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Refer to: LIGO-T1200375-v2

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From: M. Zucker, R. Weiss

Re: **Transient response of a beam tube during leak checking**

Abstract: Suppose we need to localize a vacuum leak somewhere along a beam tube using a helium mass spectrometer. This is not a textbook leak hunting application, since the volume is large (4.5 million liters) while the effective tube conductance is modest (177 liters/second for He at 2 km). The resulting long time constant and diminished flux, combined with an uncontrolled environment around the tube, suggest many false background possibilities. These may include foreline backstreaming, or transport of spilled He to distant areas far from the intended test zone. To help veto false positives, it's useful to predict the transient signature expected from an actual leak.

Test method: A localized candidate area is sealed off from ambient with tape and plastic film ("bagged"). We inflate the bag, displacing or diluting residual air, with 1 atm of helium for a duration W, after which we stop and slit the bag open to exhaust the helium.

Our leak detector is connected to the beam tube at some axial distance z through a coupling and sampling valve with characteristic He conductance C_{LD} (typically of order 10 l/s). Since the leak detector captures only a fraction of the sample gas, the remainder dissipating down the tube, its reported "leak rate" $L(t)$ (torr-liters/second) really corresponds to the local partial pressure of helium multiplied by the sampling conductance,

$$L(t) = P(t,z)C_{LD} \ .$$

A good leak detector should have nominal stability better than $L_{\min} \sim 10^{-10}$ Tl /s. Helium partial pressure changes of order 10^{-11} torr should thus be discernible, if there is no fluctuating background to interfere.

Diffusion: The pressure evolution $P(t,z)$ is governed by the 1-d diffusion equation,

$$\frac{\partial^2 P}{\partial z^2} = \frac{1}{c}\frac{\partial P}{\partial t}$$

where

$$c = \frac{2}{3}R\langle v\rangle$$

is the molecular diffusion constant of a long tube of internal radius $R \approx 0.6$ m, for molecules having mean thermal speed $\langle v\rangle \approx 1200$ m/s (corresponding to helium at room temperature)[1].

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[1] O'Hanlon, J. F., *A User's Guide to Vacuum Technology (2nd Ed.)*. Wiley (1989).

If present, the leak admits helium flux Q for duration W. Under appropriate boundary conditions, the solution for an arbitrarily short impulse is

$$\lim_{W \to 0} P(t,z) = \frac{QW}{2\pi^{3/2} R^2 \sqrt{ct}}\, e^{-\frac{z^2}{4ct}}$$

Dividing by W and integrating this impulse response with respect to t gives the step response,

$$P(t,z)_{\text{step}} = \frac{Q}{2\pi^{3/2} R^2 c} \cdot \left\{ 2\sqrt{ct}\, e^{-\frac{z^2}{4ct}} - \sqrt{\pi}\, z \cdot \operatorname{erfc}\left(\frac{z}{2\sqrt{ct}} \right) \right\} .$$

A steady injection of arbitrary but finite duration W thus gives

$$P(t,z)_{\text{pulse}} = P(t,z)_{\text{step}}\, U(t) - P(t-W,z)_{\text{step}}\, U(t-W)$$

where $U(t)$ is the Heaviside function.

An example of this pressure evolution is plotted in Figure 1, for leak parameters given in the caption. Near the leak, pressure initially rises as $\sqrt{t}$ and then turns over to $1/\sqrt{t}$ once the source is terminated. Tens of meters away, however, the pressure change is delayed by some seconds, onset and turnover are less abrupt, and the peak pressure is attenuated.

Since injection and detector locations are known, a strong putative signal waveform failing to conform at the corresponding z may indicate a spurious flanking path, and not an actual leak. For example, a monotonic increasing signature that doesn't stabilize or turn over, or a long delay before deflection first registers, would be deemed suspicious. As a guide, some families of "expected" curves are plotted in Figure 2 and Figure 3[2].

Discussion: The $1/\sqrt{t}$ tail constrains repeat testing once He gets in the tube (whether by the found leak, or by some flanking path). False positive indications are also a function of how much He gets spilled in the testing process, and how rapidly it can be flushed from the ambient environment. For these reasons it is important to use no more He than necessary.

The examples plotted in Figure 1-Figure 3 correspond roughly to the estimated LLO Y arm air leak that was under investigation during July 2012. With a 60 second purge, the leak detector signal-to-noise ratio would be expected to exceed 1,000 at 300 meters range.

[2]These plots use the impulse approximation above for computational convenience. At times $t > 2W$ the behavior is similar.

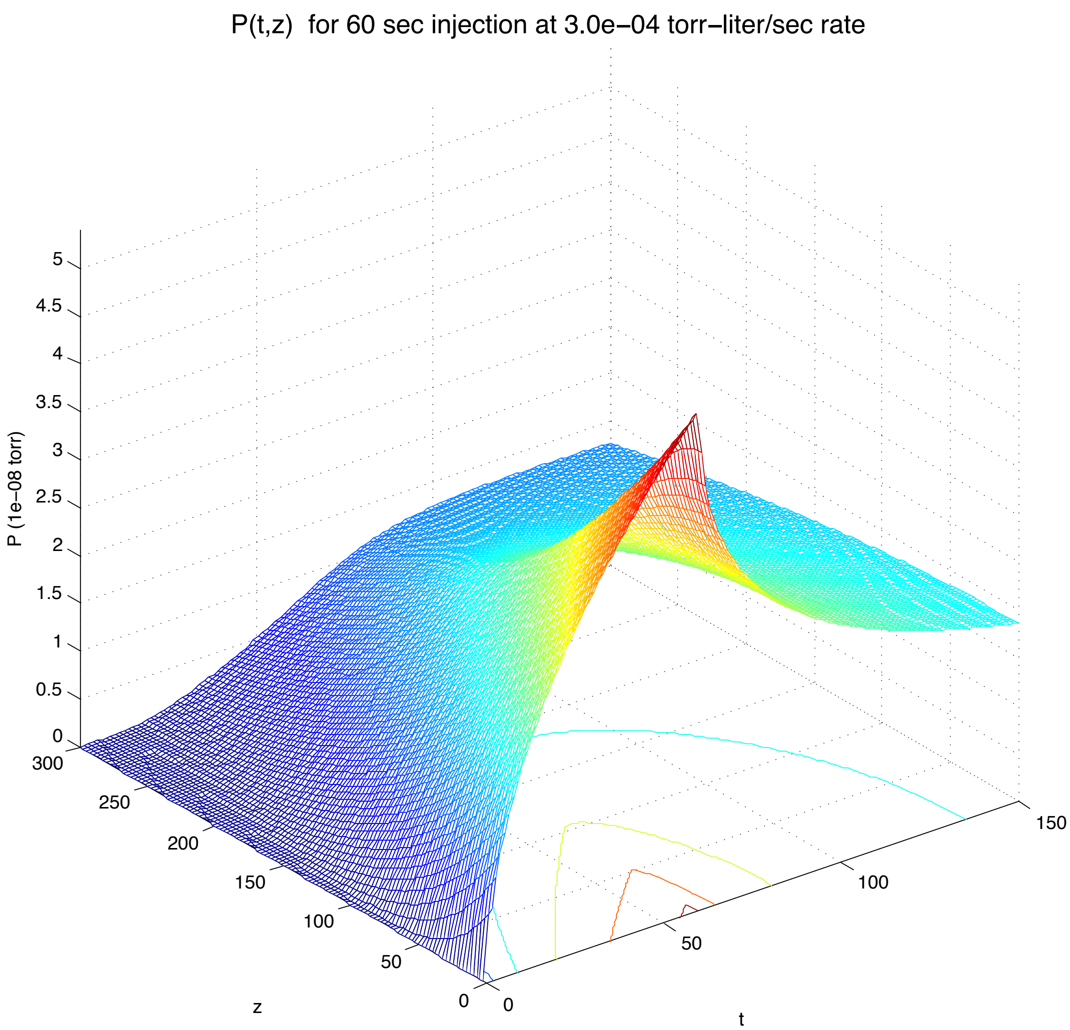

Figure 1: Tube He partial pressure vs. distance from leak *z* (meters) and time *t* (seconds), given steady $3*10^{-4}$ torr-liter/second He flux injected for *W* = 60 seconds.

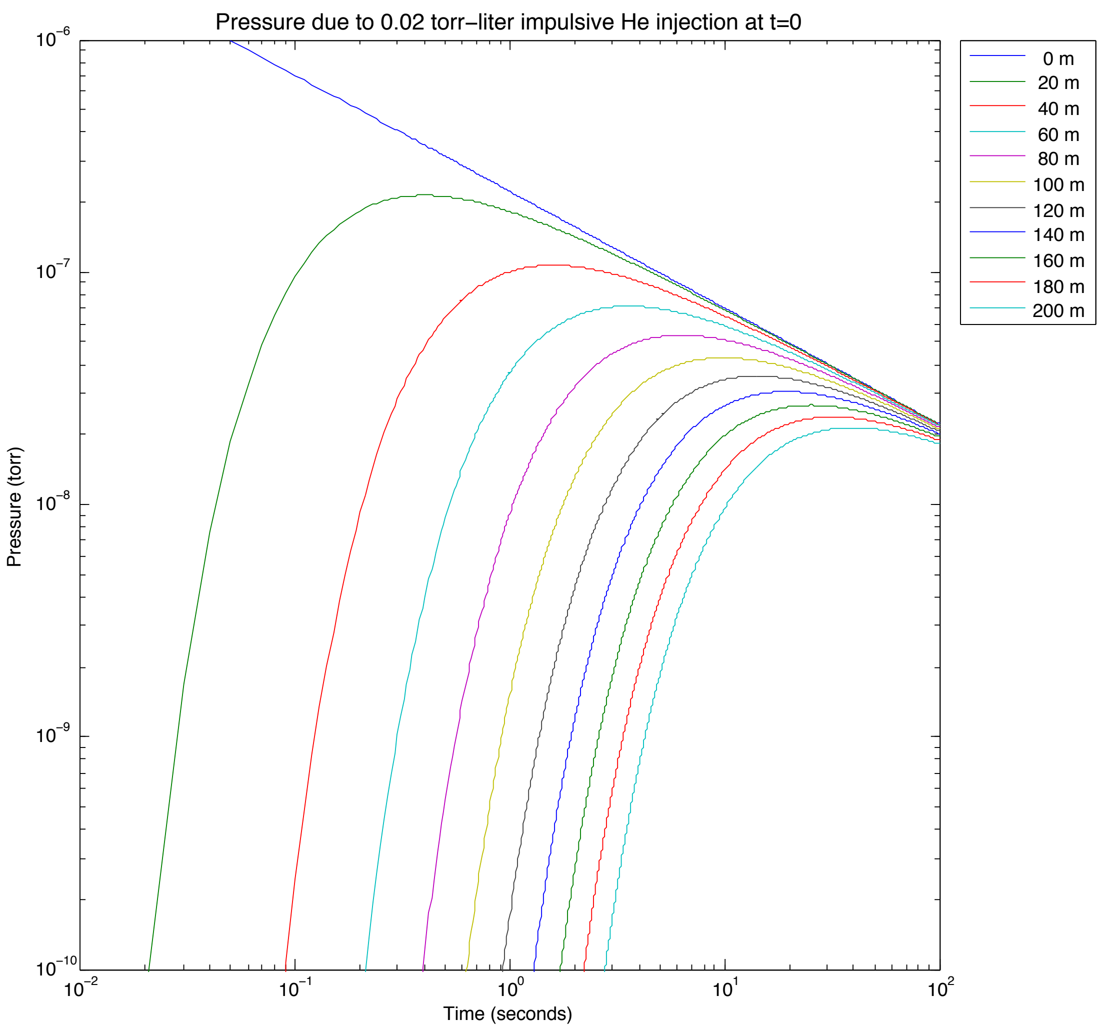

Figure 2: Pressure as a function of time delay after impulsive injection of 0.02 torr-liters of helium, as monitored at various distances z = 0m, 20m, 40m, … from the source.

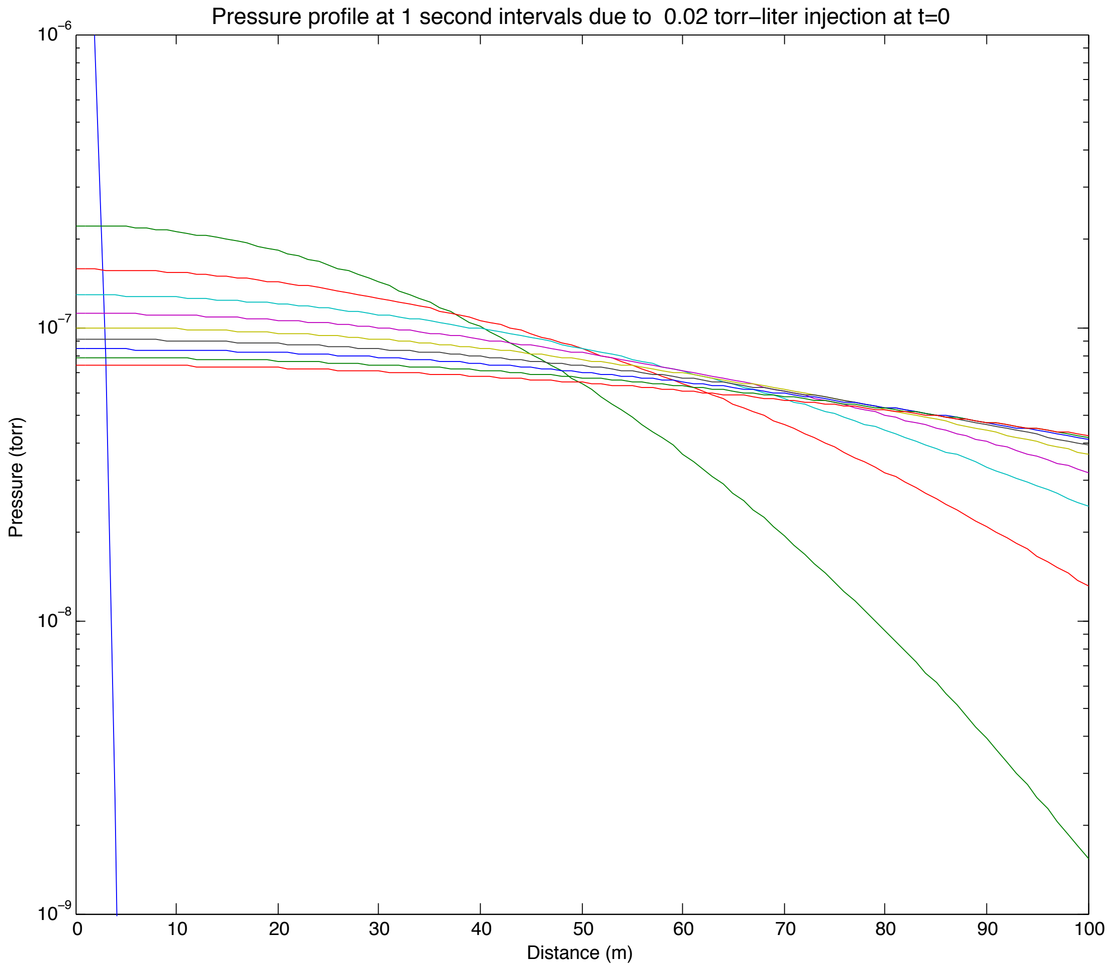

Figure 3: Pressure as a function of distance from an impulsive injection of .02 torr-liters of helium at *t* = 0, for successive sampling times *t* = 1 ms, 1 s, 2s, 3s, …

APPENDIX: Heuristic approximation. Weiss provides the following simplification to predict pressure rise during the helium injection, assuming the detector is located close to the leak.

Approximate the Gaussian spatial profile of the exact solution with a uniform square distribution, supposing all admitted gas uniformly fills a zone of length $\Delta z(t)$. Let this zone grow by diffusion as $\Delta z(t) = \sqrt{2ct}$ after injection begins, where *c* is the diffusion constant as above (about 500 m^2/s).

The instantaneous pressure is then roughly the amount of gas admitted up to *t* divided by the volume of the zone at *t*, or

$$P(t) \approx \frac{Qt}{\pi R^2 \Delta z} \approx \frac{Q}{\pi R^2}\sqrt{\frac{t}{2c}} \approx 6 \cdot 10^{-8}\,\text{torr} \cdot \left(\frac{Q}{3 \cdot 10^{-4}\,\text{Tl/s}}\right) \cdot \left(\frac{t}{60\,\text{s}}\right)^{\frac{1}{2}}.$$

This expression differs from the exact solution above, evaluated at *z* = 0, by a factor of $\sqrt{\pi/2}$ or about 25%.