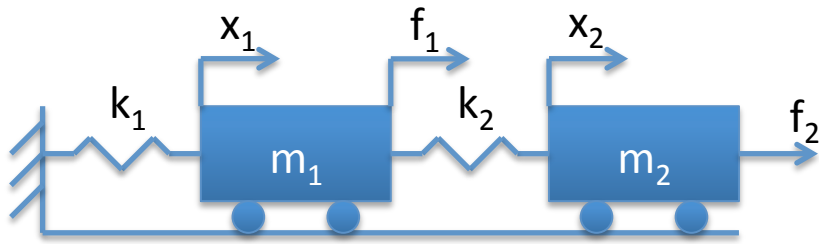
Plugging in sus parameters for \mathbf{N} :

$$\mathbf{N} = 2 \left(\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{2} \frac{-(k_{x1}+k_{x2})m_{y1}+(k_{y1}+k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2}-k_{y2} & 0 & 0 & -\frac{1}{2} \frac{m_{x1}-m_{y1}}{m_{x1}m_{y1}} & 0 \\ k_{x2}-k_{y2} & \frac{-k_{x2}m_{y2}+k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 & 0 & \frac{m_{x2}-m_{y2}}{m_{x2}m_{y2}} \end{array} \right] \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] \left[\begin{array}{c} 1 \\ 0 \end{array} \right] \mathbf{G}_{damp} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array} \right] \dots \right)^{-1} \left(\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \dots s\mathbf{I} - \frac{1}{2} \frac{-(k_{x1}+k_{x2})m_{y1}-(k_{y1}+k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2}+k_{y2} & 0 & 0 & +\frac{1}{2} \frac{m_{x1}+m_{y1}}{m_{x1}m_{y1}} & 0 \\ k_{x2}+k_{y2} & \frac{-k_{x2}m_{y2}-k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 & 0 & \frac{m_{x2}+m_{y2}}{m_{x2}m_{y2}} \end{array} \right] \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] \left[\begin{array}{c} 1 \\ 0 \end{array} \right] \mathbf{G}_{damp} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array} \right] \frac{1}{2} \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] + \frac{1}{2} \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] \left[\begin{array}{c} 0 \\ 0 \end{array} \right] \mathbf{G}_{damp} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array} \right] \right)$$

**CHANGE IN TOP MASS MODES FROM
CAVITY CONTROL – SIMPLE TWO MASS
SYSTEM EXAMPLE.**

Change in top mass modes from cavity control – simple two mass ex.

Question: What happens to x_1 response when we control x_2 with f_2 ?



Mass Matrix

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

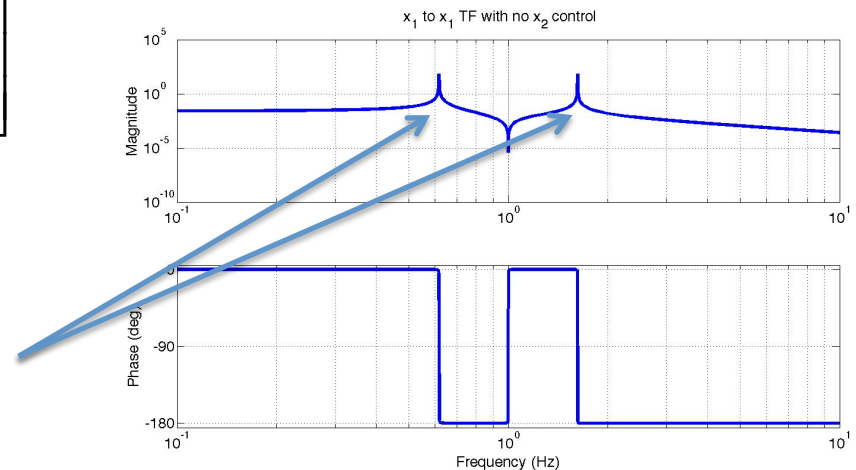
Stiffness Matrix

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

Equation of Motion

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

When $f_2 = 0$,
The f_1 to x_1 TF has two modes



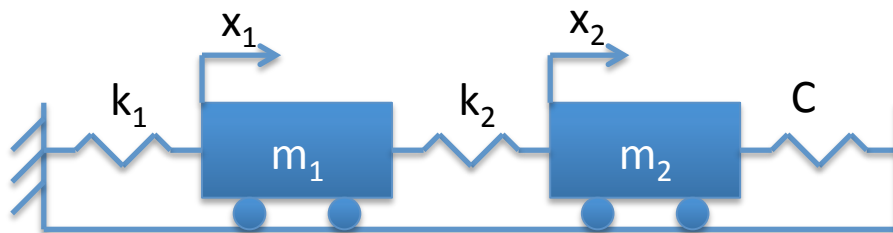
Change in top mass modes from cavity control – simple two mass ex.

If we feedback x_2 to f_2 with control C

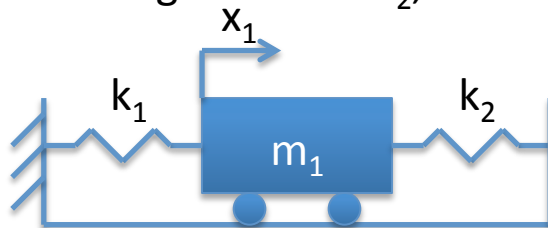
$$f_2 = -Cx_2$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

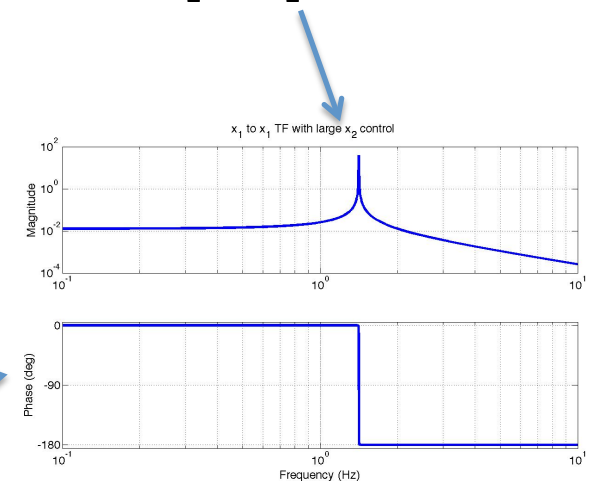
This is equivalent to



As we get to $C \gg k_2$, then x_1 approaches this system



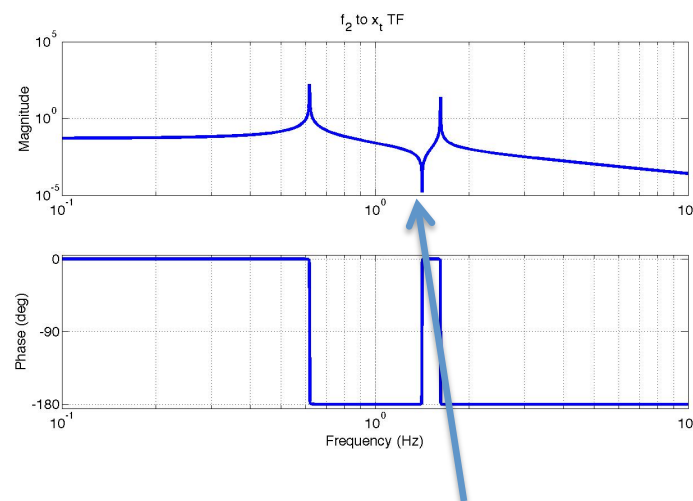
The f_1 to x_1 TF has one mode. The frequency of this mode happens to be the zero in the TF from f_2 to x_2 .



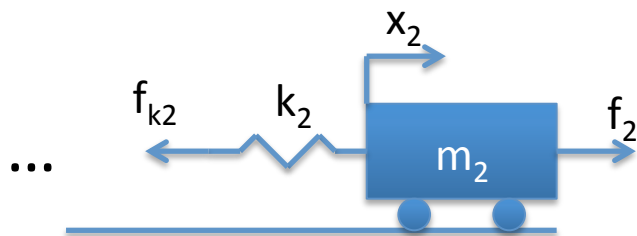
Change in top mass modes from cavity control – simple two mass ex.

Discussion of why the single x_1 mode frequency coincides with the f_2 to x_2 TF zero:

- The f_2 to x_2 zero occurs at the frequency where the k_2 spring force exactly balances f_2 . At this frequency any energy transferred from f_2 to x_2 gets sucked out by x_1 until x_2 comes to rest. Thus, there must be some x_1 resonance to absorb this energy until x_2 comes to rest. However, we do not see x_1 'blow up' from an f_2 drive at this frequency because once x_2 is not moving, it is no longer transferring energy. Once we physically lock, or control, x_2 to decouple it from x_1 , this resonance becomes visible with an x_1 drive.



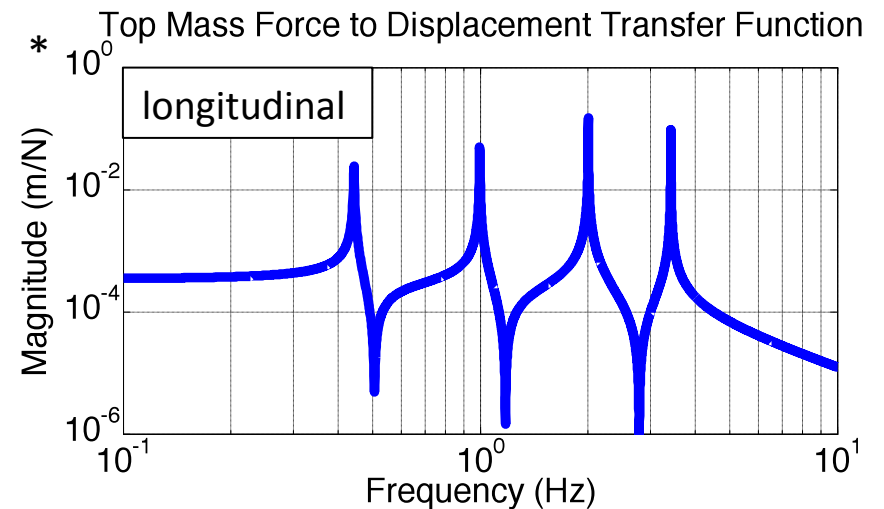
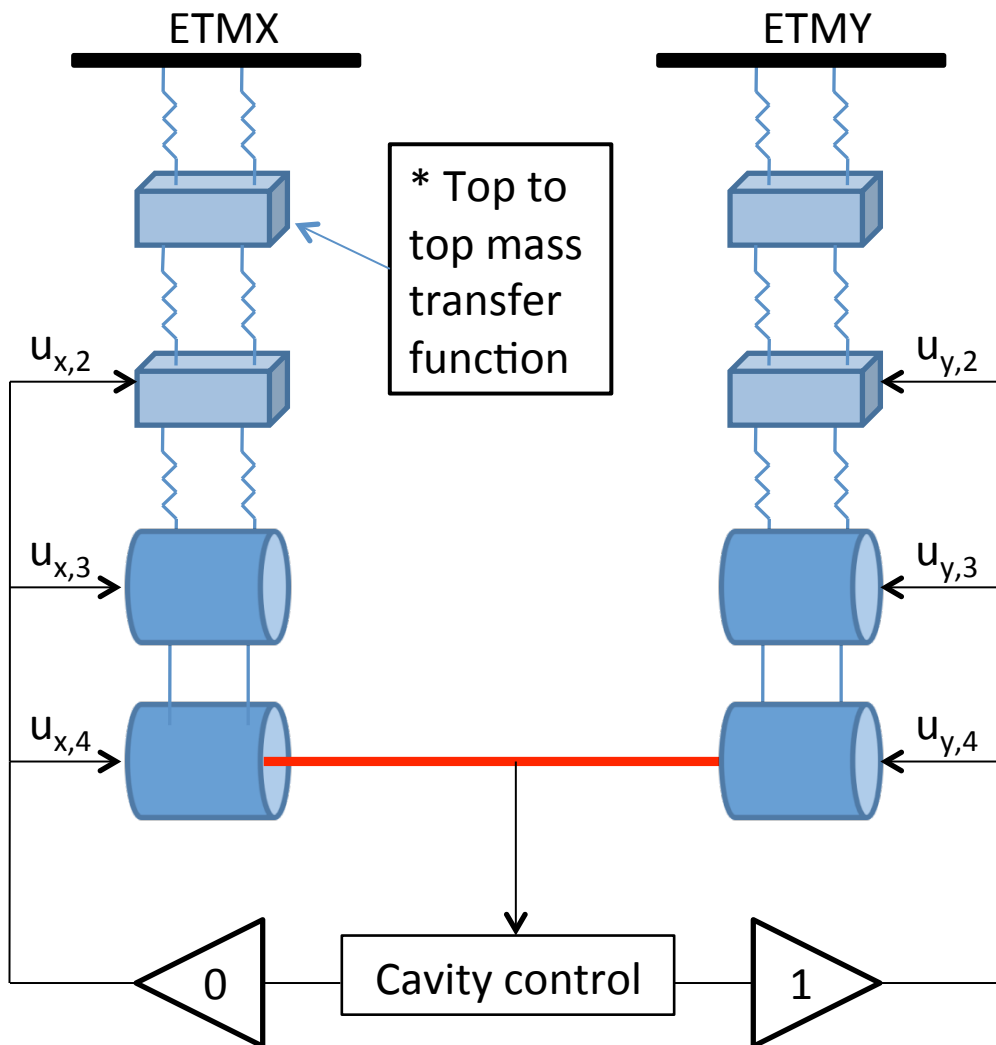
The zero in the TF from f_2 to x_2 . It coincides with the f_1 to x_1 TF mode when x_2 is locked.



**CHANGE IN TOP MASS MODES FROM
CAVITY CONTROL – FULL QUAD
EXAMPLE.**

Cavity Control Influence on Damping

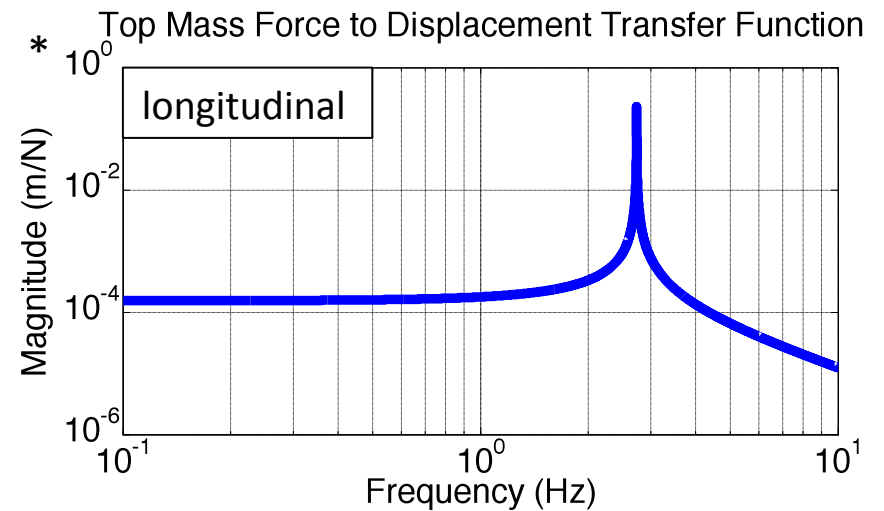
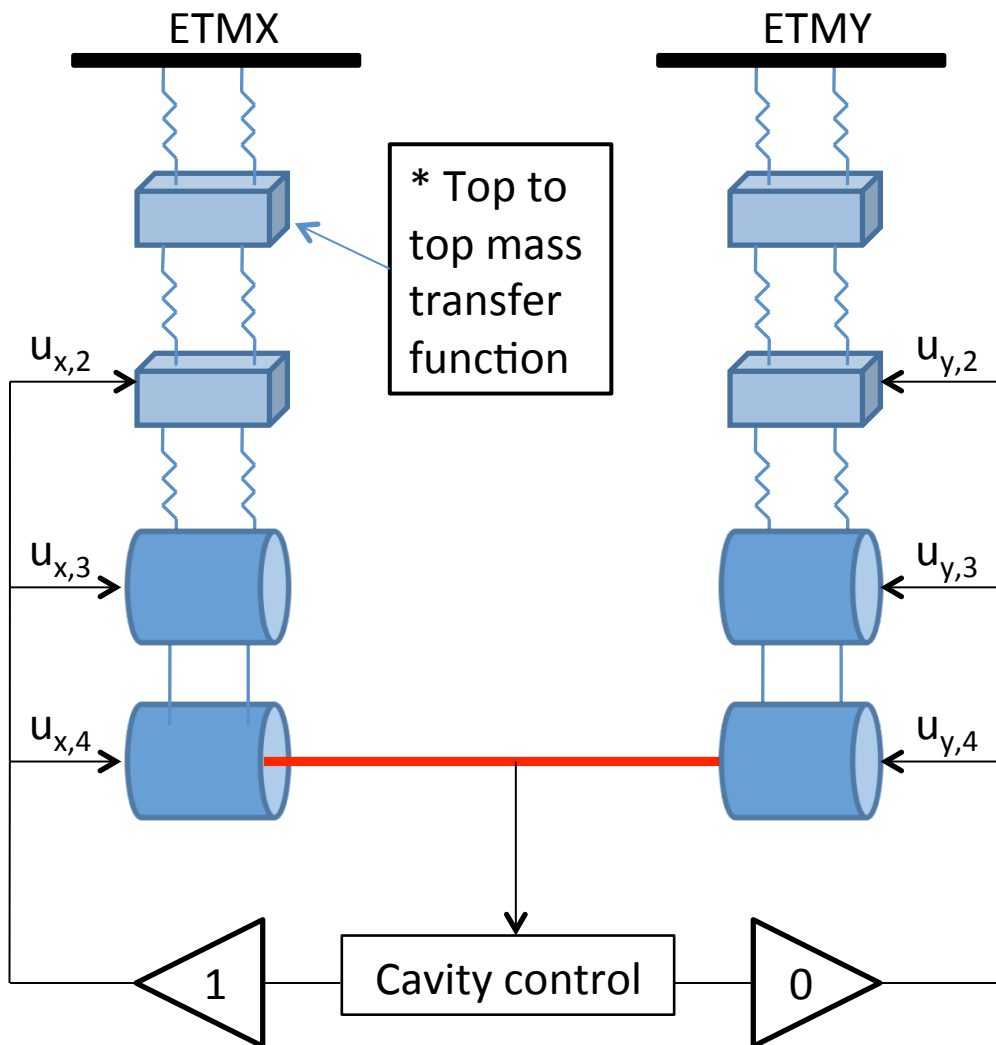
- Case 1: All cavity control on Pendulum 2



- What you would expect – the quad is just hanging free.
- Note: both pendulums are identical in this simulation.

Cavity Control Influence on Damping

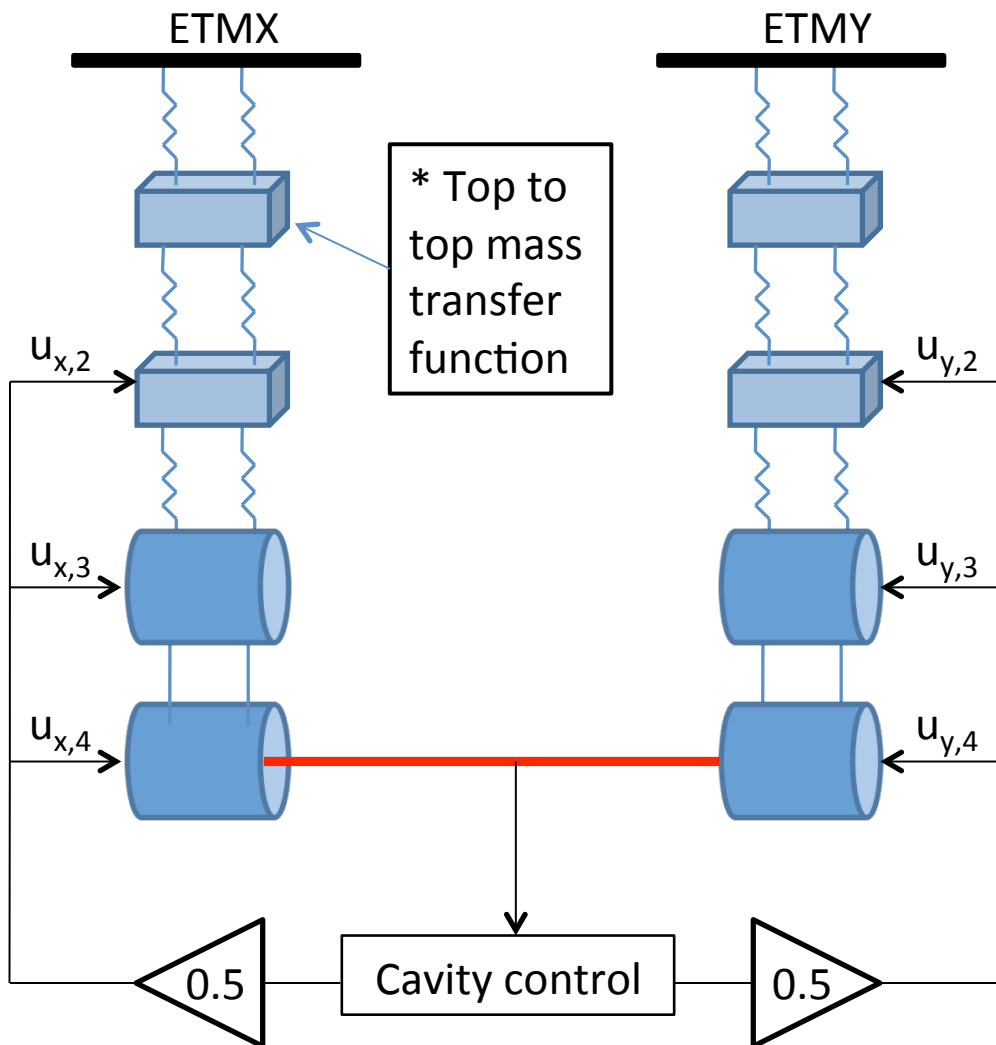
- Case 2: All cavity control on Pendulum 1



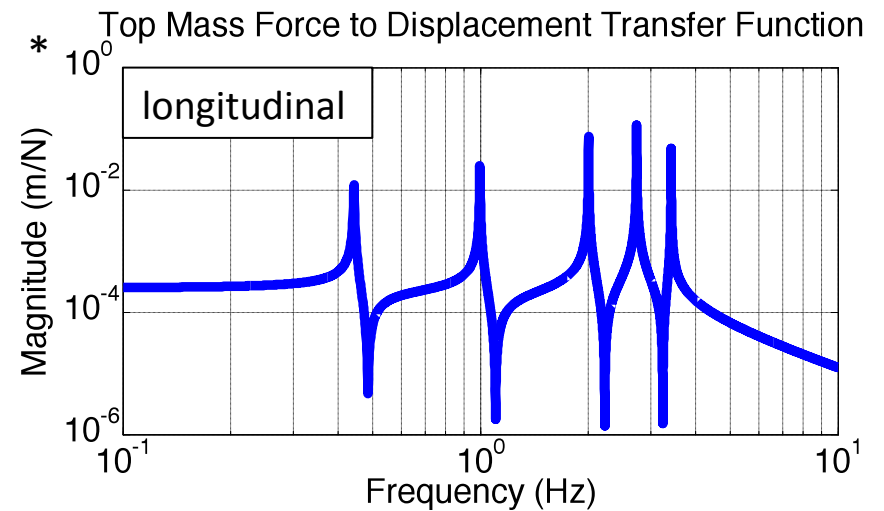
- The top mass of pendulum 1 behaves like the UIM is clamped to gnd when its ugf is high.
- Since the cavity control influences modes, you can use the same effect to apply damping (more on this later)

Cavity Control Influence on Damping

- Case 3: Cavity control split evenly between both pendulums

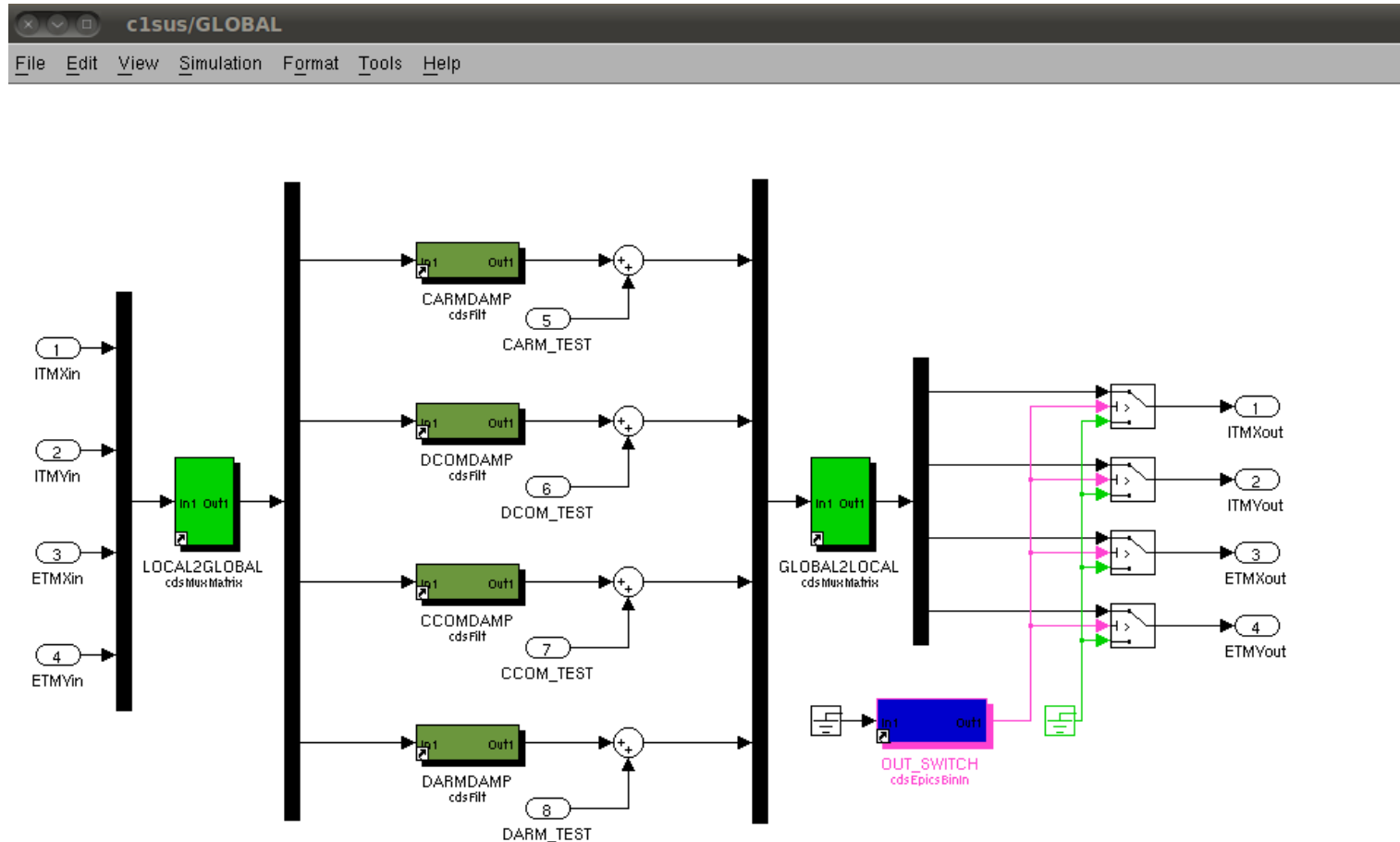


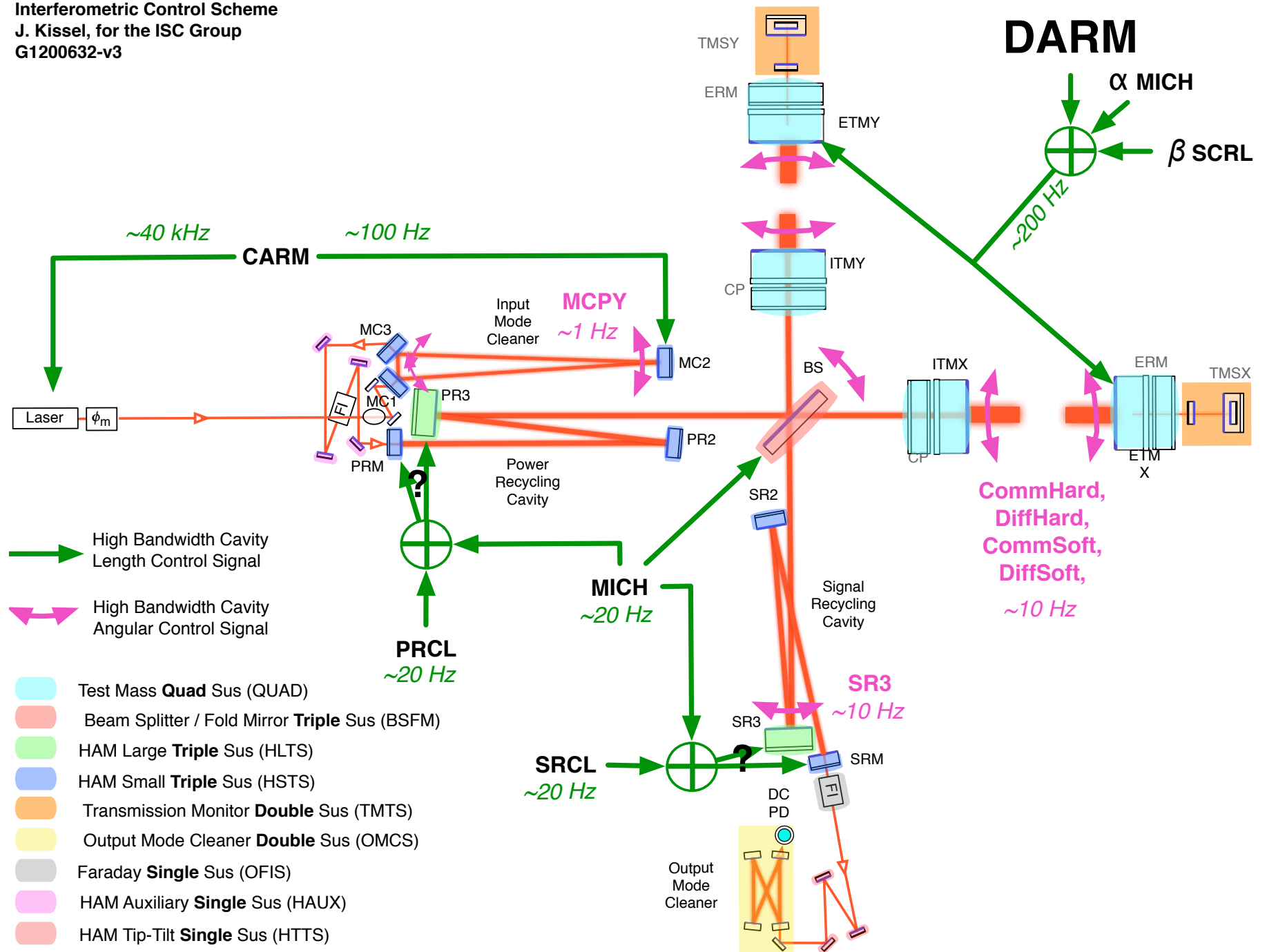
* Top to top mass transfer function



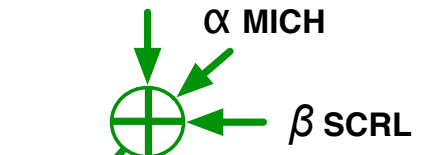
- The top mass response is now an average of the previous two cases -> 5 resonances to damp.
- Control up to the PUM, rather than the UIM, would yield 6 resonances.
- aLIGO will likely behave like this. 56

Global Damping RCG Diagram



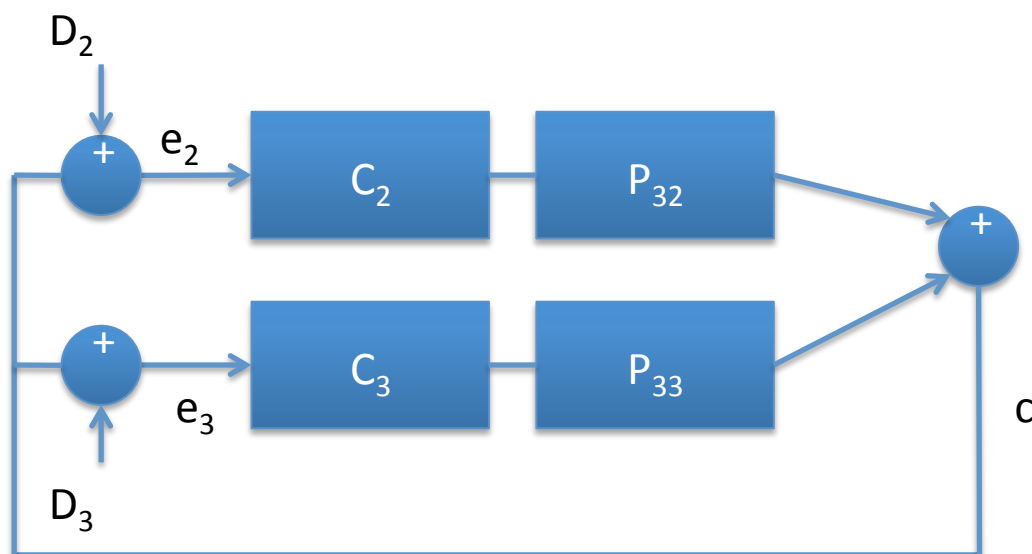


DARM



CommHard,
 DiffHard,
 CommSoft,
 DiffSoft,
 ~ 10 Hz

Cavity loop gain measurements on a triple with 2 stages of actuation



If we measure the stage 3 loop gain by driving D_3 , we get

$$c = C_3 P_{33} e_3 + C_2 P_{32} c$$

$$c(1 - C_2 P_{32}) = C_3 P_{33} e_3$$

$$\frac{c}{e_3} = \frac{C_3 P_{33}}{1 - C_2 P_{32}}$$

If we measure the stage 2 loop gain by driving D_2 , we get

$$c = C_3 P_{33} c + C_2 P_{32} e_2$$

$$c(1 - C_3 P_{33}) = C_2 P_{32} e_2$$

$$\frac{c}{e_2} = \frac{C_2 P_{32}}{1 - C_3 P_{33}}$$

Scratch

$$\mathbf{N} = \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 2s & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2s & 0 & -1 \\ \hline \frac{-(k_{x1} + k_{x2})m_{y1} + (k_{y1} + k_{y2})m_{x1} - (m_{x1} - m_{y1})\mathbf{G}_{damp}}{m_{x1}m_{y1}} & k_{x2} - k_{y2} & 0 & 0 & \frac{(k_{x1} + k_{x2})m_{y1} + (k_{y1} + k_{y2})m_{x1} + (m_{x1} + m_{y1})\mathbf{G}_{damp}}{m_{x1}m_{y1}} & -k_{x2} - k_{y2} & 2s & 0 \\ k_{x2} - k_{y2} & \frac{-k_{x2}m_{y2} + k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 & -k_{x2} - k_{y2} & \frac{k_{x2}m_{y2} + k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 2s \end{array} \right]^{-1} \mathbf{B} + \tilde{\mathbf{B}} \begin{bmatrix} \mathbf{G}_{damp} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2s & 0 & -1 & 0 \\ 0 & 2s & 0 & -1 \\ \hline \frac{(k_{x1} + k_{x2})m_{y1} + (k_{y1} + k_{y2})m_{x1} + (m_{x1} + m_{y1})\mathbf{G}_{damp}}{m_{x1}m_{y1}} & -k_{x2} - k_{y2} & 2s & 0 \\ -k_{x2} - k_{y2} & \frac{k_{x2}m_{y2} + k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 2s \end{bmatrix} = \begin{bmatrix} \mathbf{J} & \mathbf{K} \\ \mathbf{L} & \mathbf{M} \end{bmatrix}$$

$$\mathbf{L}^{-1} = \begin{bmatrix} \frac{k_{x2}m_{y2} + k_{y2}m_{x2}}{m_{x2}m_{y2}} & k_{x2} + k_{y2} \\ k_{x2} + k_{y2} & \frac{(k_{x1} + k_{x2})m_{y1} + (k_{y1} + k_{y2})m_{x1} + (m_{x1} + m_{y1})\mathbf{G}_{damp}}{m_{x1}m_{y1}} \end{bmatrix} \frac{m_{x1}m_{y1}m_{x2}m_{y2}}{[k_{x2}m_{y2} + k_{y2}m_{x2}][[(k_{x1} + k_{x2})m_{y1} + (k_{y1} + k_{y2})m_{x1} + (m_{x1} + m_{y1})\mathbf{G}_{damp}] - m_{x1}m_{y1}m_{x2}m_{y2}(k_{x2} + k_{y2})^2]}$$

Scratch: Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 2\tilde{\mathbf{A}} \\ 0.5\tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & 2\tilde{\mathbf{B}} \\ 0.5\tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

\mathbf{G}_{cavity} = cavity control
 \mathbf{G}_{damp} = damping control
 $\mathbf{R}_{s,cavity}$ = cavity sensing matrix, $\mathbf{R}_{a,cavity}$ = cavity actuation matrix
 $\mathbf{R}_{s,damp}$ = damping sensor matrix, $\mathbf{R}_{a,damp}$ = damping actuation matrix

$$\mathbf{u}_d = -\mathbf{R}_{a,cavity} \mathbf{G}_{cavity} (\mathbf{R}_{s,cavity} \mathbf{d} + n_x / 2 - n_y / 2)$$

$$\mathbf{u}_c = -\mathbf{R}_{a,damp} \mathbf{G}_{damp} (\mathbf{R}_{s,damp} \mathbf{c} + n_x / 2 + n_y / 2)$$

Now, substitute in the feedback and transform to Laplace space:

$$\begin{bmatrix} s\mathbf{Id} \\ s\mathbf{Ic} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BR}_{a,cavity} \mathbf{G}_{cavity} \mathbf{R}_{s,cavity} & 2(\tilde{\mathbf{A}} - \mathbf{R}_{a,damp} \tilde{\mathbf{B}} \mathbf{G}_{damp} \mathbf{R}_{s,damp}) \\ 0.5(\tilde{\mathbf{A}} - \tilde{\mathbf{B}} \mathbf{R}_{a,cavity} \mathbf{G}_{cavity} \mathbf{R}_{s,cavity}) & \mathbf{A} - \mathbf{R}_{a,damp} \mathbf{B} \mathbf{G}_{damp} \mathbf{R}_{s,damp} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} -\mathbf{BR}_{a,cavity} \mathbf{G}_{cavity} & -2\tilde{\mathbf{B}} \mathbf{R}_{a,damp}^T \mathbf{G}_{damp} \\ -0.5\tilde{\mathbf{B}} \mathbf{R}_{a,cavity} \mathbf{G}_{cavity} & -\mathbf{B} \mathbf{R}_{a,damp}^T \mathbf{G}_{damp} \end{bmatrix} \begin{bmatrix} n_x - n_y \\ n_x + n_y \end{bmatrix} / 2$$

For DARM we measure the test masses with the global cavity readout, no local sensors are involved. The cavity readout must also have very low noise to measure GWs. So make the assumption that $n_x - n_y = 0$ for cavity control and simplify to:

$$\begin{bmatrix} s\mathbf{Id} \\ s\mathbf{Ic} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BR}_{a,cavity} \mathbf{G}_{cavity} \mathbf{R}_{s,cavity} & 2(\tilde{\mathbf{A}} - \tilde{\mathbf{B}} \mathbf{R}_{s,damp} \mathbf{G}_{damp} \mathbf{R}_{s,damp}) \\ 0.5(\tilde{\mathbf{A}} - \tilde{\mathbf{B}} \mathbf{R}_{a,cavity} \mathbf{G}_{cavity} \mathbf{R}_{s,cavity}) & \mathbf{A} - \mathbf{BR}_{s,damp} \mathbf{G}_{damp} \mathbf{R}_{s,damp} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} -2\tilde{\mathbf{B}} \mathbf{R}_{a,damp}^T \mathbf{G}_{damp} \\ -\mathbf{B} \mathbf{R}_{a,damp}^T \mathbf{G}_{damp} \end{bmatrix} (n_x + n_y) / 2$$