An Overview of Advanced LIGO Interferometry

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Summary

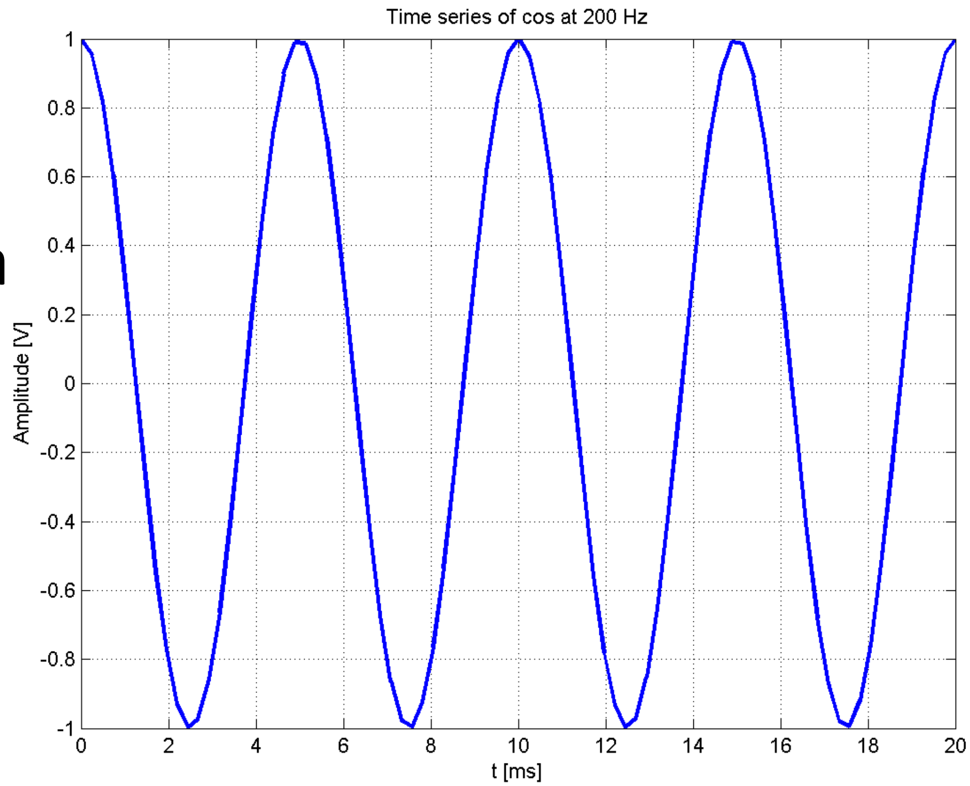
- Automatic Alignment and Wavefront sensors
  - The amount of first-order TEMs (01 or 10) provides alignment information
- Input Mode Cleaner
  - Suspended triangular cavity
  - Spatially filters incoming laser beam (non-TEM00 modes rejected)
  - Provides additional frequency noise and beam jitter suppression
- Output Mode Cleaner
  - Four-mirror bow tie configuration
  - Sidebands are rejected along with non-TEM00 modes
- Thermal Compensation System (TCS)
  - Compensates for thermal induced deformations ($\sim 800\ kW$ stored in arms)
  - Optimizes IFO coupling to TEM00 (light that carries GW information)

Noise budgeting

- Amplitude Spectral Density and Power Spectral Density
- Linear system can be described in terms of a TF
- TF poles dictate time-response of system
- Control System
  - Manages and regulates a set of variables in a system
  - A quantity is measured then controlled
- General stability criteria
- Noise propagation throughout control system
- eLIGO noise budget sample

Power Spectral Density (PSD)

- Need to work in frequency space
- PSD: a graphical representation to easily determine the power of a signal over a particular frequency band.
- Uses the `fft` algorithm

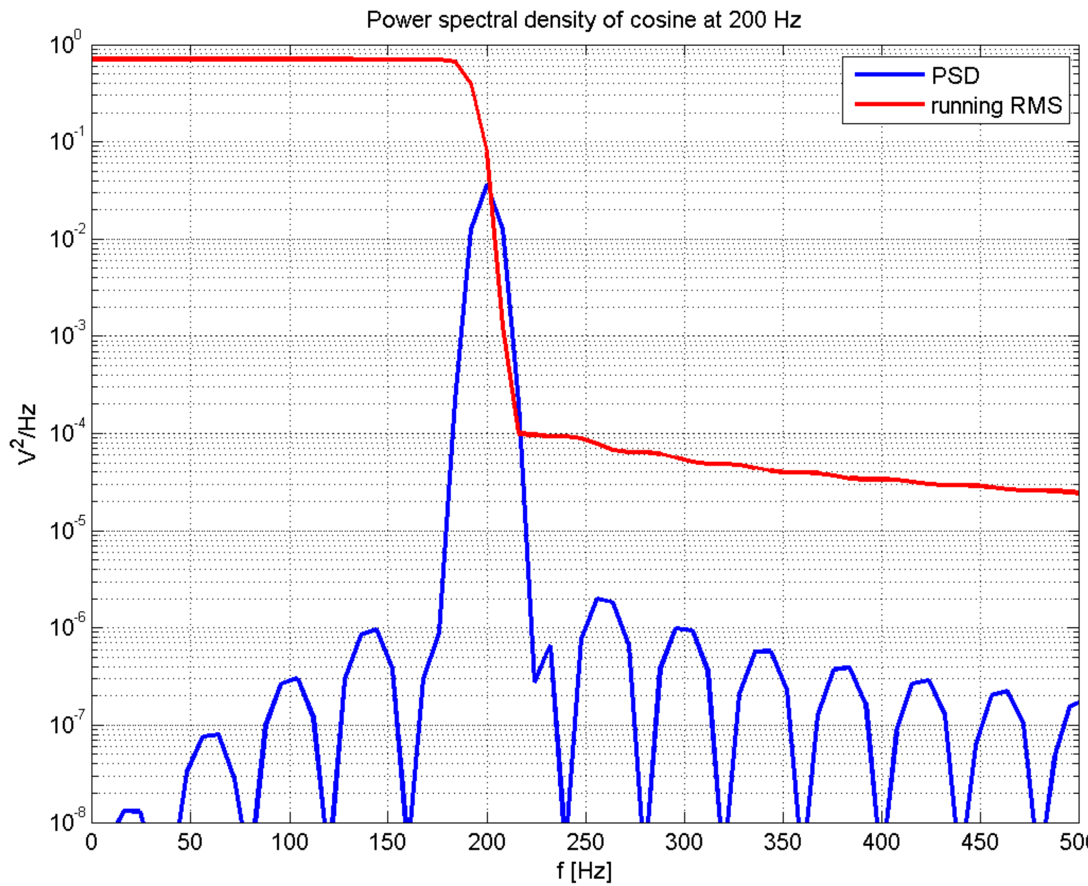


spectraldensity_example.m

Power Spectral Density (PSD)

- In this example, power is computed using
  - `w=hamming(length(x))`
  - `[Pxx,f]=periodogram(x,wi,'onesided',NFFT,Fs)`
- Data windowing
  - In the fft process, power in one frequency bin “leaks” to nearby bins.
  - Filter (with a window filter) the input data stream
- The (running) RMS computed using the PSD (and shown in red)

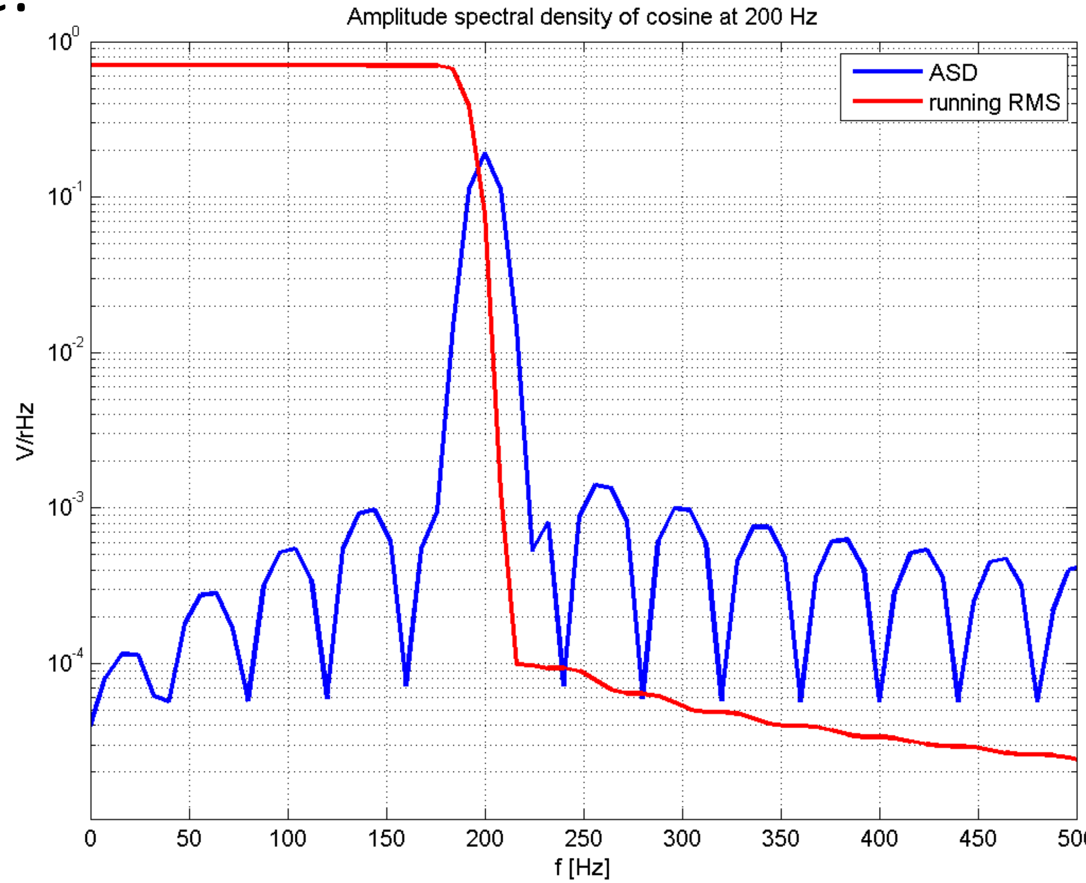
$$RMS = \sqrt{\sum P_{xx} \cdot \Delta f}$$



spectraldensity_example.m

Amplitude Spectral Density (ASD)

- Plotting the amplitude:
  - simply the square root of the power spectral density $\sqrt{P_{xx}}$

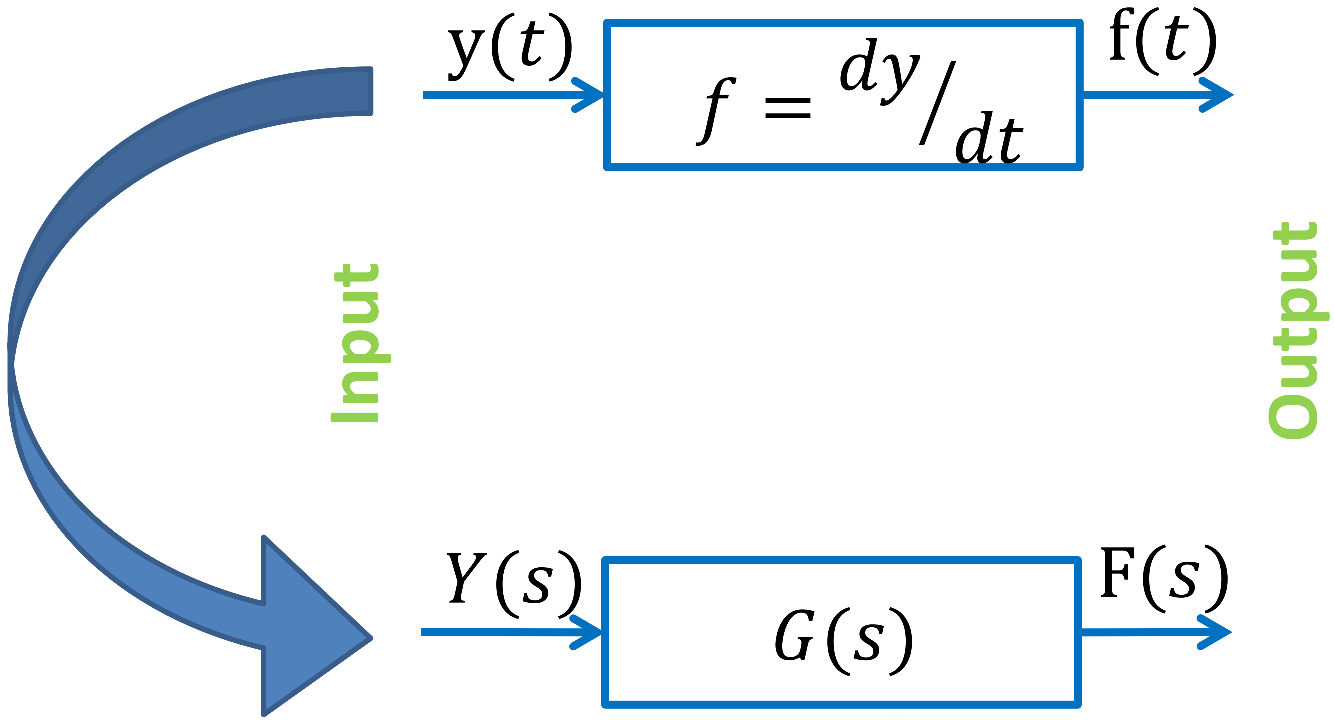


spectraldensity_example.m

Noise budget

- Need to measure the Amplitude Spectral Density of various noise terms
- Project them onto the sensitivity curve

Time domain ↔ Laplace domain



Transform variable $s = j\,\omega$
(complex frequency)

A linear system can be represented as a Transfer Function

Transfer function, poles and zeros

- Convenient to express G(s) in terms of its poles and zeros:
  - Roots of the numerator (zeros) and denominator (poles)

$$G(s) = \frac{Q(s)}{P(s)}$$

$$= k \cdot \frac{(s - z_1) \cdot (s - z_2) \ldots (s - z_m)}{(s - p_1) \cdot (s - p_2) \ldots (s - p_n)}$$

  - where *k* is the gain of the transfer function

Summary of pole characteristics

- ***Real distinct poles (often negative)***

$$\frac{c_i}{s - p_i} \quad \leftrightarrow \quad c_i\, e^{p_i t}$$

- ***Real poles, repeated m times (often negative)***

$$\left[\frac{c_{i,1}}{s - p_{i,1}} + \frac{c_{i,2}}{\left(s - p_{i,2}\right)^2} + \cdots + \frac{c_{i,3}}{\left(s - p_{i,3}\right)^3} + \frac{c_{i,m}}{\left(s - p_{i,m}\right)^m}\right]$$

$$\updownarrow$$

$$\left[c_{i,1} + c_{i,2}t + \frac{1}{2!}c_{i,3}t^2 + \cdots + \frac{c_{i,m}}{(m-1)!}t^{m-1}\right] \cdot e^{p_i t}$$

Summary of pole characteristics

- ***Complex-conjugate poles***

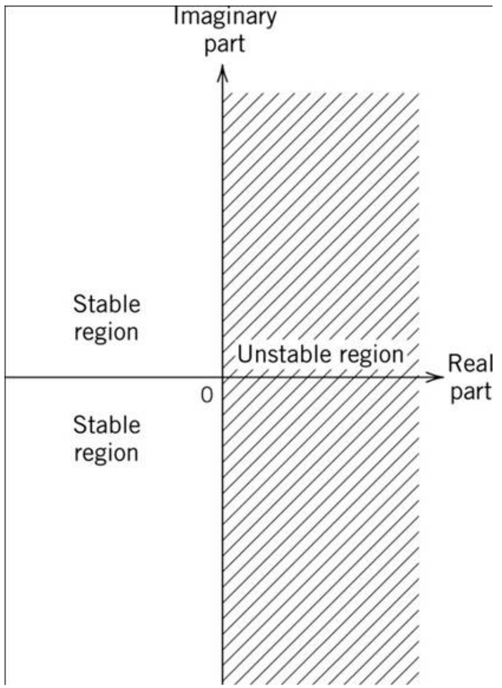
$$\frac{c_i}{s - p_i} + \frac{(c_i)^*}{s - (p_i)^*} \quad \leftrightarrow \quad c_i\, e^{p_i t} + (c_i)^* e^{(p_i)^* t}$$

often re-written as a second-order term

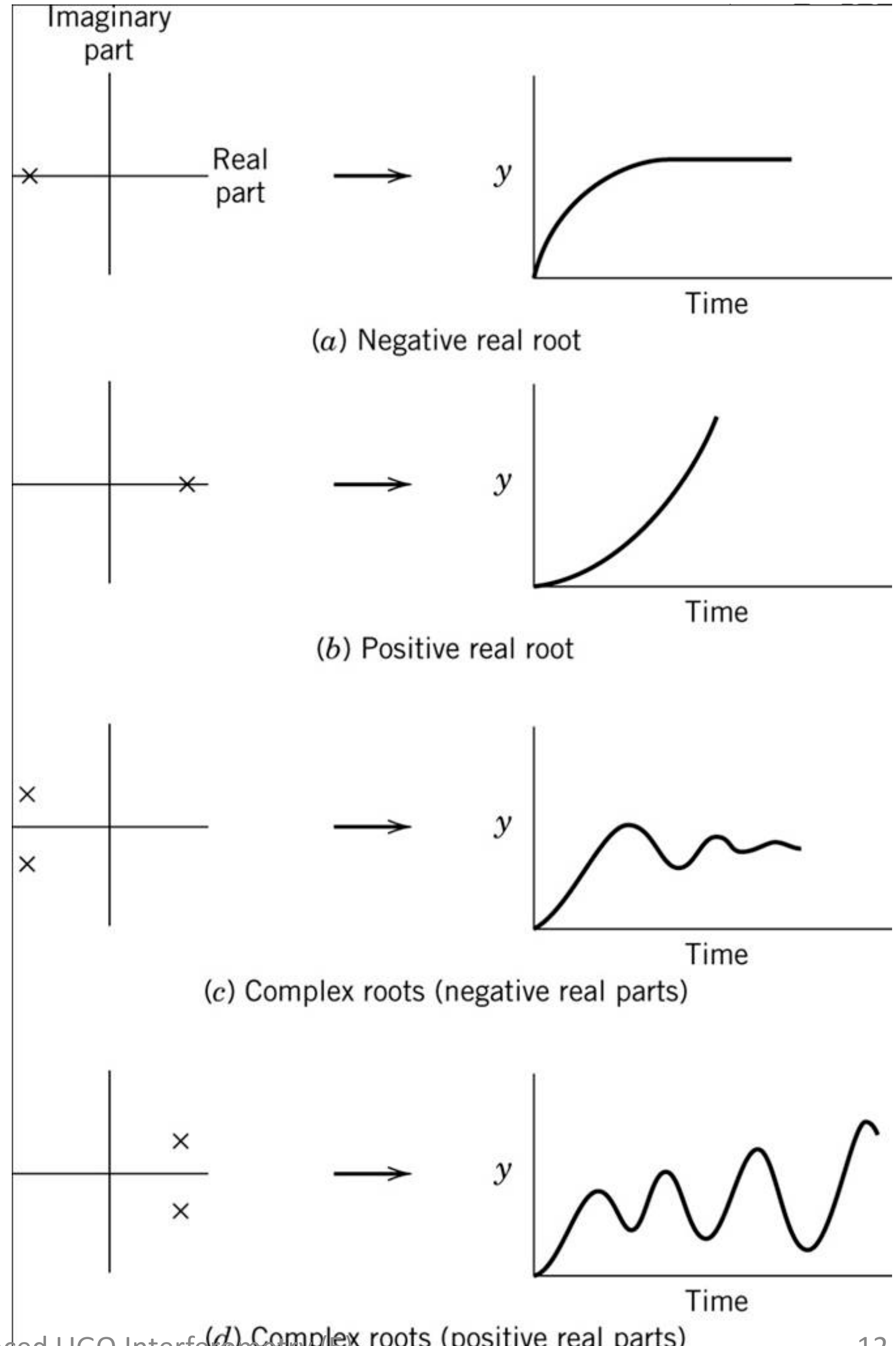
$$\frac{\omega^2}{s^2 + 2\delta\omega\, s + \omega^2} \quad \leftrightarrow \quad \sim e^{\alpha t} \cdot \sin(\beta t + \varphi)$$

- ***Poles on imaginary axis***
  – Sinusoid
  – Pole at zero: step function
- ***Poles with a positive real part***
  – Unstable time-domain solution

Time domain response



$$\frac{c_i}{s - p_i} \leftrightarrow c_i\, e^{p_i t}$$



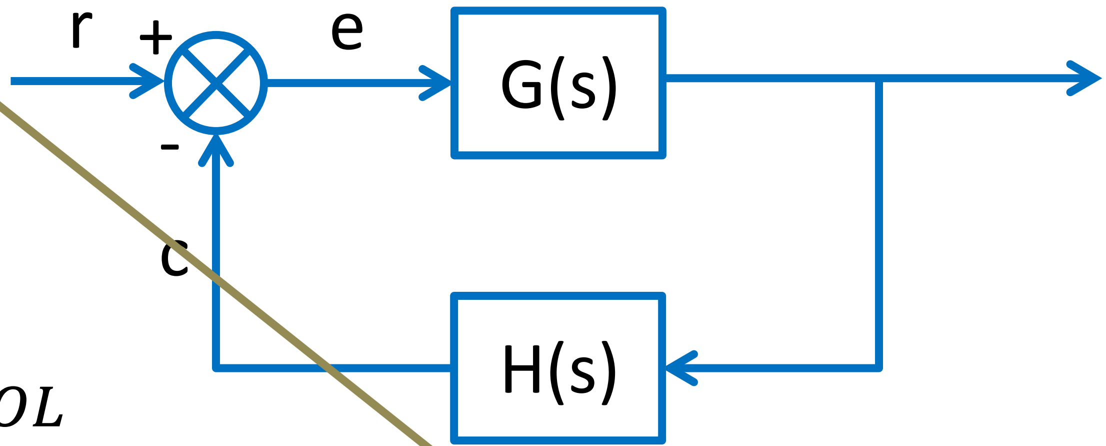
Comments

$$F(s) = \sum_i \frac{\alpha_i}{s + a_i} \Longleftrightarrow f(t) = \sum_{i=0}^{n} \alpha_i \, e^{-a_i t}$$

1. Poles of *F(s)* determine the time evolution of *f(t)*
2. Zeros of *F(s)* affect coefficients
3. Poles closer to origin → larger time constants

Negative feedback

$$e = \frac{1}{1 + GH} r$$



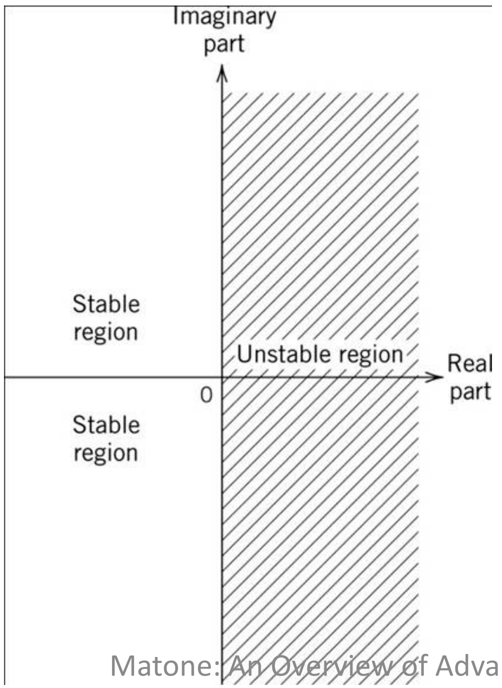
Open loop gain G_{OL}

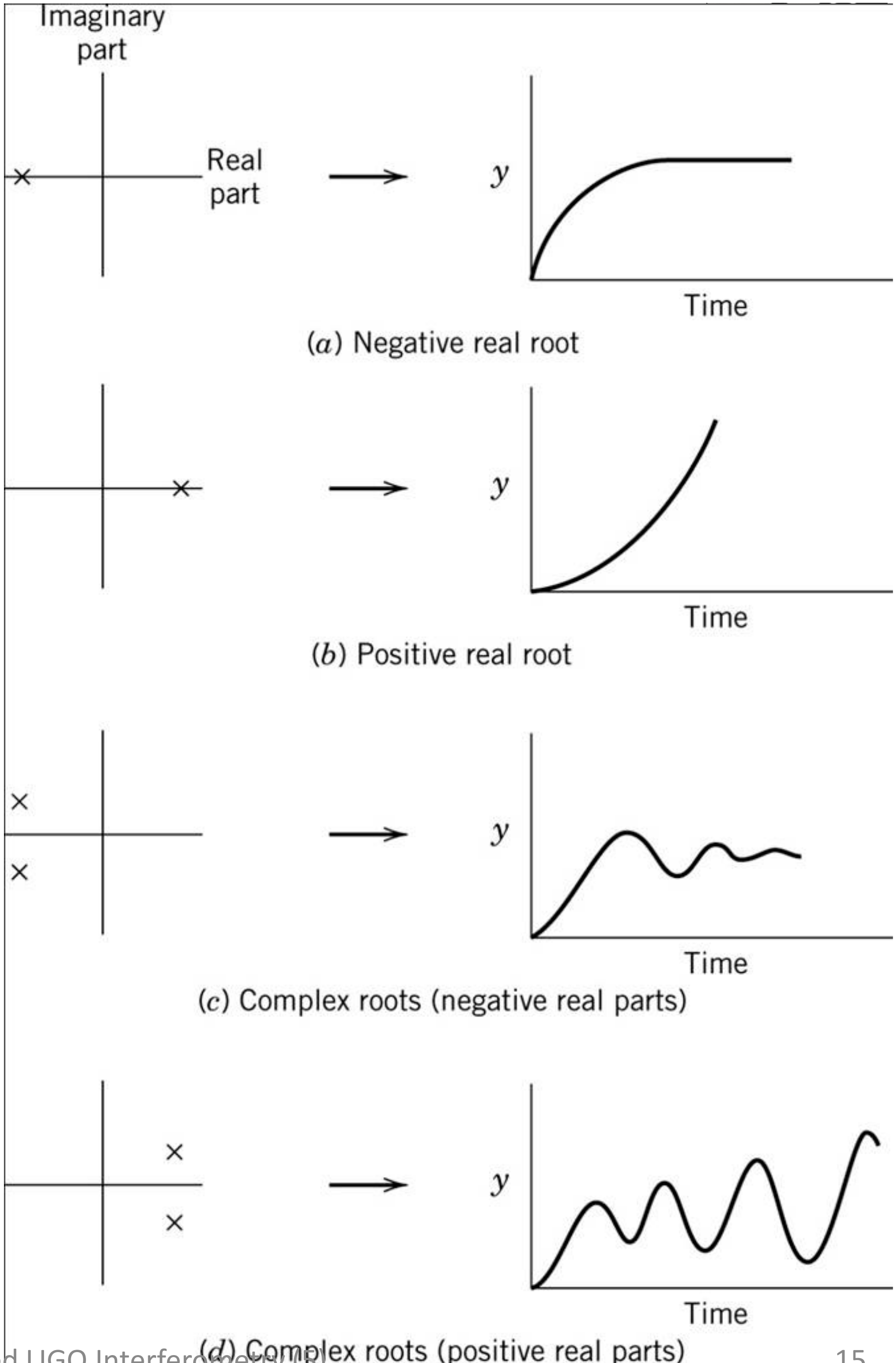
Closed loop gain G_{CL}

Stability: the poles' real part of G_{CL} must be negative

General Stability Criterion

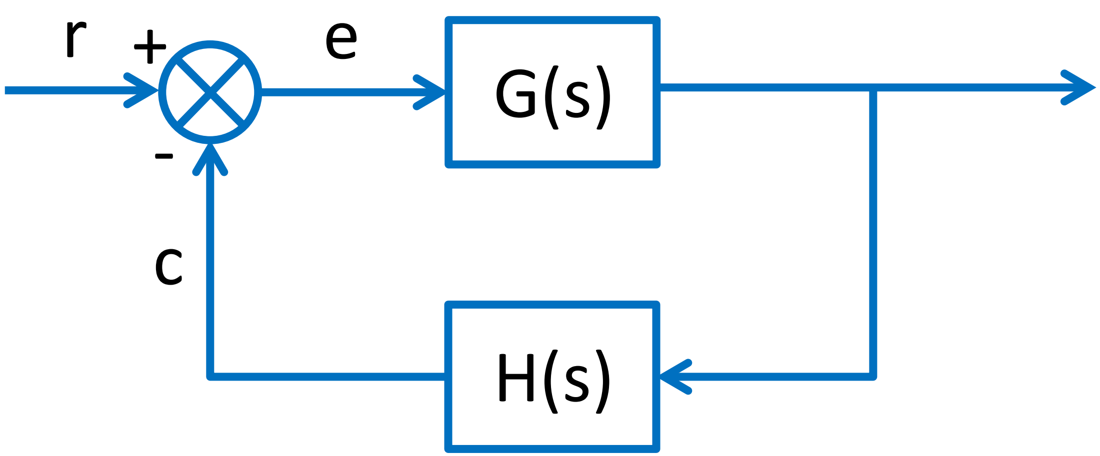
The feedback control system is <u>stable</u> if and only if all the *poles of the closed loop transfer function* G_{CL} have a negative real part. Otherwise the system is unstable.





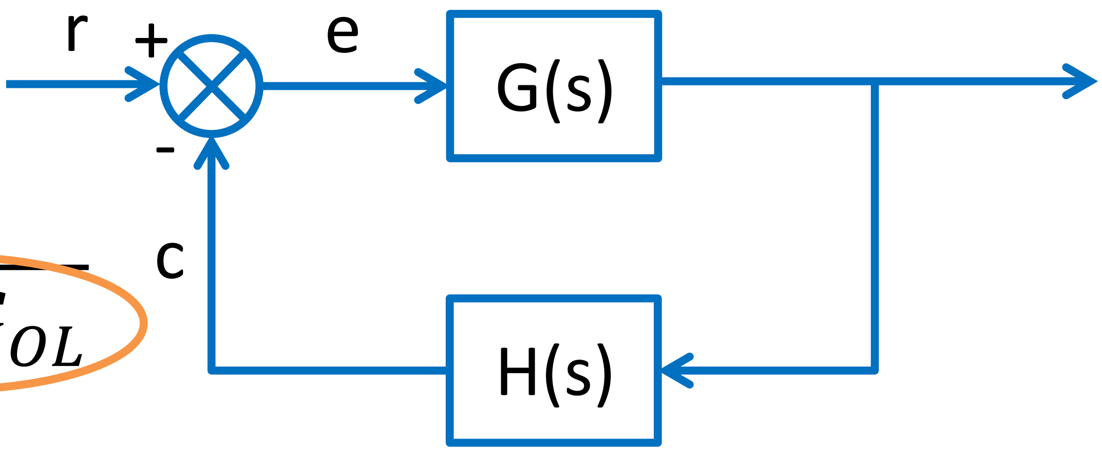
Loop stability and design

- If the system is unstable,
  - We can't change $G(s)$ but
  - We can design a different controller H so as to make the system stable
- But how should we change H? Let's look closely at the root of the problem



The problem

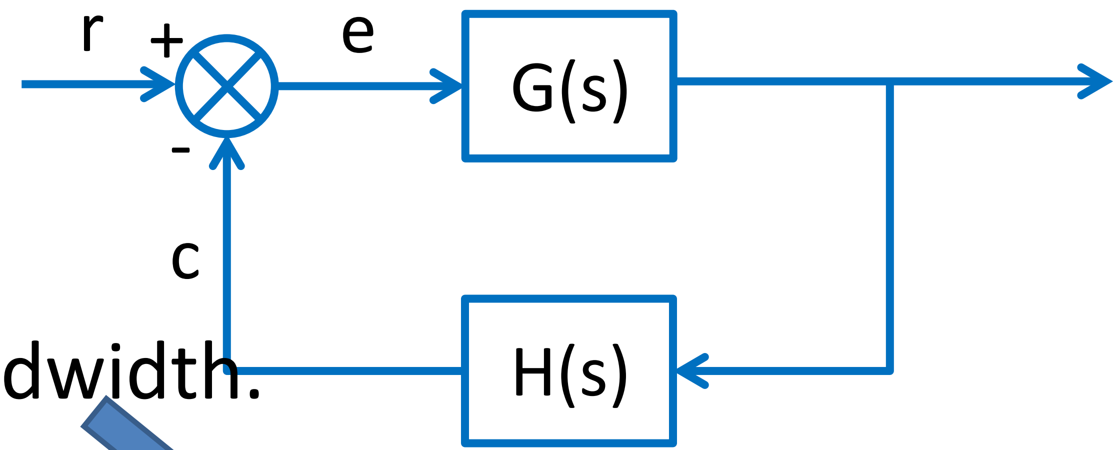
$$\frac{e}{r} = \frac{1}{1+GH} = \frac{1}{1+G_{OL}}$$



If G_{OL} becomes -1 then system is unstable

The general shape of G_{OL}

$$\frac{e}{r} = \frac{1}{1 + G_{OL}}$$



G_{OL} has a limited bandwidth.

Within bandwidth:

$$G_{OL} \gg 1$$

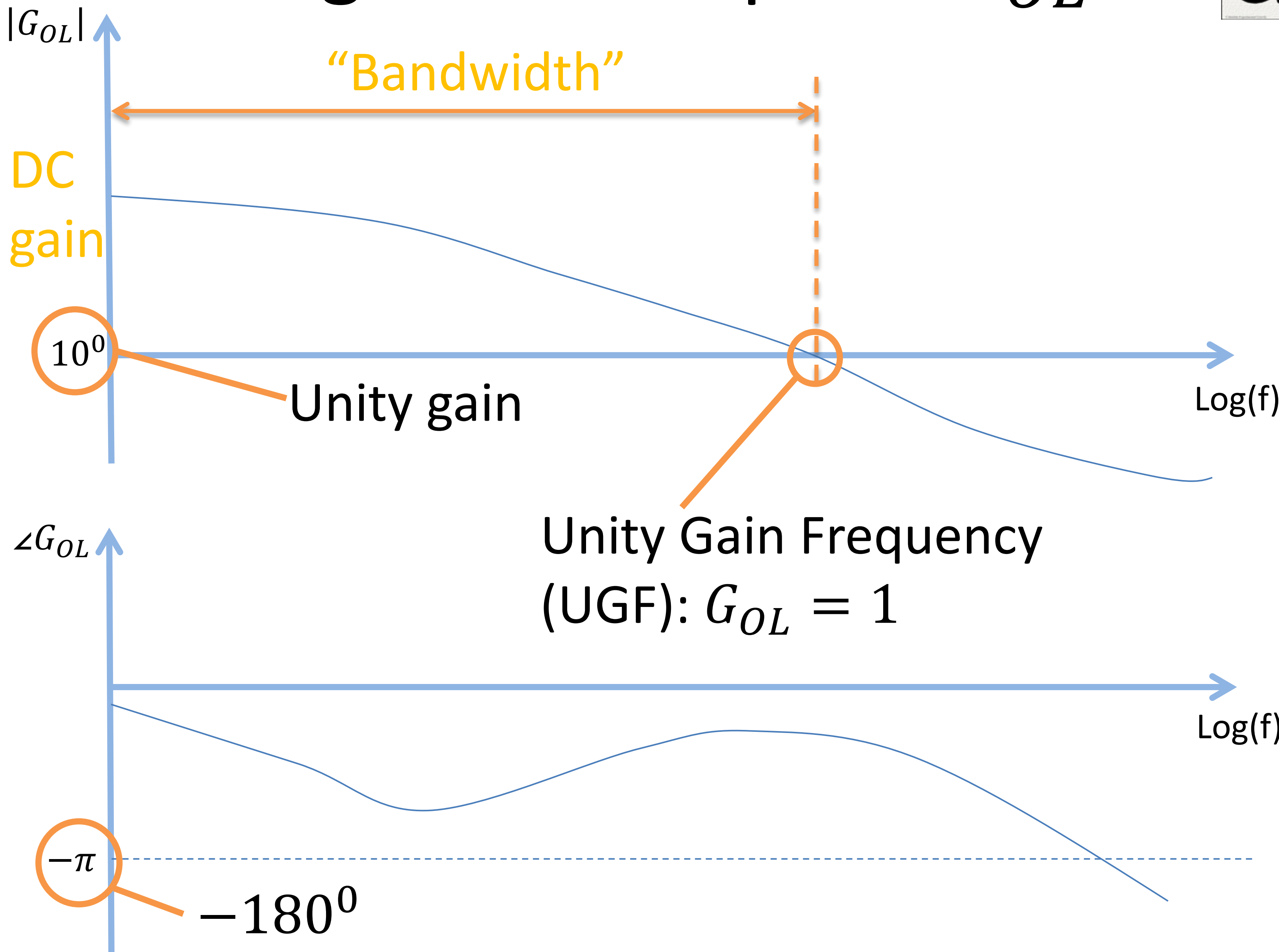
$$\frac{e}{r} \cong 0$$

Outside bandwidth:

$$G_{OL} \ll 1$$

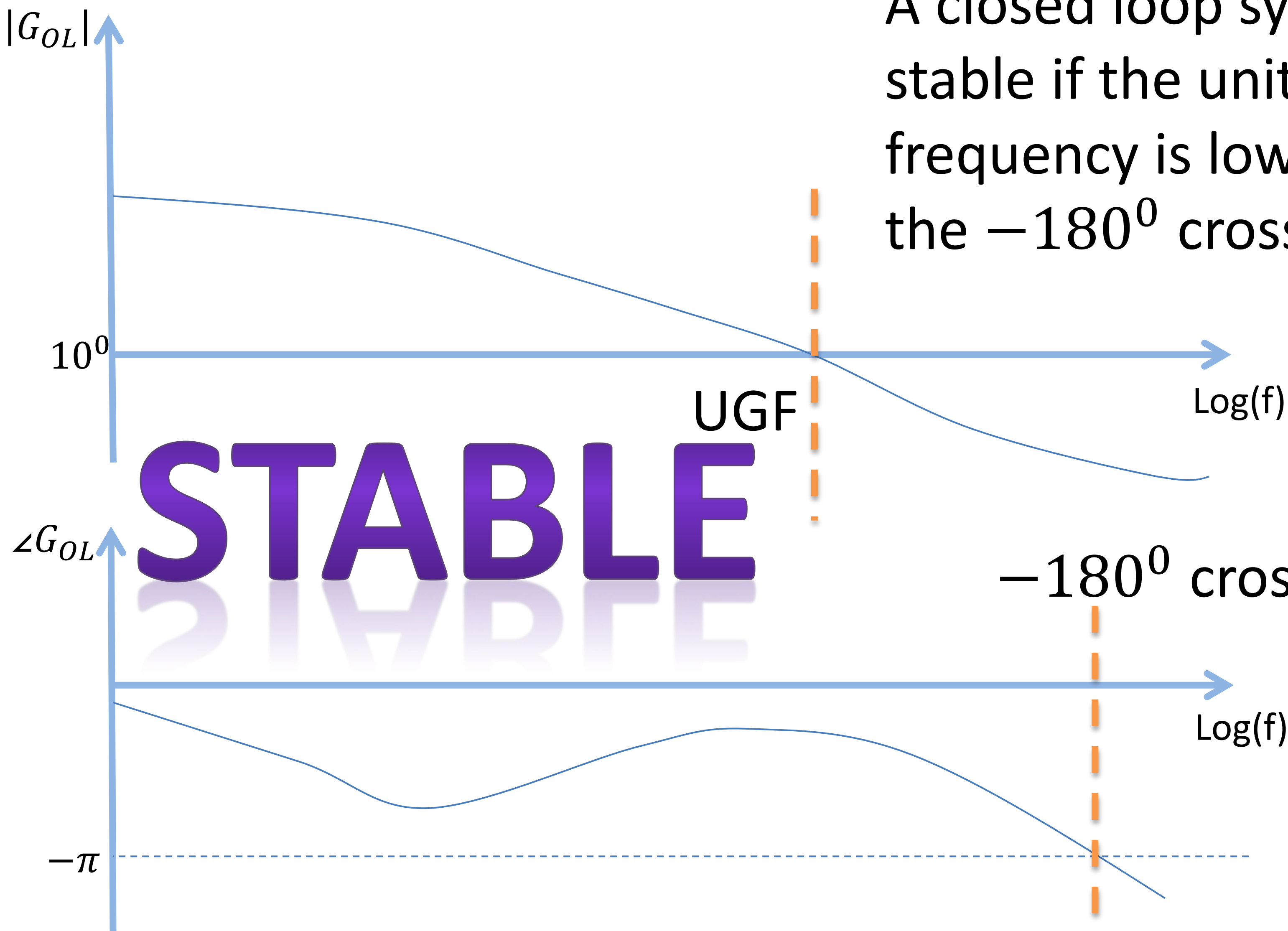
$$\frac{e}{r} \cong 1$$

The general shape of G_{OL}



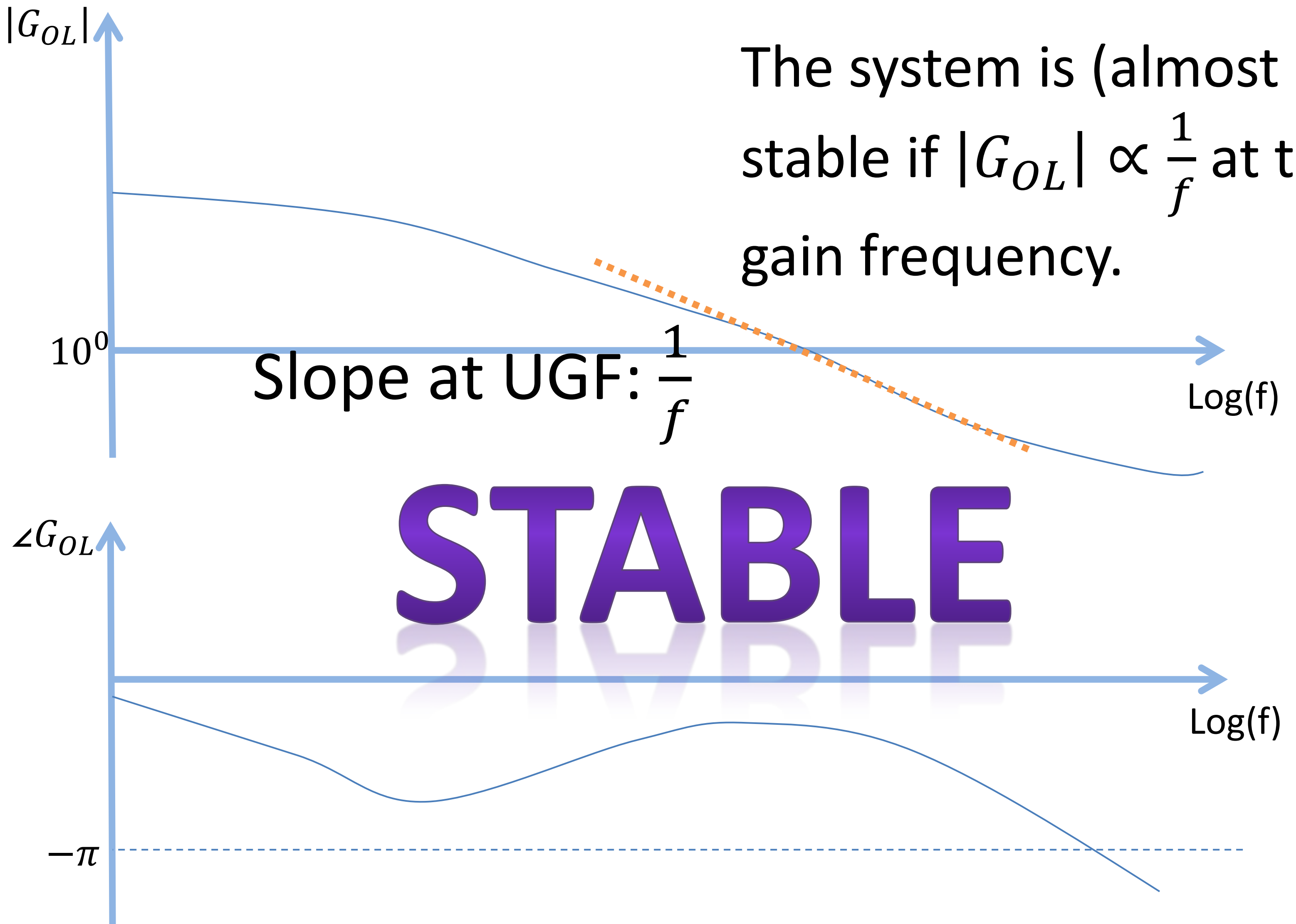
Stability Criteria

A closed loop system is stable if the unity gain frequency is lower than the -180^0 crossing.

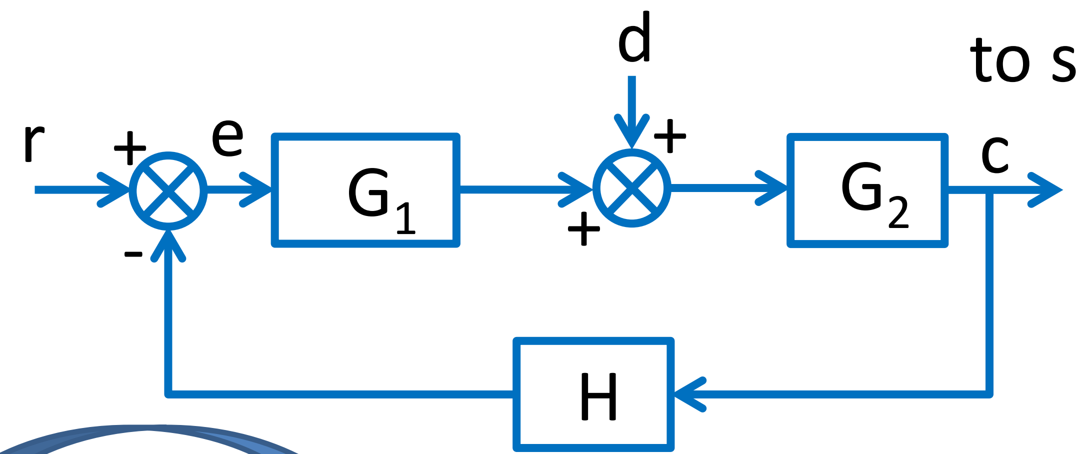


Stability Criteria: Rule of Thumb

The system is (almost always) stable if $|G_{OL}| \propto \frac{1}{f}$ at the unity gain frequency.



Performance to noise input d: with feedback



Noise contribution to signal c

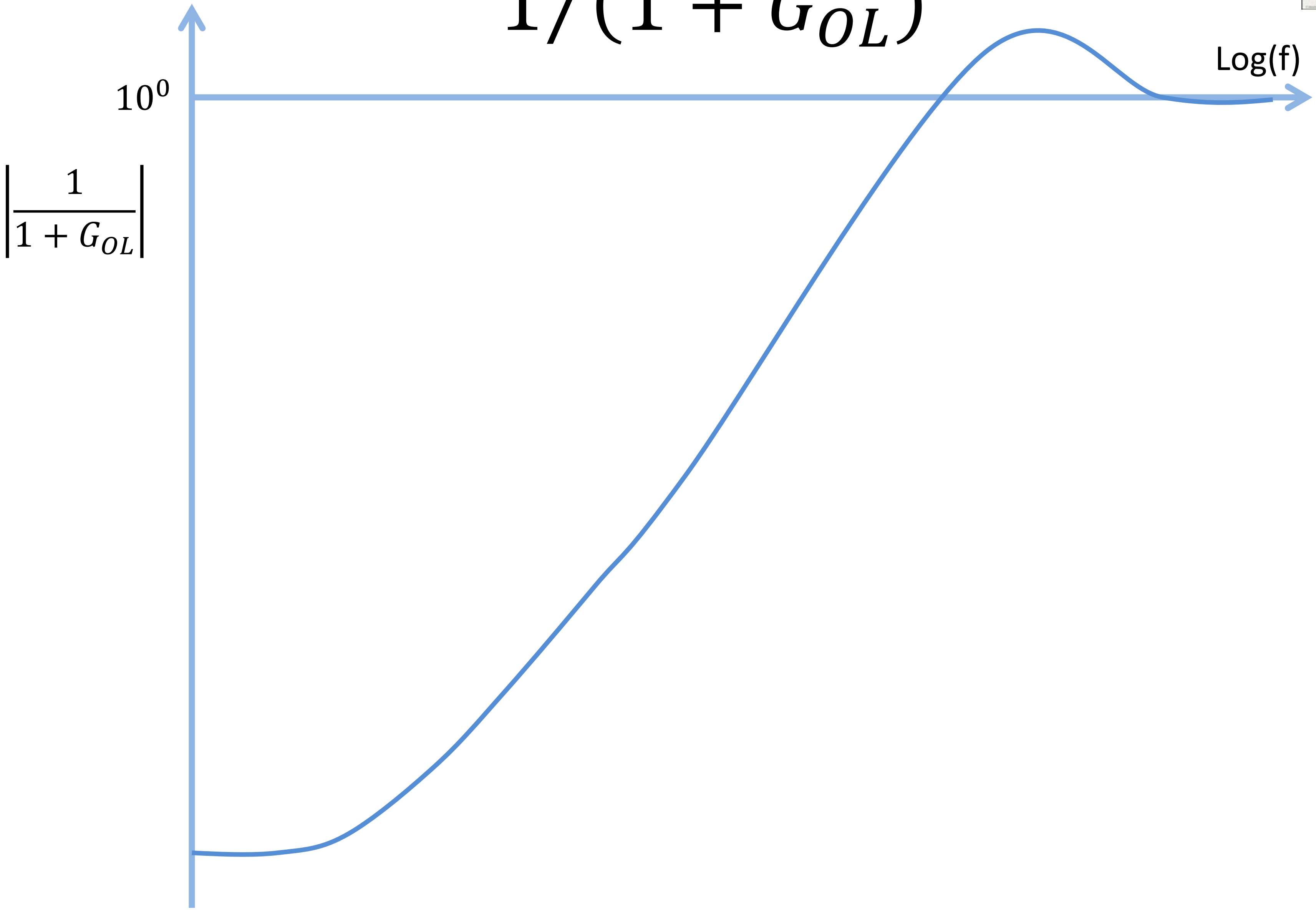
Controlled signal

$$c = \left(\frac{G_1 G_2}{1 + G_{OL}}\right) r + \left(\frac{G_2}{1 + G_{OL}}\right) d$$

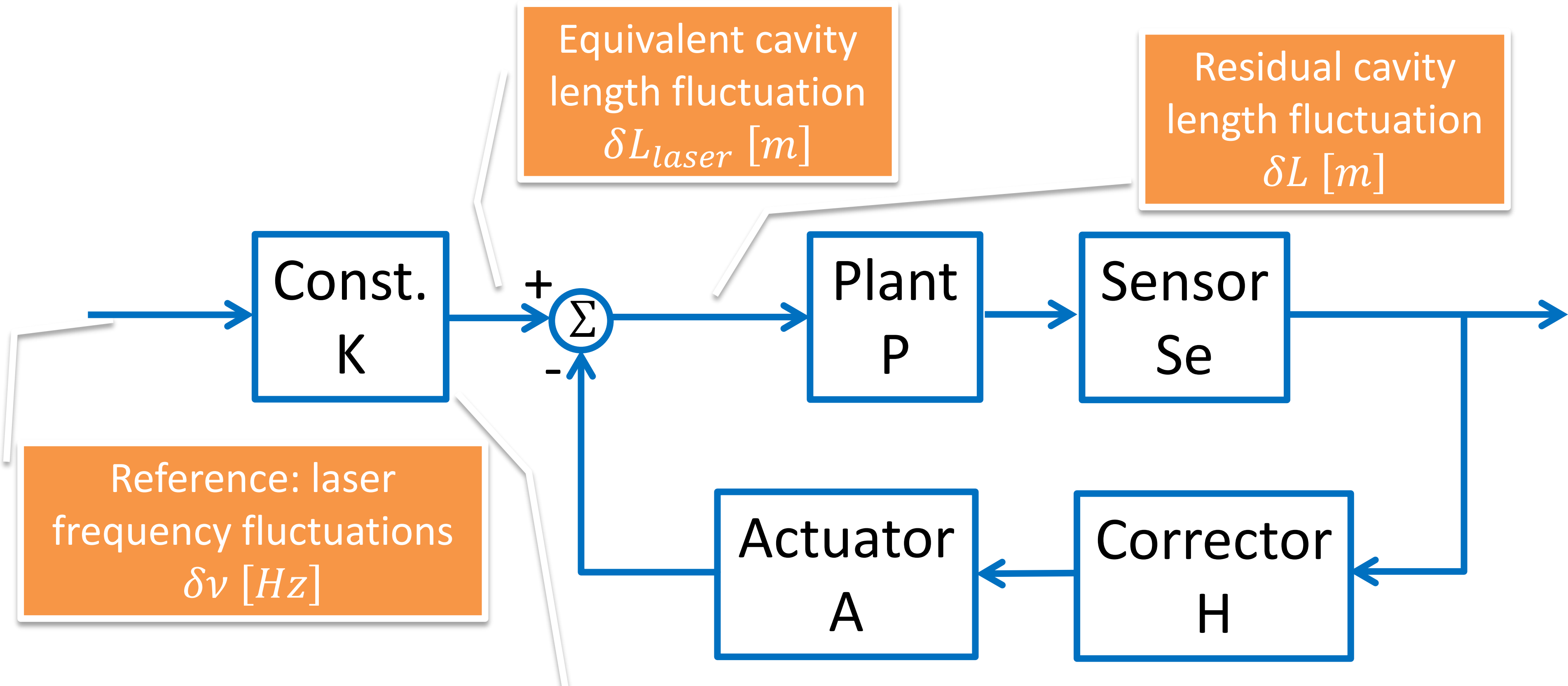
$$e = \left(\frac{1}{1 + G_{OL}}\right) r + \left(\frac{H G_2}{1 + G_{OL}}\right) d$$

Suppression factor

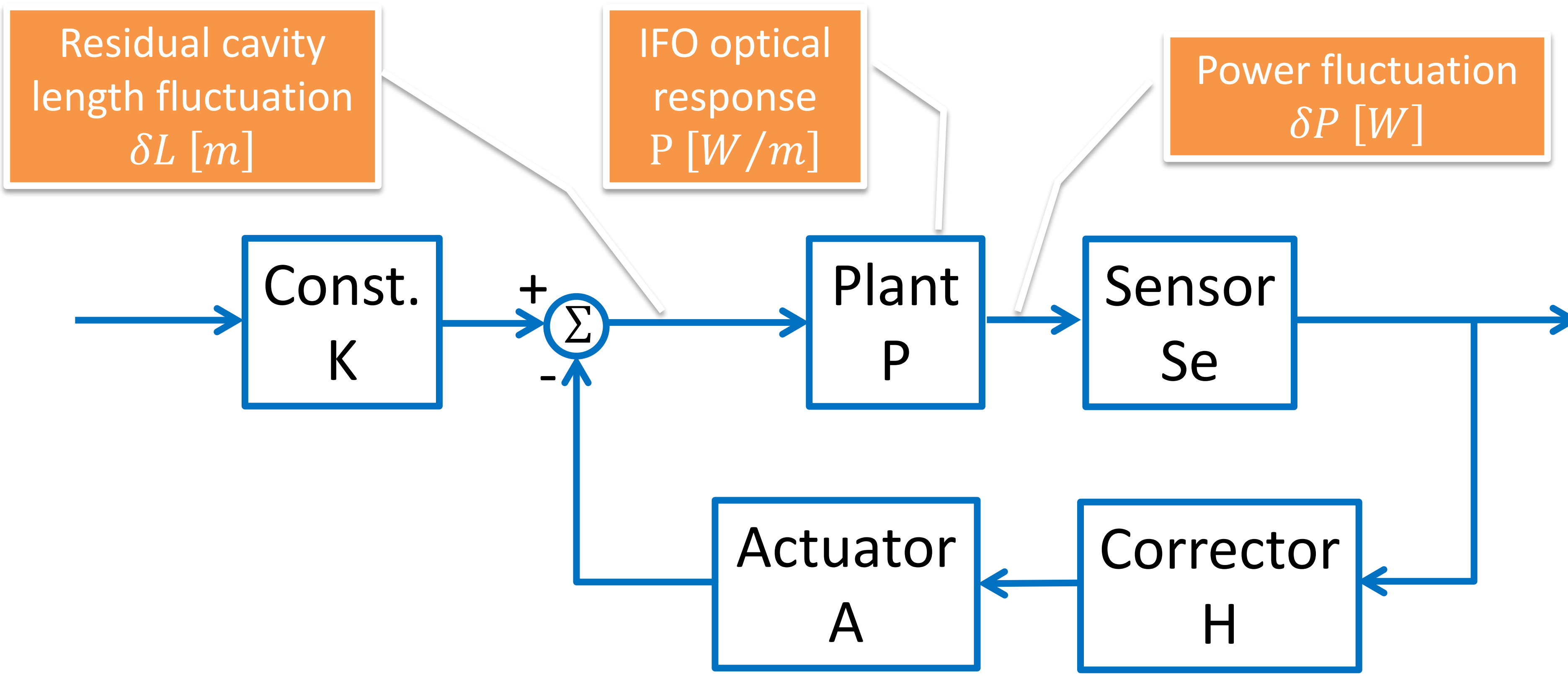
The general shape of $1/(1+G_{OL})$

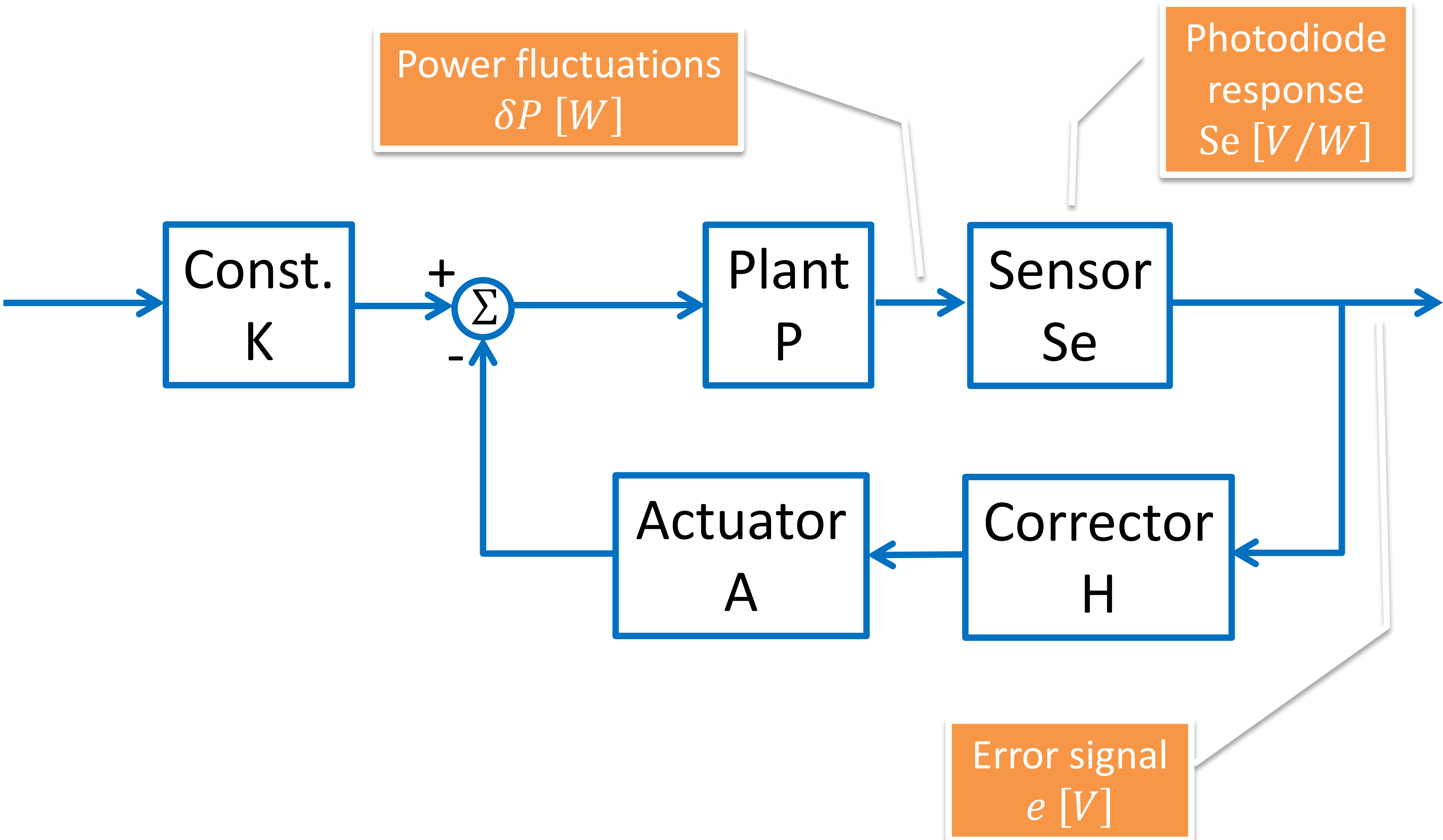


Toy example: Locking FP arm to laser frequency

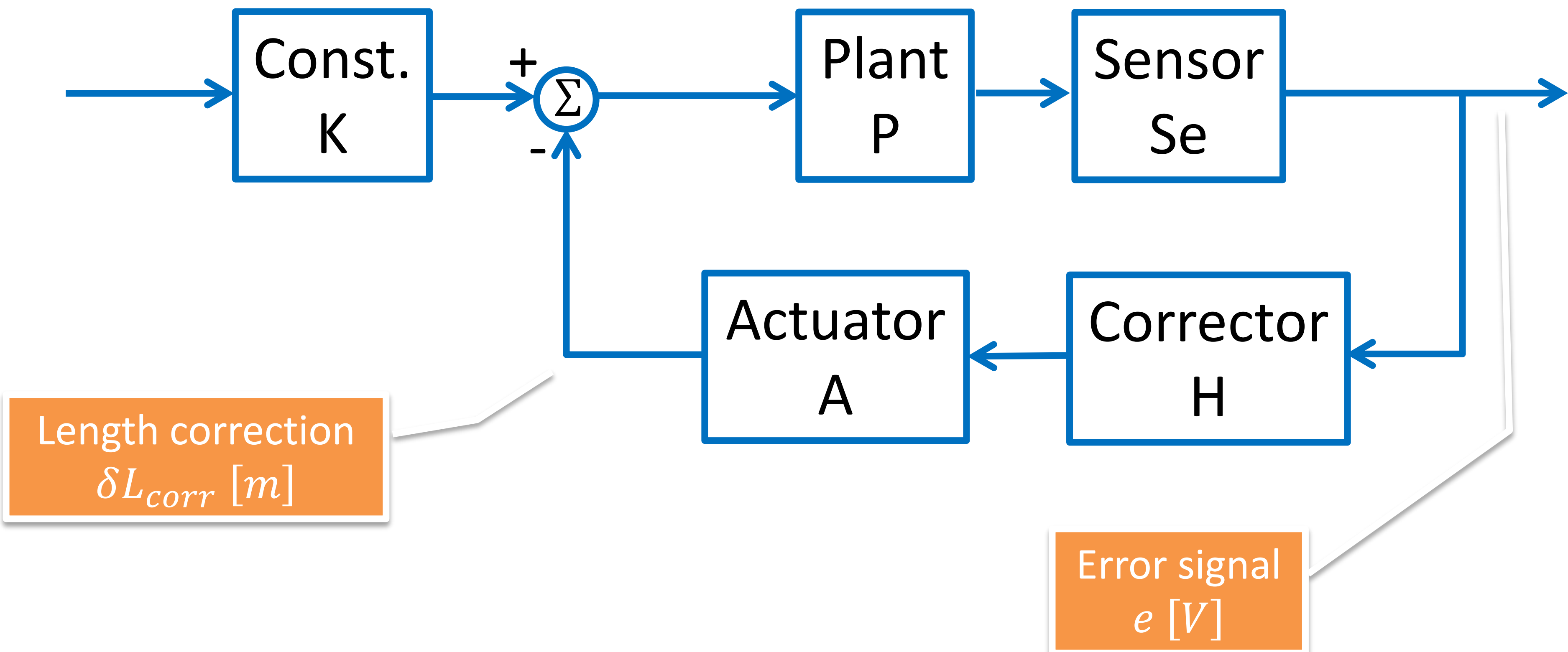


$$\frac{\delta L}{L} = \frac{\delta \nu}{\nu} \Rightarrow \delta L = \frac{L}{\nu}\delta \nu \Rightarrow \boldsymbol{K} = \frac{\boldsymbol{L}}{\boldsymbol{\nu}}$$

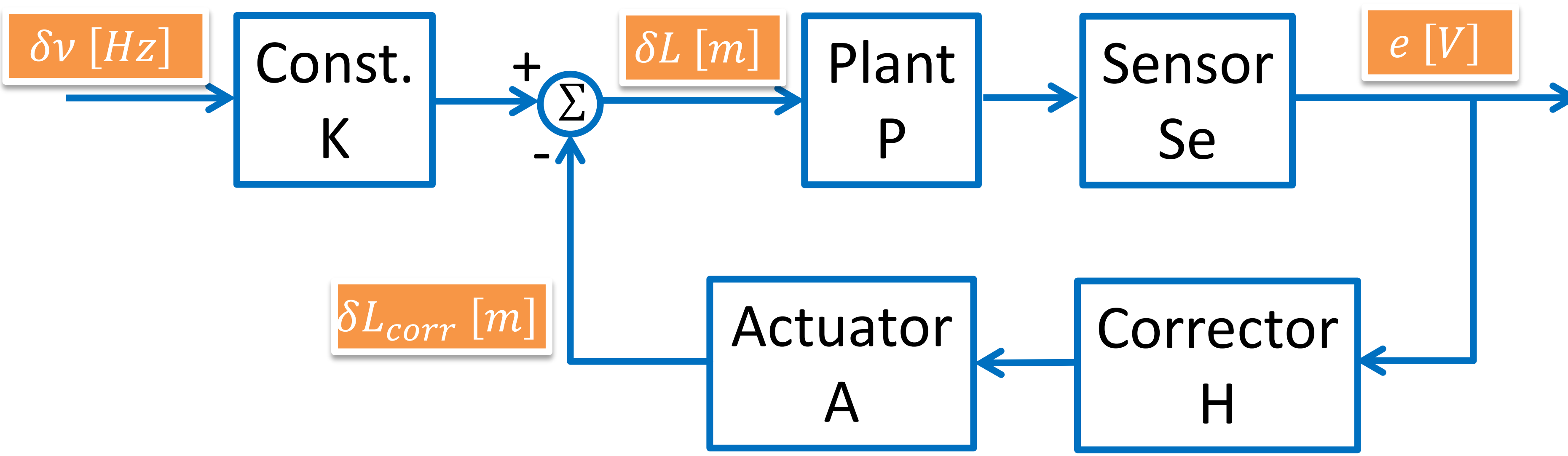

Residual cavity length fluctuation $\delta L\ [m]$
IFO optical response $P\ [W/m]$
Power fluctuation $\delta P\ [W]$
Const. K
+
-
Σ
Plant P
Sensor Se
Actuator A
Corrector H



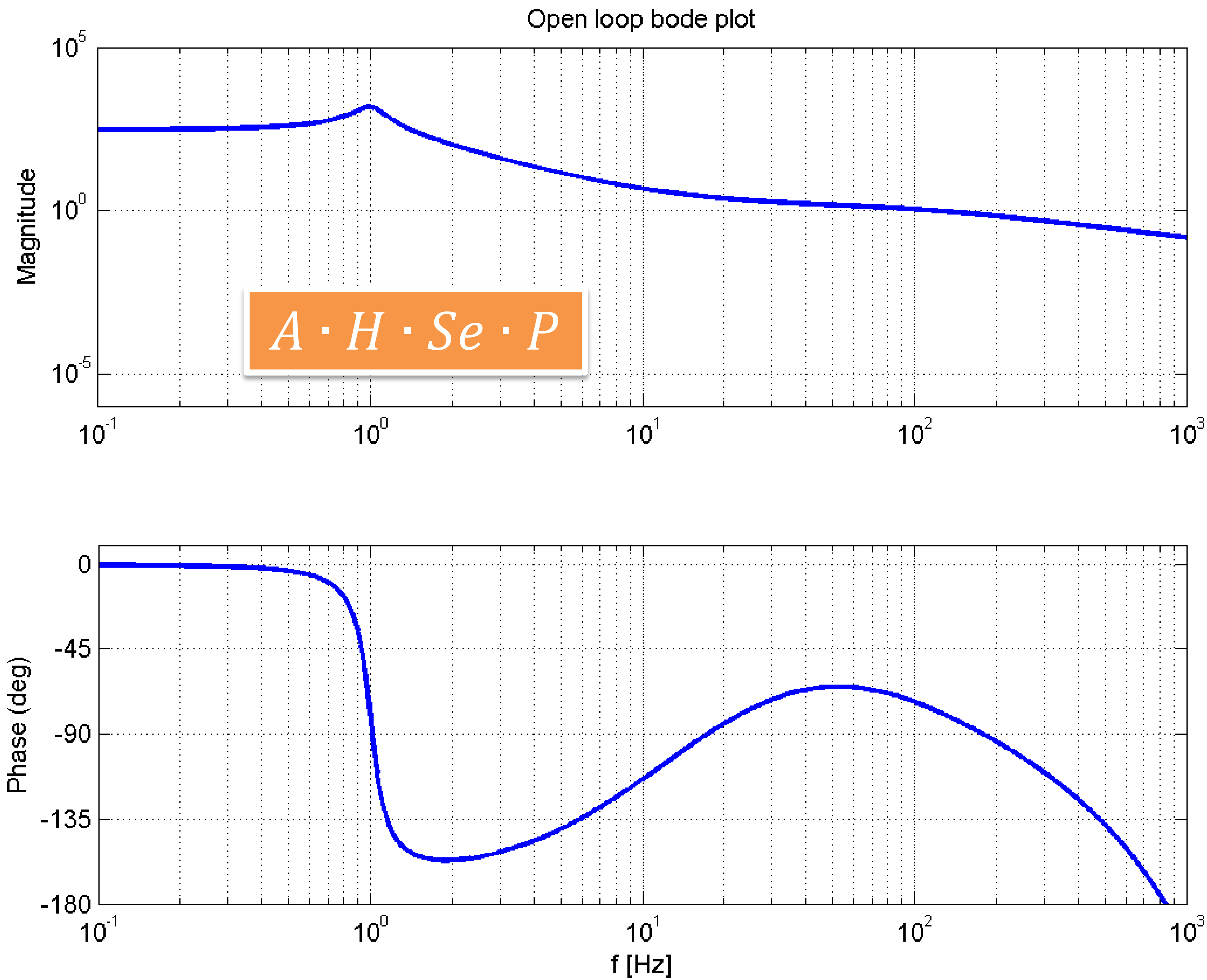
Power fluctuations
$\delta P\ [W]$
Photodiode response
Se $[V/W]$
Const.
K
+
-
Σ
Plant
P
Sensor
Se
Actuator
A
Corrector
H
Error signal
$e\ [V]$


Const.
K
+
Σ
-
Plant
P
Sensor
Se
Actuator
A
Corrector
H
Length correction
δL_{corr} [m]
Error signal
e [V]

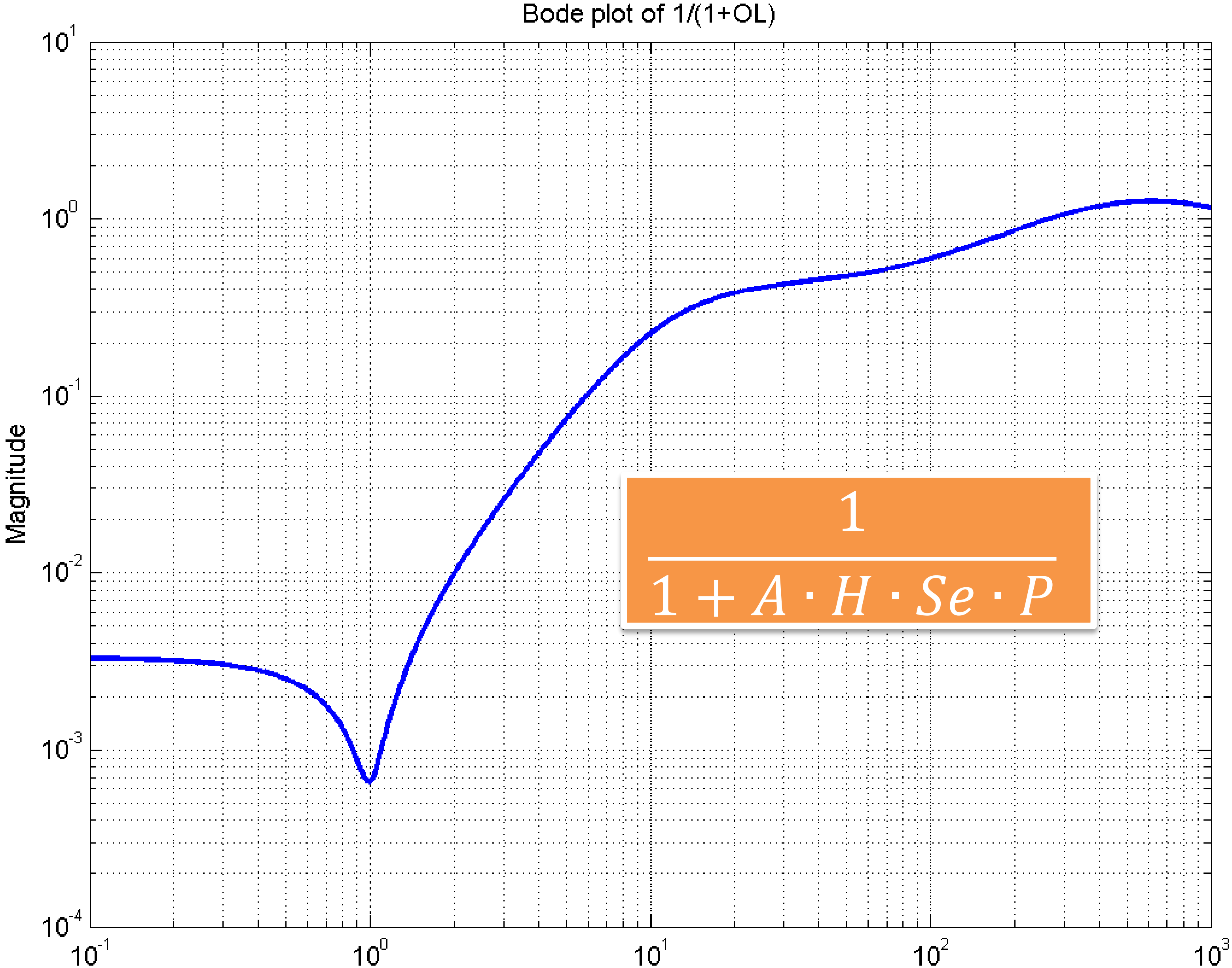
Toy example:
Locking FP arm to laser frequency



$$\delta L = \frac{1}{1 + A \cdot H \cdot Se \cdot P} K\, \delta\nu$$



Open loop bode plot
Magnitude
10^{5}
10^{0}
10^{-5}
$A \cdot H \cdot Se \cdot P$
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Phase (deg)
0
-45
-90
-135
-180
f [Hz]



$$\frac{1}{1 + A \cdot H \cdot Se \cdot P}$$

Noise budget

Project noise contribution

Amplitude Spectral Density [m/rHz]

Measure noise contribution

Amplitude Spectral Density [Hz/rHz]

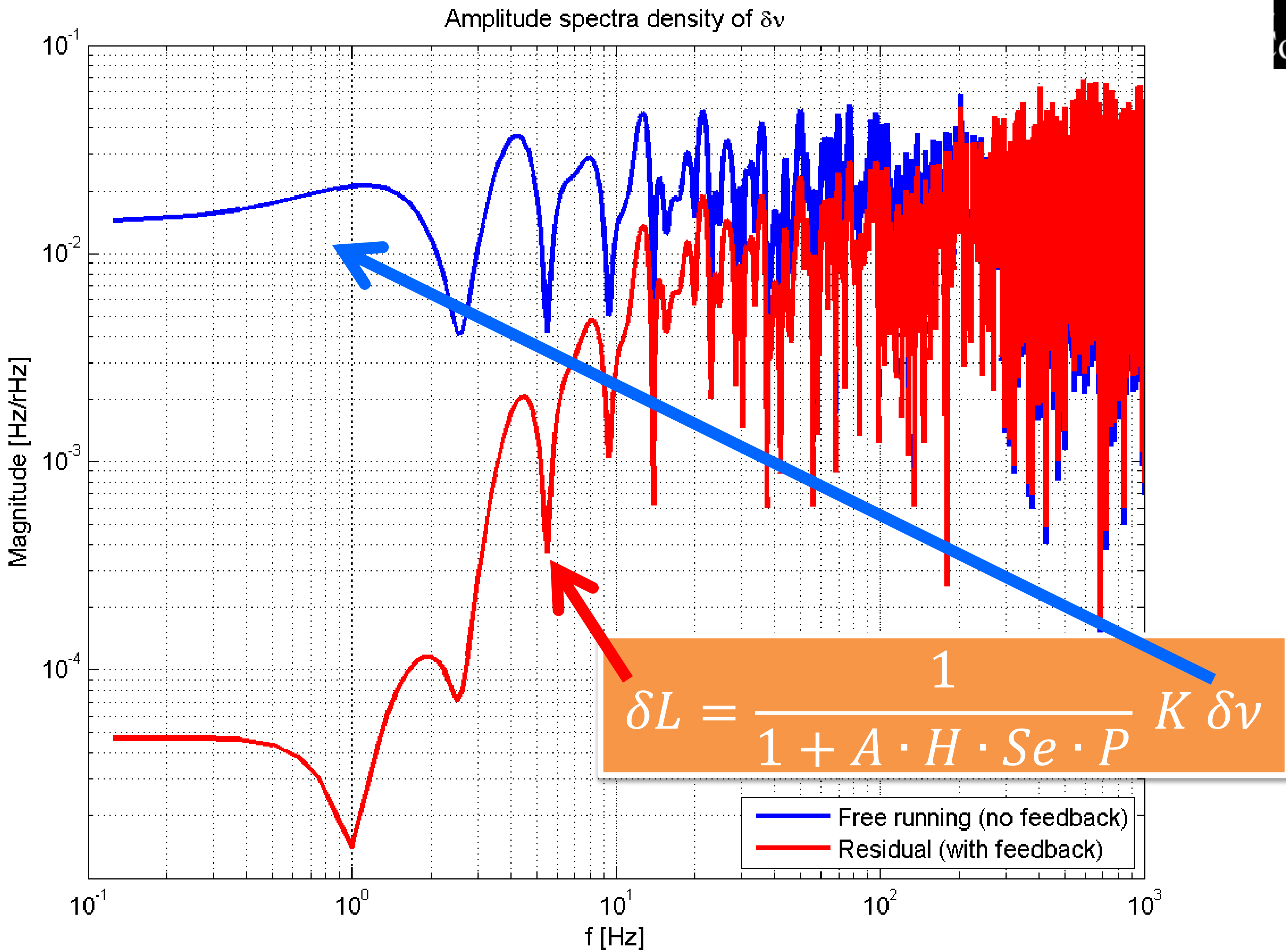
BUDGET

$$\delta L = TF \cdot \delta\nu$$

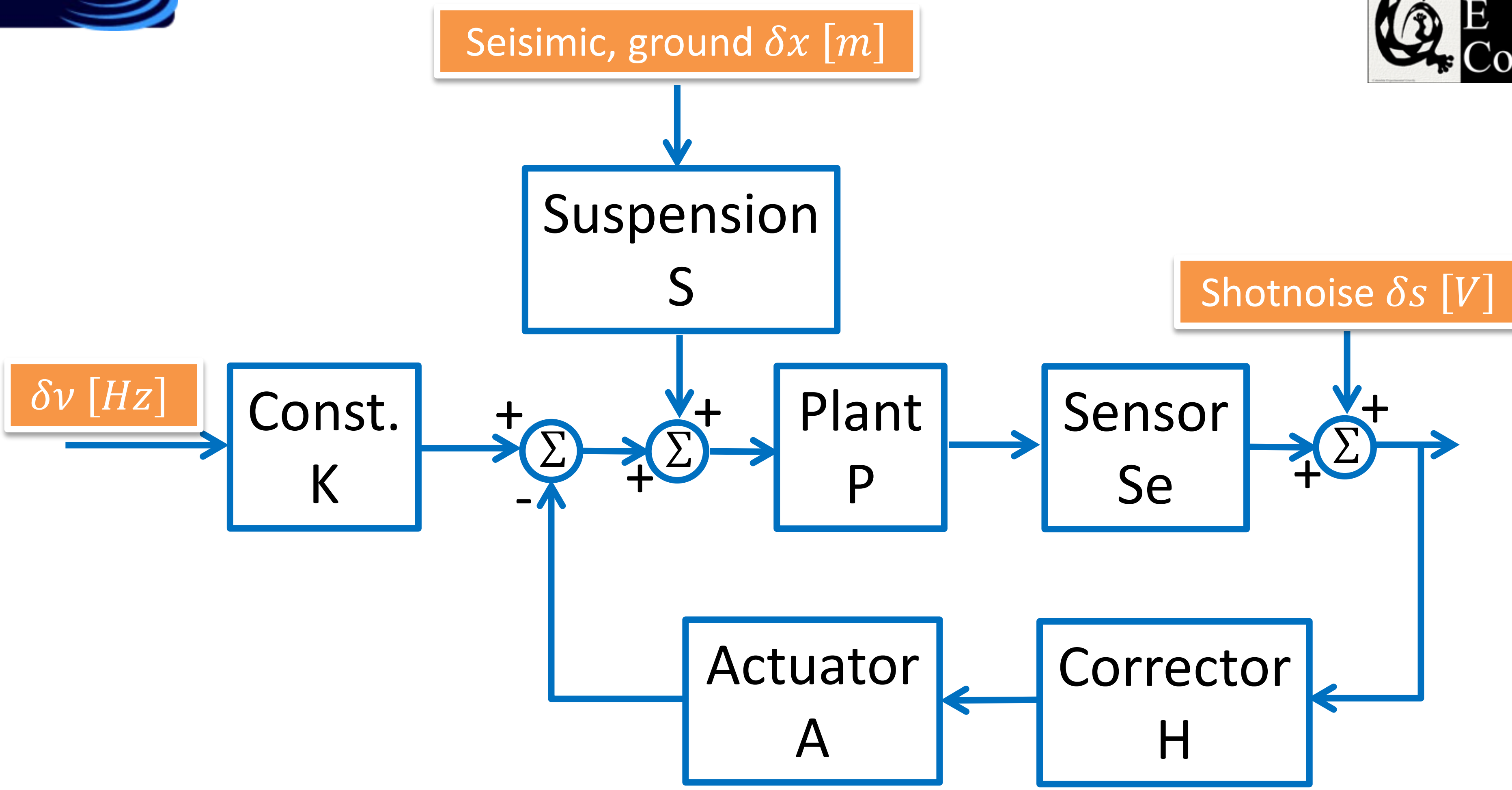
Measure signal

Amplitude Spectral Density [m/rHz]

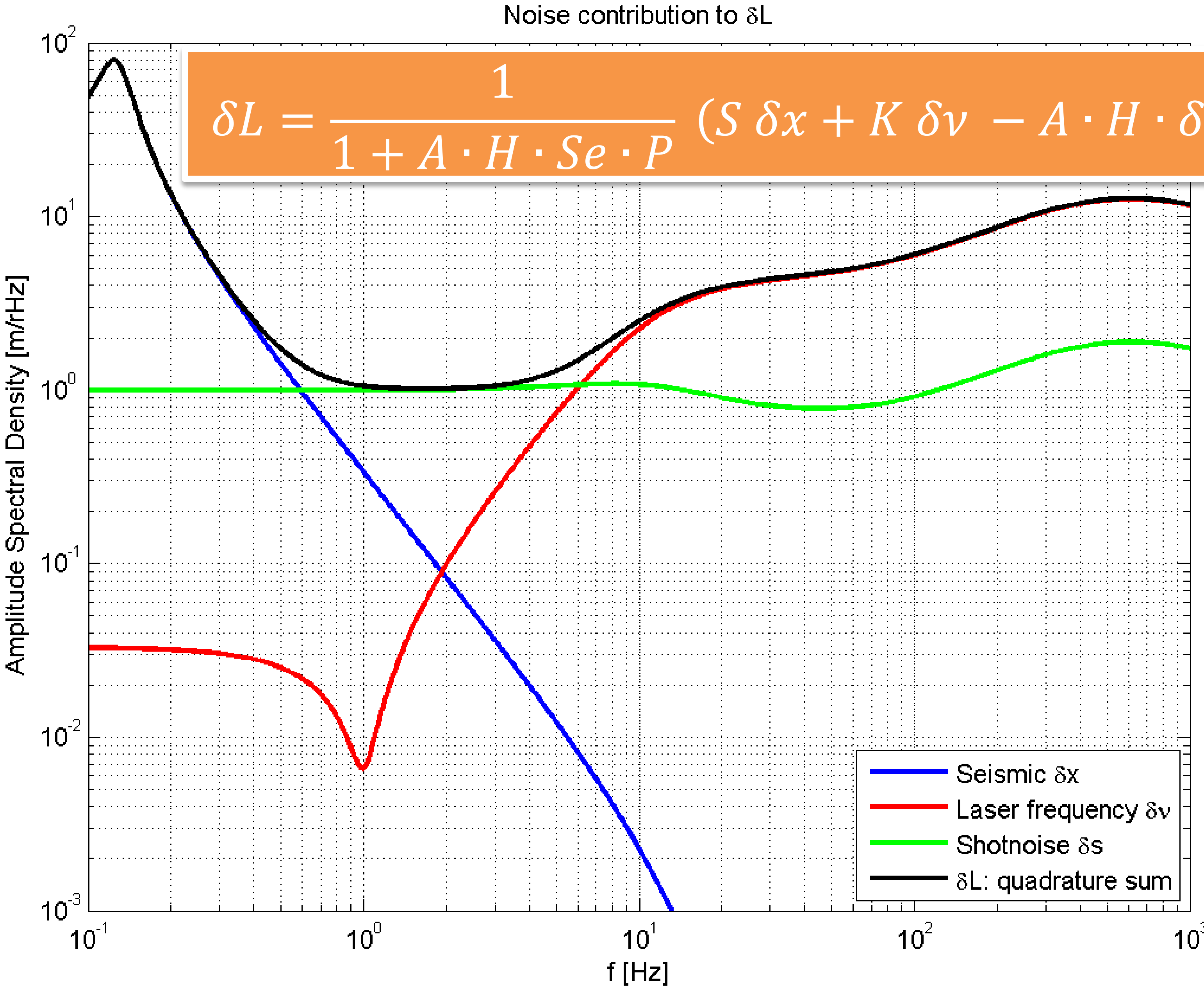
Measure Transfer Function TF relating the two signals



Amplitude spectra density of δν
Magnitude [Hz/rHz]
f [Hz]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Free running (no feedback)
Residual (with feedback)
$\delta L = \frac{1}{1 + A \cdot H \cdot Se \cdot P} K\, \delta\nu$



$$\delta L = \frac{1}{1 + A \cdot H \cdot Se \cdot P} \left(S\ \delta x + K\ \delta v\ - A \cdot H \cdot \delta s \right)$$



$$\delta L = \frac{1}{1 + A \cdot H \cdot Se \cdot P} \left(S\, \delta x + K\, \delta \nu \, - A \cdot H \cdot \delta s \right)$$

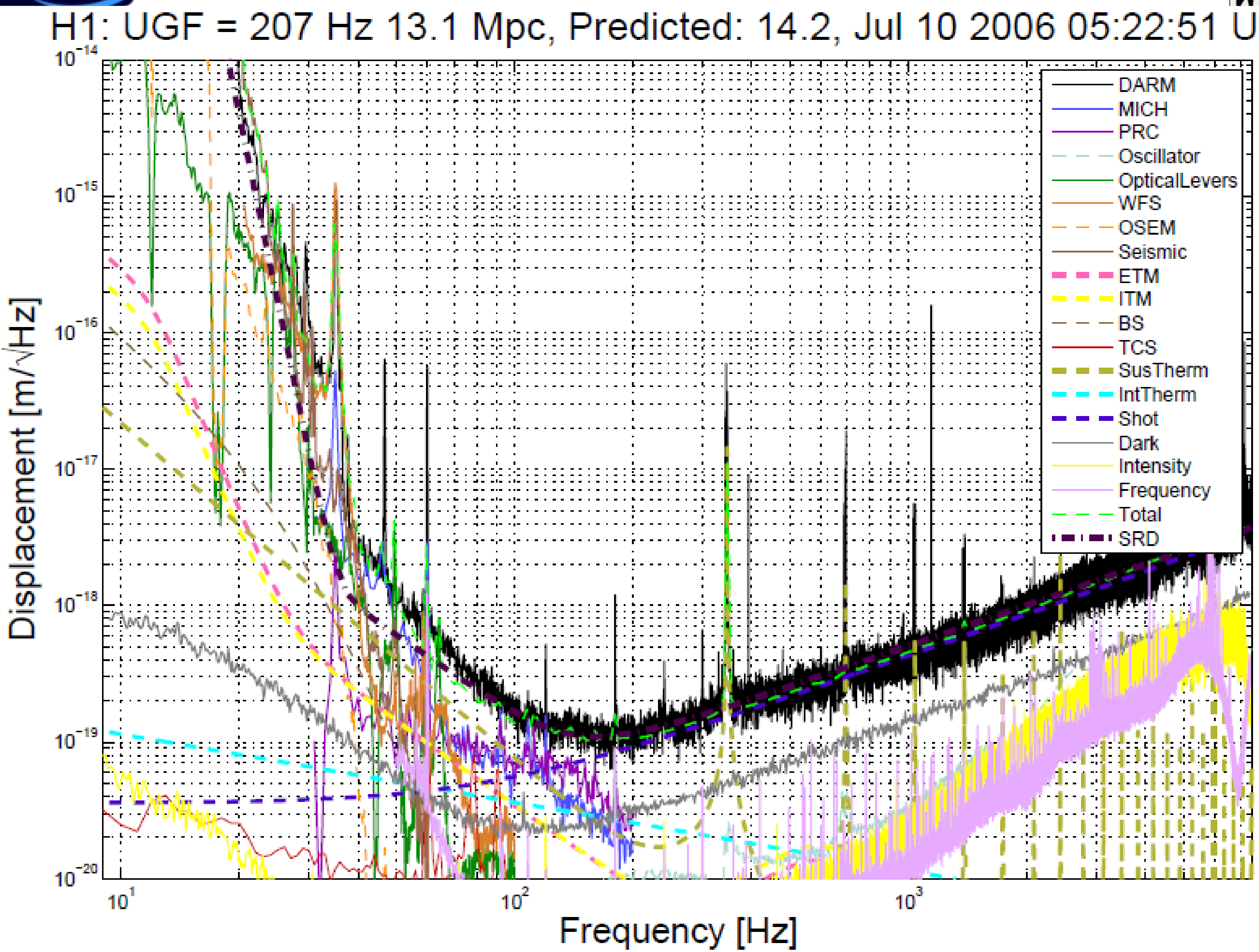
Noisebudget_example1.m

Noise budgeting

- Noise term (for example $\delta\nu$) is measured/estimated in frequency space (ASD)
- To project this noise term, need to measure/model/estimate the system's TFs
- Noise budgeting
  - Noise projection: multiply noise term (in this case $\delta\nu$) by the TF

$$\delta L_{expected} = TF \cdot \delta\nu$$

  - Compare (budget) projection $\delta L_{expected}$ with measured δL
    - If in agreement: sensitivity limited by that one noise term
    - If not in agreement: other noise terms are at play
- eLIGO noise budget sample
  - Contribution sum of all noise terms: in quadrature
  - Quadrature sum of noise terms is compared to detector's sensitivity



From T060156

MISC

Advanced LIGO Reference Design M060056

- Sensitivity and Reference Design Configuration
  - $h \sim 10^{-22}$ RMS integrated over $100\ Hz$ bandwidth
  - Tunings:
    - NS-NS: greatest 'reach', optimization at $100\ Hz$
    - BH-BH: low frequency optimization
    - Pulsars: narrow-band tuning, SRM swap

Advanced LIGO Reference Design M060056

- Quantum noise limited IFO