An Overview of Advanced LIGO Interferometry

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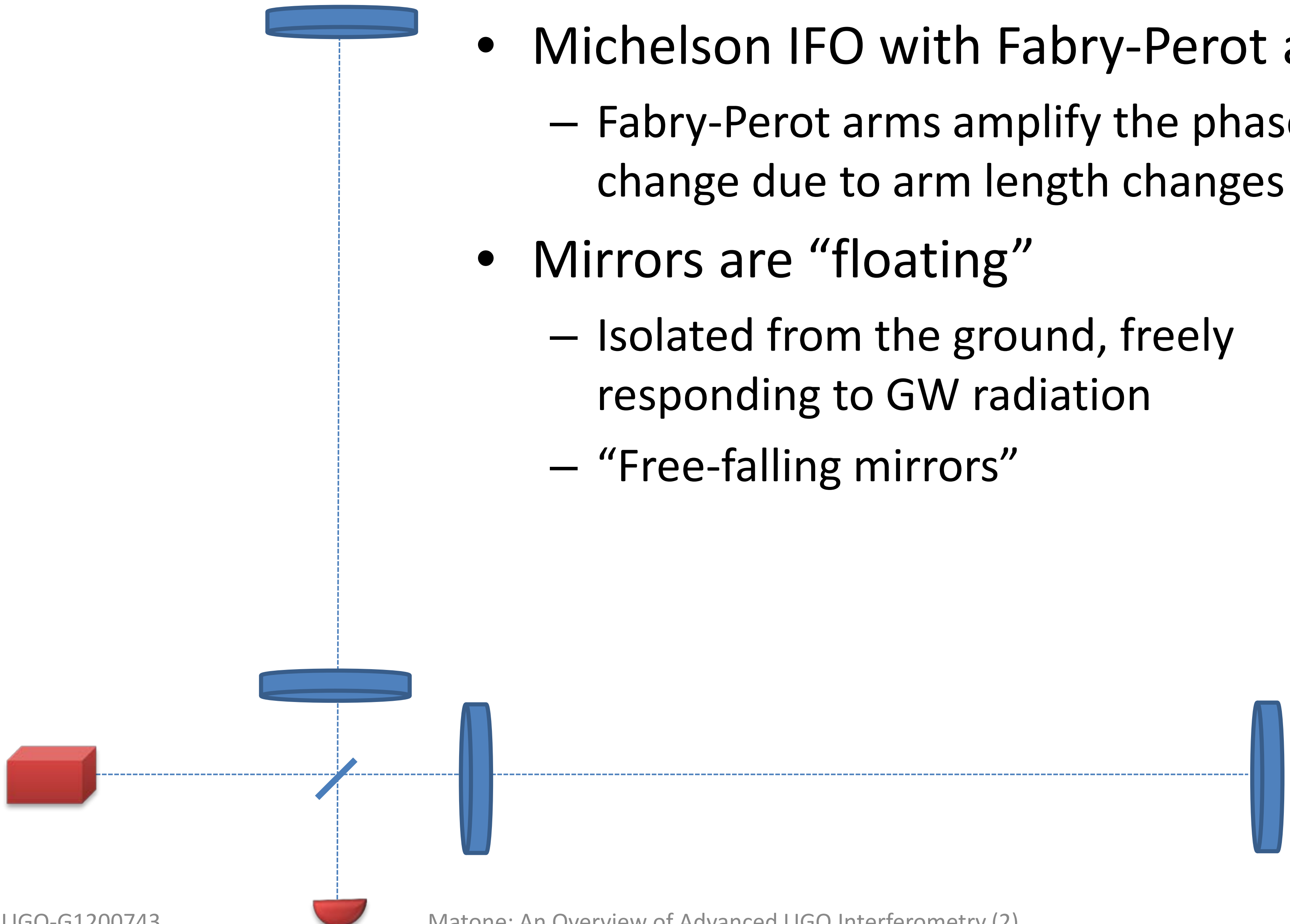
So far…

- Michelson IFO with Fabry-Perot arms
  - Fabry-Perot arms amplify the phase change due to arm length changes
- Mirrors are "floating"
  - Isolated from the ground, freely responding to GW radiation
  - "Free-falling mirrors"



- Strategy
  - Amplify signal
  - Decrease and/or control noise contribution
- Noise sources: in general two categories
  - Displacement noise
    - Ground seismic excitation
    - Thermal excitation of optical elements and suspensions
    - Radiation pressure
  - Phase noise
    - Amplitude and frequency fluctuations of the incoming laser beam
    - Shot-noise, the quantum limit to the *counting* of photons

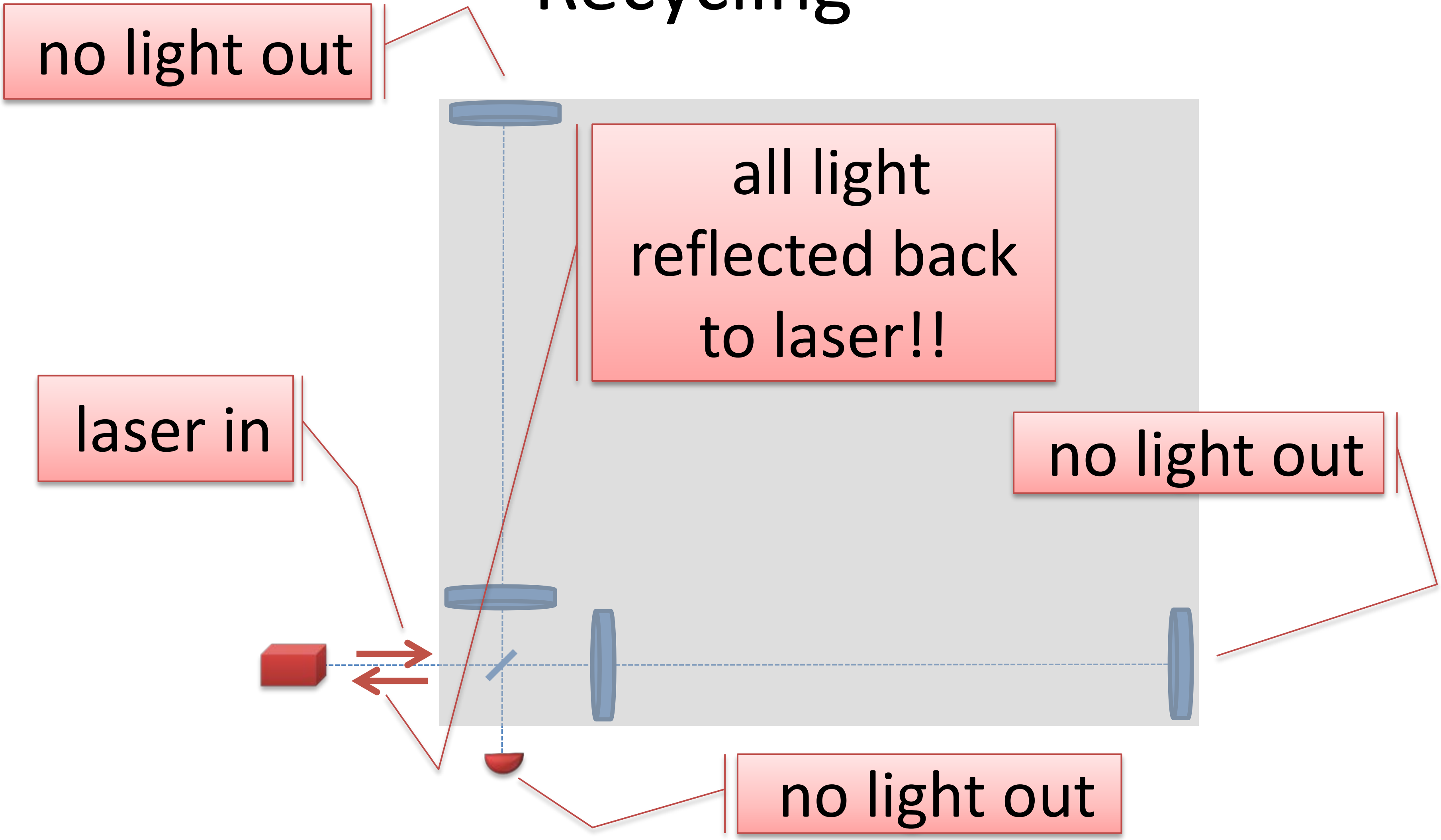
Addressing phase noise…

- Shot-noise
  - The fundamental limit to the interferometer sensitivity
  - Caused by the inevitable fluctuations in the number of photons in the laser beam
  - Follows Poisson statistics
  - Phase noise

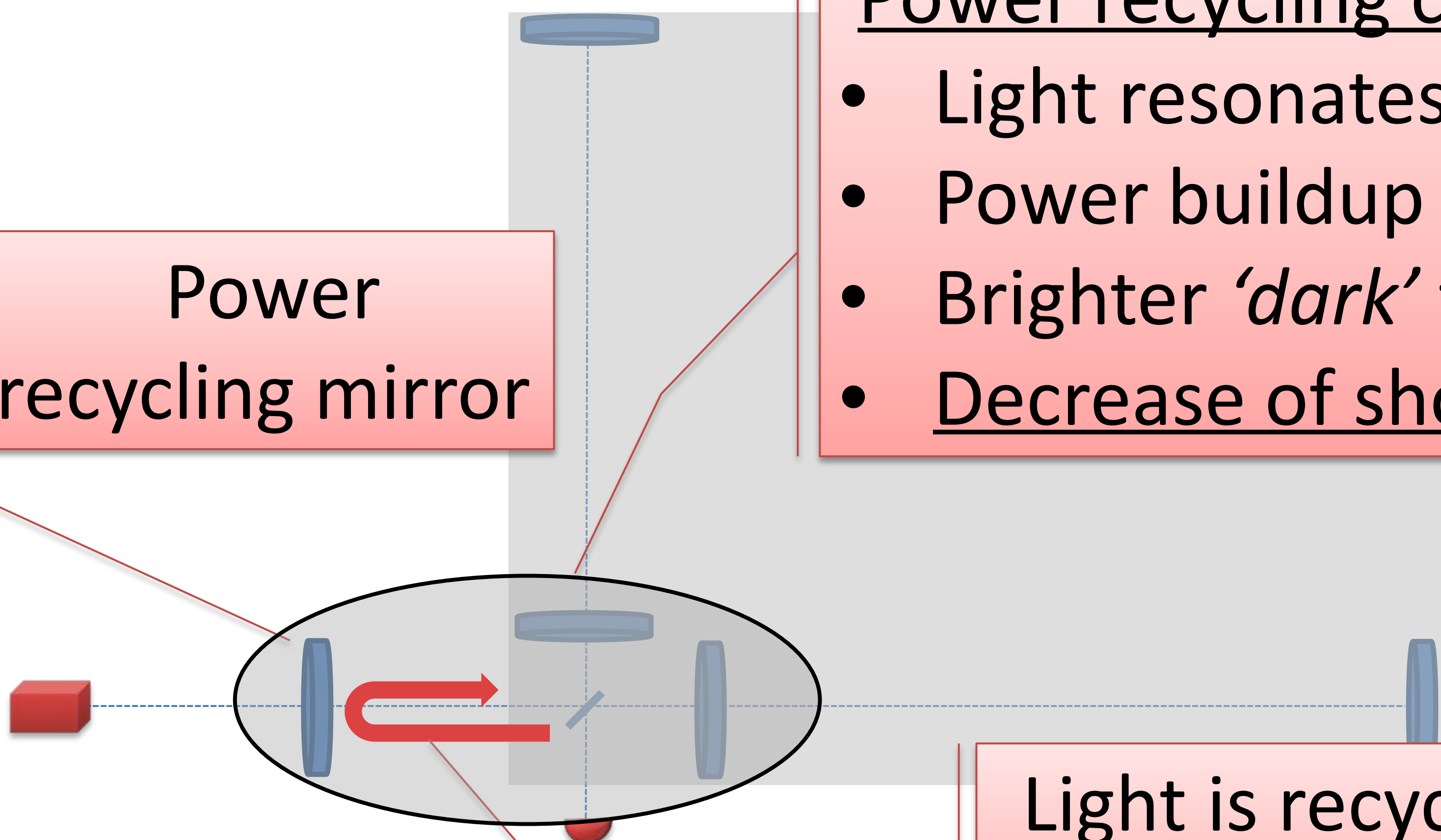
$$\delta\varphi_{shot} \propto \frac{1}{\sqrt{P_{laser}}}$$

  - <u>Need to use the most light power possible to lower this noise contribution!</u>

Laser power and Power Recycling



Laser power and Power Recycling



Power recycling cavity

- Light resonates
- Power buildup
- Brighter *'dark'* fringe
- Decrease of shot noise

Power recycling mirror

Light is recycled and sent back to the IFO

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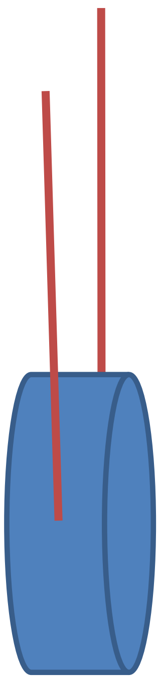
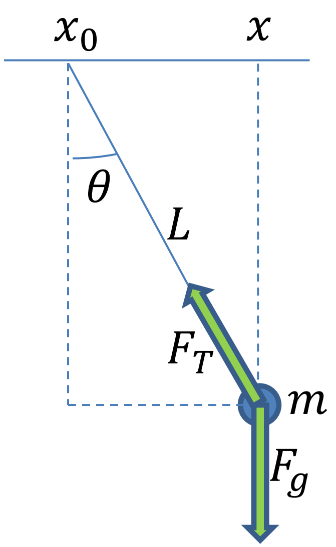
Need to ground IFO

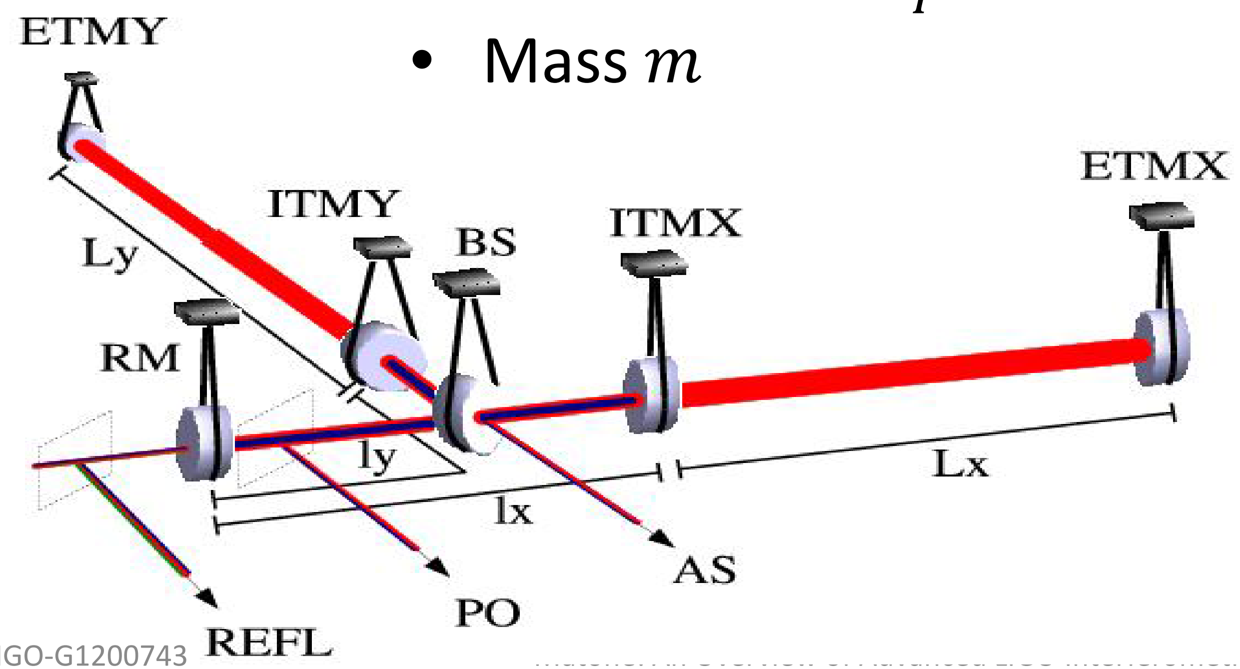


- Inevitable coupling to ground
  - displacement noise due to seismic excitation
- To limit the coupling
  - Mirrors are suspended like pendula
  - High-frequency
    - Mirrors are isolated from the ground
  - Low-frequency
    - Mirrors are inevitably coupled to the ground
  - Let’s have a closer look

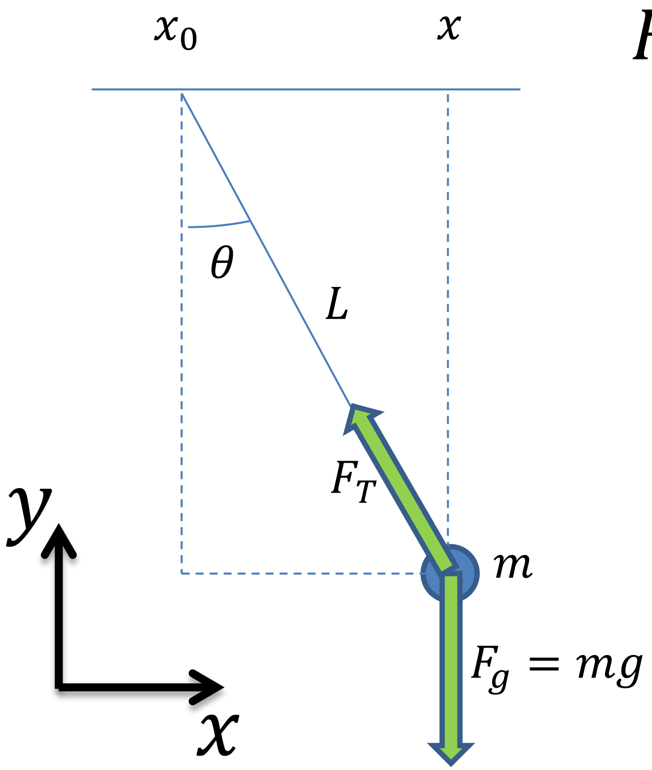
Equation of motion

- Suspension point position x_0
- Mirror position x
- Pendulum length L
- Small angle: $\theta \ll 1$
- No losses
- Force due to gravity F_g
- Tension force F_T
- Mass m





Equation of motion



$$F_T \simeq m\,g \qquad \begin{cases} y: & -F_g + F_T \cos\theta = 0 \\ x: & -F_T \sin\theta = m\,a \end{cases}$$

$$-F_T \sin\theta = -m\,g\,\frac{x - x_0}{L} = m\,\ddot{x}$$

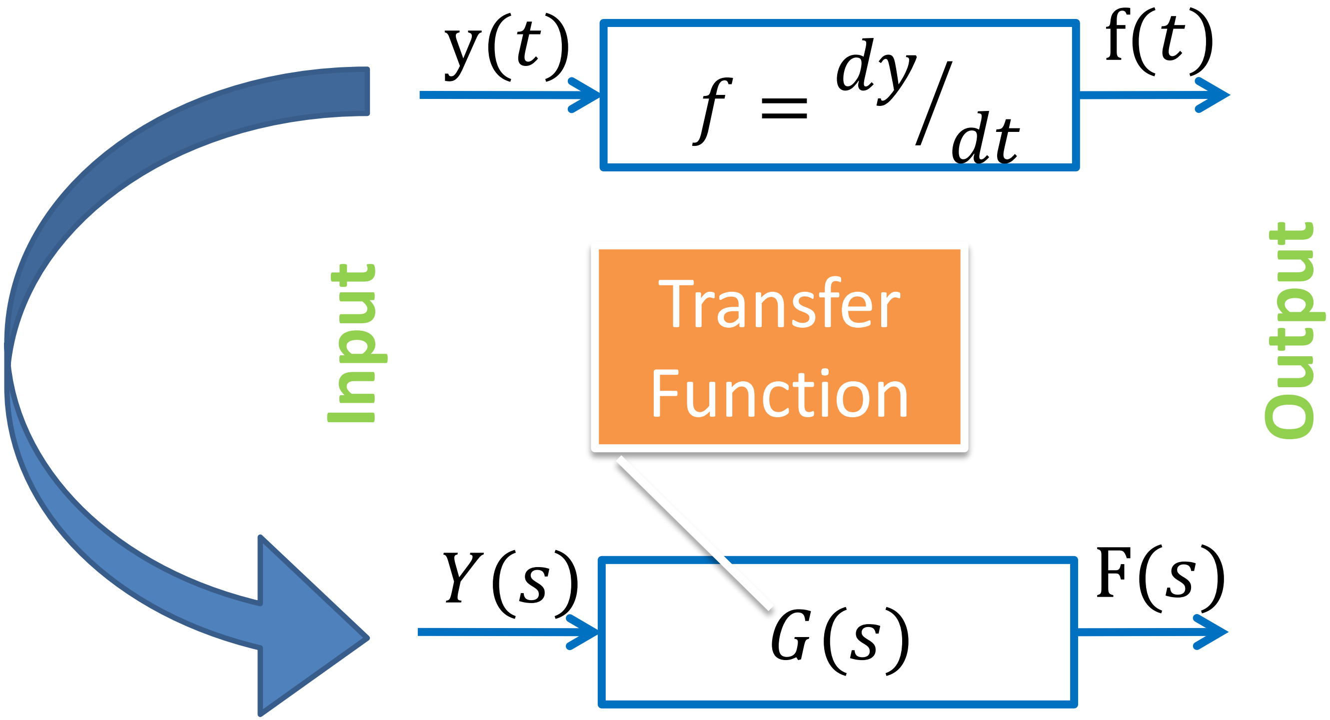
$$-\frac{g}{L}(x - x_0) = \ddot{x}$$

Time domain: solve for differential equation

Laplace Transforms

- A technique to facilitate the solution of ordinary differential equations.
- Transformation from the time-domain to the frequency-domain.
- Functions are complex, often described in terms of magnitude and phase → Transfer Functions

Time domain ↔ Laplace domain



Transform variable $s = j\,\omega$ (complex frequency)

Laplace transform properties

- Linearity

$$\mathcal{L}[c_1 f_1(t) + c_2 f_2(t)] = c_1 F_1(s) + c_2 F_2(s)$$

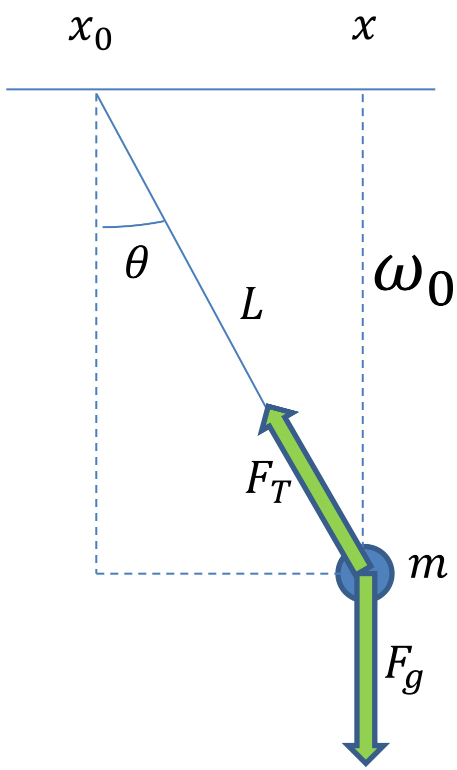
- Derivatives
  - First-order: $\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s)$
  - Second-order: $\mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 F(s)$
- Integral

$$\mathcal{L}\left[\int_0^t f(t)dt\right] = \frac{1}{s}F(s)$$

Equation of motion

$$-\frac{g}{L}(x - x_0) = \ddot{x}$$

Time domain



$$\omega_0{}^2 = \frac{g}{L}$$

$$x = \frac{\omega_0{}^2}{s^2 + \omega_0{}^2}\, x_0$$

Frequency domain

Sample Laplace transform pairs

	$f(t)$	$F(s)$
Unit step	$u(t)$	$1/s$
Unit ramp	t	$1/s^2$
Exponential	e^{at}	$1/(s-a)$
Sinusoid	$\sin(\omega_0 t)$	$\omega/(s^2+\omega_0{}^2)$
	$(1/a)(1-e^{-at})$	$1/s(s+a)$
SHO	$\frac{\omega_0}{\sqrt{1-\delta^2}} e^{-\delta\omega_0 t} \times \times \sin(\sqrt{1-\delta^2}\,\omega_0 t)$	$\frac{\omega_0{}^2}{s^2+2\delta\omega_0\, s+\omega_0{}^2}$

Seismic Attenuation

Mirror position

$$x(s) = G(s)\ x_0(s)$$

Suspension point

$$x = \frac{{\omega_0}^2}{s^2 + {\omega_0}^2}\, x_0$$

Attenuation

$$\simeq \begin{cases} \dfrac{1}{s^2}\, x_0 & \text{for high frequency} \quad s \gg \omega_0 \\ x_0 & \text{for low frequency} \quad s \ll \omega_0 \end{cases}$$

No attenuation – mirror follows ground

Simple Harmonic Motion (SHM)

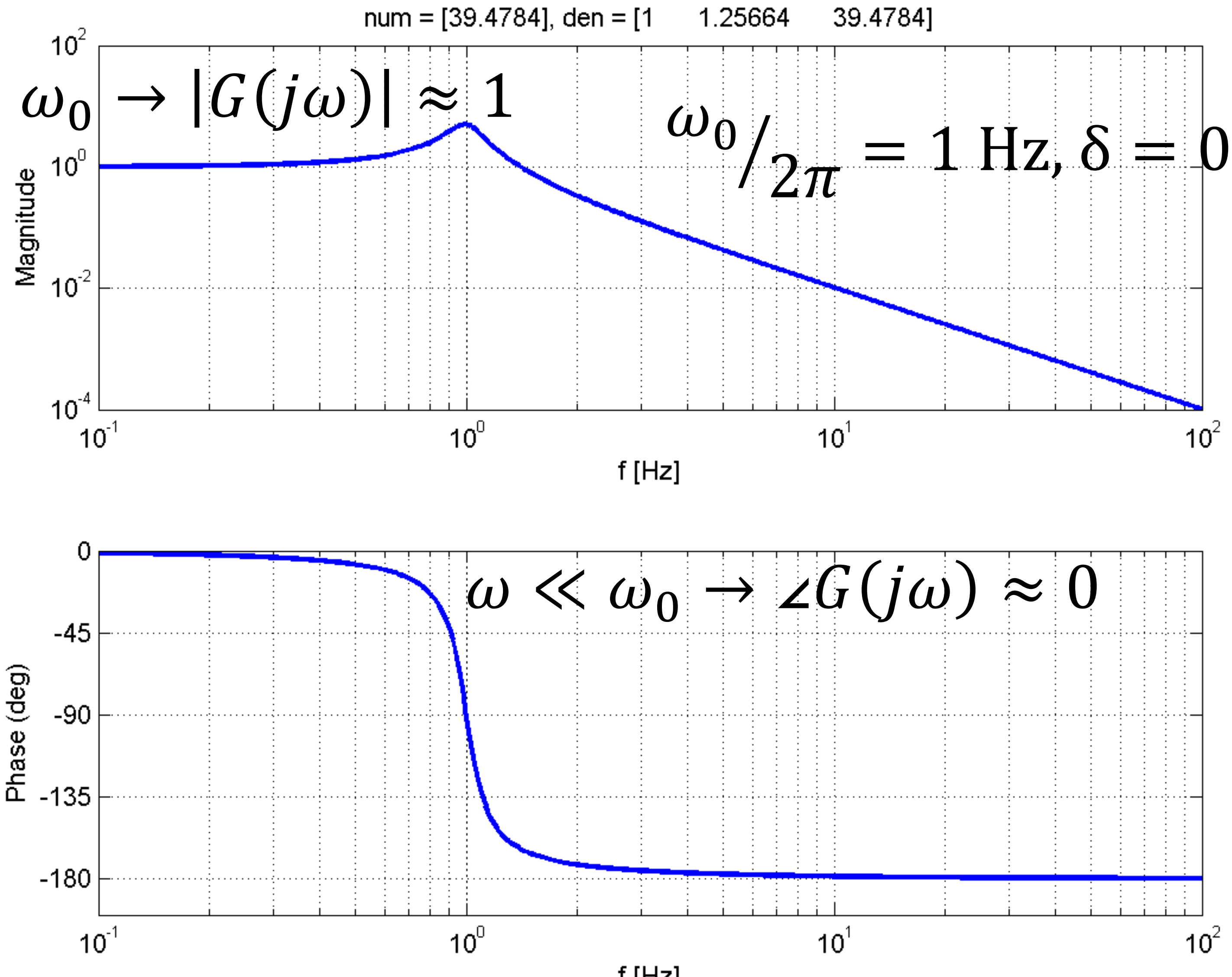
- Including damping terms

$$G(s) = \frac{(\omega_0)^2}{(s^2 + 2\delta\omega_0 \cdot s + (\omega_0)^2)}$$

- With quality factor

$$Q = \frac{1}{2\delta}$$

Bode plot: $G(s) = \frac{(\omega_0)^2}{(s^2 + 2\delta\omega_0 \cdot s + (\omega_0)^2)}$



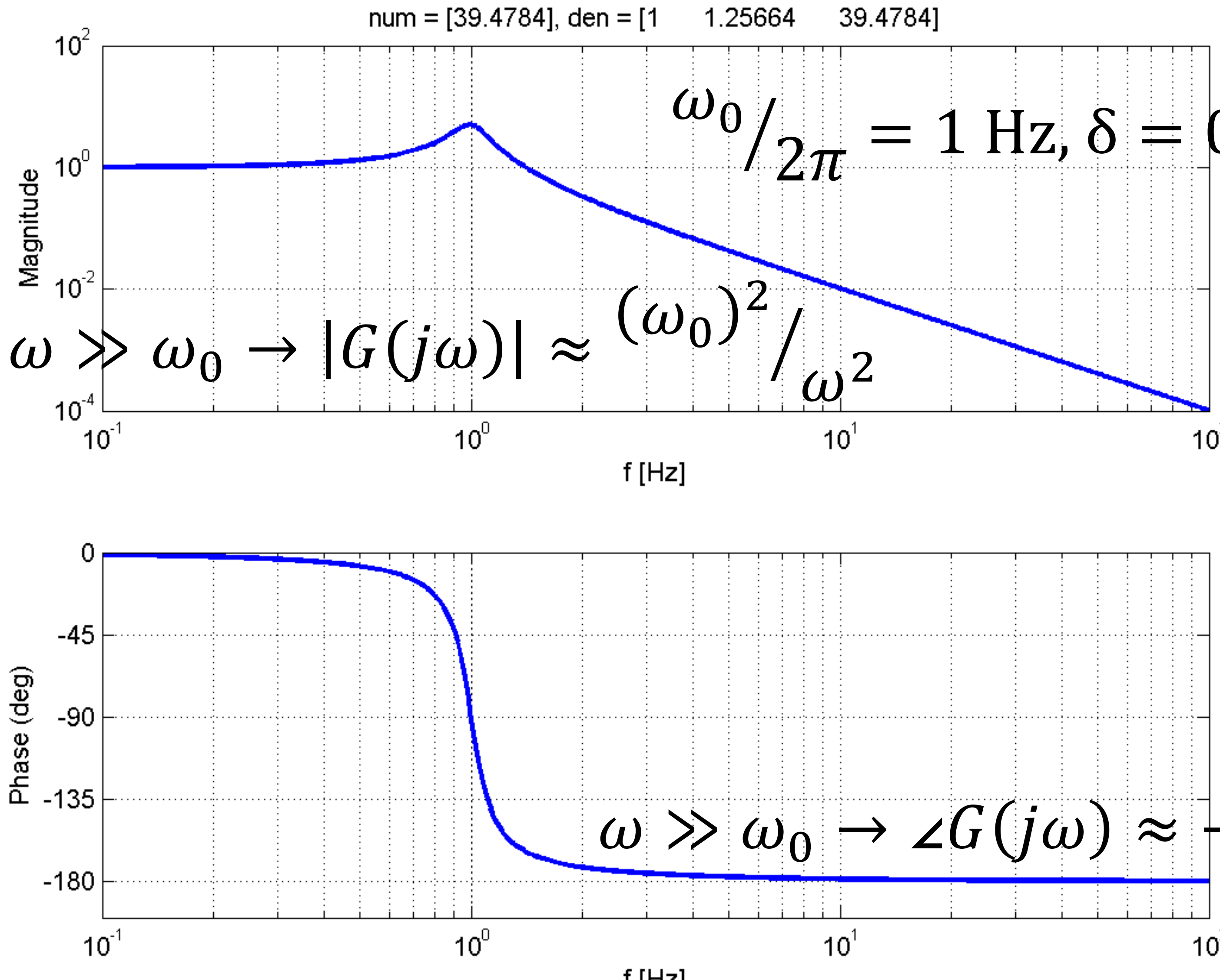
$\omega \ll \omega_0 \rightarrow |G(j\omega)| \approx 1$

$\omega_0/2\pi = 1\ \text{Hz}, \delta = 0.1$

$\omega \ll \omega_0 \rightarrow \angle G(j\omega) \approx 0$

bodeexamples.m

Bode plot: $G(s) = \frac{(\omega_0)^2}{(s^2 + 2\delta\omega_0 \cdot s + (\omega_0)^2)}$



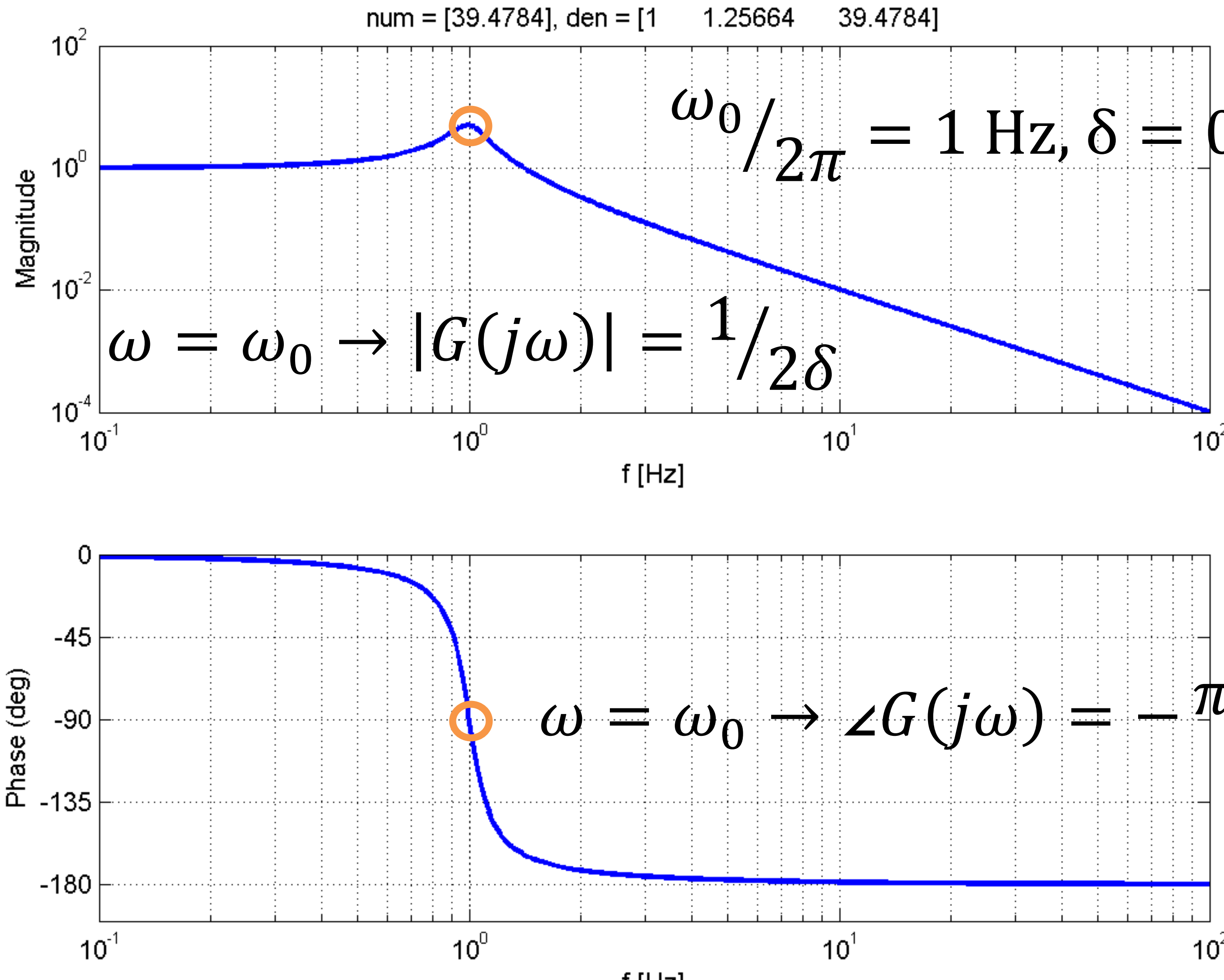
$\omega_0 / 2\pi = 1\ \mathrm{Hz},\ \delta = 0.1$

$\omega \gg \omega_0 \rightarrow |G(j\omega)| \approx (\omega_0)^2 / \omega^2$

$\omega \gg \omega_0 \rightarrow \angle G(j\omega) \approx -\pi$

bodeexamples.m

Bode plot: $G(s) = \dfrac{(\omega_0)^2}{(s^2 + 2\delta\omega_0 \cdot s + (\omega_0)^2)}$



$\omega_0/_{2\pi} = 1\ \text{Hz}, \delta = 0.1$

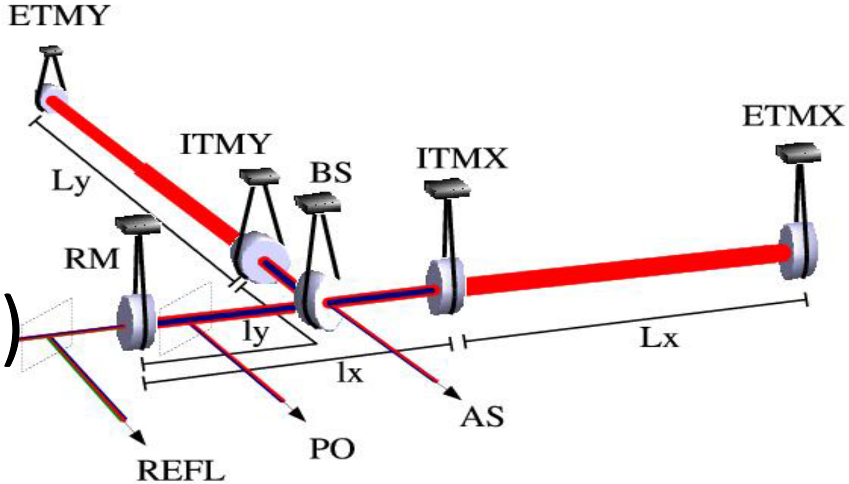
$\omega = \omega_0 \rightarrow |G(j\omega)| = {}^1/_{2\delta}$

$\omega = \omega_0 \rightarrow \angle G(j\omega) = -{}^{\pi}/_{2}$

bodeexamples.m

IFO as a reference

- At high frequencies ($s \gg \omega_0$)
  - IFO is very stable
    - used as a reference
  - Masses are “free-falling”
  - Decoupled from the ground
- At low frequencies ($s \lesssim \omega_0$)
  - Seismic noise creeps in
  - Cavities will not hold resonance unless there is control



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Holding the cavity on resonance

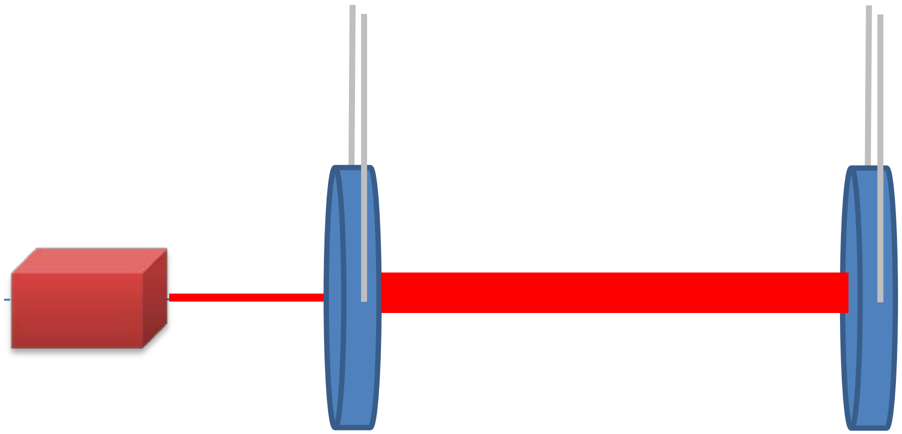
- Mirrors are in motion
  - Low-quality factors ($Q \lesssim 10$, with local controls) and low frequency ($\sim 1\ Hz$) resonances
  - Test mass residual motion of $\sim 1\ \mu m$ RMS
    - Compare with $^{\lambda}/_{2}$ FSR
    - Several fringe crossings
  - "Narrow" optical resonance

$$FWHM = \frac{FSR}{\mathcal{F}} = \frac{\lambda/2}{450} \simeq 1\ nm$$

- How to keep light resonating in cavity?

The Pound-Drever-Hall (PDH) Locking Scheme

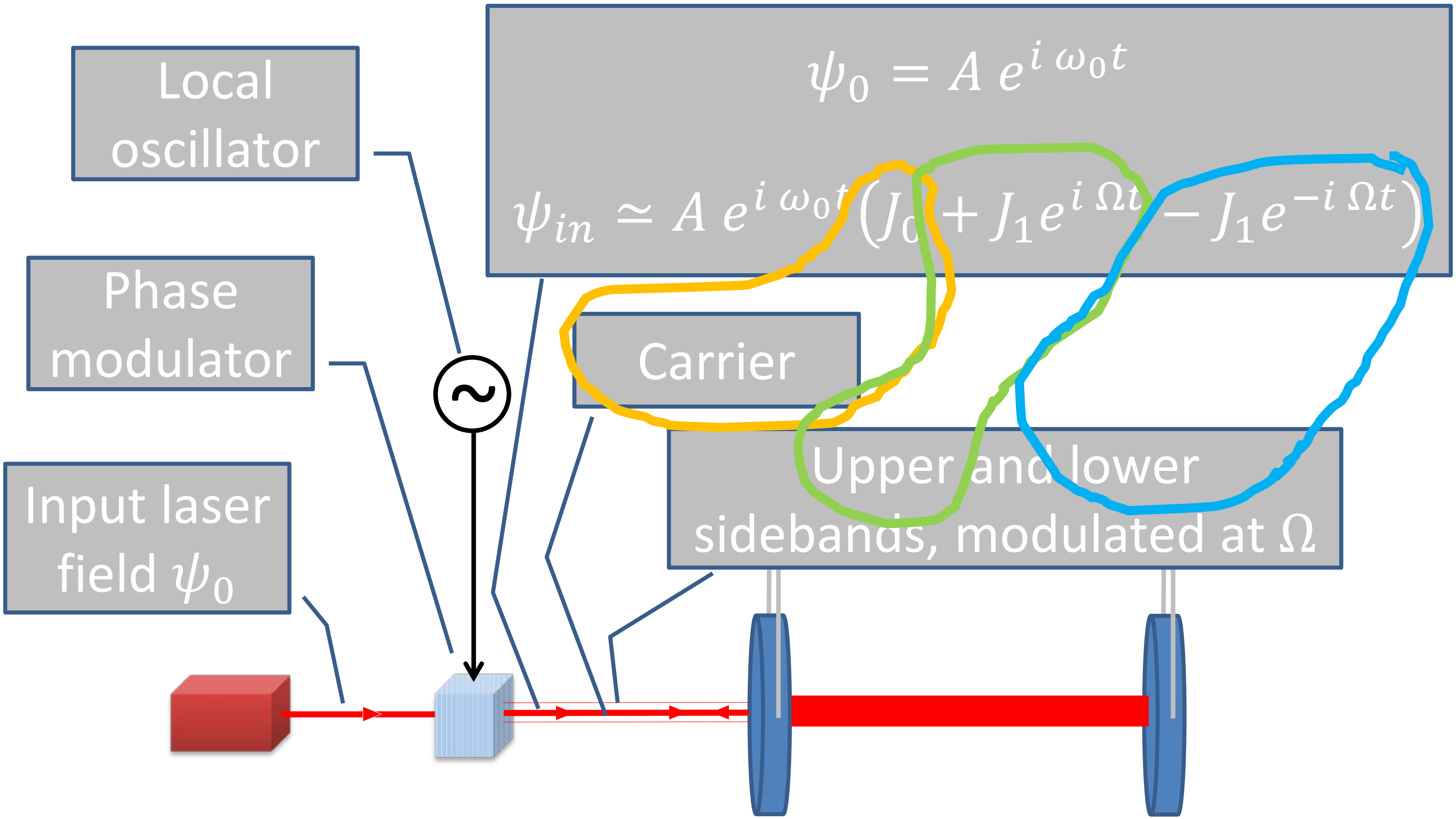
- Two options
  1. Force the laser frequency ν to track the cavity length L
     - Freely moving mirrors with adjustments to the laser frequency: light on resonance
  2. Force the cavity length L to track the laser frequency ν
     - Laser frequency ν is uncontrolled but mirrors are driven: light on resonance



The Pound-Drever-Hall (PDH) Locking Scheme

$$\psi_0 = A\, e^{i\,\omega_0 t}$$

$$\psi_{in} \simeq A\, e^{i\,\omega_0 t}\left(J_0 + J_1 e^{i\,\Omega t} - J_1 e^{-i\,\Omega t}\right)$$



The Pound-Drever-Hall (PDH) Locking Scheme

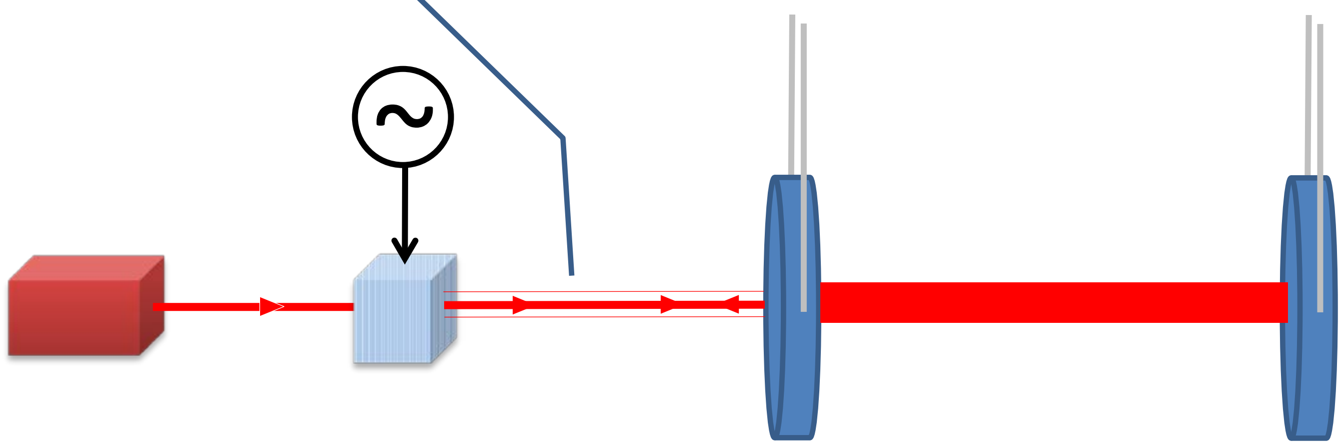
DC power

$$|\psi_R|^2 \simeq \left|J_0\psi_{R,0}\right|^2 + \left|J_1\psi_{R,+}\right|^2 + \left|J_1\psi_{R,-}\right|^2$$
$$+ 2J_0J_1\Re\left[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*\right]\cos\Omega t$$
$$- 2J_0J_1\Im\left[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*\right]\sin\Omega t$$

Modulated components

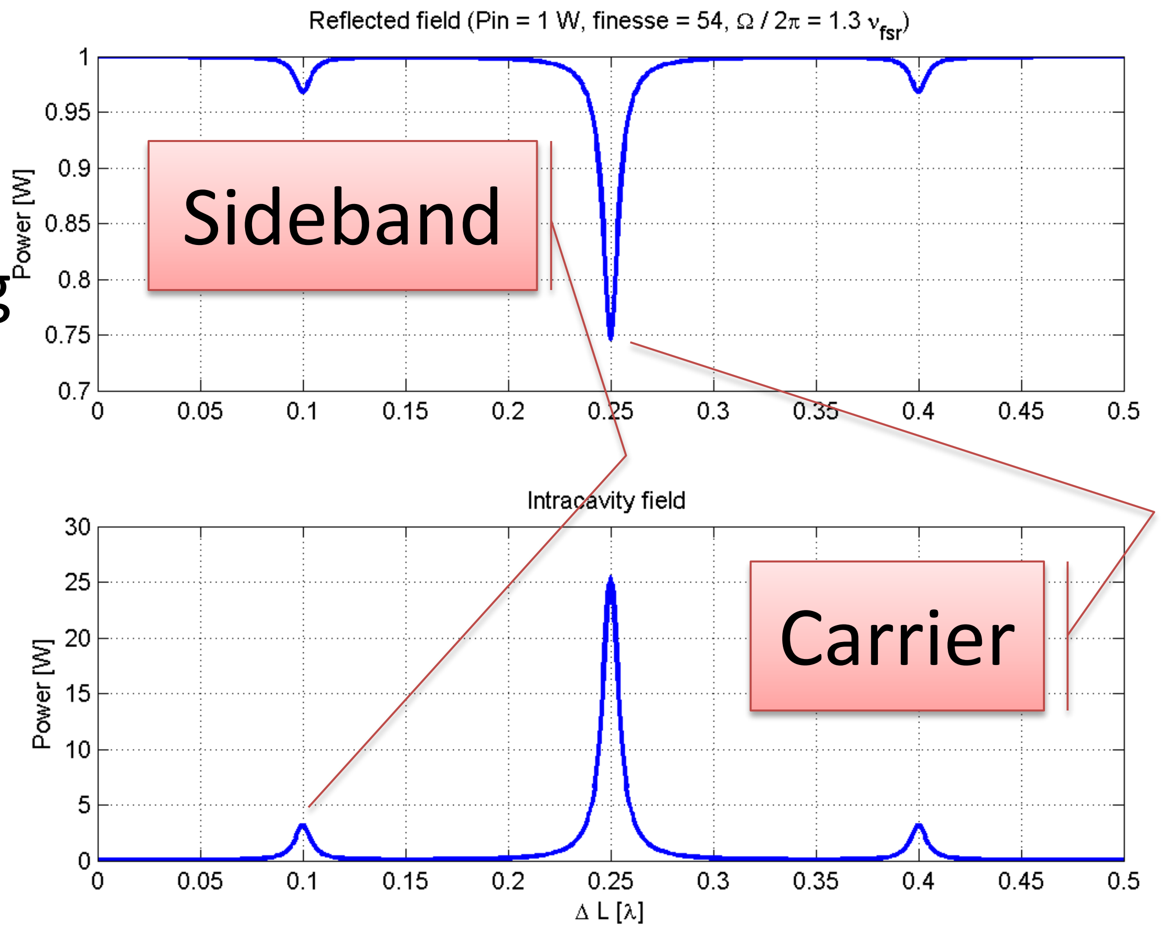
- The beat between the carrier and the sidebands
- Contains the PDH error signal

$$\psi_R \simeq J_0\,\psi_{R,0} + J_1\,\psi_{R,+}\,e^{i\,\Omega t} - J_1\psi_{R,-}\,e^{-i\,\Omega t}$$



DC power

- Sweeping cavity length L (but keeping laser frequency ν fixed)
- When carrier resonates, sidebands anti-resonate
  - completely reflected

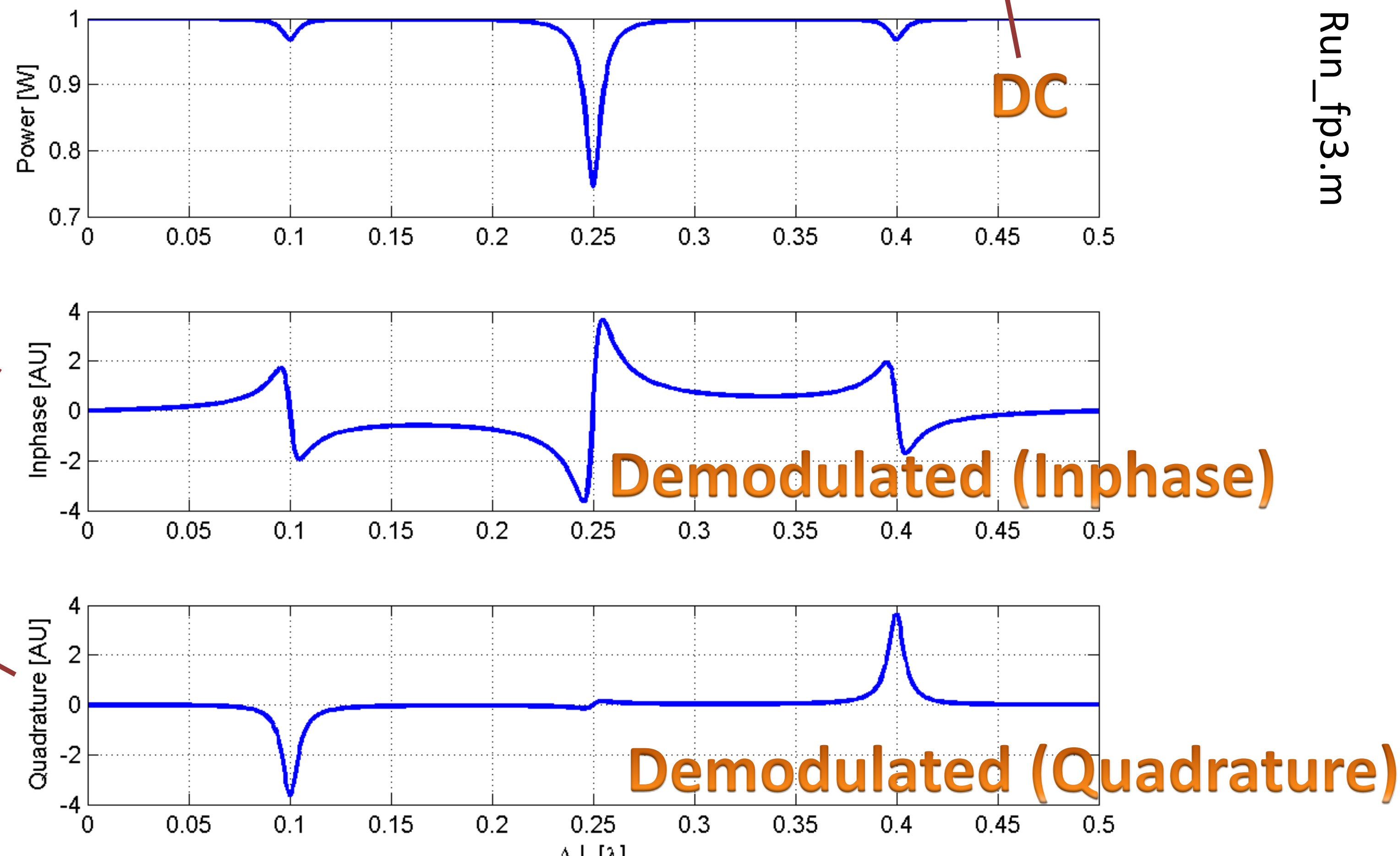


Run_fp3.m

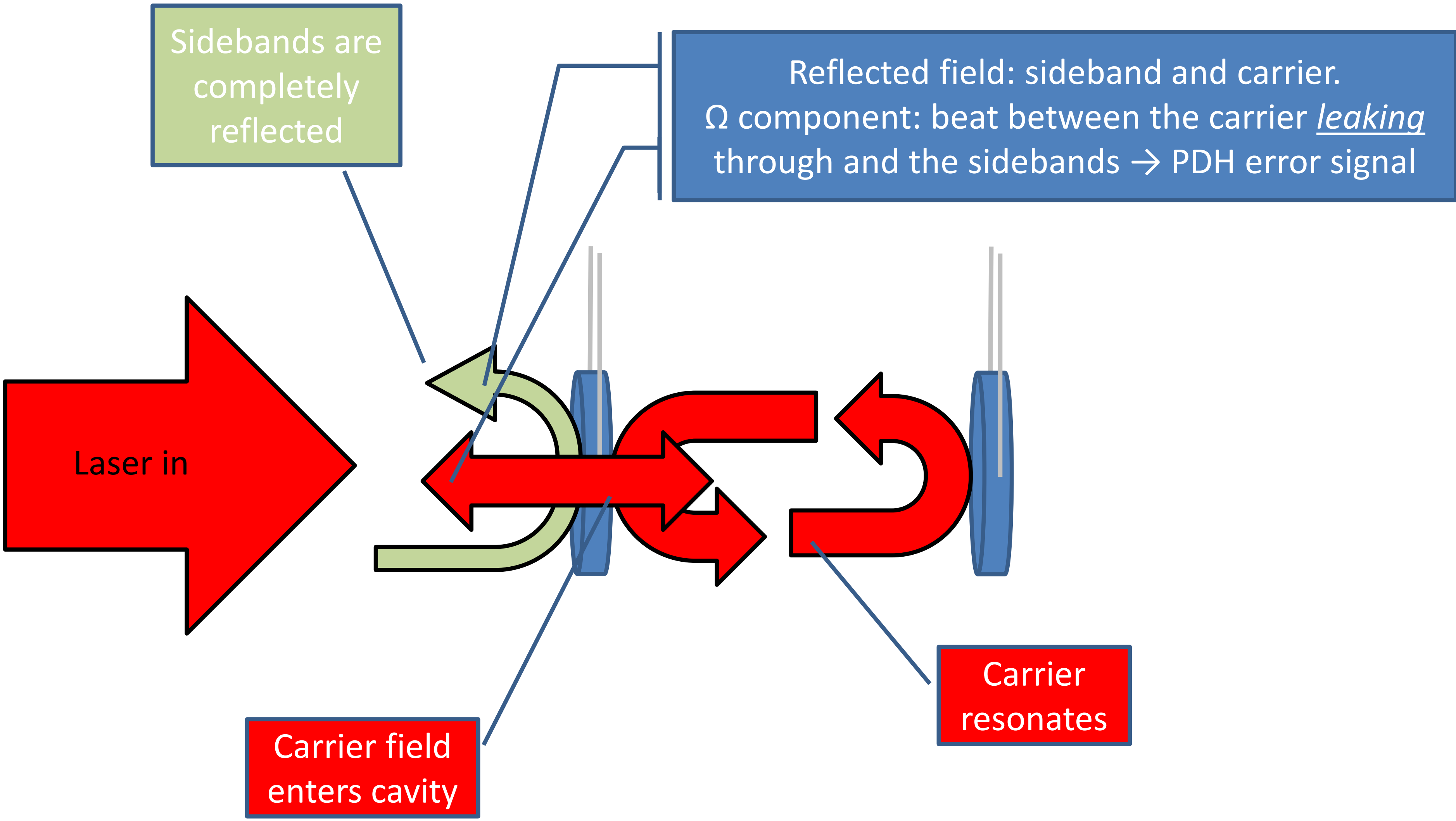
$$|\psi_R|^2 \simeq |J_0\psi_{R,0}|^2 + |J_1\psi_{R,+}|^2 + |J_1\psi_{R,-}|^2$$
$$+ 2J_0J_1\Re[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*]\cos\Omega t$$
$$- 2J_0J_1\Im[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*]\sin\Omega t$$

Reflected field

Run_fp3.m

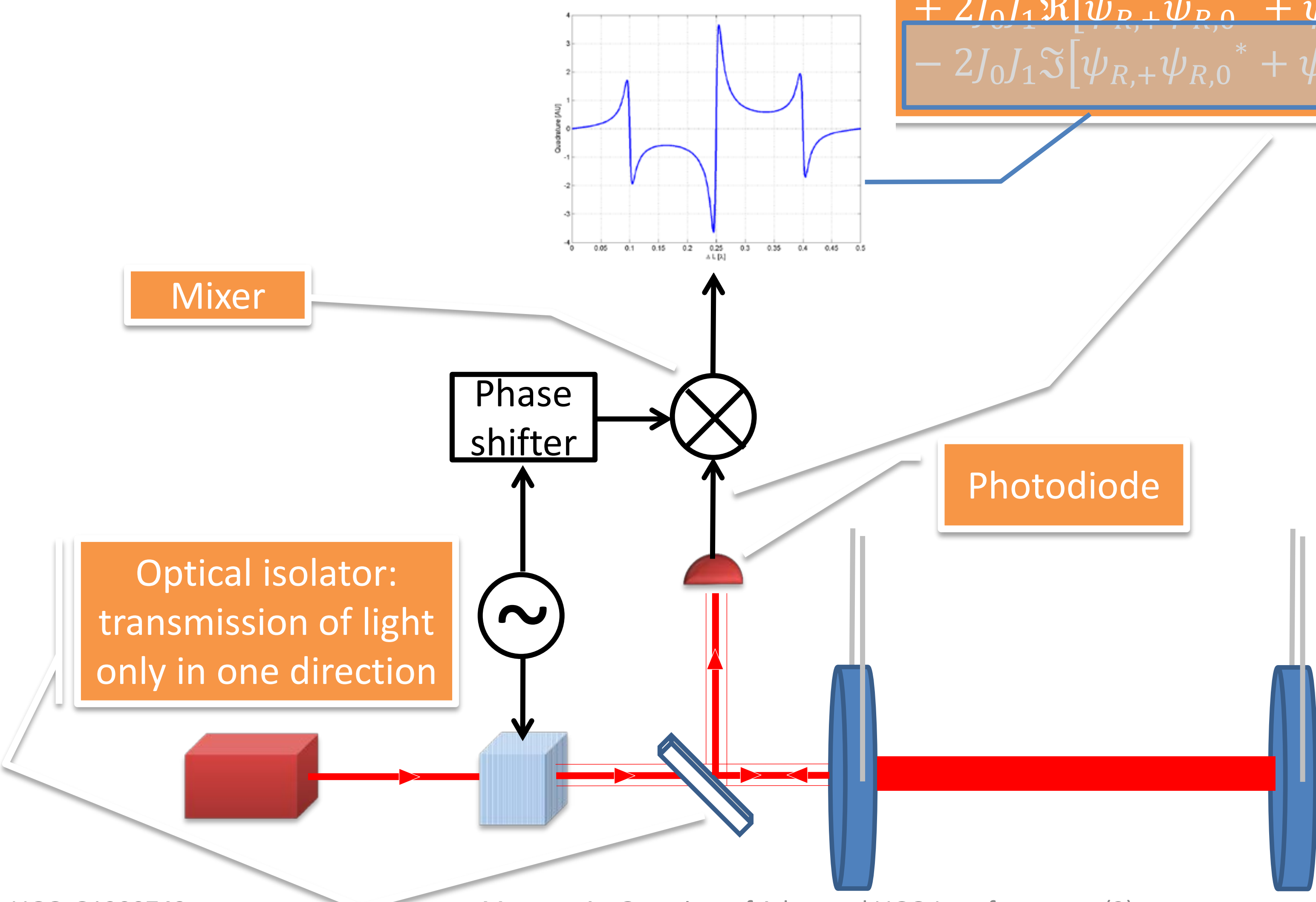


Conceptualizing

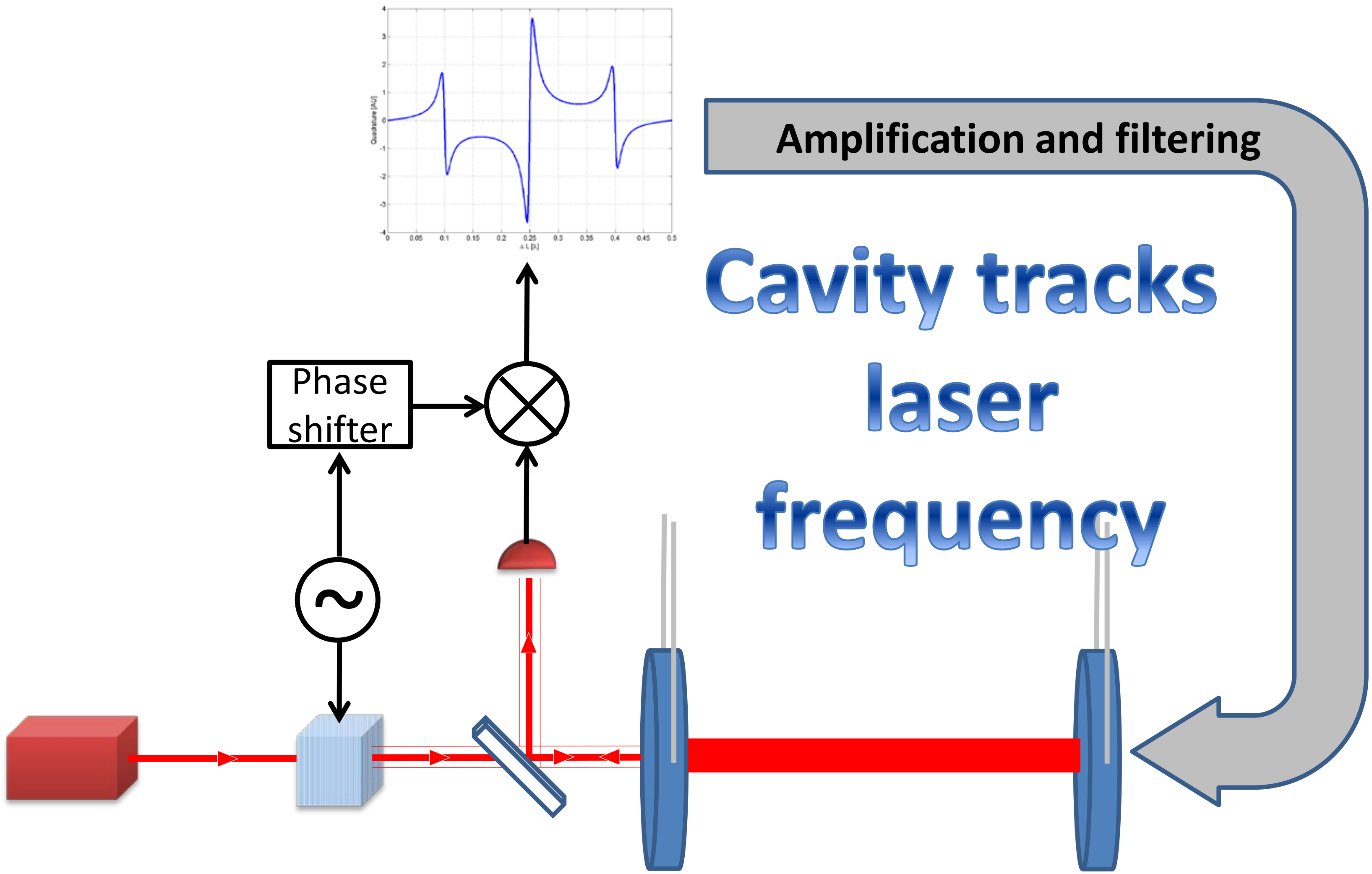


Measuring the error signals

$$
\begin{aligned}
|\psi_R|^2 \simeq\ & |J_0\psi_{R,0}|^2 + |J_1\psi_{R,+}|^2 + |J_1\psi_{R,-}|^2 \\
& + 2J_0J_1\Re\left[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*\right]\cos\Omega t \\
& - 2J_0J_1\Im\left[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*\right]\sin\Omega t
\end{aligned}
$$



Locking the arm to the laser



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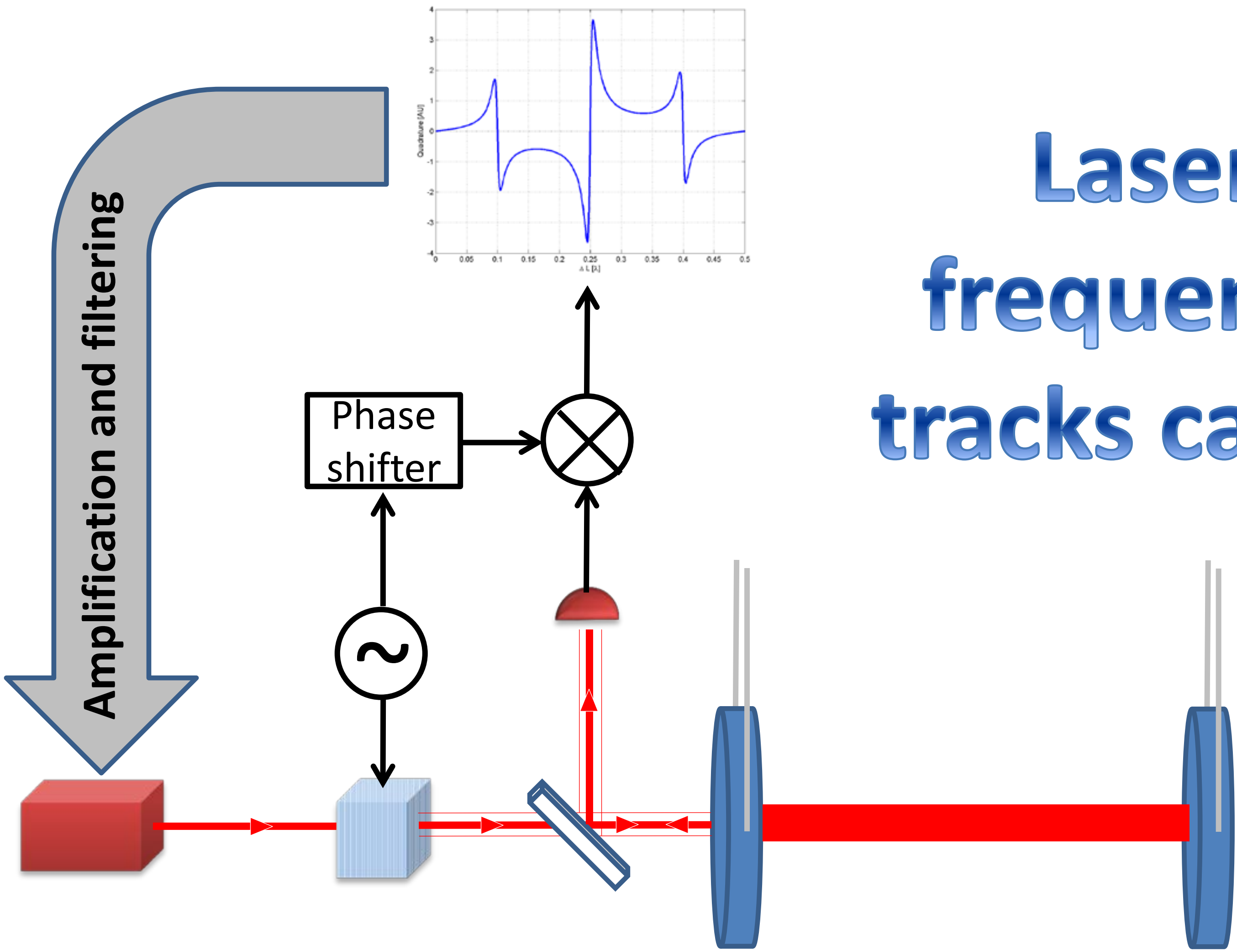

Laser frequency tracks cavity

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As of now …

- Power Recycled Michelson IFO with Fabry-Perot arms
  - Fabry-Perot arms amplify the phase change due to ΔL
  - Power Recycling mirror re-injects light back into IFO, generating power build-up and lowering phase noise due to shot noise
- Displacement noise
  - Mirrors are suspended to isolate IFO from the ground
  - At high frequencies ($s \gg \omega_0$)
    - IFO is very stable, Test Masses are "free-falling", decoupled from the ground
  - At low frequencies ($s \lesssim \omega_0$)
    - Seismic noise creeps in, cavities will not hold resonance unless there is control
- Pound-Drever-Hall Locking technique
  - Phase modulate laser beam to create sidebands
  - Requiring the sidebands not to resonate in cavity
  - Demodulating reflected signal to retrieve signal for longitudinal locking

Group Activities

1. Form groups of two or three
2. Using Matlab, write a *function* that calculates the reflected EM field from an arbitrary Fabry-Perot
3. Use this function to plot the demodulated signals – also using arbitrary settings

$$2J_0J_1\Re\left[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*\right]\cos\Omega t \\ - 2J_0J_1\Im\left[\psi_{R,+}\psi_{R,0}{}^* + \psi_{R,0}\psi_{R,-}{}^*\right]\sin\Omega t$$

3. Generate the corresponding plots for the demodulated signal using the specifications in Table 2 of T1000298 (AdvLIGO LSC final design document)
4. Just for fun – let a single mirror move sinusoidally around its equilibrium position with an amplitude of 2 μm. Plot the demodulated signals.

From "The AdvLIGO Length Sensing and Control Final Design" T1000298

Quantity	*Non-Folded IFOs*	*Folded IFO*
Finesse	446	446
ITM transmission	0.014	0.014
PRM transmission	0.030	0.030
SRM transmission	0.200	0.200
Schnupp asymmetry	0.050	0.050
ETM radius of curvature	2245 m	2245 m
ITM radius of curvature	1934 m	1934 m
l_{PRC}	57.6557 m	60.4112 m
l_{SRC}	56.0084 m	62.1372 m
l_{IMC} (round trip)	32.9461 m	34.5207 m
l_{EX}	3994.50 m	3996.00 m
l_{EY}	3994.50 m	3996.00 m
Free Spectral Range (FSR)	37.526 kHz	37.512 kHz
Transverse mode spacing	32.453 kHz	32.462 kHz
Lower mod. frequency	9'099'471 Hz	8'684'428 Hz
Upper mod. frequency	45'497'355 Hz	43'422'140 Hz

Table 2: Basic interferometer parameters for both folded and non-folded case.

Preparation for tomorrow

Read the following papers

- K. A. Strain et al., “Sensing and control in dual-recycling laser interferometer gravitational-wave detectors,” Appl. Opt. **42**, 1244-1256 (2003)
- Rollins J. et al., “Multi-color Cavity Metrology” P1200019-v5
- Mullavey A. J. et al, “Arm-length stabilization for interferometric gravitational-wave detectors using frequency-doubled auxiliary lasers,” Optics Express, **20**, 81-89 (2011)

Prepare questions to post on board