Noise in the LIGO Interferometers

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The Fourier Domain

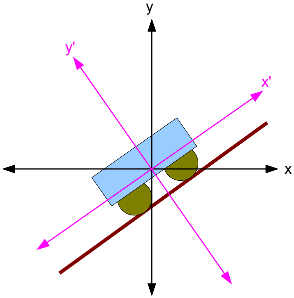
The Fourier transformation.

- Definition:

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt. \tag{1}$$

- Perhaps more easily thought of as a dot product:

$$F(\omega) = f(\vec{t}) \cdot e^{-\vec{i}\omega t}. \tag{2}$$



The Fourier Domain

Some of my favorite properties:

- The Pythagorean theorem still holds (Parseval's theorem);

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega. \tag{3}$$

- A delta function in the time domain has equal contributions at all frequencies in the Fourier domain;

$$\frac{1}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(t) e^{-i\omega t} dt. \tag{4}$$

The Fourier Domain

A more practical way to work in the Fourier domain.

- When faced with a differential equation

  $$\frac{d^2}{dt^2}x(t) = \frac{g}{\ell}x(t) + \frac{1}{m\ell^2}d(t),$$

  assume that the time dependence (or the space dependence) is sinusoidal

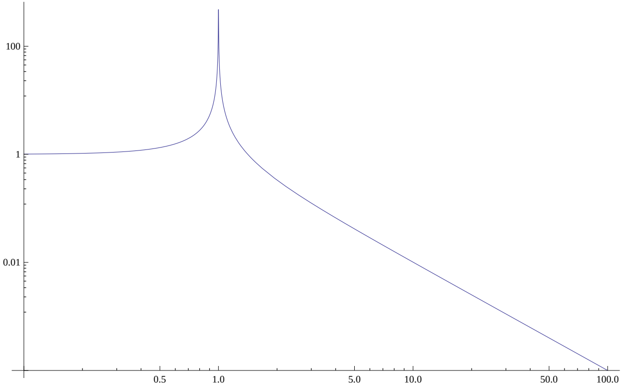
  $$x(t) = X(\omega)e^{-i\omega t} \quad \& \quad d(t) = D(\omega)e^{-i\omega t},$$

  and the differential equation reduces to a simple algebraic equation

  $$\begin{aligned} -\omega^2 X(\omega) &= \frac{g}{\ell}X(\omega) + \frac{1}{m\ell^2}D(\omega) \\ \Rightarrow \quad \frac{X(\omega)}{D(\omega)} &= \frac{1}{m\ell^2}\frac{1}{\frac{g}{\ell} - \omega^2} \end{aligned}$$

The Fourier Domain

The simple pendulum transfer function.



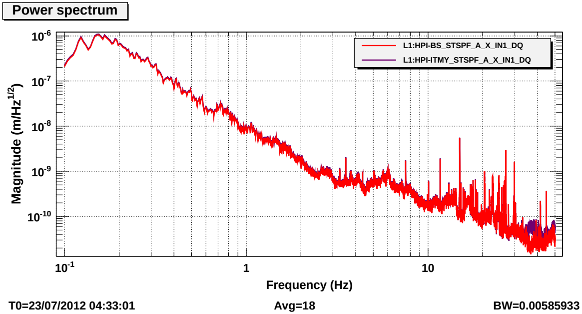
The Fourier Domain

The Fourier Domain in other areas of physics.

- Wavenumber and spatial coordinates share the same relationship as frequency and time ($k \leftrightarrow x$ and $\omega \leftrightarrow t$).
- Quantum mechanics relates Energy to frequency ($E = \hbar\omega$) and momentum to wavenumber ($p = \hbar k$). Hence, energy and time form a Fourier conjugate pair and momentum and position form a Fourier conjugate pair.
- Bloch waves in condensed matter physics.
- Fourier transform spectroscopy in chemical physics.
- Diffraction patterns are the Fourier transform of the scattering source in particle physics.

Seismic Noise

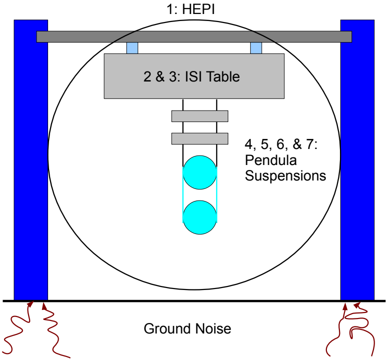
What does the seismic noise look like?



This in 14 orders of magnitude too high for LIGO.

Seismic Noise

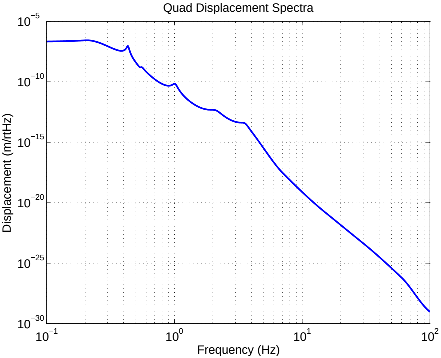
How can we isolate ourselves from it?



HEPI and ISI are capable of active sensor correction as well as passive isolation.

Seismic Noise

How well do we expect to do for Advanced LIGO?



Seismic noise will be a limiting noise source for Advanced LIGO below $\sim$ 10 Hz.

Thermal Noise

What can we do about thermal noise?

- The suspensions and the optics themselves are in thermal equilibrium (via radiation) with the surrounding vacuum system which is at room temperature.
- This internal energy excites the normal modes of the optic and suspension, some of which show up as a displacement of test mass surface.
- Dissipation in the materials couples all of the modes to each other so that even the modes that we do not care about become a problem.
- So; until cooling down becomes financially feasible, **the name of the game is to look for materials (and procedures) with low dissipation coefficients.**

Thermal Noise

- Test mass thermal noise[1]

$$S_x(f) \simeq \frac{2k_B T}{\pi^{3/2}\omega Y_S f}\left[\phi_S + \frac{2d_C}{\sqrt{\pi}\omega}\phi_C\right].$$

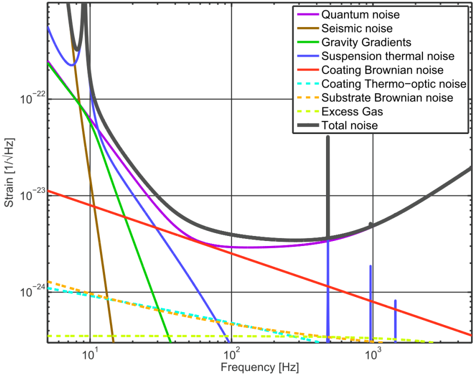
- Suspension thermal noise[2]

$$S_x(f) \simeq \frac{k_B T}{2\pi^3 m f}\left[\frac{f_0^2\phi_w}{f_0^4\phi_w^2 + (f_0^2 - f^2)^2}\right].$$

- Losses in the substrate, coating, and suspension wires ($\phi_S,\ \phi_C,\ \phi_w$) are the driving source of thermal noise. In addition, losses at connections between materials (e.g. wire to mirror) increase thermal noise. Hence, Advanced LIGO uses monolithic suspensions.

Thermal Noise

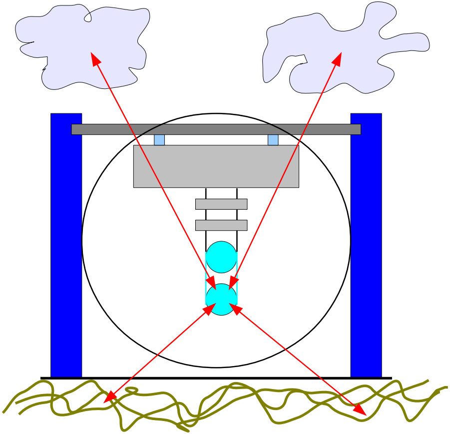
The Advanced LIGO Noise Curve



Thermal noise in the suspension and coatings will prevent Advanced LIGO from reaching the quantum limit from 10 Hz to 100 Hz

Newtonian Noise

Even if we isolate perfectly from ground motion, mass perturbations couple directly to the test masses through gravity.



Newtonian Noise

Why is this such a difficult problem?

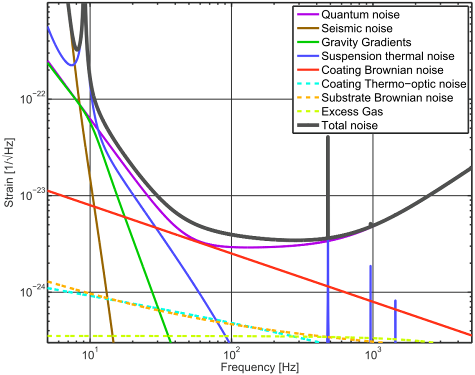
- Because mass comes in only one sign, we cannot shield from gravity.
- Even if we could shield from gravitational forces; it would likely be impossible to shield from local perturbations while still being coupled to astrophysical sources.

So, what can we do?

- Current efforts in Newtonian noise mitigation center around understanding how to predict Newtonian noise by measuring the local seismic field.
- Actuators in our seismic isolation systems allow us to feed forward once we are able to predict the motion.

Newtonian Noise

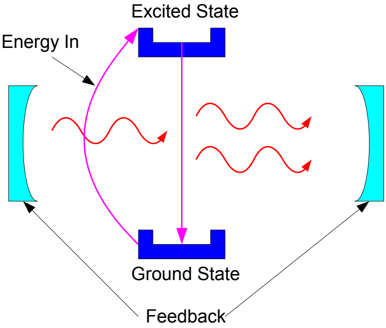
The Advanced LIGO Noise Curve



Newtonian noise is expected to be just beyond Advanced LIGO sensitivity.

Laser Frequency Noise

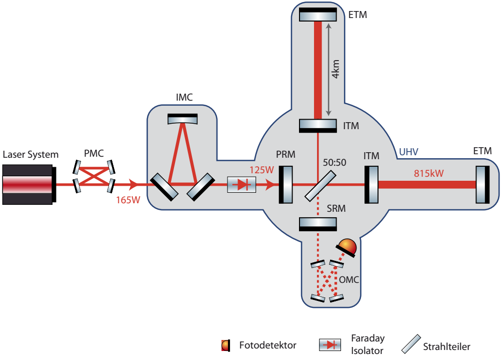
What sets the frequency of a laser?



Temperature and mechanical stress (among others) affect the transition frequency and the feedback mechanism.

Laser Frequency Noise

How do these couple into the interferometer output?



- A perfect Michelson would be insensitive to frequency noise.
- Imperfections in things such as mirror reflectivities, arm lengths, and BS to ITM distance allow frequency noise to show up as power fluctuations at the output.

Laser Frequency Noise

How can we stabilize the laser frequency?

- Actuators: temperature, resonator length, and electro-optic phase modulator.
- Sensors: ultra-stable optical cavities.
- At frequencies above ~ 1 Hz we stabilize the laser frequency to the arm cavities, and at frequencies below ~ 1 Hz we stabilize the laser frequency to a small reference cavity located on the PSL table.

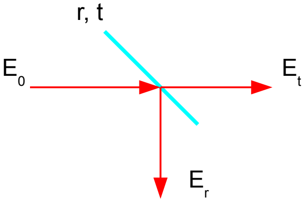
Quantum Noise

Quantum noise arises because of the granularity of the laser field which shows up in our observations in two ways.

- Shot Noise
  - Measuring power is essentially counting photons.
  - The process of counting independent events is described by Poisson statistics.
- Radiation Pressure Noise
  - The radiation pressure on the mirrors of the interferometer depends on the number of photons per unit time.
  - This fluctuating force shows up as small changes in length of the optical cavities.

Quantum Noise

A quick fact from quantum optics.

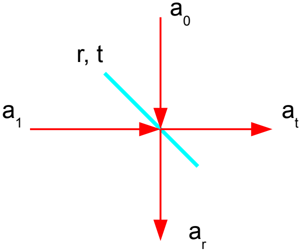


- A classical beam splitter satisfies:

$$E_t = tE_0 \qquad \& \qquad E_r = rE_0$$

Quantum Noise

A quick fact from quantum optics.



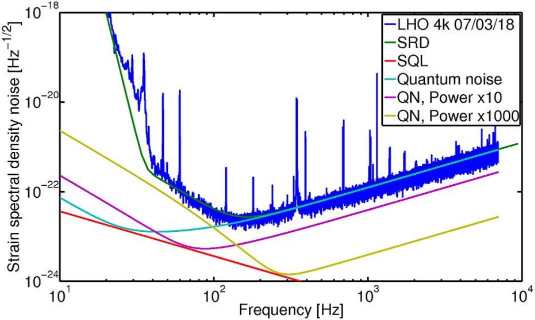
- If we treat the fields as quantum fields (which they are), then we find the classical relations violate the commutation relations, $[a_i, a_j] = \delta_{ij}$.
- The problem is solved by including the vacuum fluctuations of the electric field, a_0.

$$a_t = ta_1 - ra_0 \qquad \& \qquad a_r = ra_1 + ra_0$$

Quantum Noise

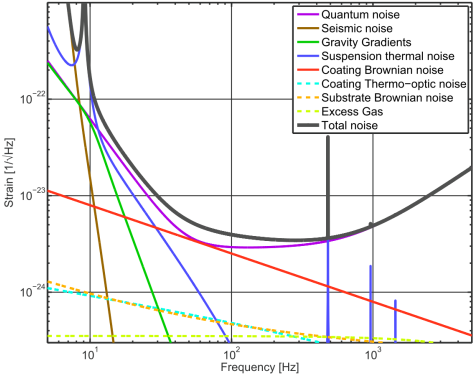
Two equivalent ways of understanding quantum noise

- The entire interferometer must also have vacuum fluctuations entering through the dark port, and these vacuum fluctuations lead to the same quantum noise derived by considering shot noise and radiation pressure.[3]
- In either case, quantum noise looks like[4]



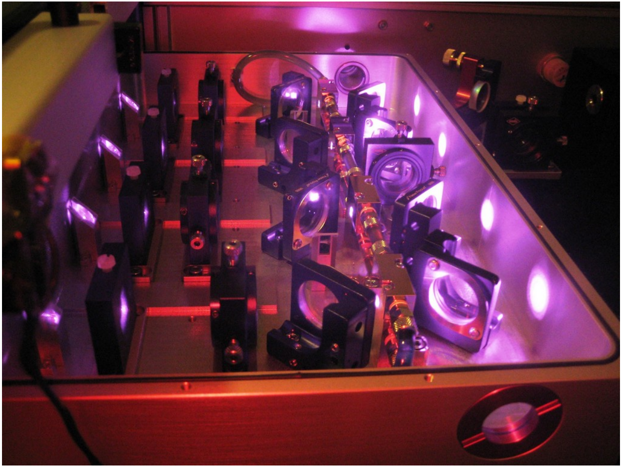
Quantum Noise

The Advanced LIGO Noise Curve

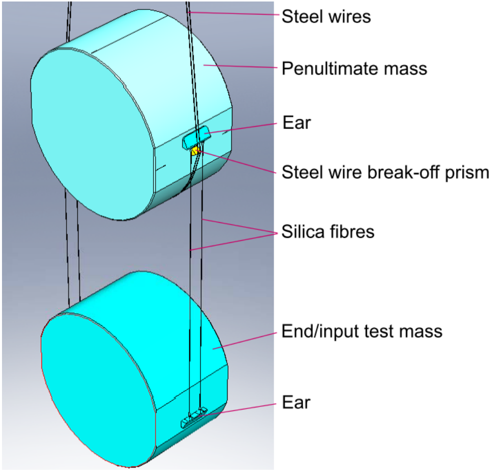


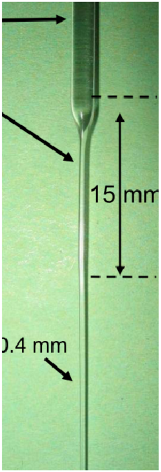
Quantum noise is a limiting noise source above $\sim$ 10 Hz.

Laser Physics

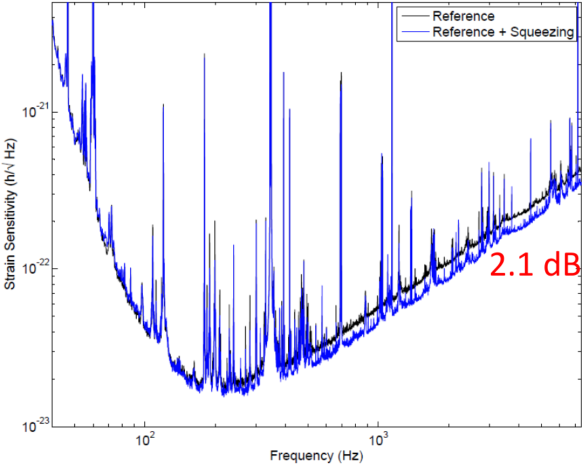


Materials Science[2]



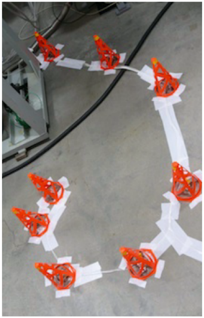


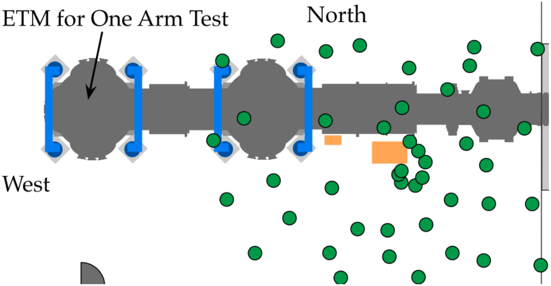
Squeezed States[5]



The Science of LIGO

Seismology[6]





References

Adhikari, Rana. “Sensitivity and Noise Analysis of 4 km Laser Interferometric Gravitational Wave Antennae.” Ph.D. Thesis Submitted to the Massachusetts Institute of Technology in July of 2004.

Hammond, G. D. et. al. “Thermal Noise, Suspensions, and New Materials.” *LIGO-P1100161-v3*. 1 December, 2011.

Kimble, H. J. et. al. “Conversion of Conventional Gravitational-Wave Interferometers into Quantum Nondemolition Interferometers by Modifying their Input and/or Output Optics.” Phys. Rev. D. **65**, 022002 .

Corbitt, Thomas R. “Quantum Noise and Radiation Pressure Effects in High Power Optical Interferometers.” Thesis Submitted to the Department of Physics at the Massachusetts Institute of Technology. August 2008.

Dwyer, Sheila. “Enhanced LIGO with Squeezing: Lessons Learned for Advanced LIGO and Beyond.” *LIGO-G1200637-v1*.

Driggers, Jenne. “Measured Newtonian Noise: Implications for Advanced Detectors.” *LIGO-G1200540-v1*.

Hammond, Giles. “aLIGO Monolithic Stage.” *LIGO-G1200209-v3*.