An Overview of Advanced LIGO Interferometry

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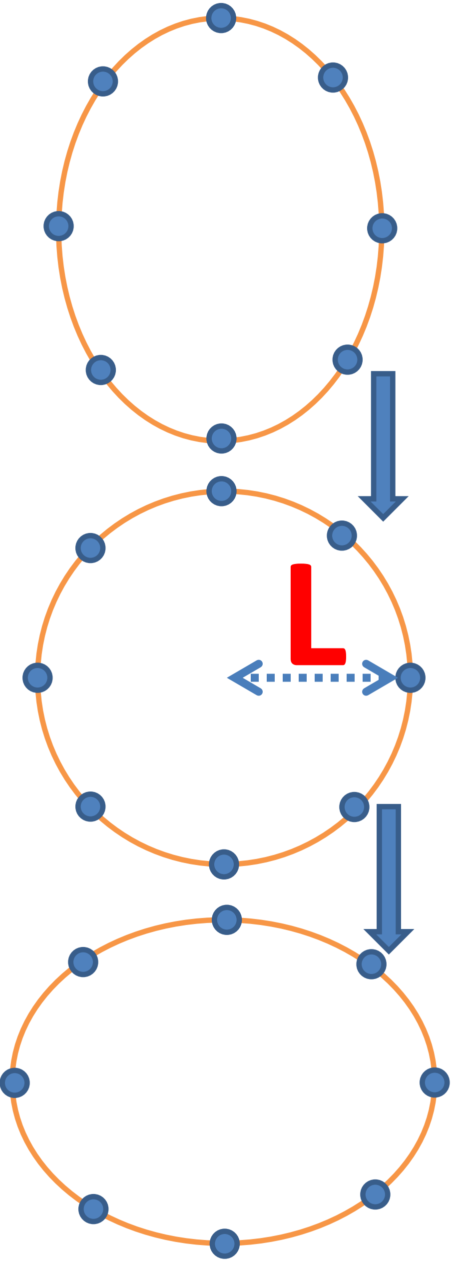
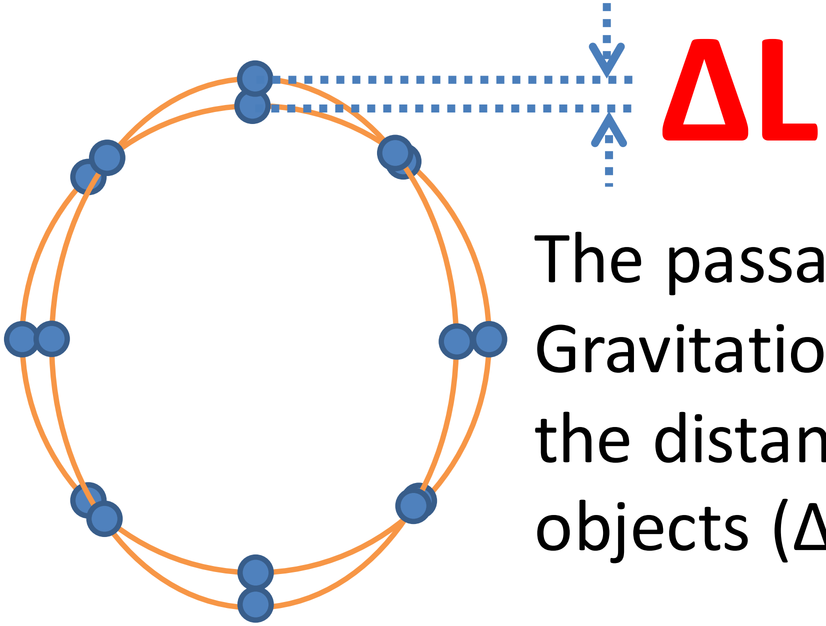
Day	Topic	References
1	Gravitational Waves, Michelson IFO, Fabry-Perot cavity, finesse, free-spectral range (FSR), Michelson with Fabry-Perot arms. Group activity: single arm numerical calculations (MATLAB) and plots.	• H. Kogelnik and T. Li, *Appl. Opt*. **5**, 1550 (1966) • K. A. Strain et al., Appl. Opt. 42, 1244-1256 (2003) • Mullavey A. J. et al, Optics Express, 20, 81-89 (2011) • Morrison E. et al., *Appl. Opt*. **33**, 5037-5040 (1994) • T970122 • P1200019 • G050091 • T070247 • T020020 • T040156 • G1101270 • M060056 • T1000298
2	Power recycled Michelson IFO, suspended TMs and equation of motion, Laplace transform, modeling of Pound-Drever-Hall locking scheme. Group activity: numerical calculations (MATLAB) of single arm cavity w/o sidebands w/o seismic noise.	
3	Initial LIGO: Schnupp asymmetry, degrees of freedom to sense and control, output ports. Advanced LIGO: Resonant Sideband Extraction (RSE), output ports, degrees of freedom to sense and control, sensing matrix, homodyne and DC readout, lock acquisition and green laser locking. Group activity: paper discussion	
4	Transverse Electro-Magnetic (TEM) modes, cavity mode, mode-matching, transverse mode spacing, Gouy phase, Wavefront sensor, Input Mode-Cleaner, frequency stabilization, Output Mode-Cleaner, Thermal Compensation System (TCS). Group activity: paper discussion	
5	Amplitude spectral noise, basics on control loops, noise budgeting, paper discussion and summary	

Gravitational Waves





The passage of a Gravitational Wave changes the distance between objects (ΔL).

The change in distance ΔL depends on

- The distance L
- The gravitational wave amplitude h

$$\Delta L = h\, L$$

How big is ΔL? Astrophysical motivation

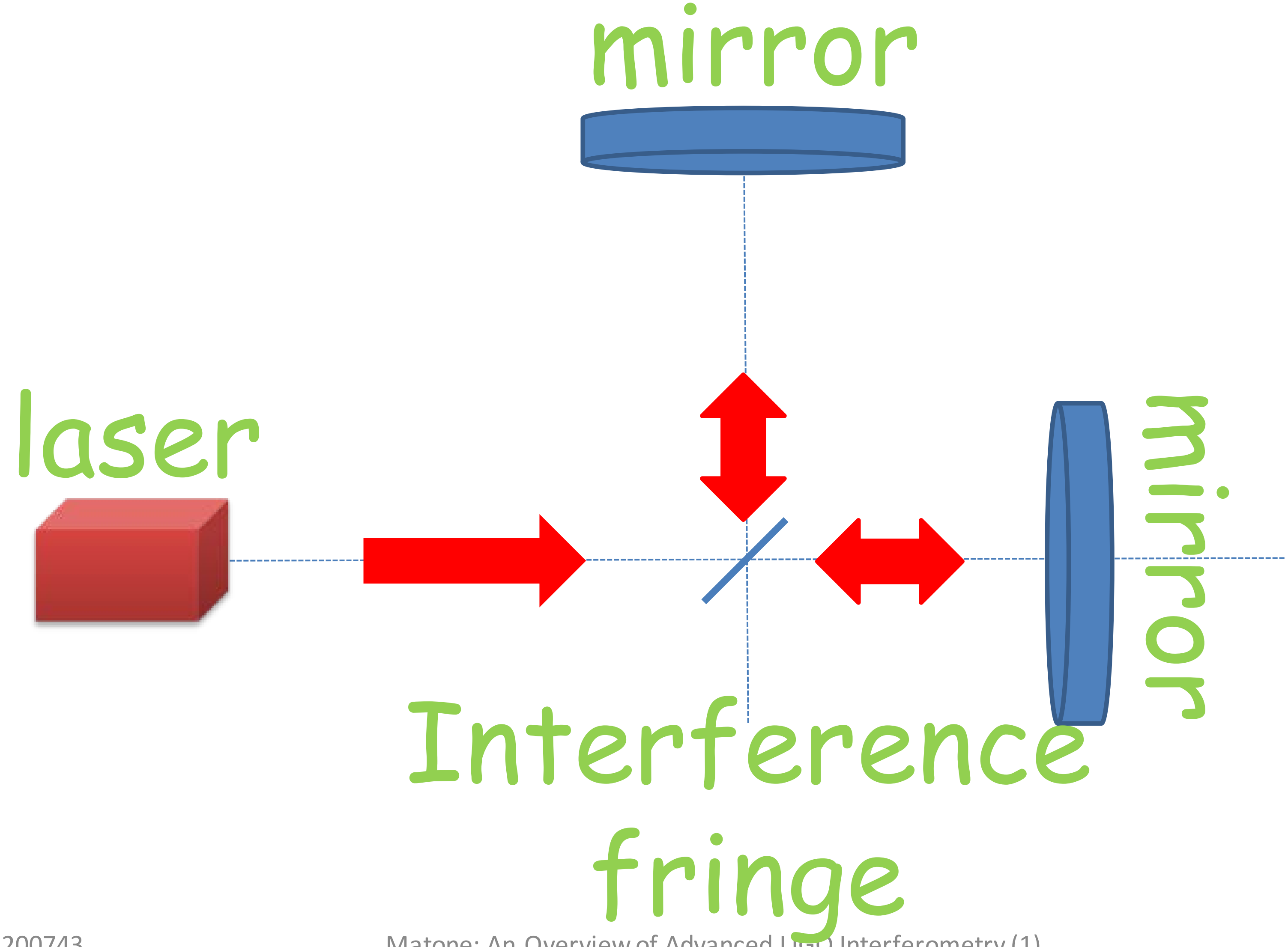
Amplitude of GWs produced by binary neutron star systems in the VIRGO cluster is expected to be

$$h \sim 10^{-21}$$

For a 4 km baseline (L)

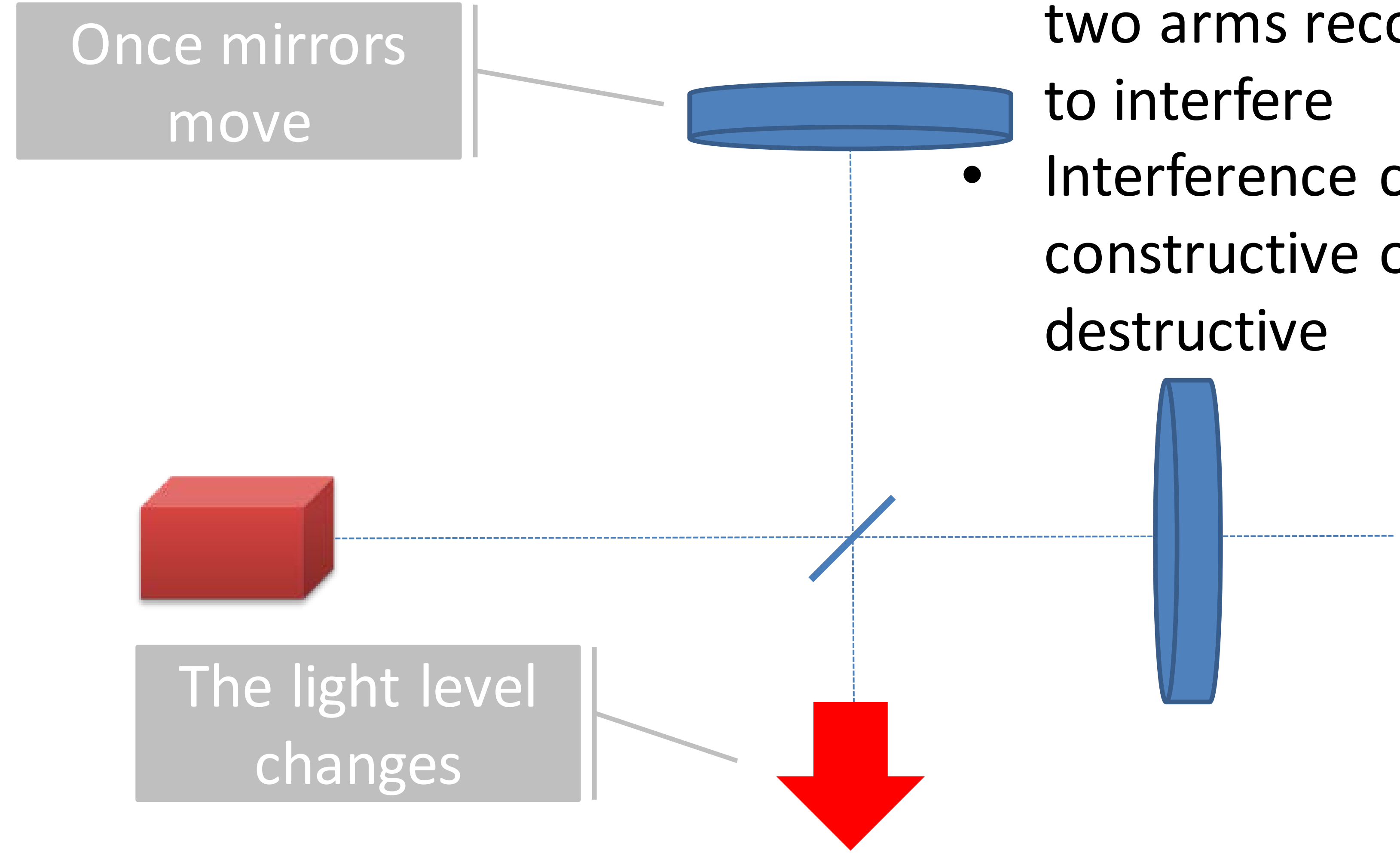
$$\Delta L = h\,L \sim 10^{-18} m$$

Idea: Michelson Interferometer (IFO) to measure ΔL



Michelson IFO to measure ΔL

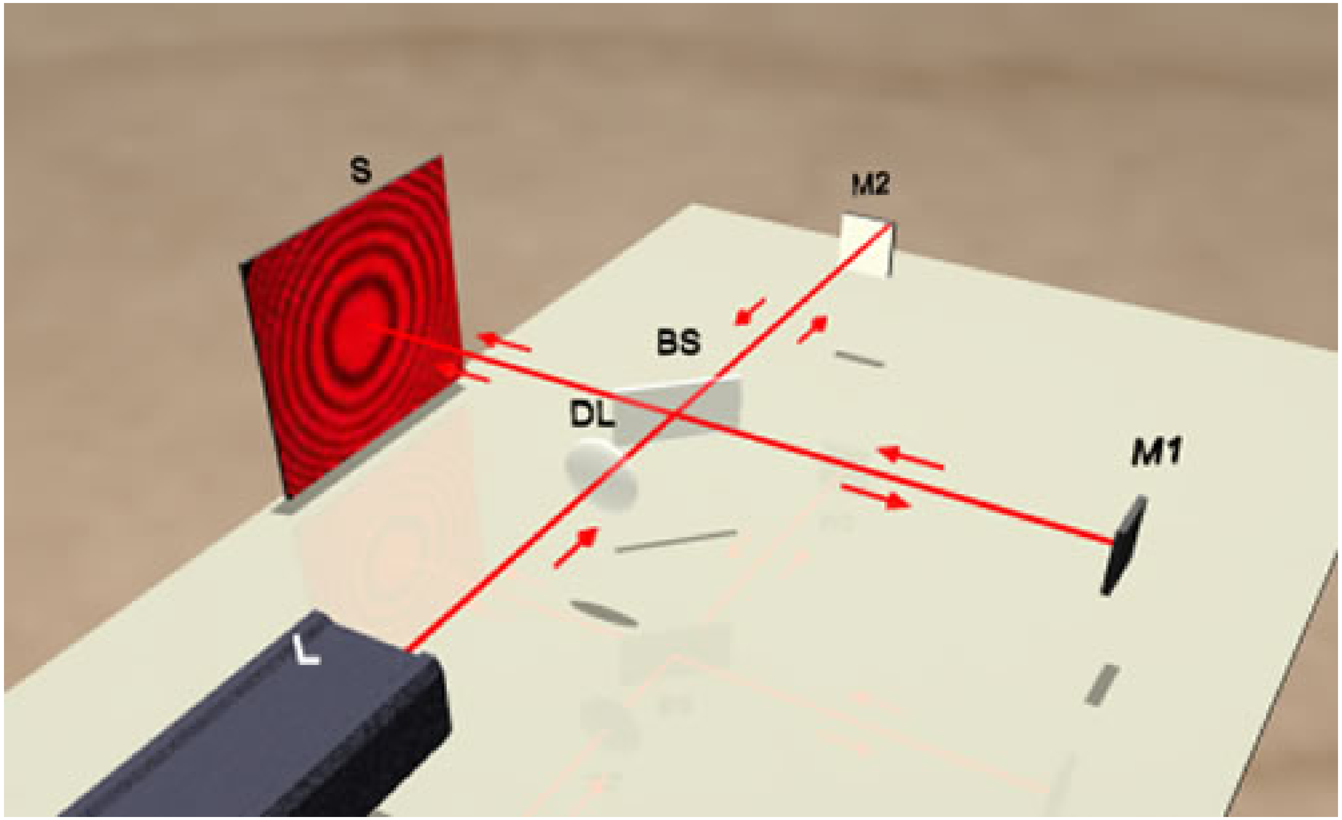
- The laser fields in the two arms recombine to interfere
- Interference can be constructive or destructive



Once mirrors move

The light level changes

How does a Michelson IFO work?



Description of a Laser Beam

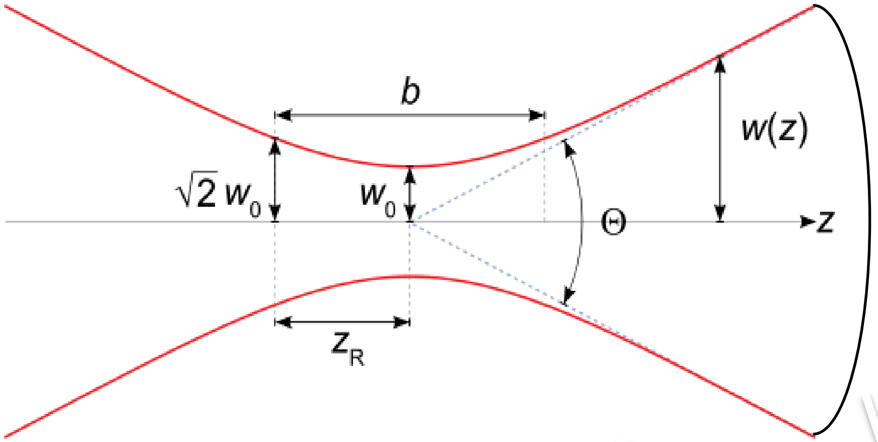
Any laser beam can be represented in terms of TEM (Propagation) modes

Gouy phase

Spot size

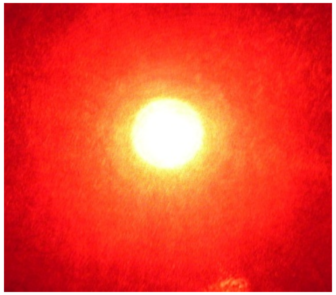
Wavefront radius

$$U_{m,n}(x,y,z) = \frac{A_{m,n}}{\boxed{w(z)}} \boxed{H_m\left(\sqrt{2}\frac{x}{w(z)}\right)} \boxed{H_n\left(\sqrt{2}\frac{y}{w(z)}\right)} \times e^{-\left(x^2+y^2\right)/w^2(z)} \times e^{-ik\left(x^2+y^2\right)/2\boxed{R(z)}} \times e^{-i\left(k\,z - \boxed{\varphi_{m,n}(z)}\right)}$$

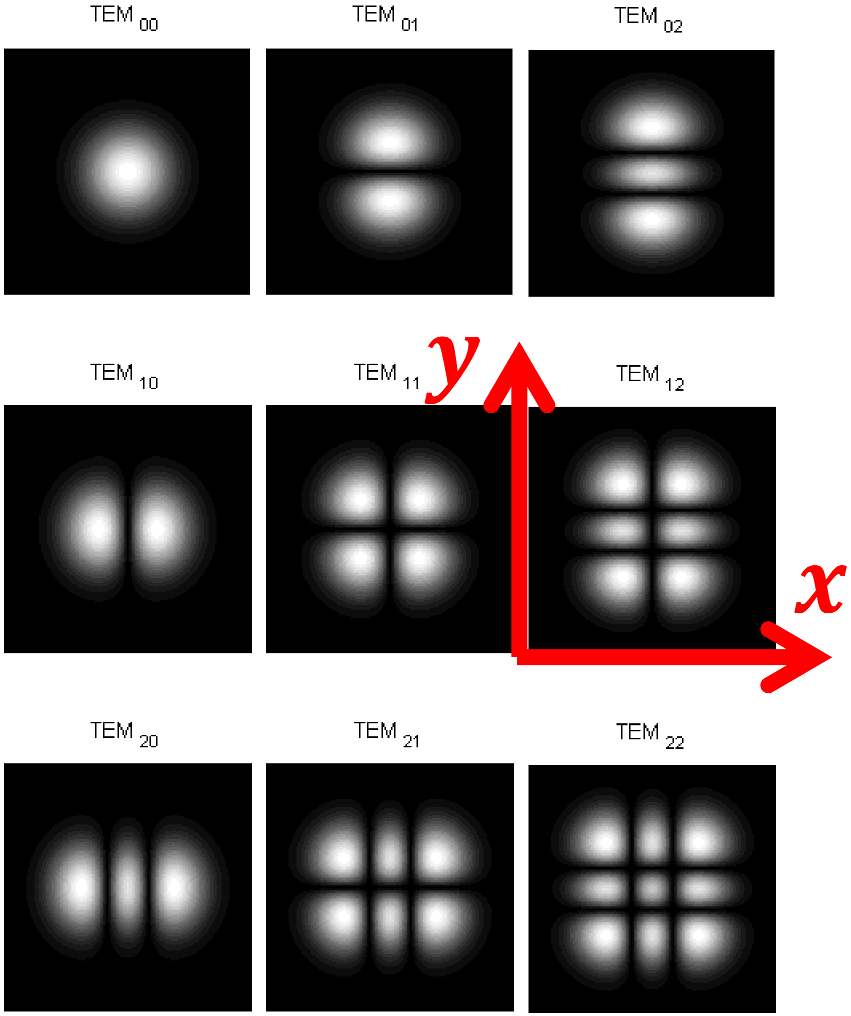


Wavefront

Hermite polynomial



Mode patterns



- TEMmn:
  - Transverse Electro-Magnetic (TEM) modes
- Figure
  - plot of $|U_{m,n}(x,y,z)|^2$ vs. position x, y

Electromagnetic wave: In general

$$\boldsymbol{\psi = A\, e^{i\varphi}}$$

Phase φ

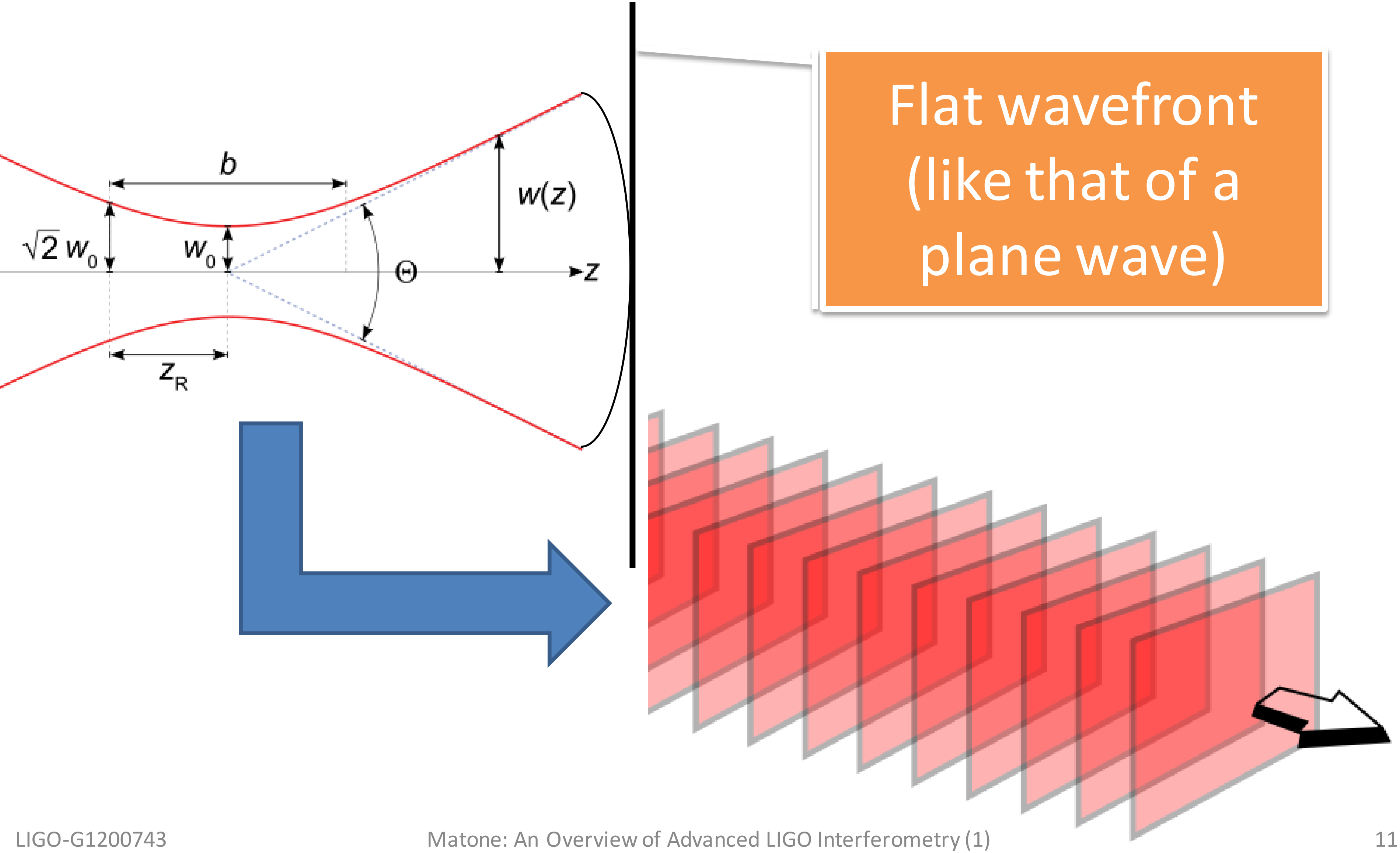
Amplitude A

Electromagnetic wave function ψ

Photodiode measures power P

$$P = |\psi|^2 = \psi\, \psi^* = A^2$$

Simplifying: Plane-Wave Approximation

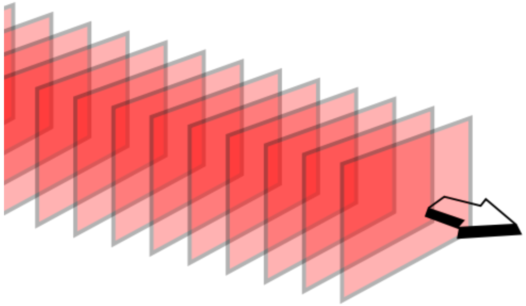


Flat wavefront (like that of a plane wave)

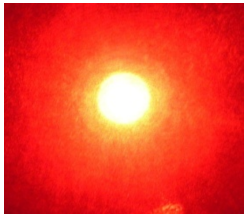
A laser beam in the plane-wave approximation

$$U_{m,n}(x,y,z) = \frac{A_{m,n}}{\cancel{w(z)}} H_m\left(\cancel{\sqrt{2}\frac{x}{w(z)}}\right) H_n\left(\cancel{\sqrt{2}\frac{y}{w(z)}}\right) \times \cancel{e^{-(x^2+y^2)/w^2(z)}} \times \cancel{e^{-ik(x^2+y^2)/2R(z)}} \times e^{-i\left(k\,z-\cancel{\varphi_{m,n}(z)}\right)}$$

$$\boxed{\Psi(z) = A\,e^{-i\,k\,z}}$$



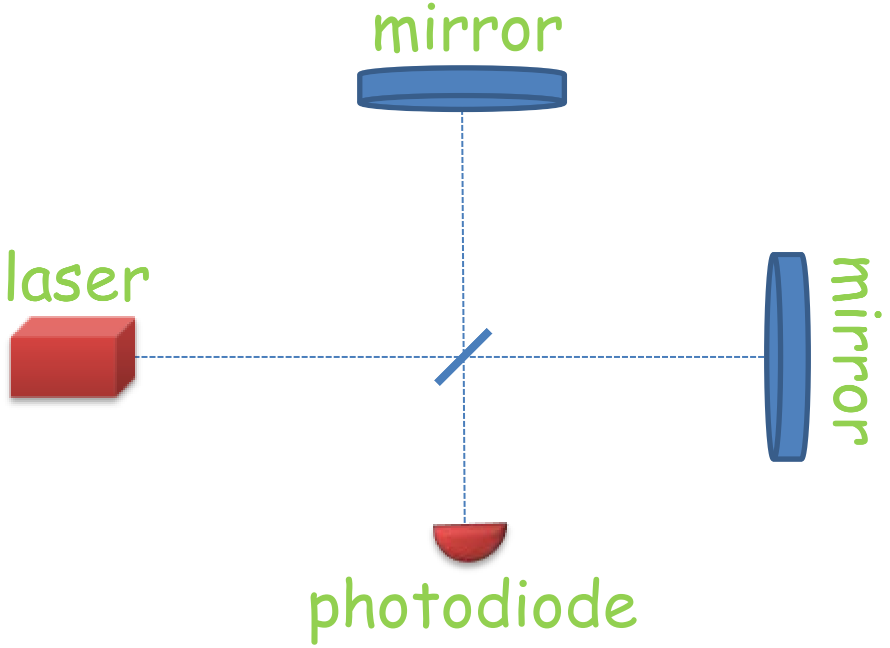
- Wave number $k = \frac{2\pi}{\lambda}$
- Propagation axis z



Modeling a Michelson IFO

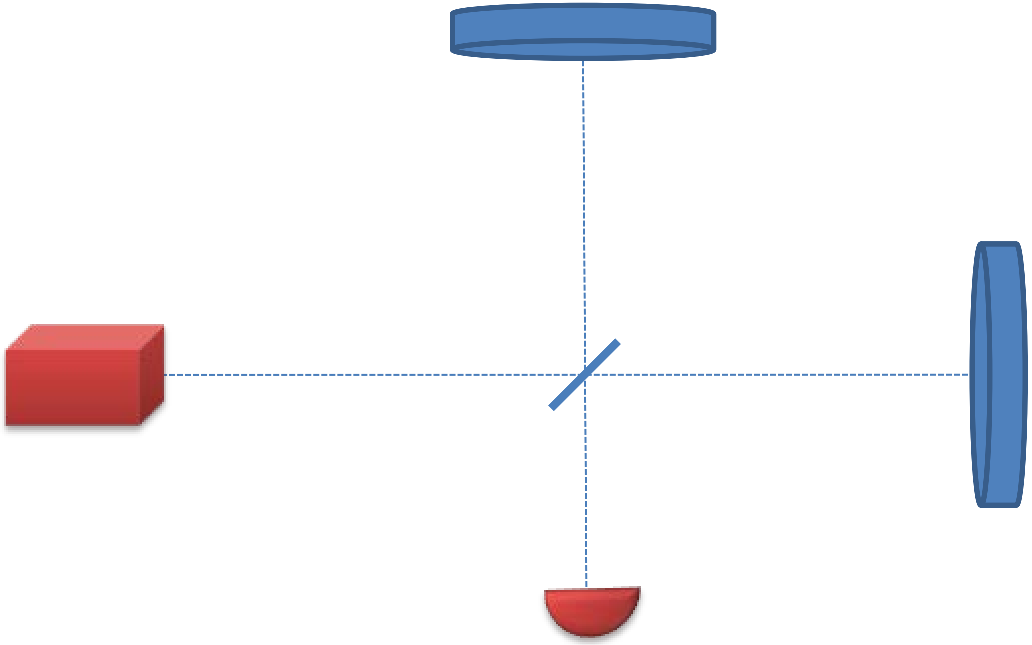
- Assuming no misalignments
- Assuming plane mirrors
- ➢ Plane-Wave approximation
  - – Helpful in conceptualizing the detector

>> <u>Powerful model</u>



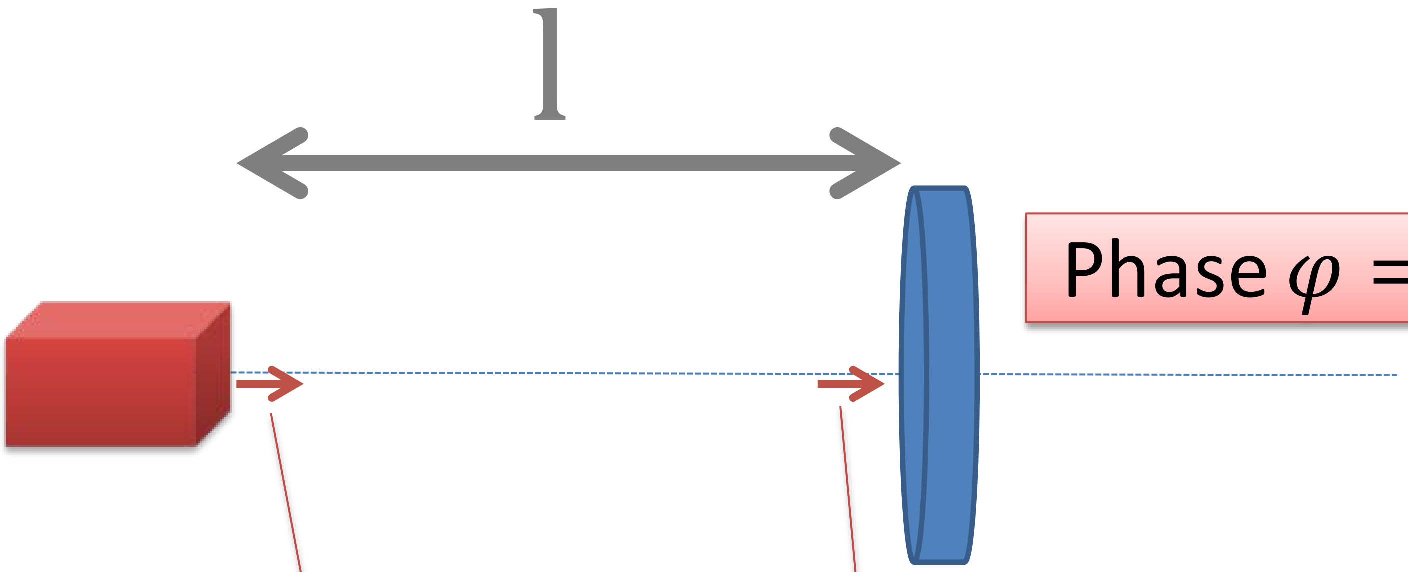
Objective:

To determine what the EM fields look like throughout the IFO



Interference port

Beam Propagation



Phase $\varphi = k\, l$

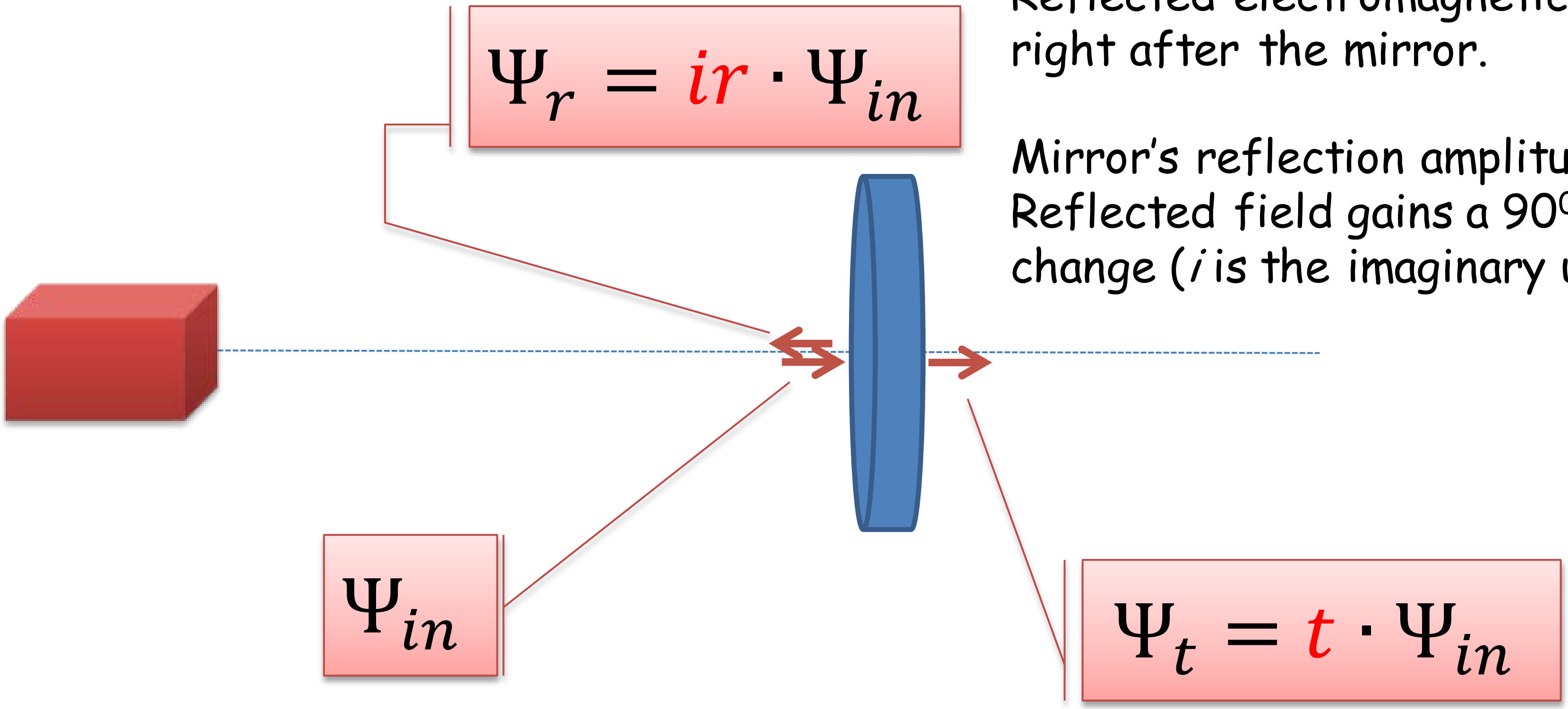
Ψ_{laser}

Electromagnetic field right after the laser

Electromagnetic field right before the mirror

$\Psi_1 = e^{-i\,k\,l} \cdot \Psi_{laser}$

Beam Reflection and Transmission



$$\Psi_r = \color{red}{ir} \cdot \Psi_{in}$$

Reflected electromagnetic field right after the mirror.

Mirror's reflection amplitude *r*. Reflected field gains a 90^0 phase change (*i* is the imaginary unit)

$$\Psi_{in}$$

Impinging electromagnetic field

$$\Psi_t = \color{red}{t} \cdot \Psi_{in}$$

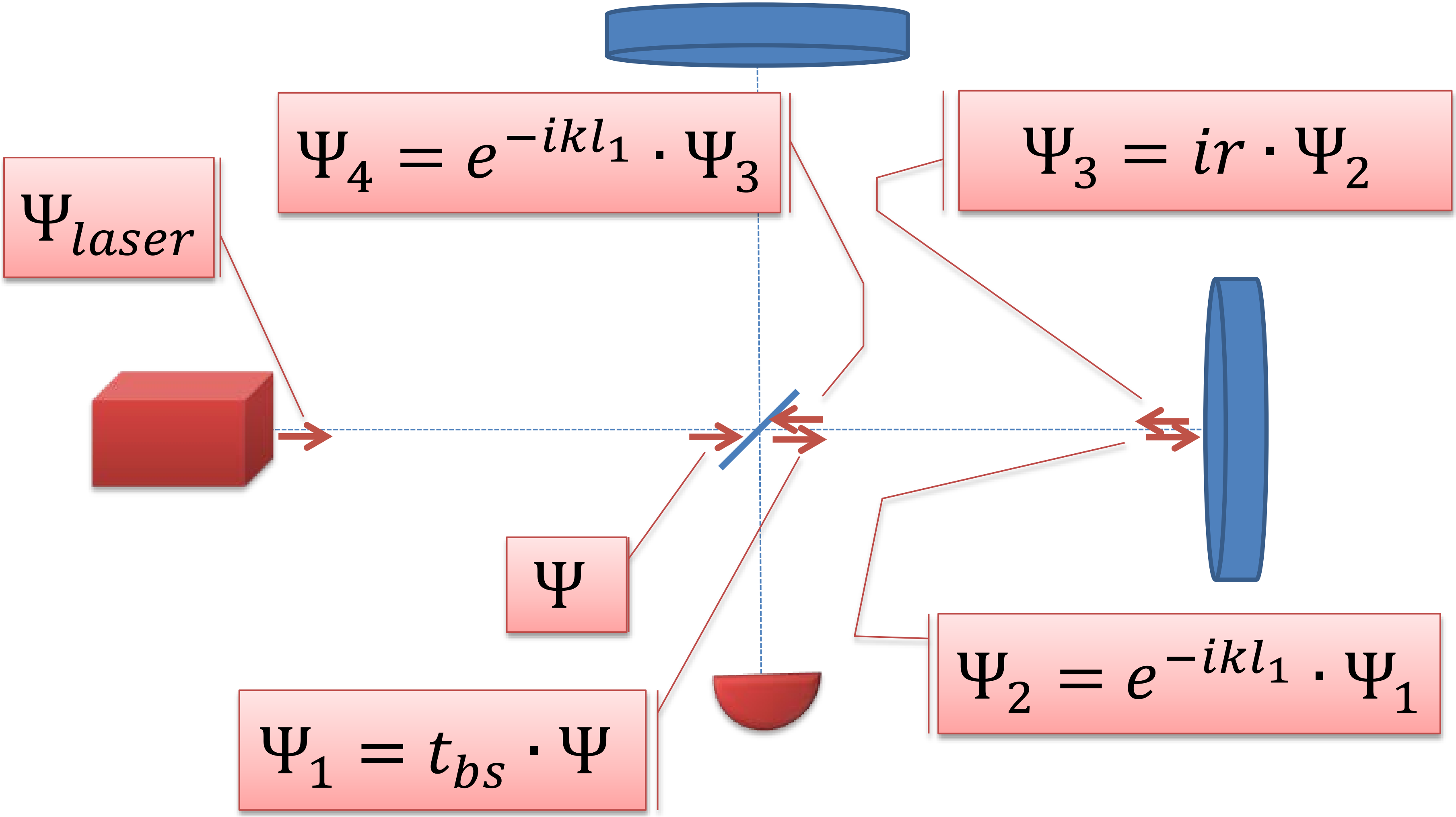
Transmitted electromagnetic field right after the mirror.

Mirror's transmission amplitude *t*

$$t^2 + r^2 = 1$$

no losses

EM fields – part 1



Ψ_{laser}

$\Psi_4 = e^{-ikl_1} \cdot \Psi_3$

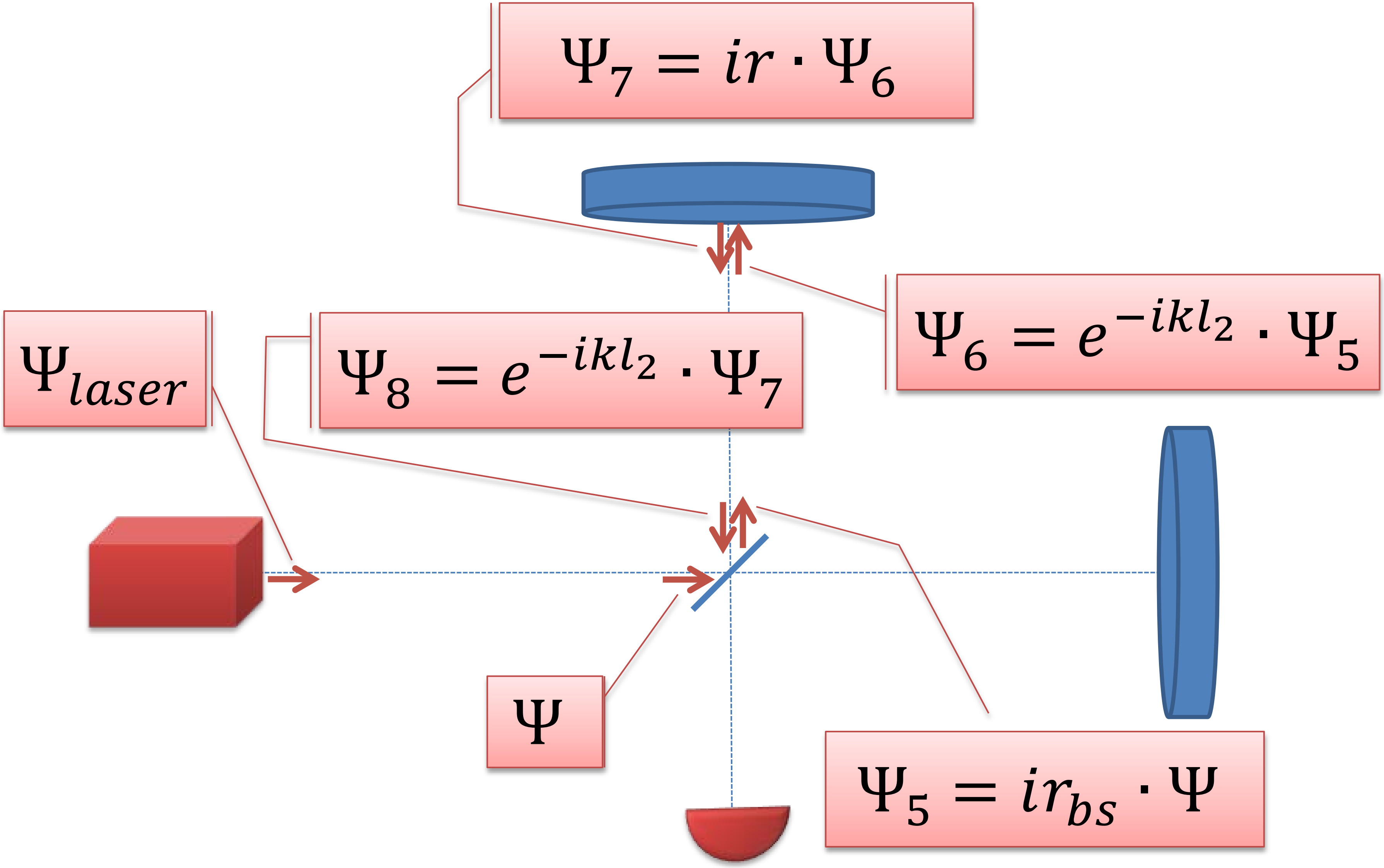
$\Psi_3 = ir \cdot \Psi_2$

Ψ

$\Psi_1 = t_{bs} \cdot \Psi$

$\Psi_2 = e^{-ikl_1} \cdot \Psi_1$

EM fields – part 2



$\Psi_7 = ir \cdot \Psi_6$

$\Psi_6 = e^{-ikl_2} \cdot \Psi_5$

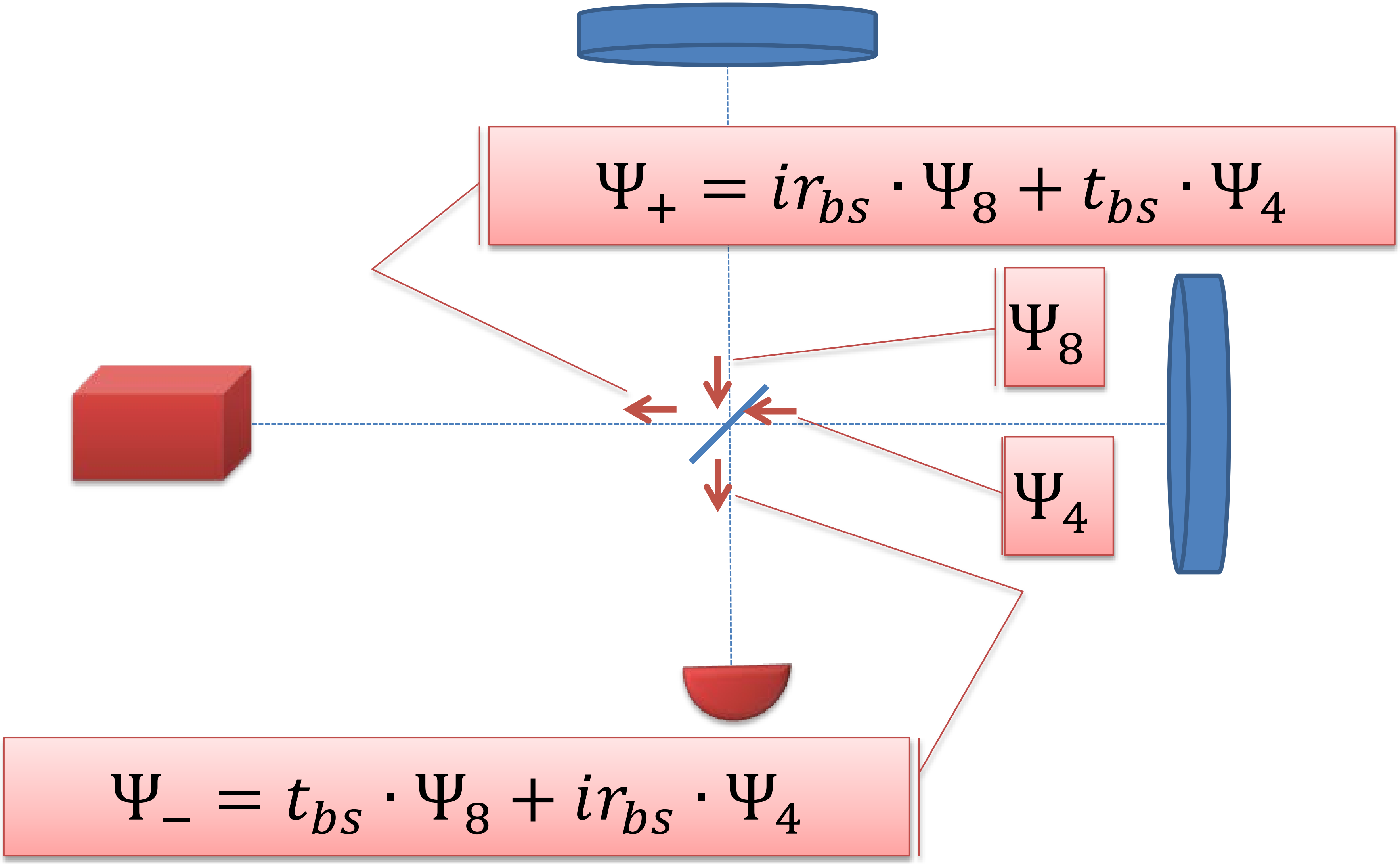
Ψ_{laser}

$\Psi_8 = e^{-ikl_2} \cdot \Psi_7$

Ψ

$\Psi_5 = ir_{bs} \cdot \Psi$

EM fields:
Common (+) and Differential (-) Ports



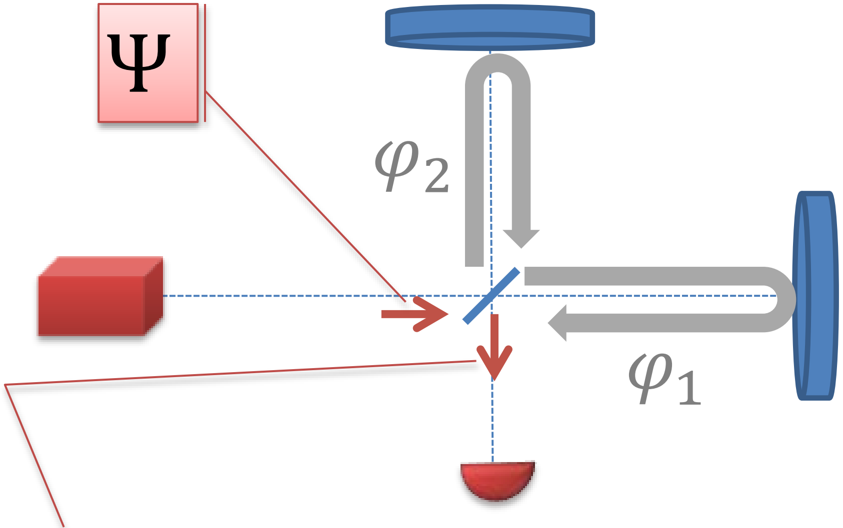
$$\Psi_+ = ir_{bs}\cdot\Psi_8 + t_{bs}\cdot\Psi_4$$

Ψ_8

Ψ_4

$$\Psi_{\_} = t_{bs}\cdot\Psi_8 + ir_{bs}\cdot\Psi_4$$

EM field at the Differential Port



Phase in arm 1

$$\varphi_1 = 2\,k\,l_1$$

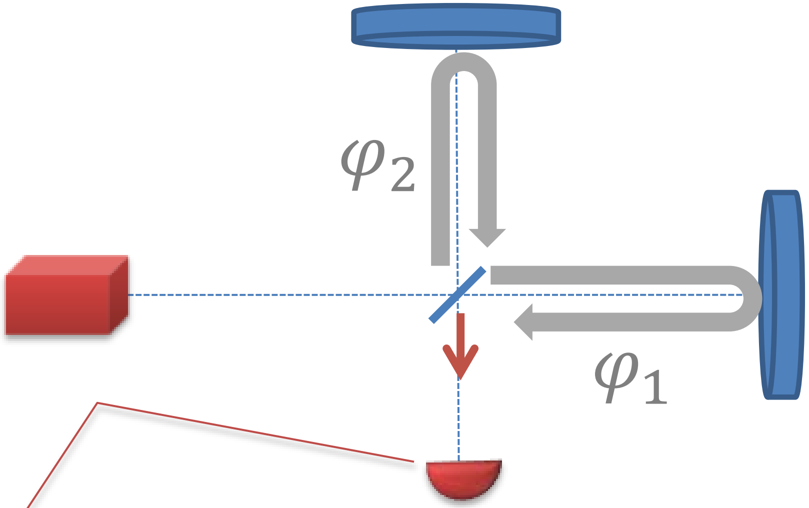
Phase in arm 2

$$\varphi_2 = 2\,k\,l_2$$

$$\Psi\_ = -\frac{1}{2}\,\Psi\left(e^{-i\,\varphi_1} + e^{-i\,\varphi_2}\right)$$

Assuming no losses, highly reflective end mirrors $(r^2 = 1)$ and a symmetric beam-splitter $({r_{bs}}^2 = {t_{bs}}^2 = 50\%\,)$

Power at the Differential Port

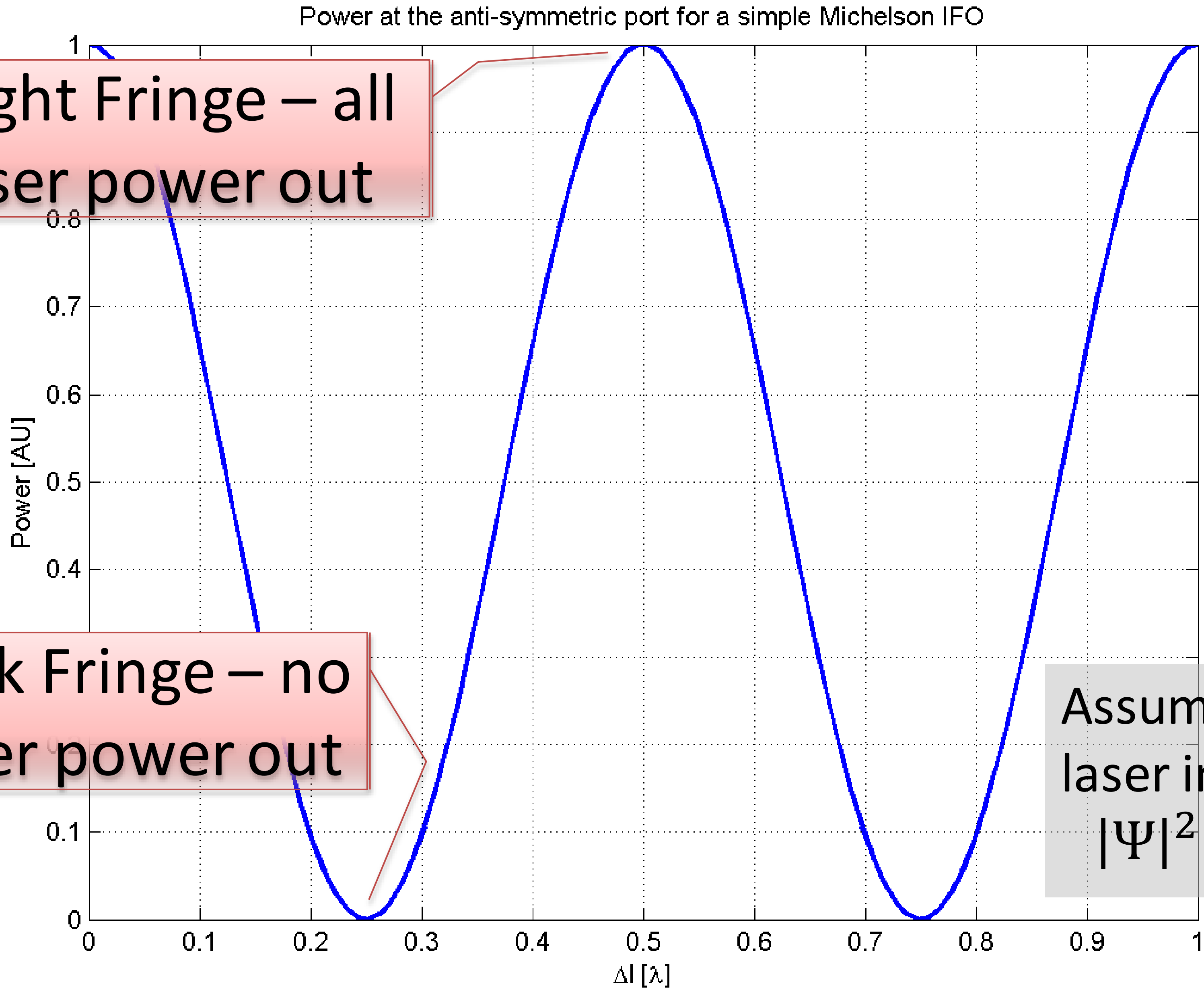


$$\Delta\varphi = \varphi_2 - \varphi_1$$

$$P_- = |\Psi_-|^2 = \frac{1}{2}\,|\Psi|^2 \cdot (1 + \cos\Delta\varphi)$$

Power level depends on the <u>phase difference $\Delta\varphi$</u> (*differential changes*) between the two arms.

Power at the Differential Port



Bright Fringe – all laser power out

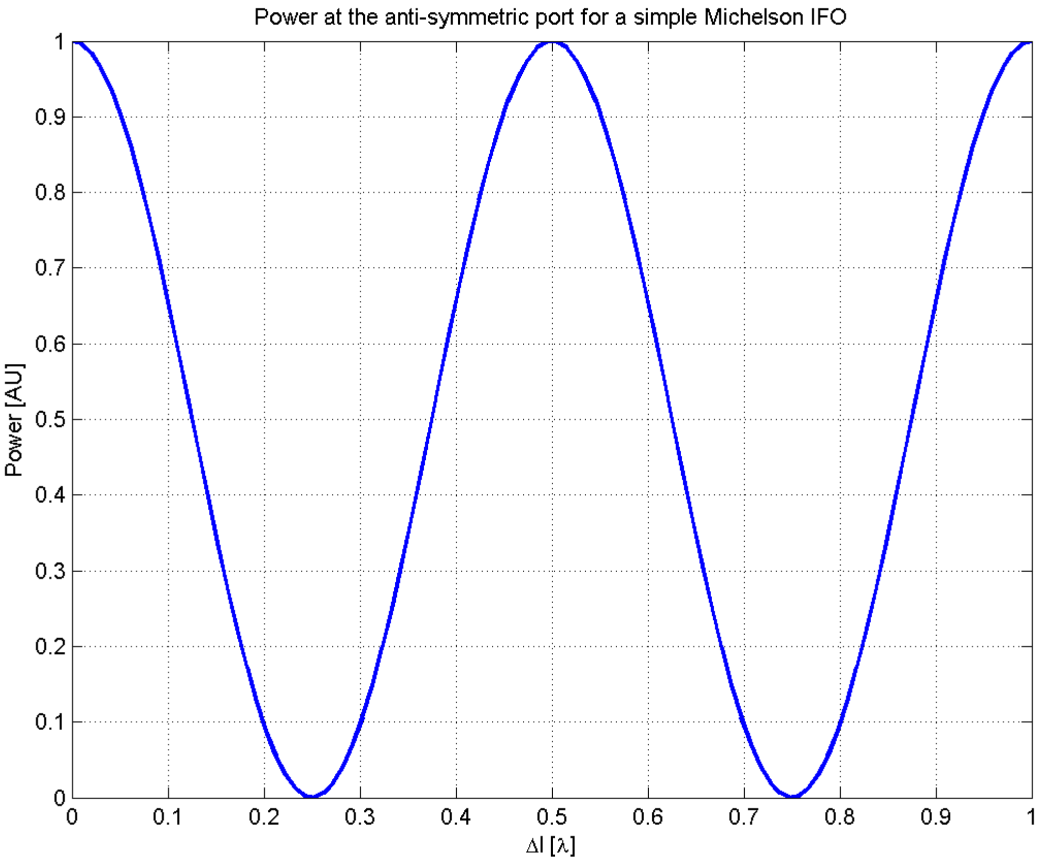
Dark Fringe – no laser power out

Assuming a laser input $|\Psi|^2 = 1$

Simplemichelson.m

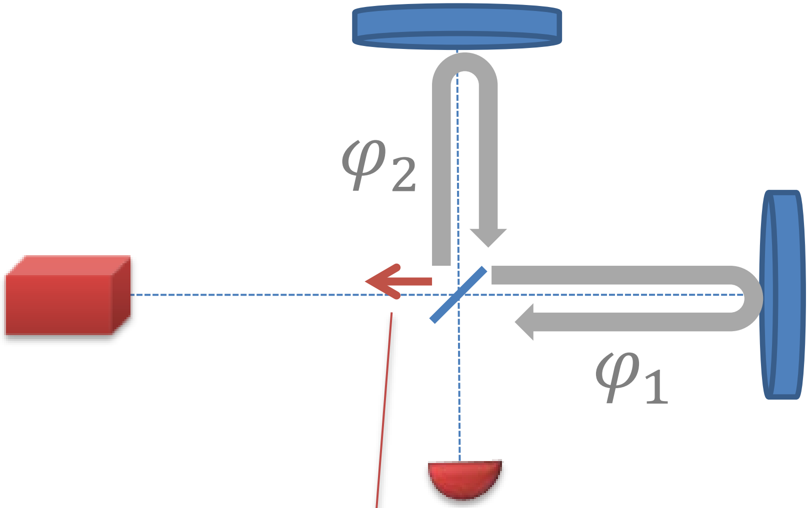
Power at the Differential Port

- Periodic function
- Power level depends on $\Delta\varphi$ (or Δl)
- General idea
  - Measure power change to determine Δl



Common Port

Power level depends on the phase sum φ_+ (*common changes*).

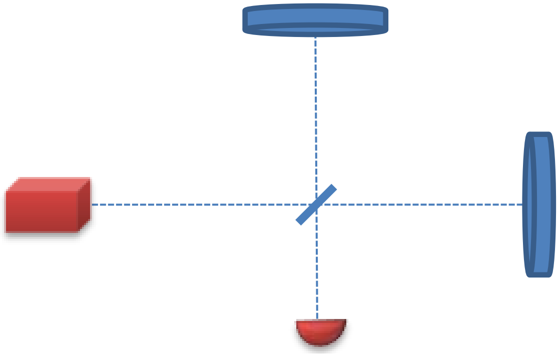


$$P_+ = \frac{1}{2} |\Psi|^2 \cdot (1 + \cos\varphi_+)$$

$$\varphi_+ = \varphi_2 + \varphi_1$$

So far…

- The passage of a GW induces a change in the arm lengths Δl
- The arm length change Δl generates a change in the beam phases $\Delta\varphi$
- This phase change $\Delta\varphi$, in turn, causes a change in the light power ΔP.
- A photodiode then converts this light power to an electrical signal.



$\Rightarrow$ Measurement of gravitational wave amplitude h!

But…

- Simple Michelson IFO not enough to measure

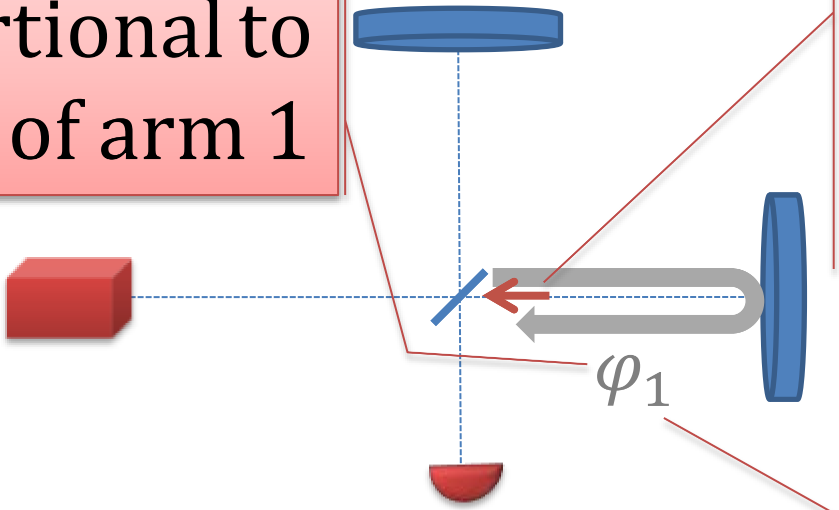
$$\Delta L \sim 10^{-18}\ m$$

- Strategy
  - Amplify signal
  - Decrease and/or control noise contribution
- Noise sources: two categories
  - Displacement noise
    - Ground seismic excitation
    - Thermal excitation of optical elements and suspensions
    - Radiation pressure
  - Phase noise
    - Amplitude and frequency fluctuations of the incoming laser beam
    - Shot-noise, the quantum limit to the *counting* of photons

Signal amplification

- Let’s look at how to amplify the effect of a GW onto a Michelson IFO
- Recall:

φ_1 is proportional to length of arm 1



Beam coming back to BS has phase φ_1:

$$\Psi_4 = i\, r\, t_{bs} e^{-i\varphi_1}$$

where

$$\varphi_1 = 2\, k\, l_1$$

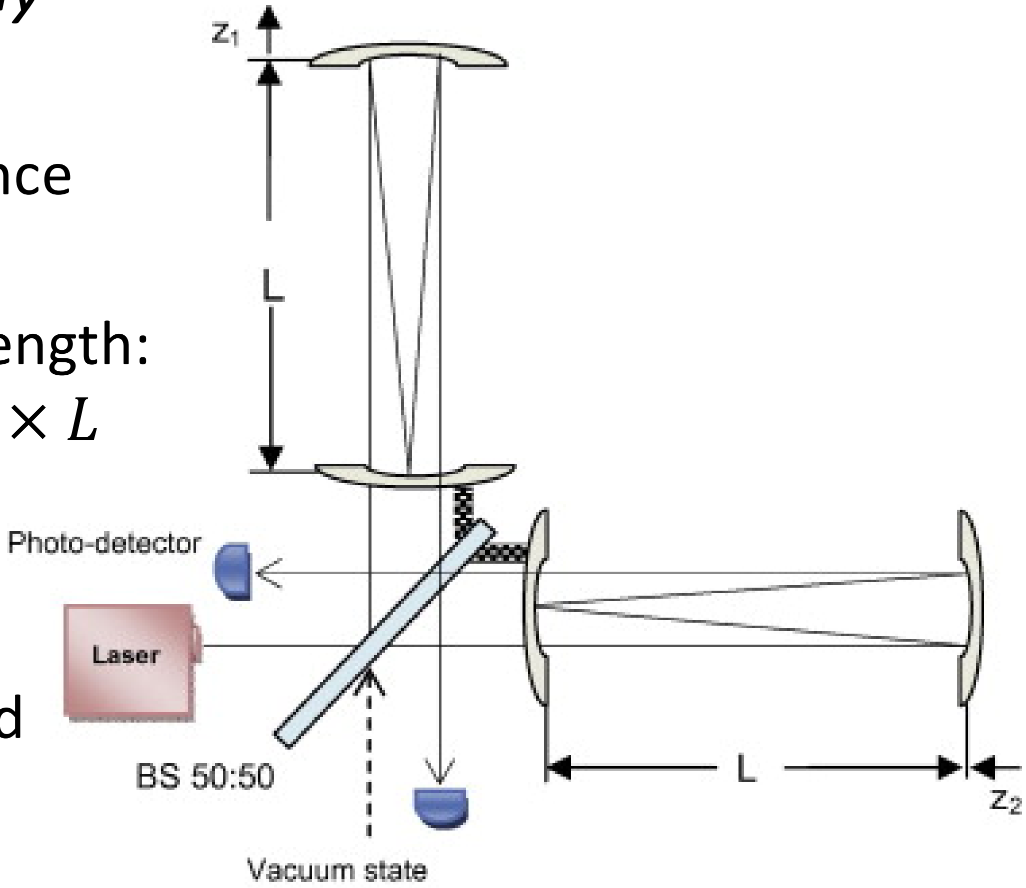
Phase φ_1 carries information about arm length 1

Michelson arms with Delay-Lines

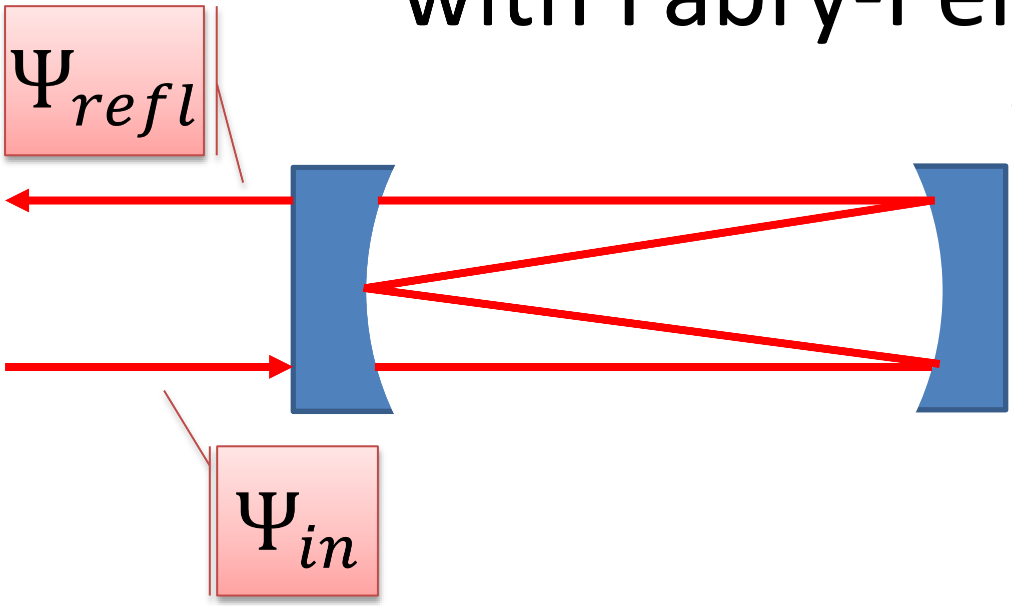
- Solution: Michelson arms replaced with optical *delay-lines*
  - Laser beam forced to bounce several times in one arm
  - *Effective* increase of arm length:

$$L_{effective} = n.of\ bounces\ \times L$$

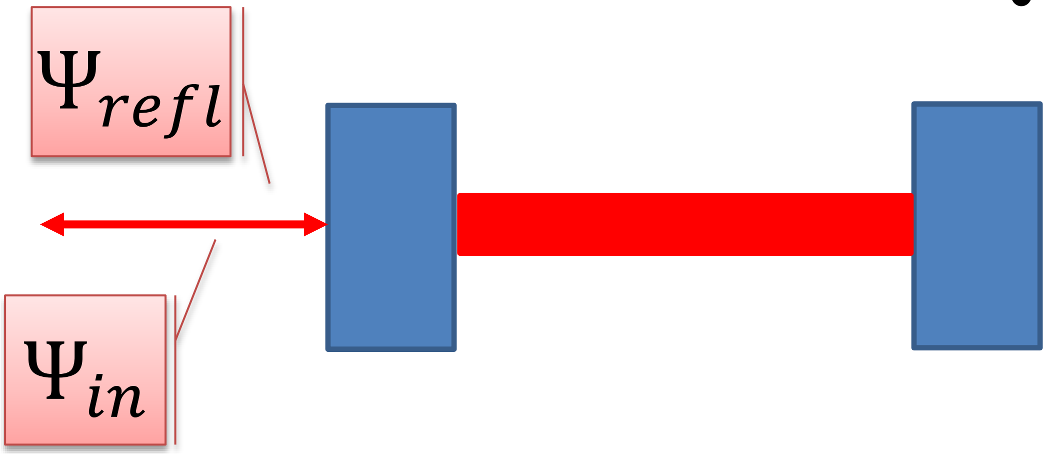
- But
  - Difficult to manufacture
  - Number of bounces limited



Alternative: Michelson arms with Fabry-Perot cavities

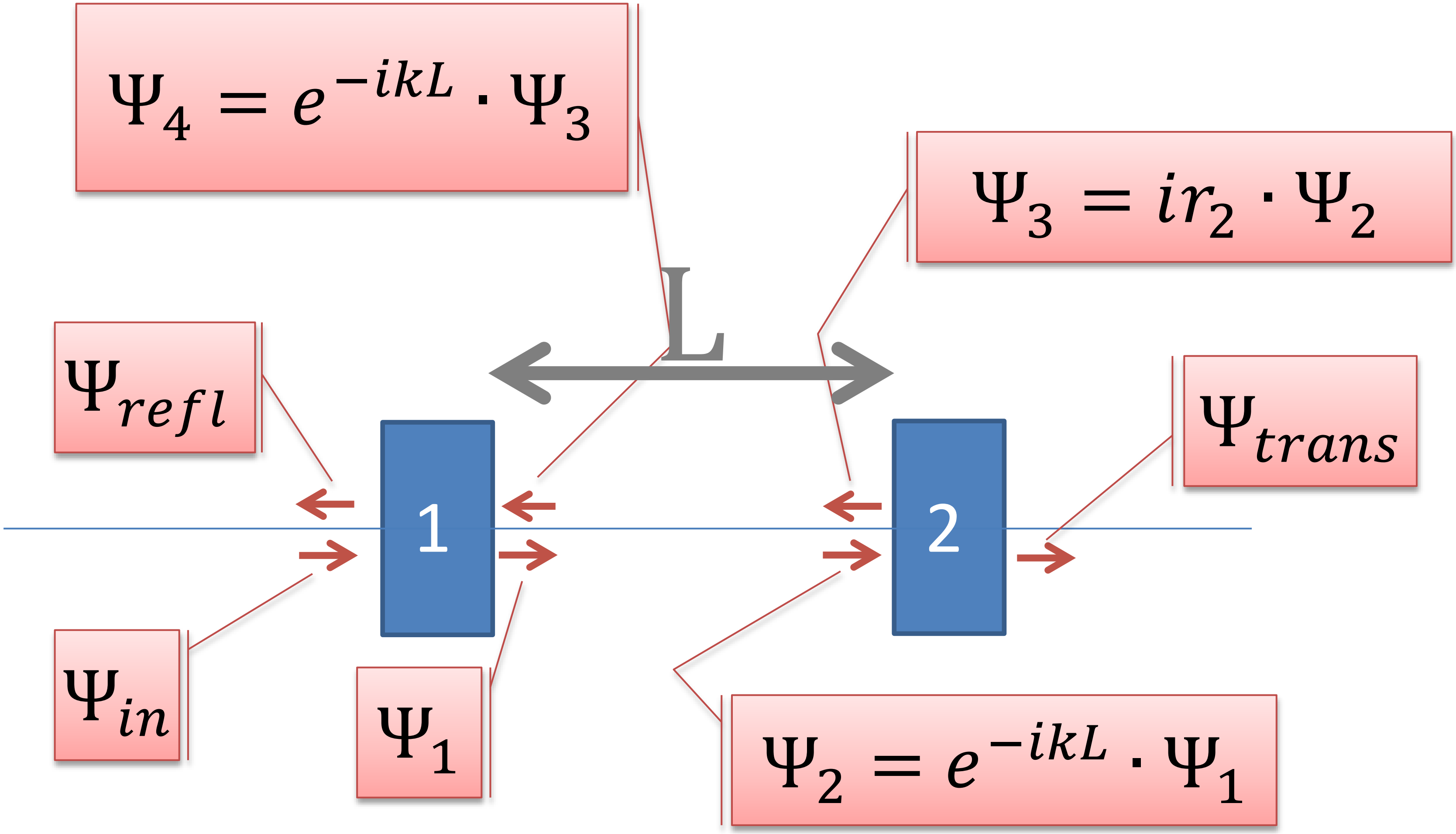


- Delay-line
  - Beams bounce multiple times (4x in diagram)
  - Beams do not overlap



- Fabry-Perot
  - Partly reflective mirrors
  - Photons bounce multiple times
  - Beams overlap in space

What's the advantage?



$\Psi_4 = e^{-ikL} \cdot \Psi_3$

$\Psi_3 = ir_2 \cdot \Psi_2$

Ψ_{refl}

Ψ_{trans}

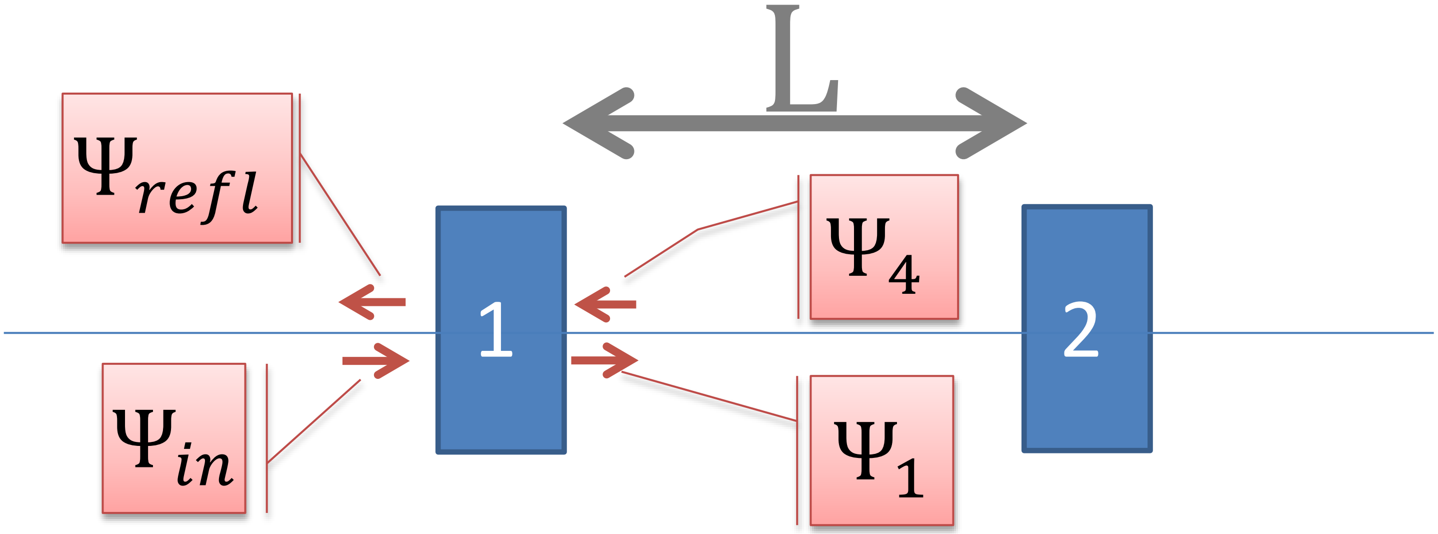
Ψ_{in}

Ψ_1

$\Psi_2 = e^{-ikL} \cdot \Psi_1$

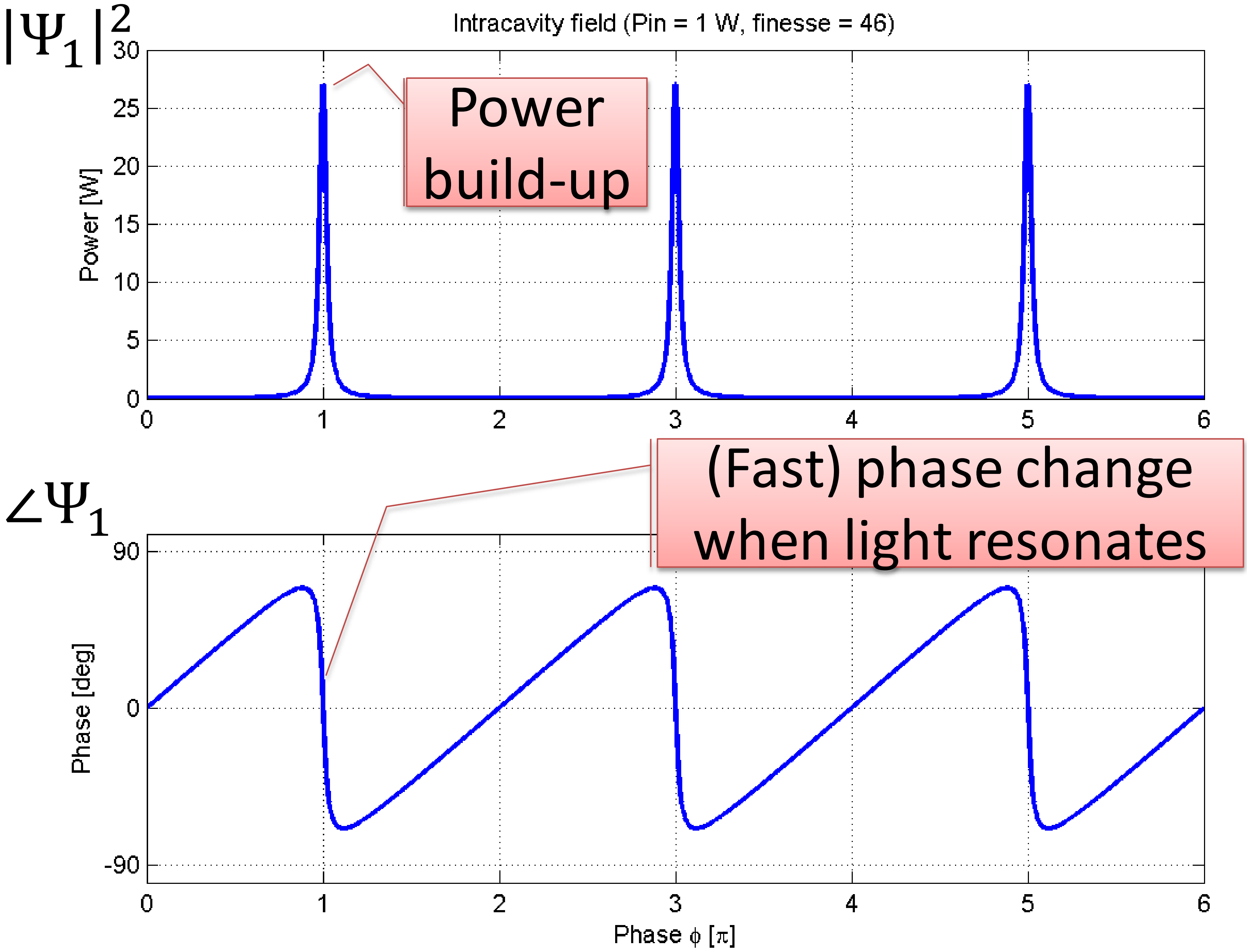
$$\Psi_1 = ir_1\Psi_4 + t_1\Psi_{in}$$

$$\begin{aligned}\Psi_1 &= ir_1\Psi_4 + t_1\Psi_{in} = ir_1\left(e^{-ikL}\cdot\Psi_3\right) + t_1\Psi_{in}\\ &= ir_1\left(e^{-ikL}\cdot ir_2\cdot\Psi_2\right) + t_1\Psi_{in} =\\ &ir_1\left(e^{-ikL}\cdot ir_2\cdot e^{-ikL}\cdot\Psi_1\right) + t_1\Psi_{in}\end{aligned}$$



$$\Psi_1 = \frac{t_1}{1 + r_1 r_2 e^{-i\varphi}}\Psi_{in}$$

$$\varphi = 2\,k\,L$$



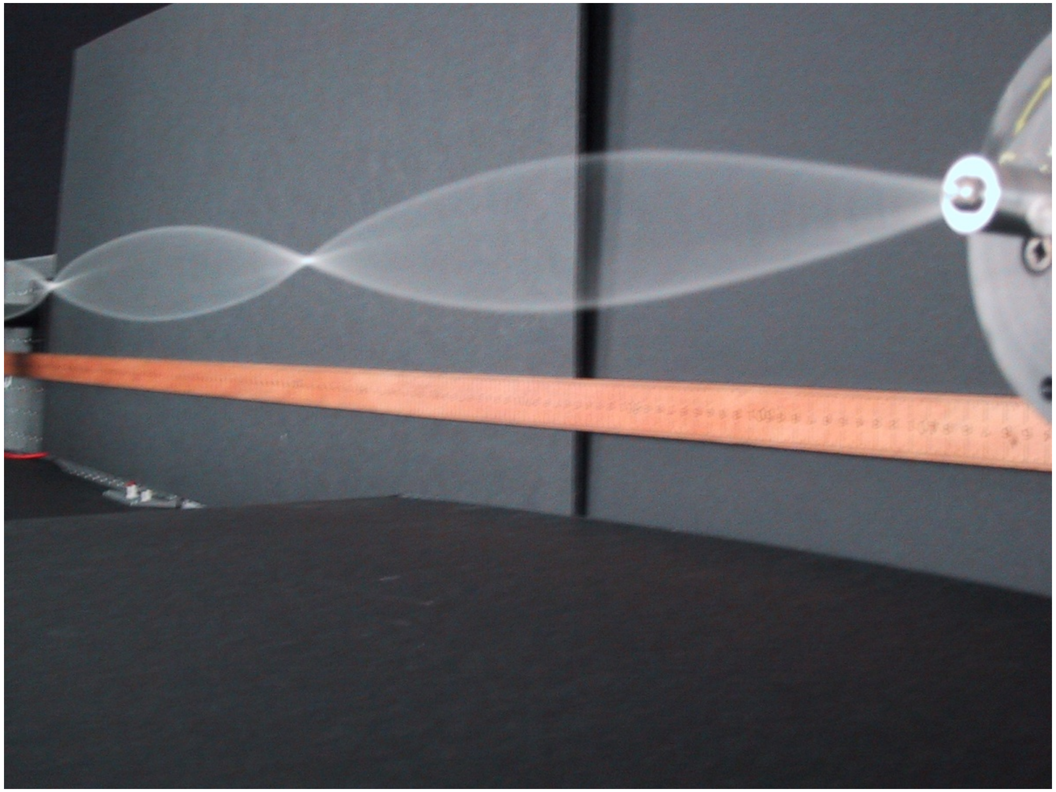
$|\Psi_1|^2$

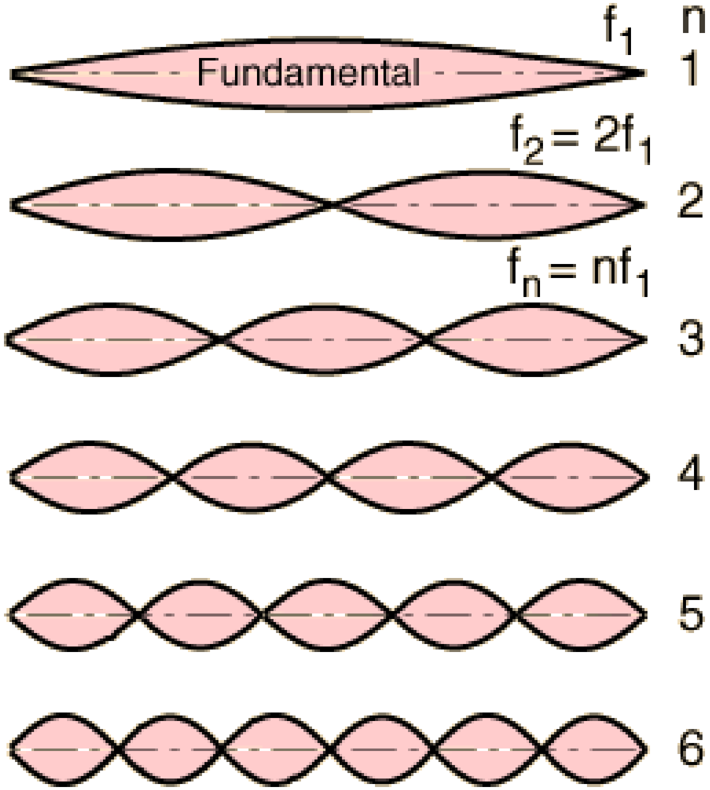
Power build-up

$\angle\Psi_1$

(Fast) phase change when light resonates

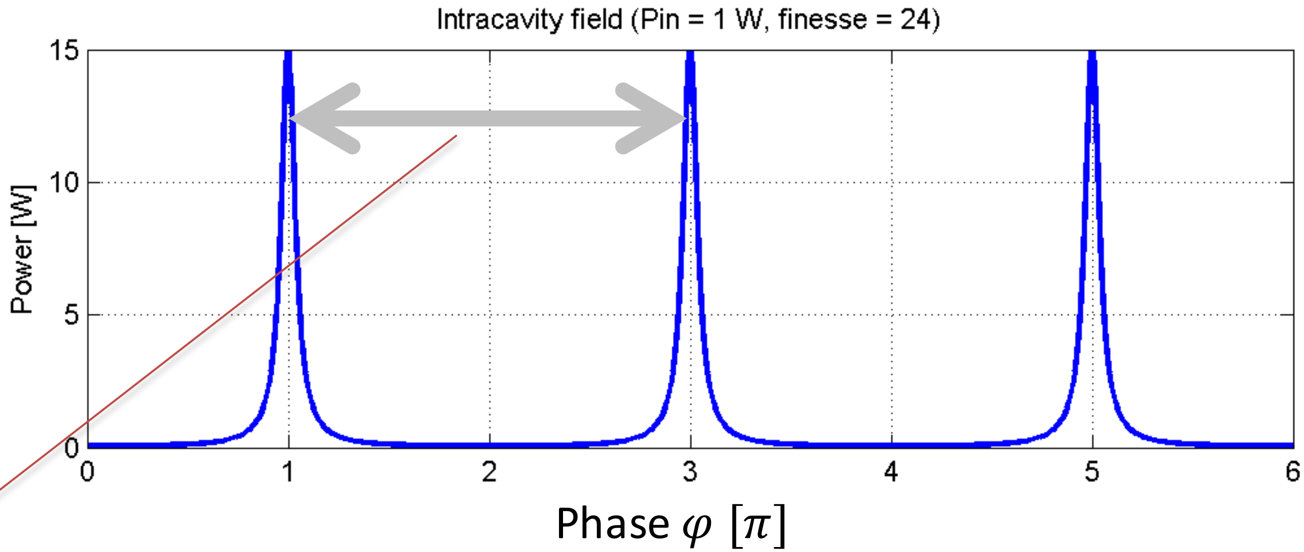
Power build-up:
Just like standing waves





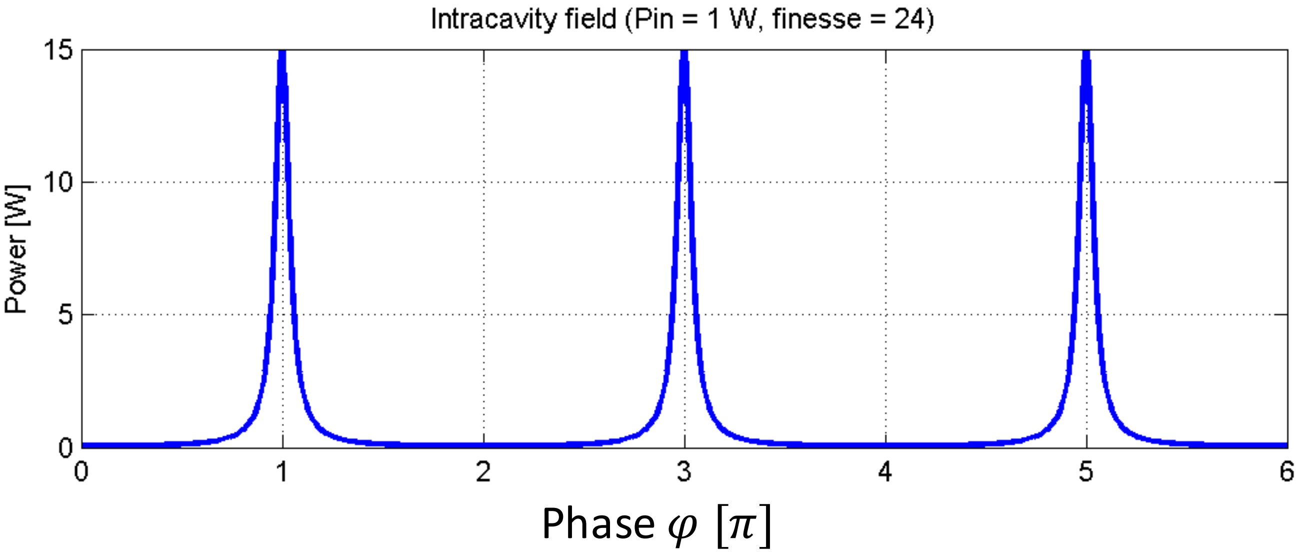
Free Spectral Range (FSR)

The spacing in *frequency* ν_{fsr} or *wavelength* between successive resonances



$2\,\pi$ (phase) or $^{\lambda}/_{2}$ (length) or
$\nu_{fsr} = {}^{c}/_{2\,L}$ (frequency)

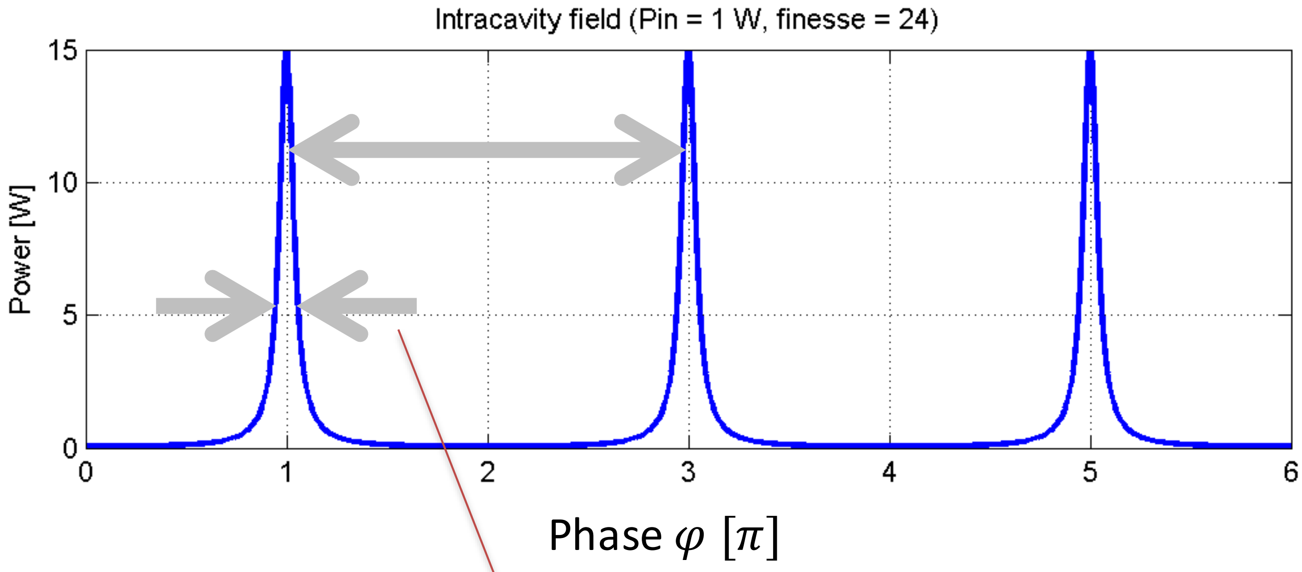
Note



$$\varphi = 2\,k\,L = 2\,\pi\,\frac{\nu}{\nu_{fsr}}$$

where ν is the laser frequency

Finesse $\mathcal{F}$



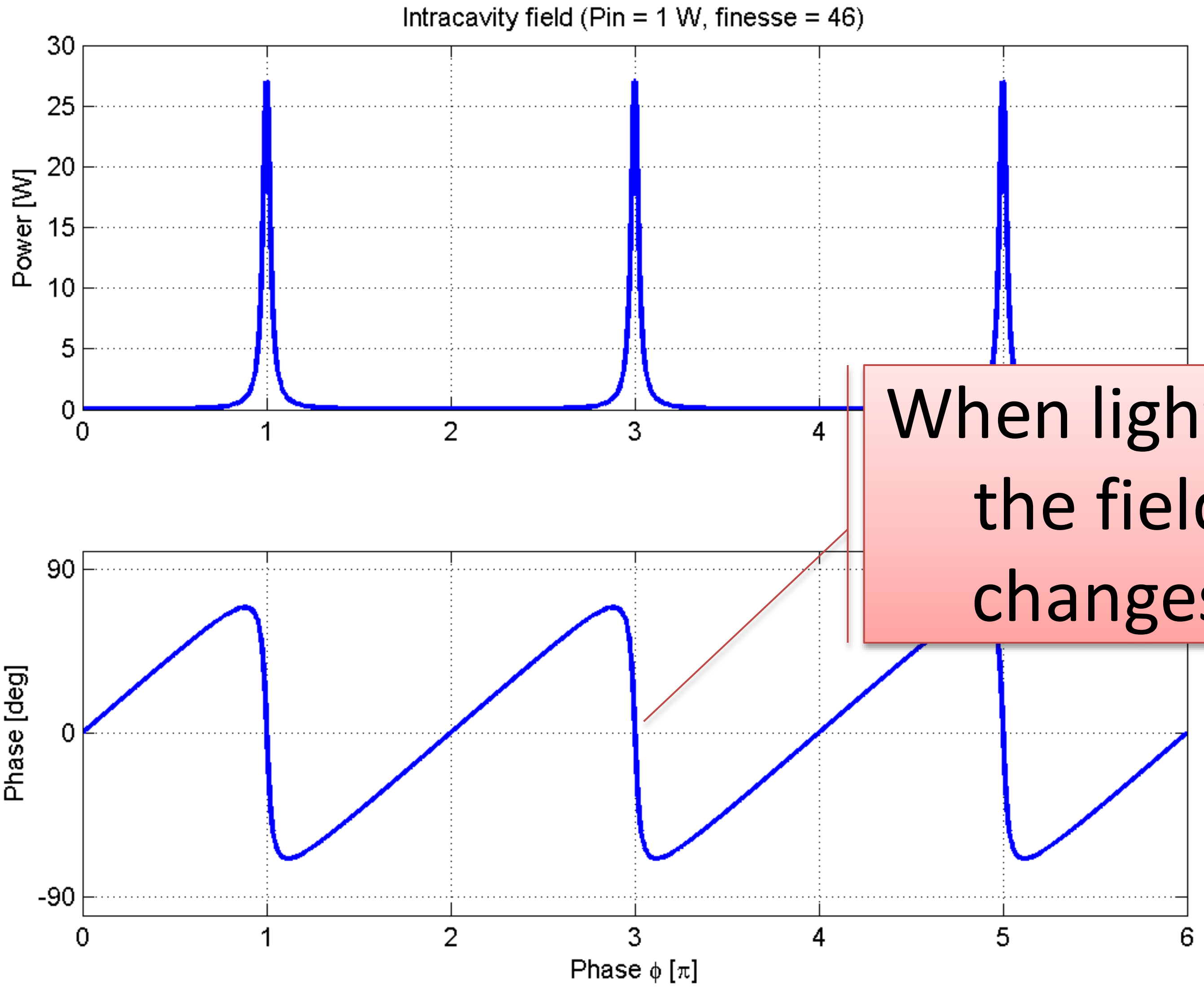
Defined as

$$\mathcal{F} = \frac{FSR}{FWHM} \simeq \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Full-Width-at-Half-Maximum (FWHM)

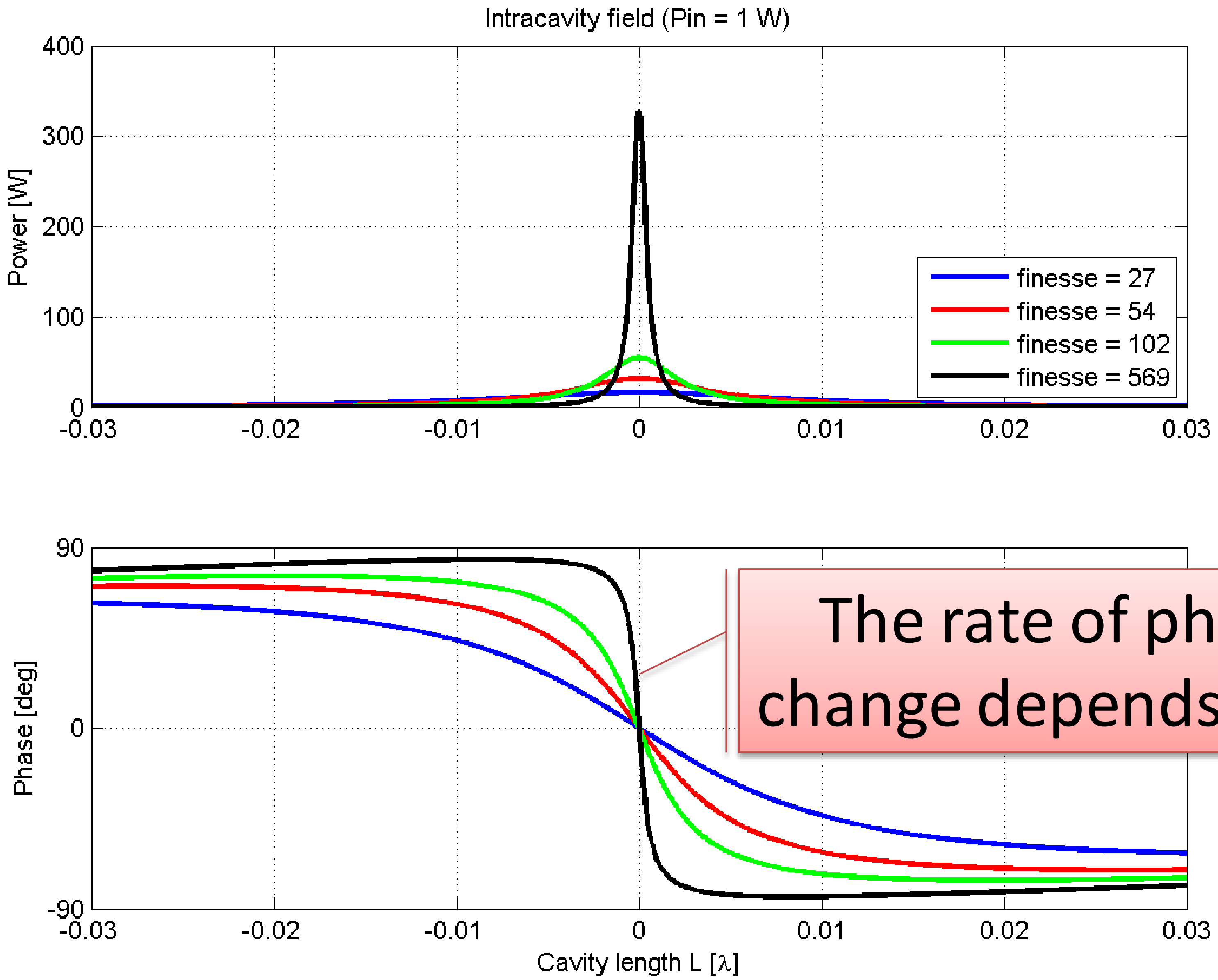
- Like quality factor Q
- Determines power build-up
- *Average* number of photon bounces

So – what is the advantage of having Fabry-Perot cavities?



When light resonates, the field's phase changes rapidly!

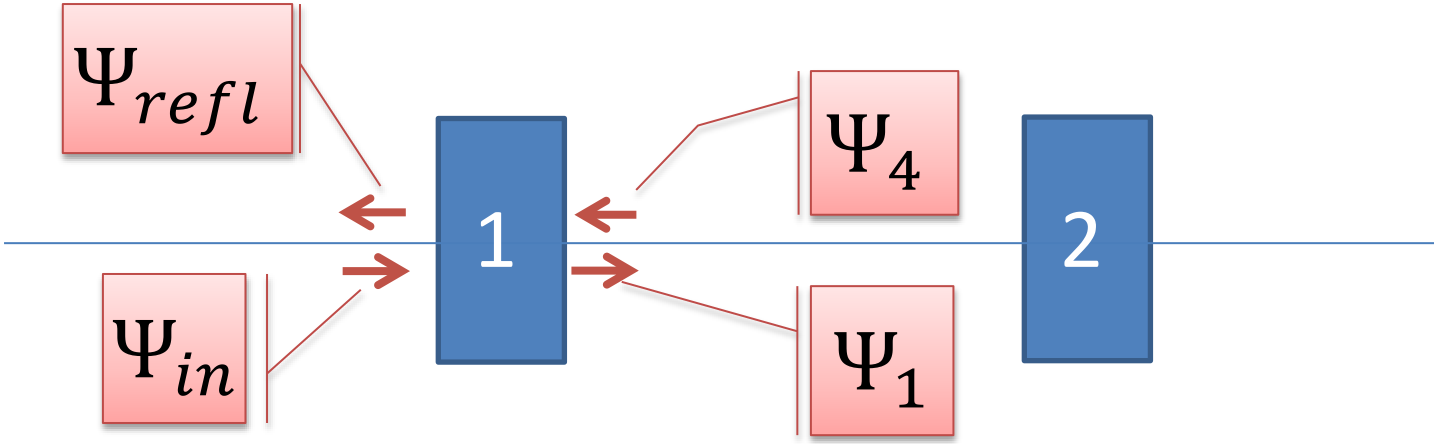
Intra-cavity field as a function of $\mathcal{F}$



The rate of phase change depends on $\mathcal{F}$

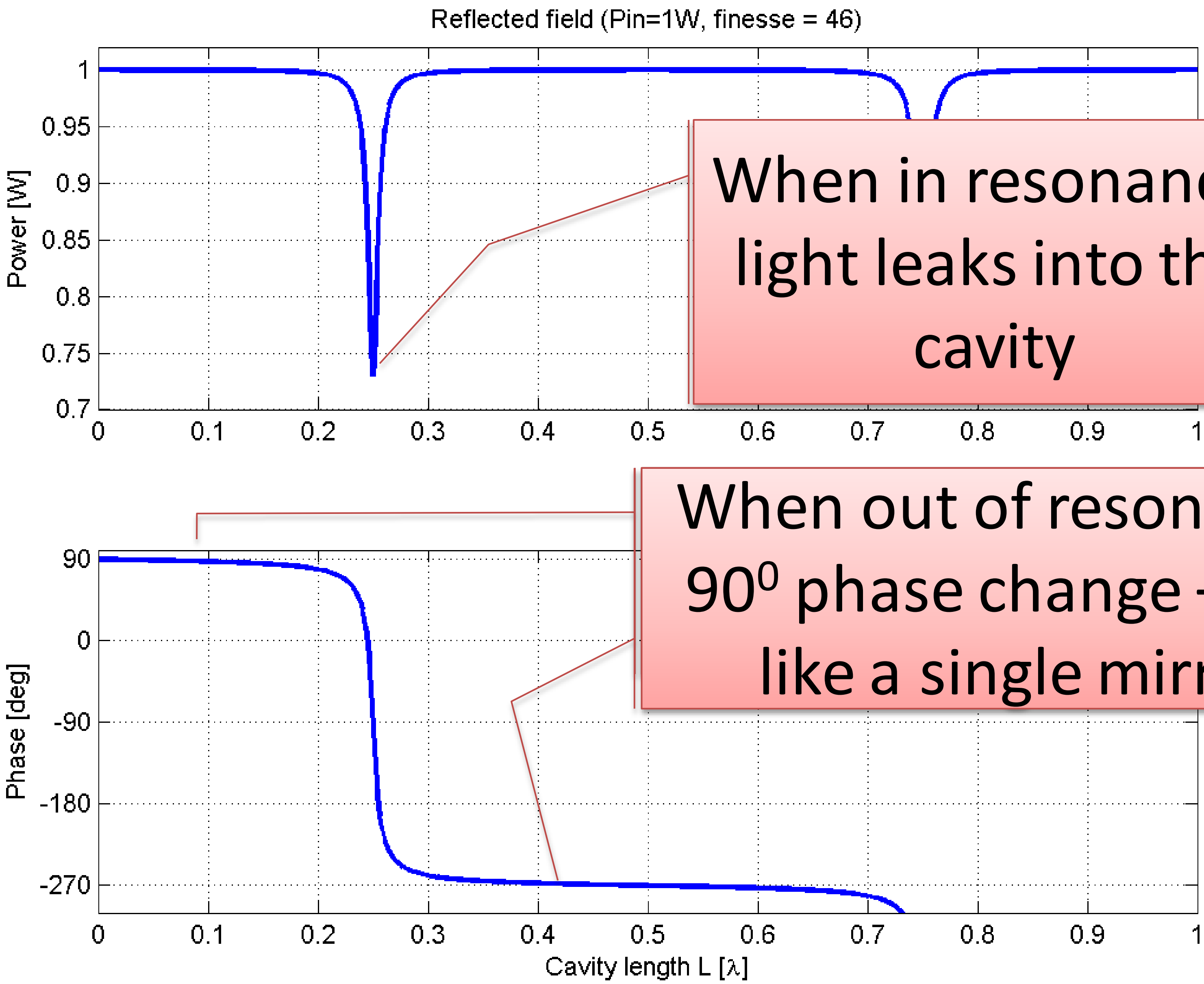
Run_fp.m

$$\Psi_{refl} = i r_1 \Psi_{in} + t_1 \Psi_4$$



$$\boxed{\Psi_{refl} = i\,\frac{r_1 + r_2\, e^{-i\varphi}}{1 + r_1 r_2 e^{-i\varphi}}\,\Psi_{in}}$$

Reflected field

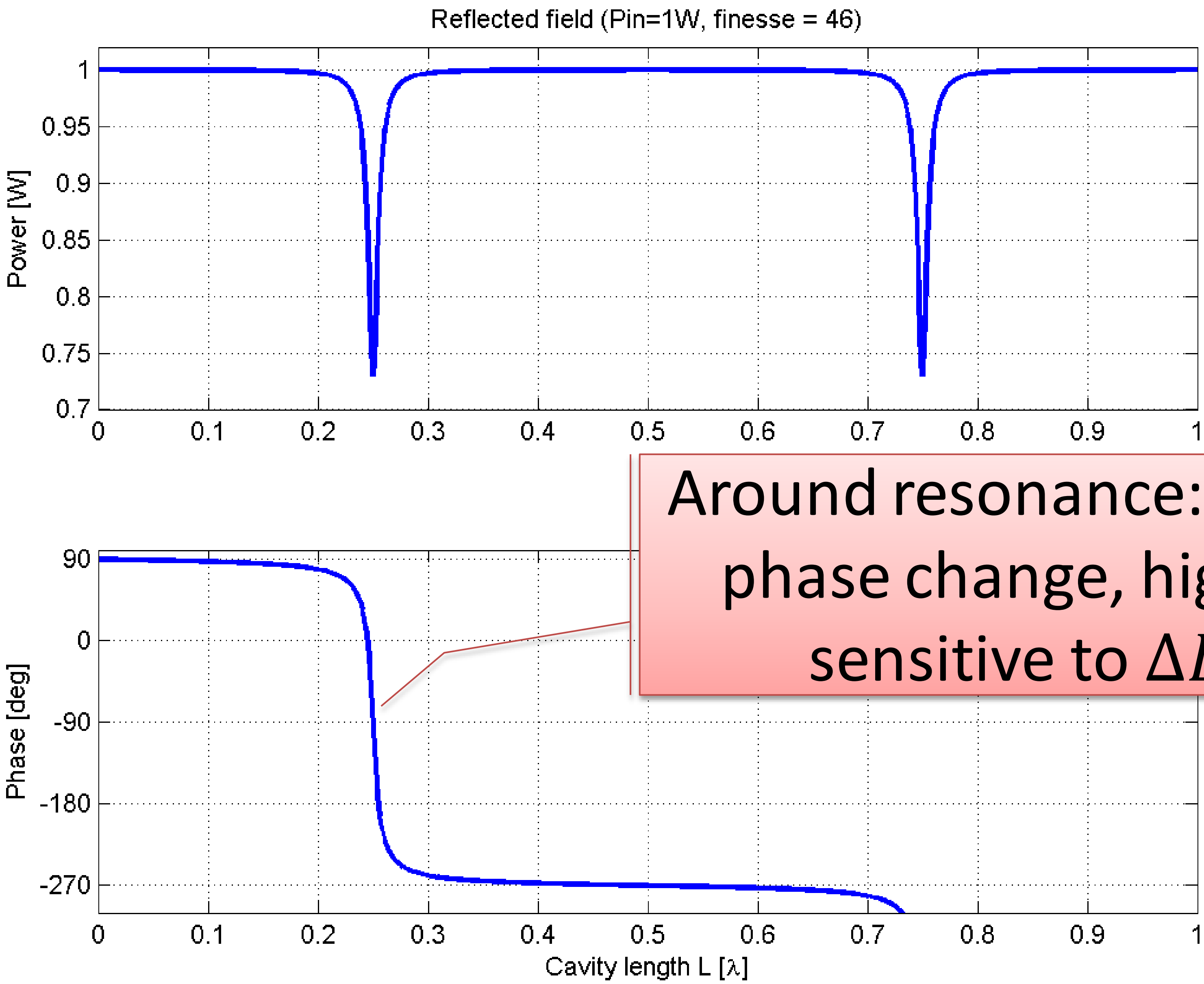


When in resonance, light leaks into the cavity

When out of resonance: 90^0 phase change – just like a single mirror

Run_fp.m

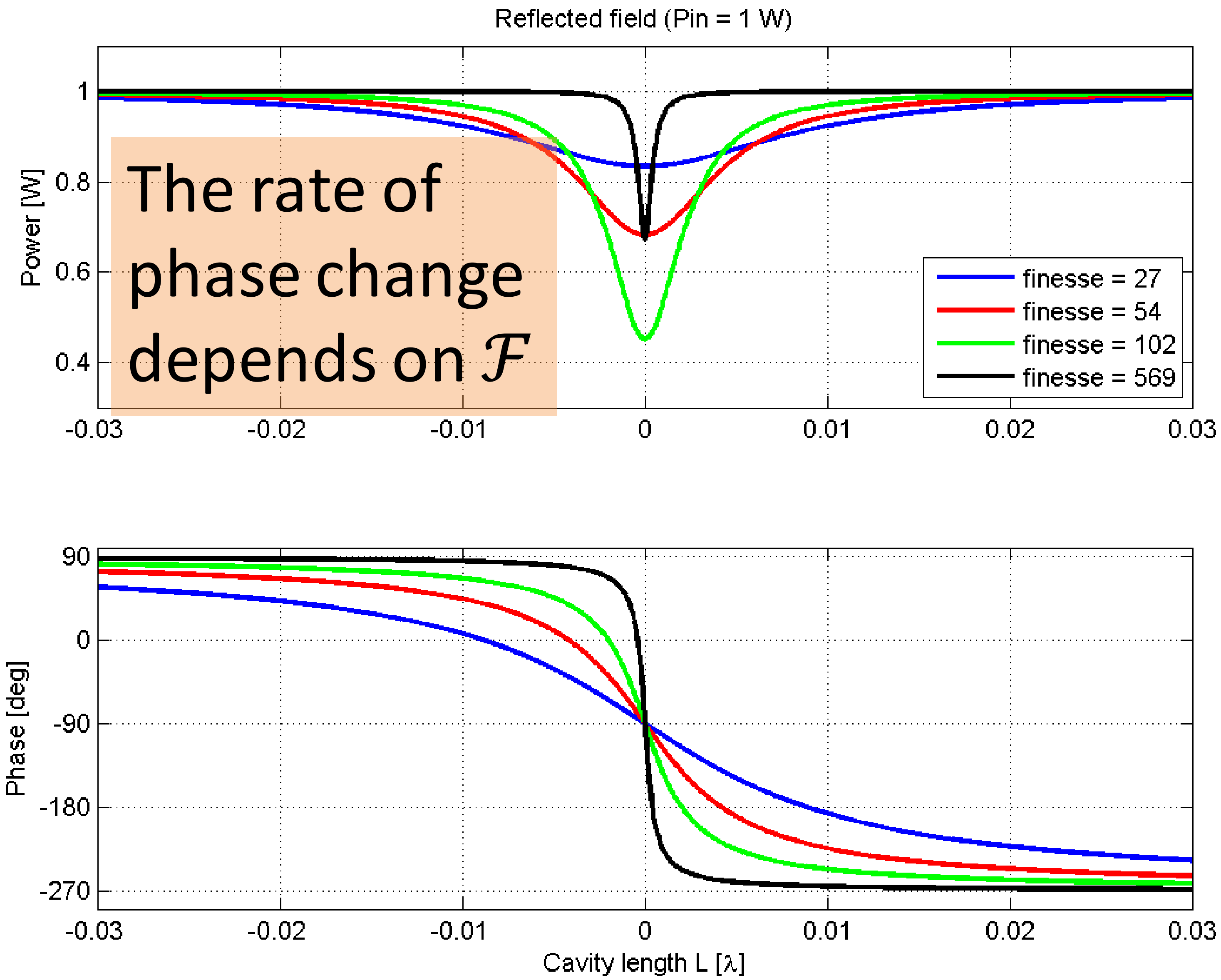
Reflected field



Around resonance: rapid phase change, highly sensitive to ΔL

Run_fp.m

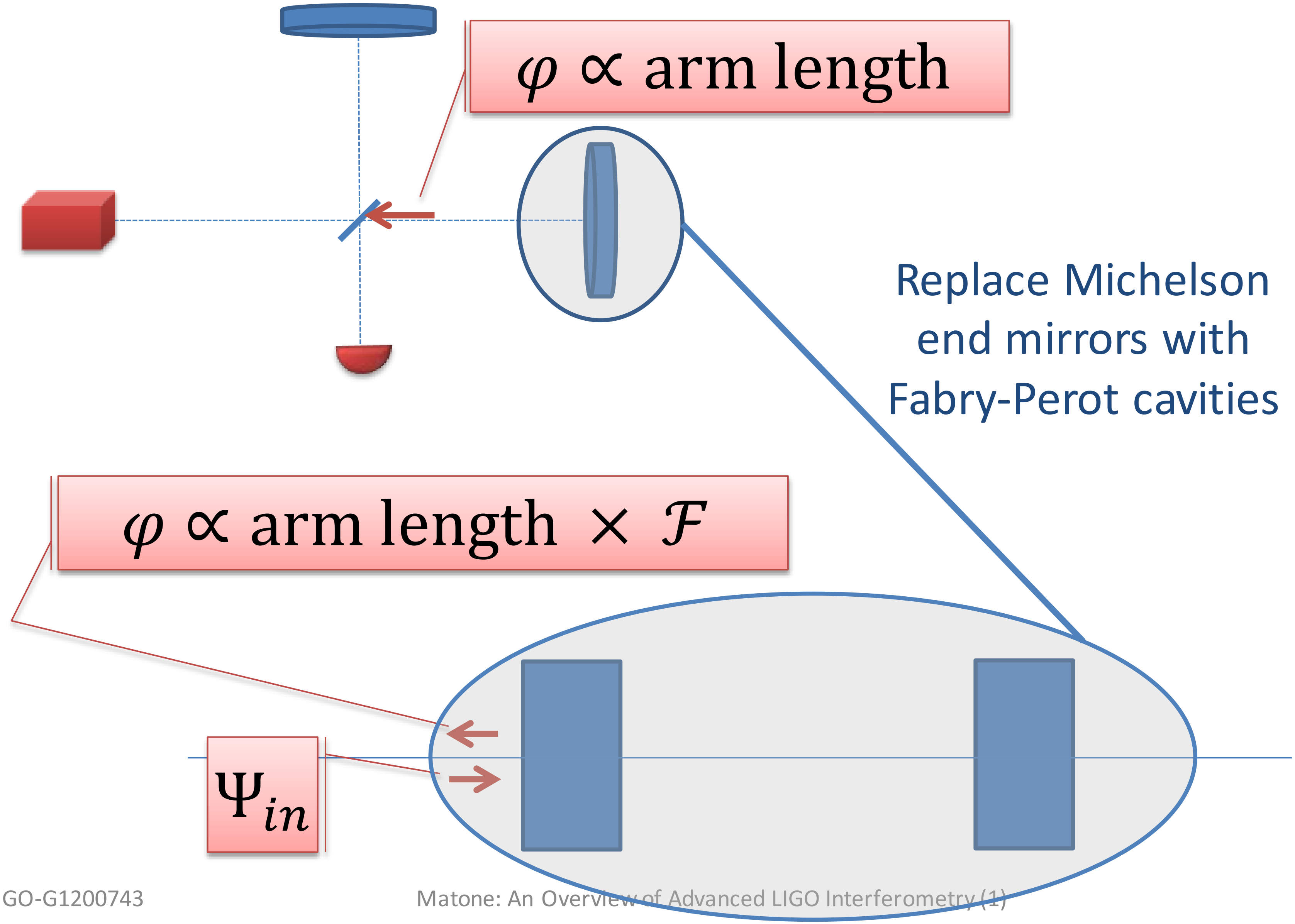
Reflected field as a function of $\mathcal{F}$



The rate of phase change depends on $\mathcal{F}$

Run_fp.m

Phase sensitivity



$\varphi \propto \text{arm length}$

Replace Michelson end mirrors with Fabry-Perot cavities

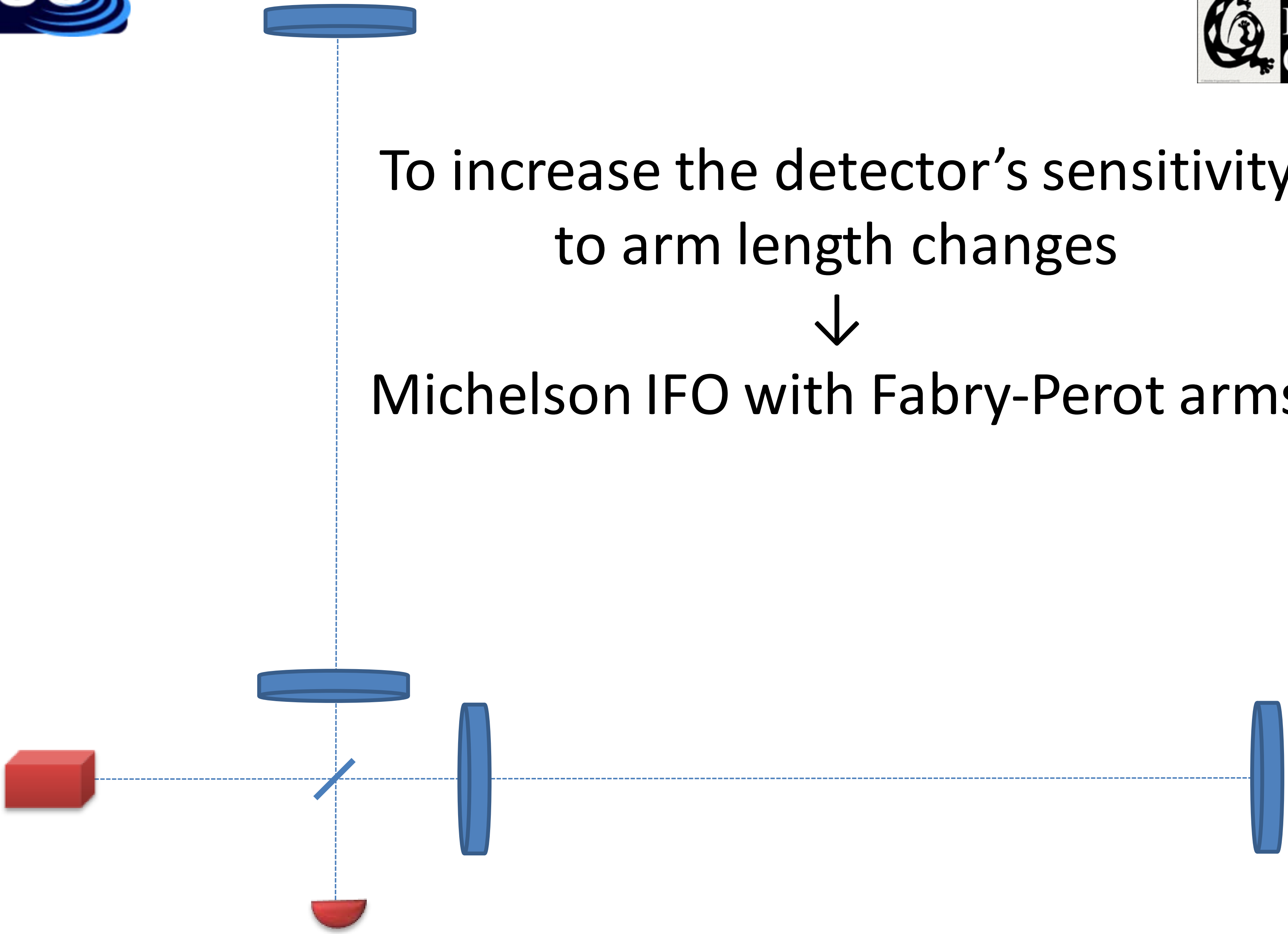
$\varphi \propto \text{arm length} \times \mathcal{F}$

Ψ_{in}

To increase the detector’s sensitivity to arm length changes

↓

Michelson IFO with Fabry-Perot arms

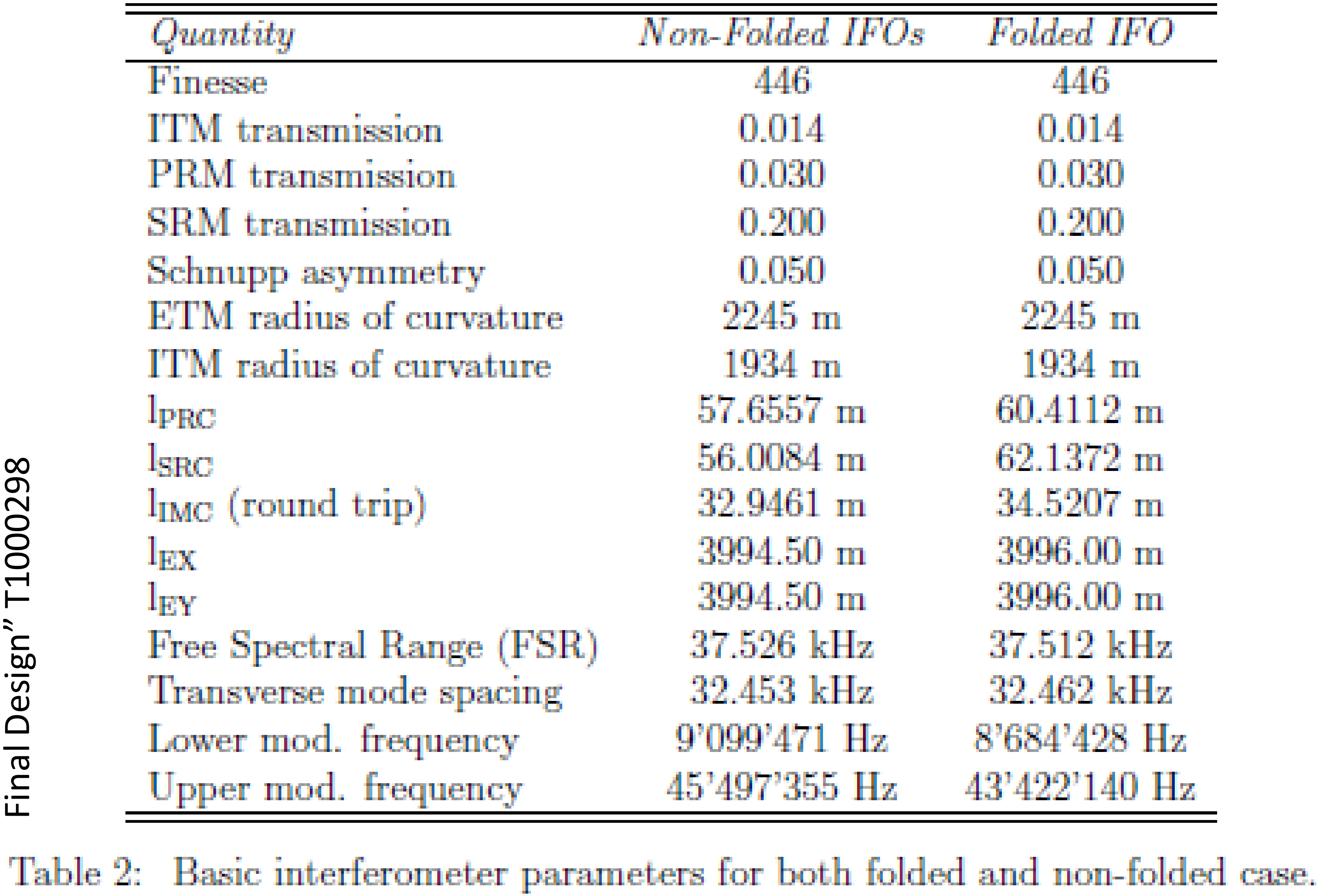


- Strategy
  - Amplify signal ✓
  - Decrease and/or control noise contribution
- Noise sources: in general two categories
  - Displacement noise
    - Ground seismic excitation
    - Thermal excitation of optical elements and suspensions
    - Radiation pressure
  - Phase noise
    - Amplitude and frequency fluctuations of the incoming laser beam
    - Shot-noise, the quantum limit to the *counting* of photons

Group Activities

1. Form groups of two or three
2. Using table 2 of T1000298 (AdvLIGO LSC final design document), verify
    - AdvLIGO's arm cavity finesse $\mathcal{F}$ and FSR
3. Using Matlab and the field equations shown, plot
    a. The dark port power vs. arm length change for a Michelson IFO
    b. The reflected and intra-cavity power (with phases) for one of LIGO's arms as a function of cavity length (assume 1W of light in)

From “The AdvLIGO Length Sensing and Control Final Design” T1000298



Quantity	*Non-Folded IFOs*	*Folded IFO*
Finesse	446	446
ITM transmission	0.014	0.014
PRM transmission	0.030	0.030
SRM transmission	0.200	0.200
Schnupp asymmetry	0.050	0.050
ETM radius of curvature	2245 m	2245 m
ITM radius of curvature	1934 m	1934 m
l_{PRC}	57.6557 m	60.4112 m
l_{SRC}	56.0084 m	62.1372 m
l_{IMC} (round trip)	32.9461 m	34.5207 m
l_{EX}	3994.50 m	3996.00 m
l_{EY}	3994.50 m	3996.00 m
Free Spectral Range (FSR)	37.526 kHz	37.512 kHz
Transverse mode spacing	32.453 kHz	32.462 kHz
Lower mod. frequency	9’099’471 Hz	8’684’428 Hz
Upper mod. frequency	45’497’355 Hz	43’422’140 Hz

Table 2: Basic interferometer parameters for both folded and non-folded case.