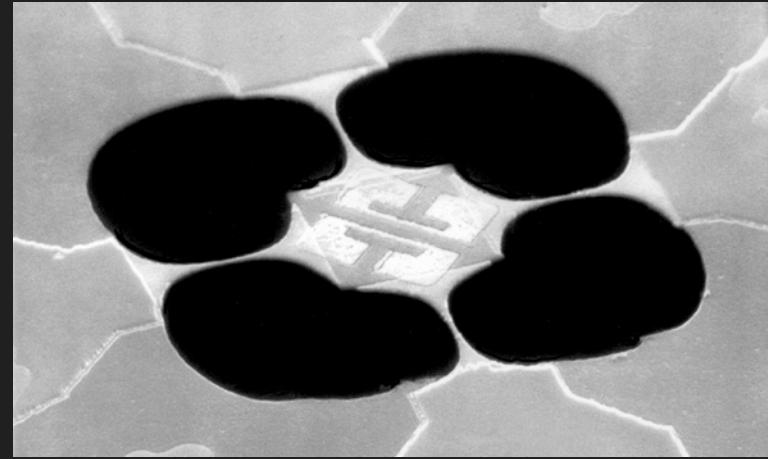
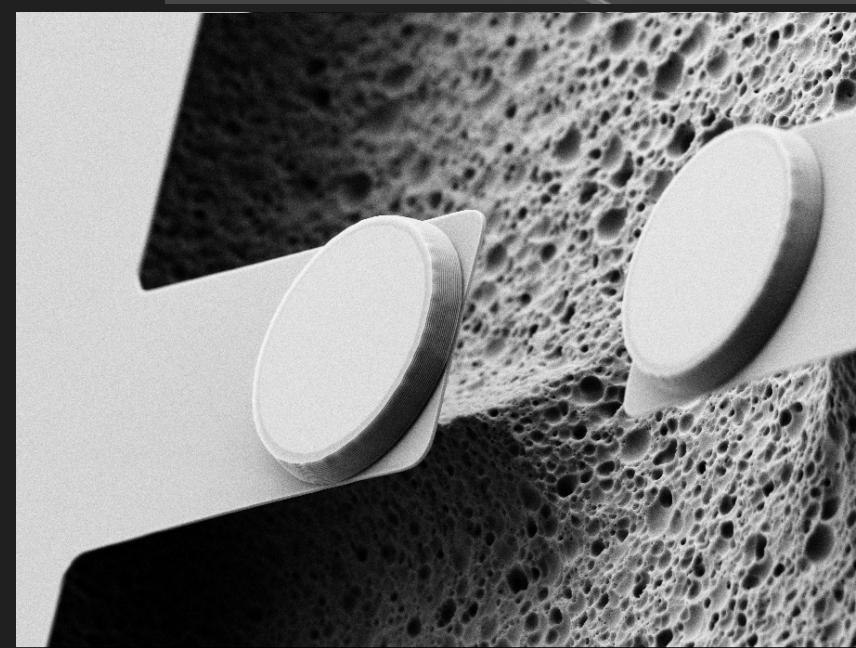
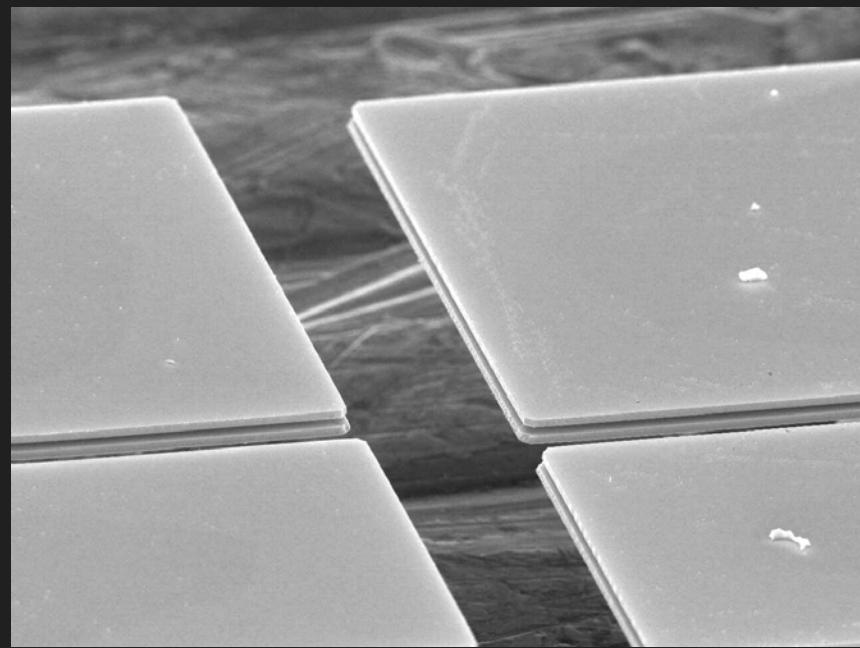
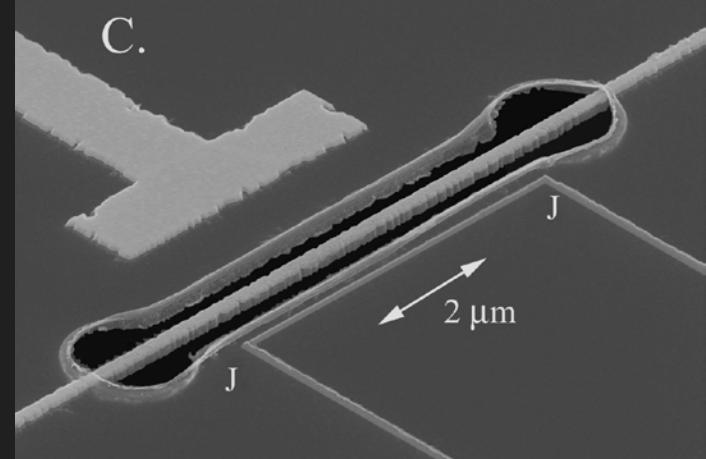


Development of an Ultra-low loss Superfluid He-4 Acoustic Resonator



Keith Schwab
and
Laura De Lorenzo

Applied Physics
Caltech
May 2012



Outline

- Motivation
- Properties of superfluid He-4, what is known
- Estimation of the loss mechanisms
 - Intrinsic loss in the fluid - phonon-phonon coupling
 - He-3 impurities
 - Container radiation losses
- Microwave cavity readout near the SQL
- Sensitivity estimates to accelerations, forces, and gravitational waves
- Other possibilities, configurations

Our work in quantum limits of nanomechanics in the past years

REPORTS

Approaching the Quantum Limit of a Nanomechanical Resonator

M. D. LaHaye,^{1,2} O. Buu,^{1,2} B. Camarota,^{1,2} K. C. Schwab^{1*}

2004: Approaching the SQL and the ground state with nanomechanics

nature
physics

ARTICLES

PUBLISHED ONLINE: 6 DECEMBER 2009 | DOI: 10.1038/NPHYS1479

Back-action-evasive measurements of nanomechanical motion

J. B. Hertzberg^{1,2}, T. Rocheleau¹, T. Ndakum¹, M. Savva¹, A. A. Clerk³ and K. C. Schwab^{4*}

2006: Observation of the back action forces due to shot noise, and cooling

2004: Approaching the SQL and the ground state with nanomechanics

LETTERS

Cooling a nanomechanical resonator with quantum back-action

A. Naik^{1,2}, O. Buu^{1,3}, M. D. LaHaye^{1,3}, A. D. Armour⁴, A. A. Clerk⁵, M. P. Blencowe⁶ & K. C. Schwab¹

2009: Back action evading measurement:

$$\Delta X_1 = 4 \cdot \Delta x_{zp}$$

$$\langle N \rangle = 4$$

2010: Sideband cooling to near the ground state:

LETTERS

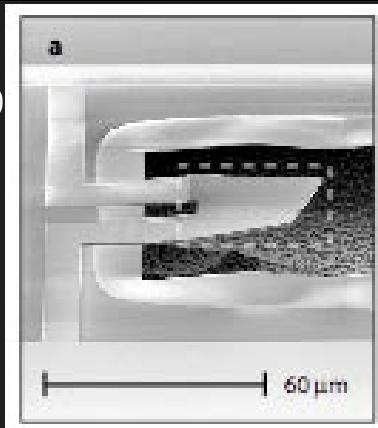
Preparation and detection of a mechanical resonator near the ground state of motion

T. Rocheleau^{1*}, T. Ndakum^{1*}, C. Macklin¹, J. B. Hertzberg², A. A. Clerk³ & K. C. Schwab⁴

Studies of motion in the quantum regime – dissipation is key

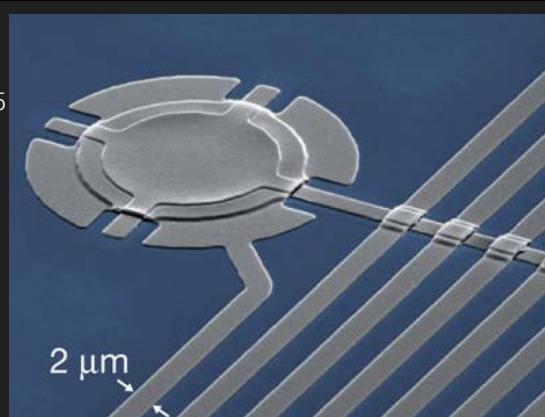
AlN

$Q = 900$



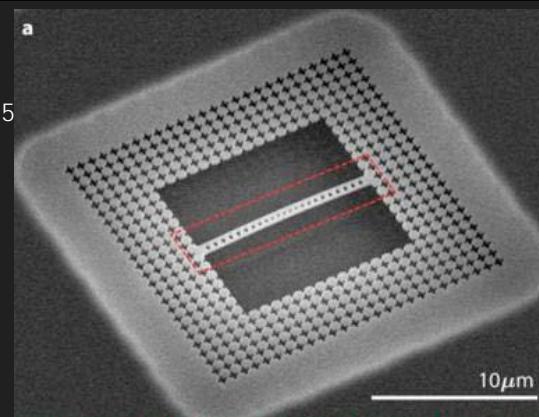
Al

$Q = 3 \cdot 10^5$



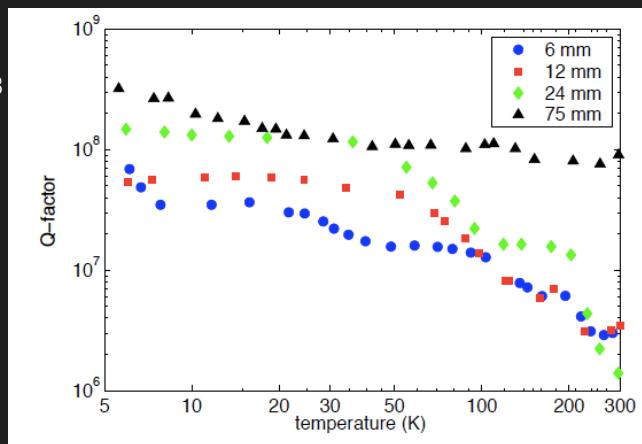
SiO₂

$Q = 10^5$



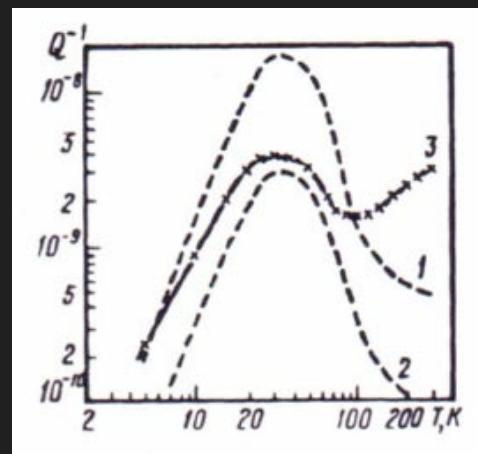
Silicon:

$Q = 4 \cdot 10^8$



Sapphire:

$Q = 3 \cdot 10^9$



O'Connell, et al, *Nature* (2010).

Teufel, Dale, Harlow, Allman, Cicak, Sirois, Whittaker, Lehnert, Simmonds, *Nature* **475**, 359 (2011).

J. Chan, T. Alegre, A. Safavi-Naeini, J. Hill, A. Krause, S. Groblacher, M. Aspelmeyer, O. Painter, *Nature* **478**, 389 (2011).

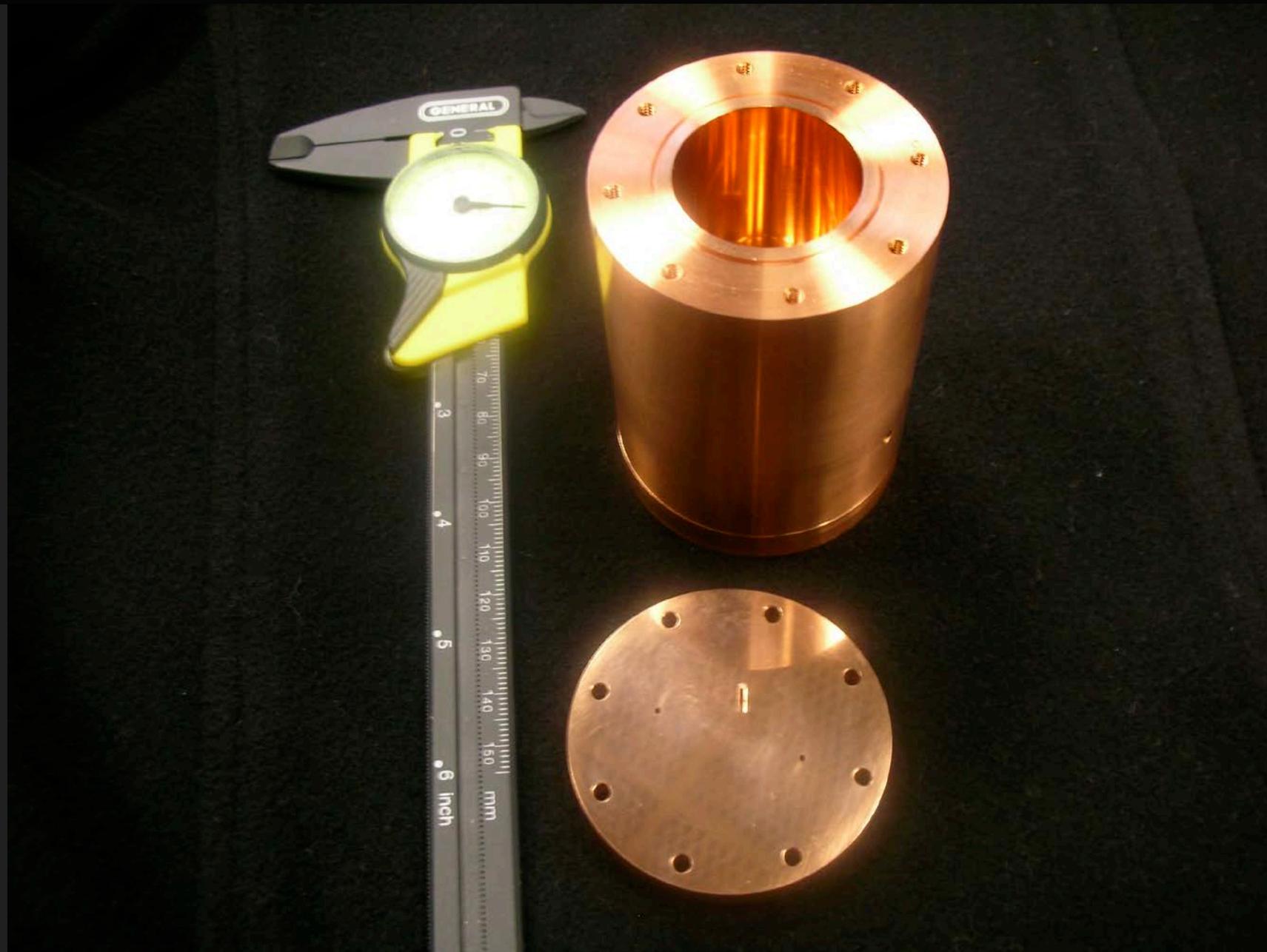
Braginsky, Mitrofanov, Panov, "Systems with Small Dissipation." Univ. of Chicago Press (1985.)

Nawrodt, et al, "High mechanical Q-factor measurements on silicon bulk samples." *J. Phys.: Conf. Ser.* 112, (2008).

Basic Properties of Superfluid He-4

- Speed of first sound: $c=240 \text{ m/s}$ ($\times 10$ smaller than most metals)
- Density: $\rho=145 \text{ Kg/m}^3$ ($\times 10$ less than most solids)
- Dielectric constant: 1.05
- Chemically pure (impurities freeze to container walls)
- Isotopic impurities: He-3, concentration of $n=10^{-7}$. Concentration of $n<10^{-14}$ has been achieved
- Transition temperature: $T_\lambda= 2.17\text{K}$, below this a macroscopic order parameter appears, Ψ
- Entropy resides in normal fluid density composed of rotons ($e^{-8K/kbT}$) and phonons (T^4)
- Shows persistent mass currents (frictionless flow) below T_λ , quantized circulation around loops.

Basic concept: superfluid acoustic resonator coupled to microwave resonator



Dissipation of first sound in He-4 – phonon-phonon coupling

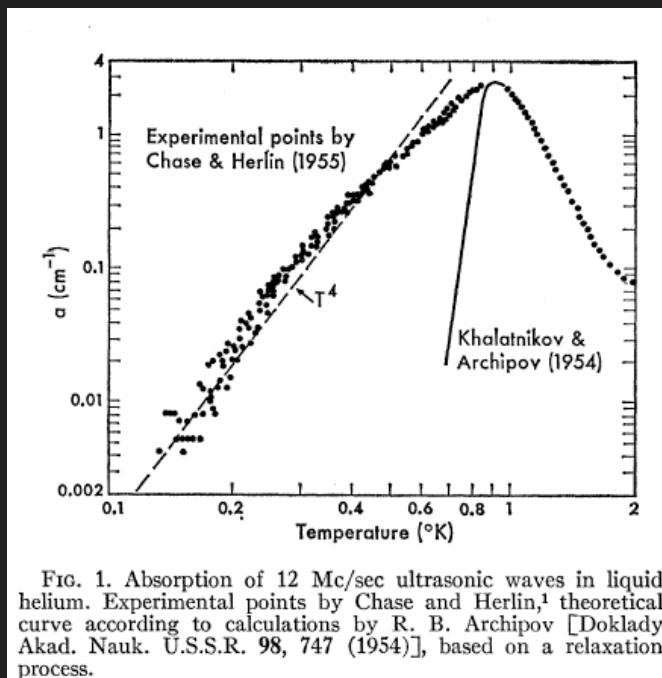
- Non-linear acoustic response:

$$G = \frac{\rho}{c} \frac{\partial c}{\partial \rho} \approx 2.78$$

- Dispersion relationship for sound:

$$e(p) = cp(1 - \gamma p^2)$$

- For $\gamma < 1$, three phonon acoustic damping process is possible where phonons scatter off of thermal phonons in fluid



- Acoustic attenuation length:

$$\alpha = \frac{\pi^3}{60} \frac{G+1}{\rho} \frac{k_B^4}{h^3 c^6} T^4 \omega$$

$$T = 10 \text{ mK}, \omega = 2\pi 3 \text{ KHz} \Rightarrow 1/\alpha = 3 \cdot 10^9 \text{ m}$$

$$T = 1 \text{ mK}, \omega = 2\pi 3 \text{ KHz} \Rightarrow 1/\alpha = 3 \cdot 10^{13} \text{ m}$$

C.E. Chase, Proc. Roy. Soc. (London) A220, 116 (1953).

Humphrey Maris, "Attenuation and Velocity of Sound in Superfluid Helium." Phys. Rev. Lett. **28**, 277 (1972).

Dissipation due to He-3 impurities

Treat He-3 atoms as dilute, viscous classical gas.

$$\bar{l} = \frac{1}{\sqrt{2}\sigma n_3} \quad \eta = \frac{1}{3} \bar{l} \rho_3 \bar{v} = \frac{2}{3\sigma} \sqrt{\frac{k_B m_3 T}{\pi}}$$

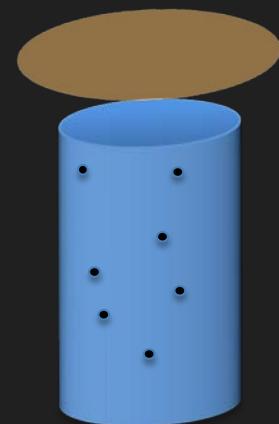
When mean free path is smaller than container, acoustic attenuation is density independent:

$$\alpha = \frac{2\pi^2 \eta}{\rho_4 c_4 \lambda^2} \left(\frac{4}{3} + (\gamma - 1) \frac{1}{\gamma} \right) = \frac{104}{45} \frac{\pi^2 \sqrt{k_B T \pi m_3}}{\sigma \rho_4 c_4 \lambda^2}$$

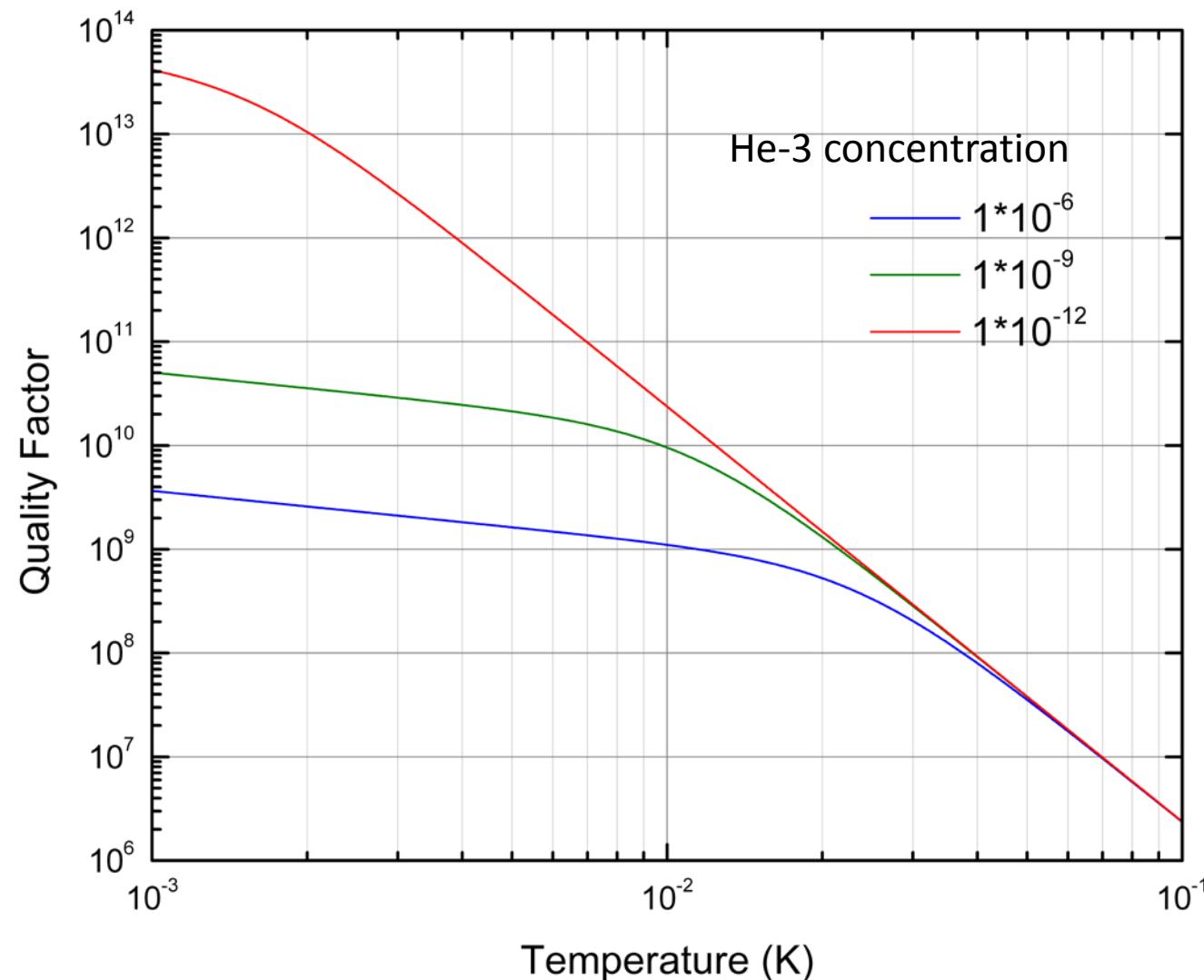
When concentration of He-3 drops to 10^{-8} , mean free path is equal to the size of the container, ~10cm.

There is some uncertainty about the nature of the scattering of the He-3 atoms on the container walls. We will assume diffuse scattering after N bounces off the walls.

$$\alpha = \frac{104\pi}{45} \sqrt{\frac{2\pi k_B T}{m_3}} \frac{n_3 N}{n_4 c_4 L}$$



Expected intrinsic dissipation of He-4: phonons and impurities



Acoustic coupling and losses in container

Copper: $c_{Cu}=3900$ m/s

Niobium: $c_{Nb}=3500$ m/s

First normal modes of container is at:

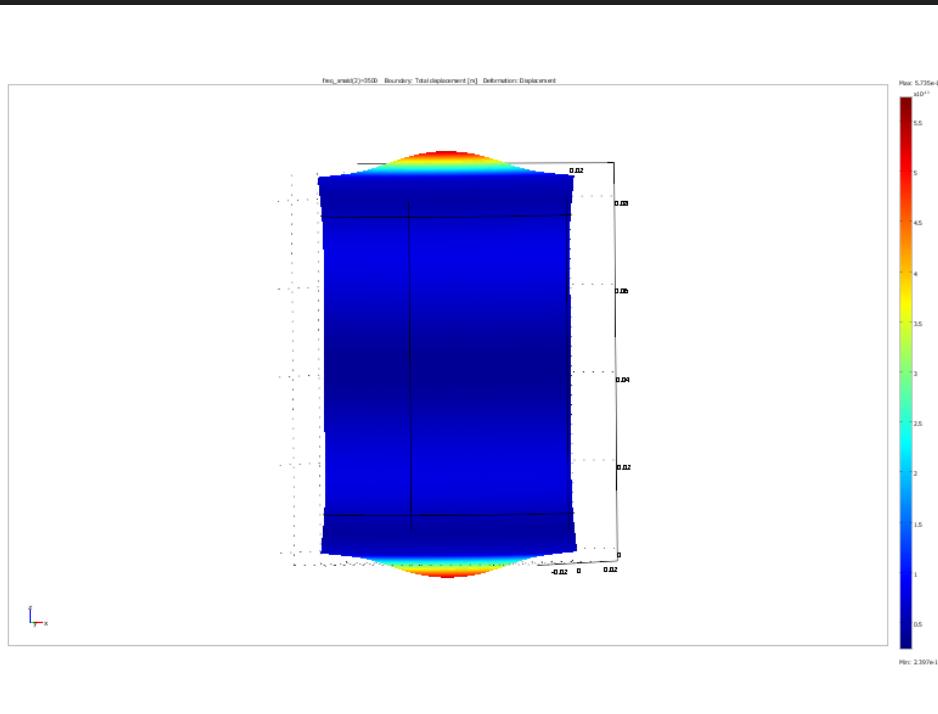
12.5KHz, 13.2KHz, 13.6KHz

Superfluid resonance is at 3.5KHz

$$\dot{E} = E_{Hr}\gamma_{Hr} + E_c\gamma_c$$

Energy stored in Cu or Nb container for design shown in photo:

$$\frac{E_c}{E_{Hr}} = 10^{-6}$$



Quality factor at 50mK:

$$Cu \text{ Q} = 10^5$$

$$BeCu \text{ Q} = 8 \cdot 10^6$$

$$Nb \text{ Q} = 40 \cdot 10^6$$

This will limit Q of superfluid resonance to

$$Cu: Q = 10^{11}$$

$$BeCu: Q = 10^{13}$$

$$Nb: Q = 10^{14}$$

Acoustic resonator coupled to a microwave resonance

L=6.8 cm

D = 3 cm

M= 7g

ω = 3.5 kHz

ω = 12 GHz

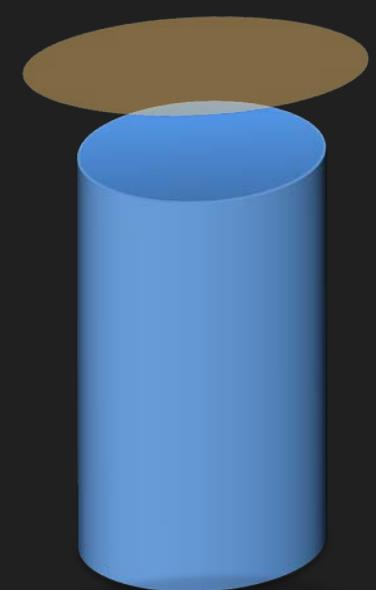
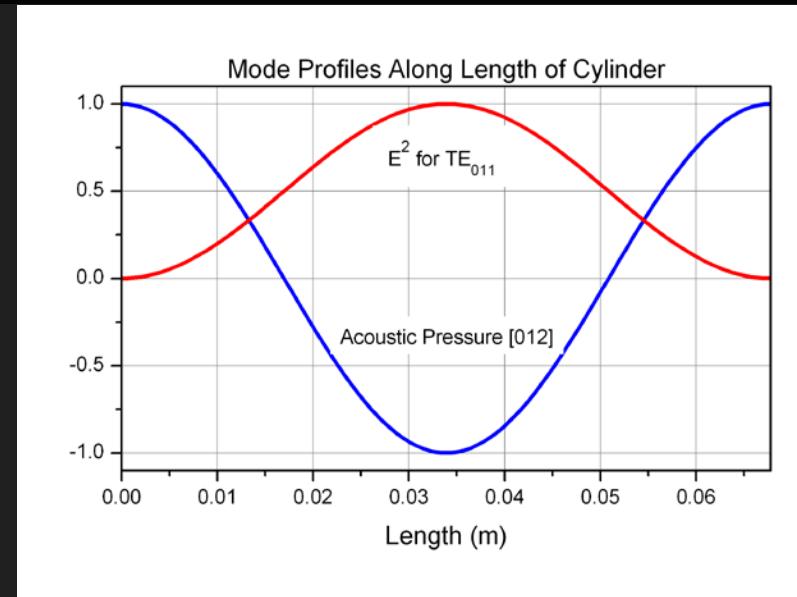
$$\frac{\Delta\omega_c}{\omega_c} = \frac{\int \Delta\mu |H|^2 + \Delta\varepsilon |E|^2 dV}{\int \mu |H|^2 + \varepsilon |E|^2 dV}$$

Opto-mechanical Hamiltonian

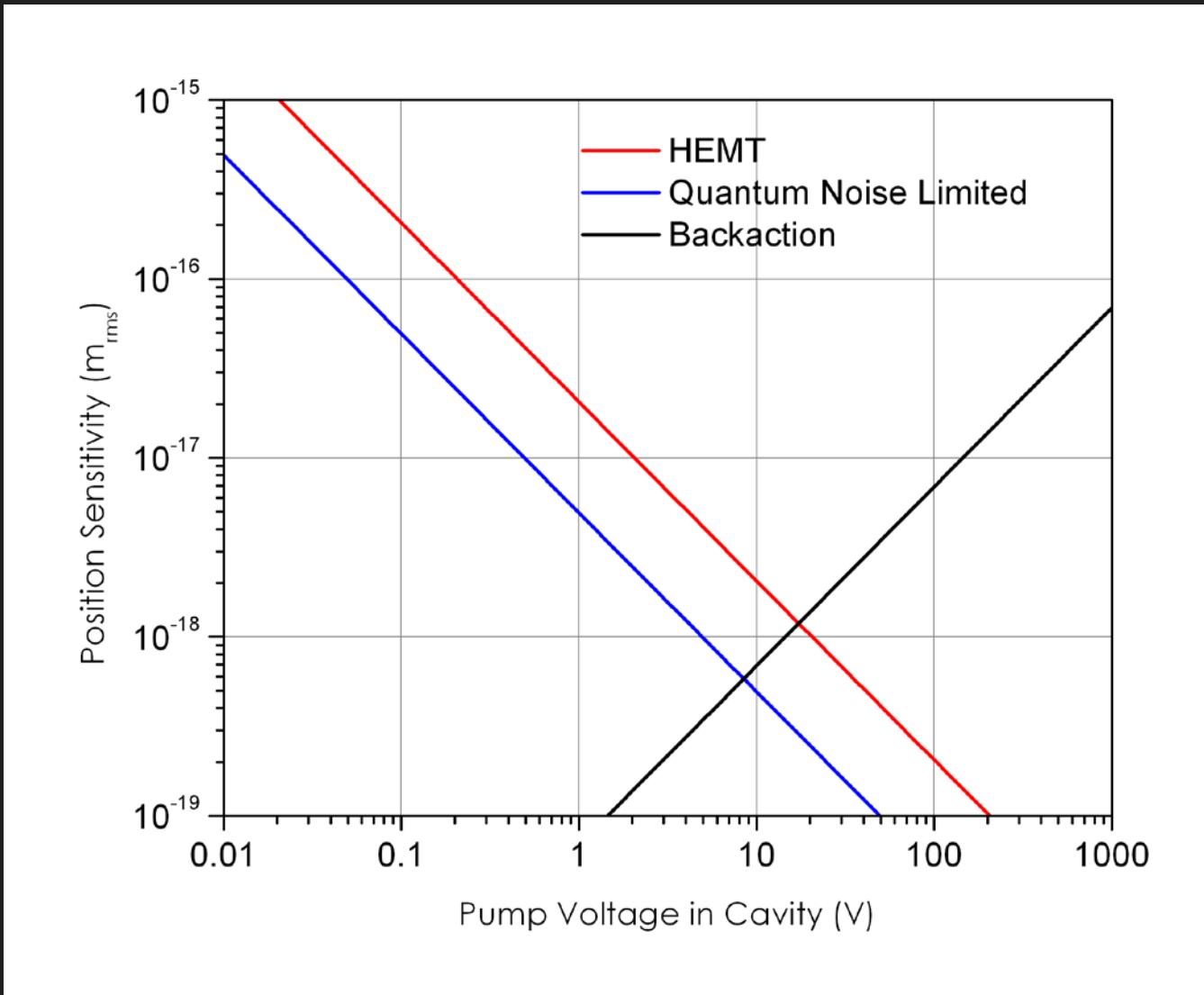
$$H = \hbar\omega_c \left(a^\dagger a + \frac{1}{2} \right) + \hbar\omega_a \left(b^\dagger b + \frac{1}{2} \right) + \hbar \frac{\partial\omega_c}{\partial x} \Delta x_\varphi (b^\dagger + b) \left(a^\dagger a + \frac{1}{2} \right)$$

Coupling per quanta...very weak

$$\frac{\partial\omega_c}{\partial x} \Delta x_\varphi = 10^{-6}$$



"Position" sensitivity



Sensitivity to sudden gravitational wave pulse

Imagine sudden pulse, of duration $|\tau_i|$: 1ms

Pulse energy density: $F(\omega) : \frac{c^3 h^2}{4\pi G}$

Energy deposited into resonant bar detector: $U_s : F(\omega) \frac{8MGV^2}{\pi c^2}$

Energy sensitivity: $U_s = \boxed{2k_B T \frac{\tau_i}{\tau_a} + \frac{|Z_{12}|^2}{2M} S_i(\omega) \tau_i + \frac{2M}{|Z_{21}|^2} \frac{S_e(\omega)}{\tau_i}}$

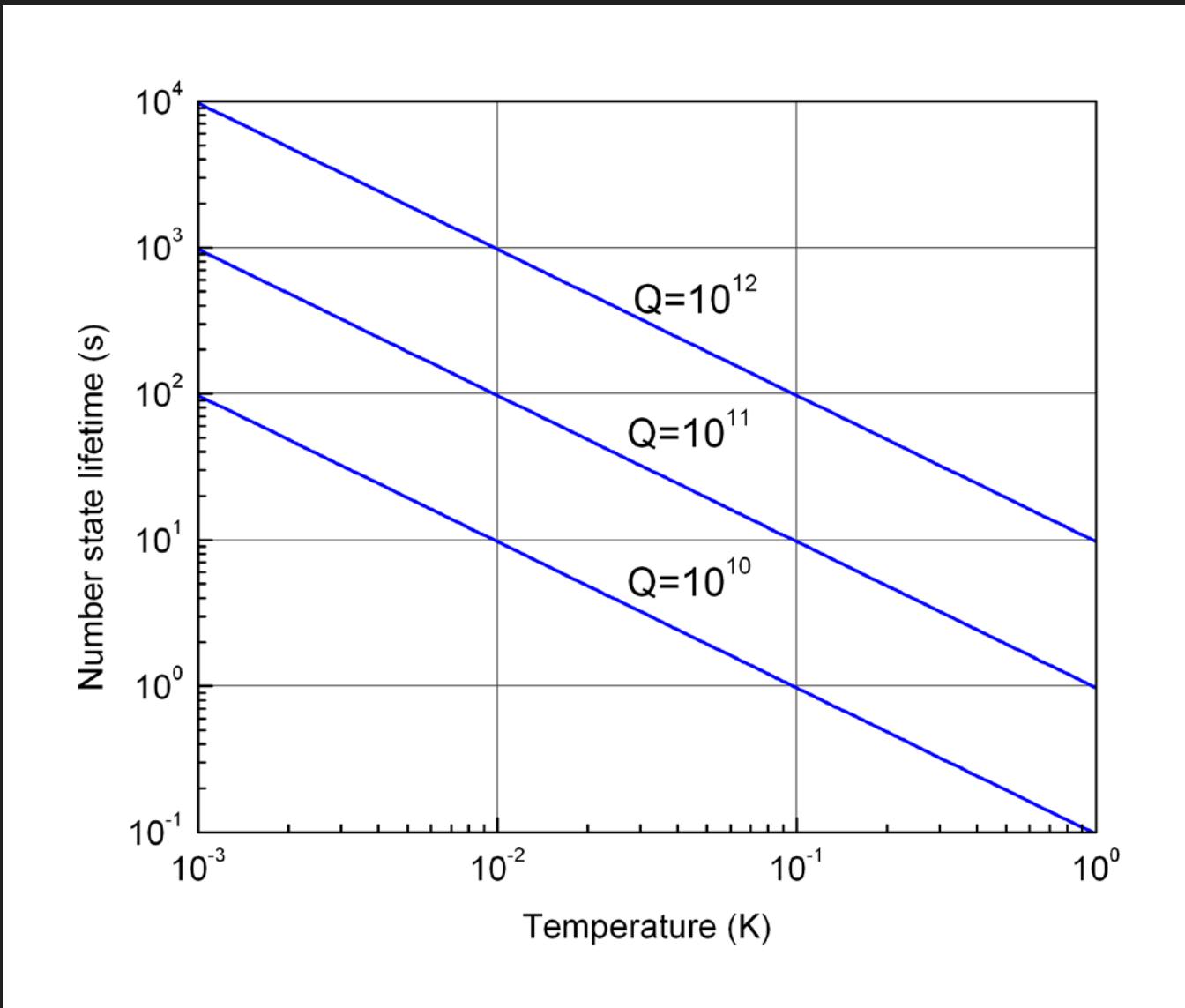
$$10^{-4} \hbar \omega$$

$$\phi$$

Strain sensitivity:

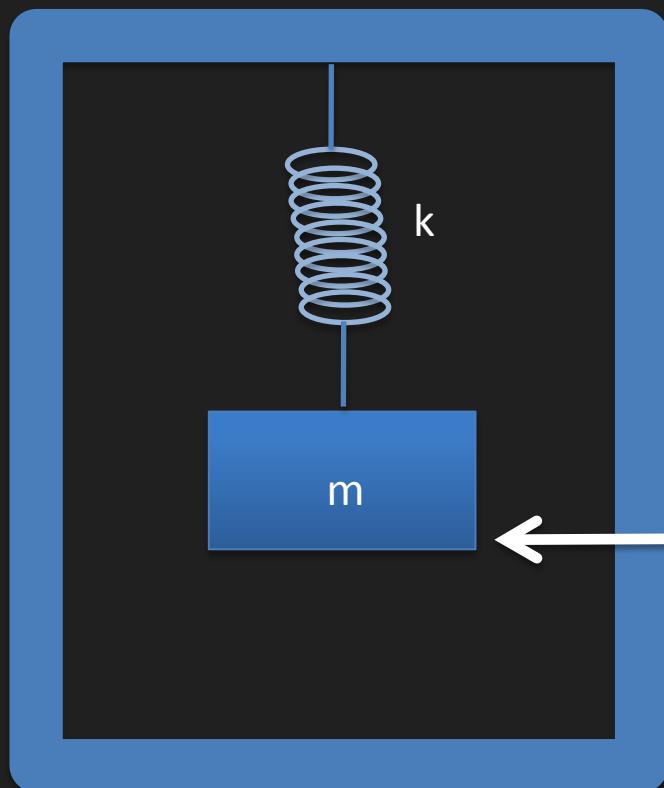
Vol (m ³)	F (Hz)	h
1	120	10 ⁻²¹
0.1	250	10 ⁻¹⁸
0.01	500	10 ⁻¹⁵
0.001	1000	10 ⁻¹²

Time for one phonon to enter acoustic resonator



$$\tau = \frac{\hbar Q}{k_B T}$$

Sensitivity to linear displacements and gravitational gradient forces



$$M=10g$$

$$F = 3.5\text{KHz}$$

$$K = 10^6 \text{ N/m}$$

$$T = 10\text{mK}$$

$$Q=10^{10}$$

$$\Delta x_{He} = Q\Delta x_c$$

$$\Delta x_{He} = \sqrt{\frac{k_B T}{m\omega^2}} = 10^{-16} m$$

$$\Delta x_c = 10^{-26} m$$

$$\Delta x_c / L = 10^{-25}$$

$$\omega^2 \Delta x_c = 10^{-18} g$$

This is the gravitational force of 10g at 10m, oscillating 1 cm

Further challenges....

Phase noise of microwave source will require superconducting cavity filter to achieve SQL.

Nb cell will be required to avoid ohmic heating to achieve less than 10mK.

Coupling the microwave circuit without spoiling the mechanical Q: antenna coupling?

Holding onto the cell and providing strong thermal link, without spoiling mechanical Q.

Filling the cell: pre-filling at 300K at 1K bar will avoid a fill line

Conclusions

~~This will never work.~~

~~Life is a dark and bitter place.~~

~~I should spend a lot more time in Hawaii and
stop worrying so much about low
temperature physics.~~

Superfluid resonators offer an extremely low dissipation material for mechanical resonators with very long coherence times and extreme sensitivity to accelerations.

This could be an excellent system to manipulate the quantum state of motion of a gram sized object.

schwab@caltech.edu

Meanwhile, at the other end of parameter space....

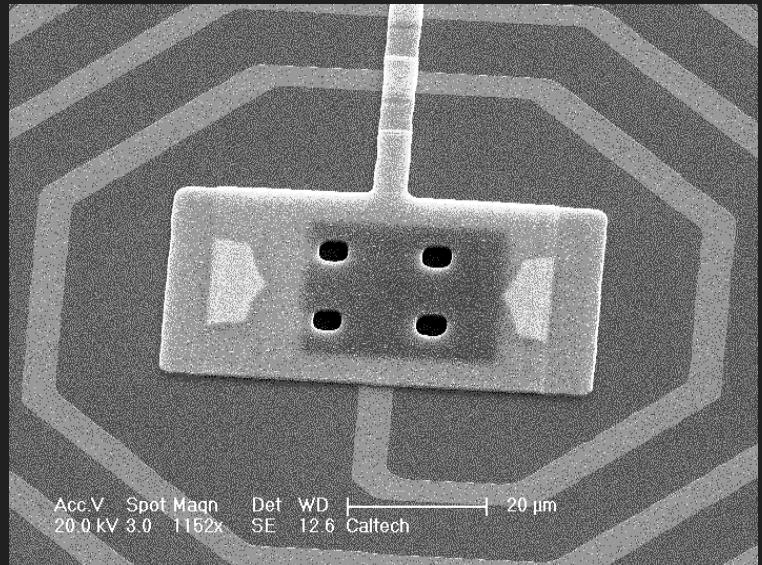
Acceleration of nanomechanical resonator

$$\omega = 2\pi \cdot 1\text{GHz}$$

$$x = 10\text{nm}$$

$$a = \omega^2 x = 4 \cdot 10^{10} g$$

Neutron star accelerations in the laboratory

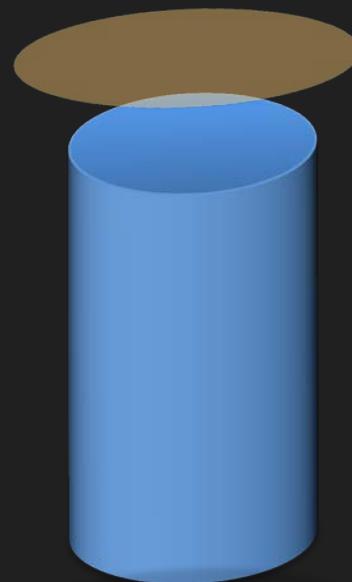


What physics can be probed with this extreme parameter?

Acoustic resonator coupled to a microwave resonance

Treat He-3 atoms as dilute, viscous classical gas.

When the mean free path is smaller than the container size:

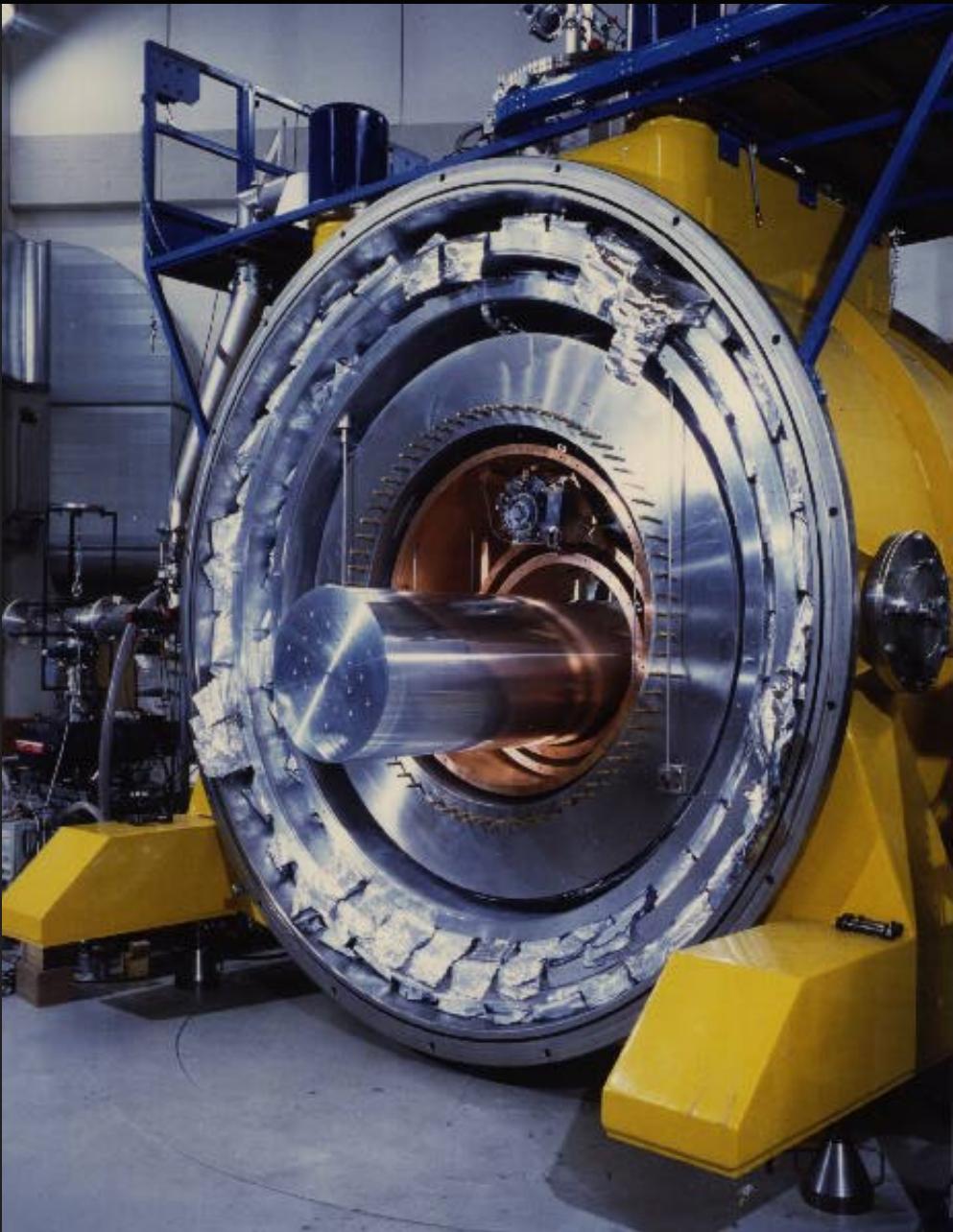


Gravitational wave antenna – Weber bars

Auriga

2 Ton Acoustic Resonators

$$\Delta x \approx 167 \Delta x_{QL}$$



Gravitational wave antenna – Interferometers



LIGO

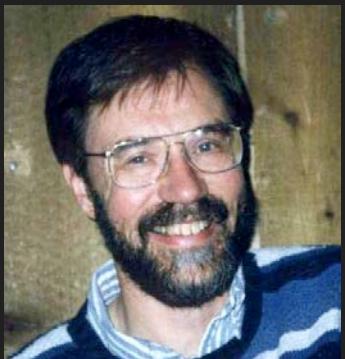
4 km Interferometer

$$\Delta x \approx 4\Delta x_{QL}$$

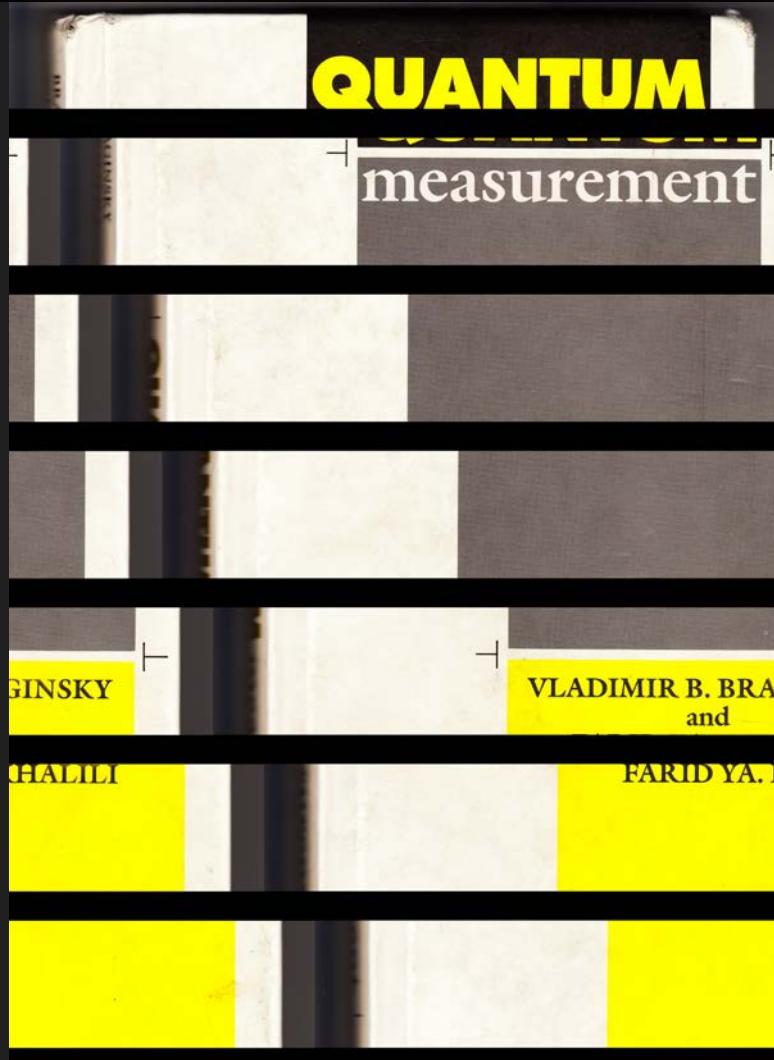
Gravitational wave antenna – Early literature starting in the 1970's



V. Braginsky



Carlton Caves



On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle*

Carlton M. Caves, Kip S. Thorne, Ronald W. P. Drever,[†] Vernon D. Sandberg,[‡] and Mark Zimmermann[§]

W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

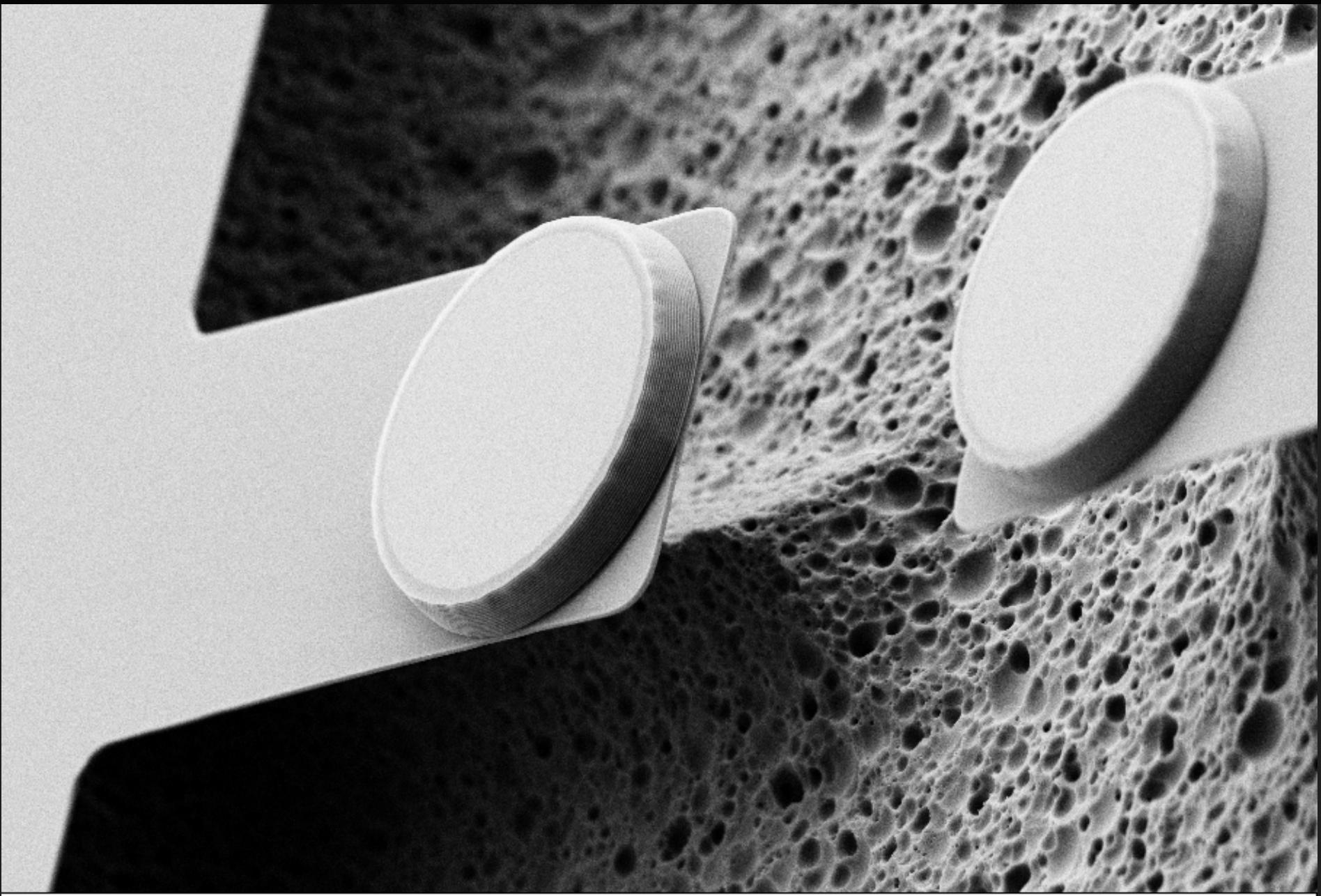


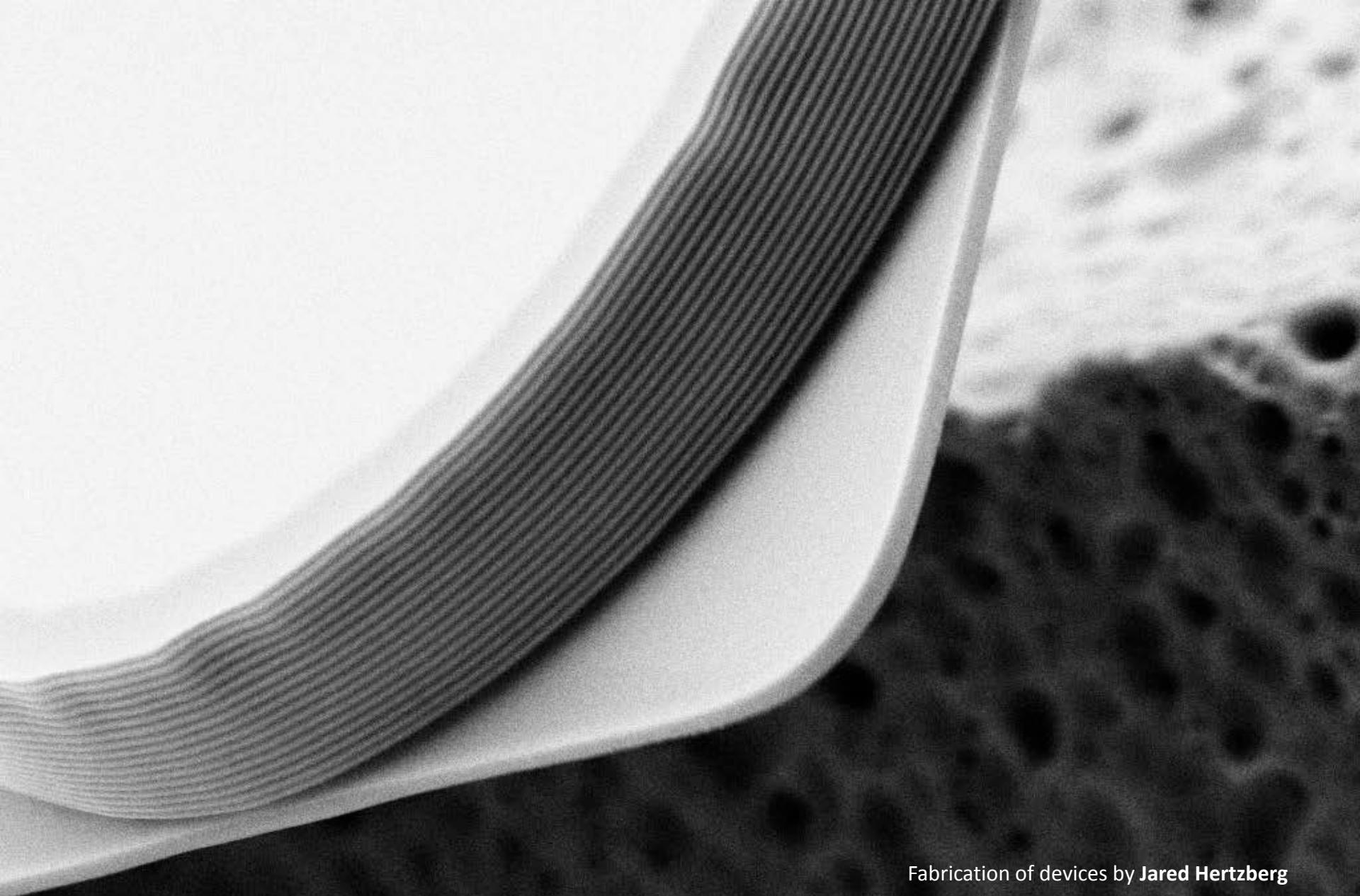
Kip Thorne



Ron Dreaver

Opto-mechanical structures





Fabrication of devices by **Jared Hertzberg**

2 μ m

WD = 4 mm
Mag = 10.31 K X

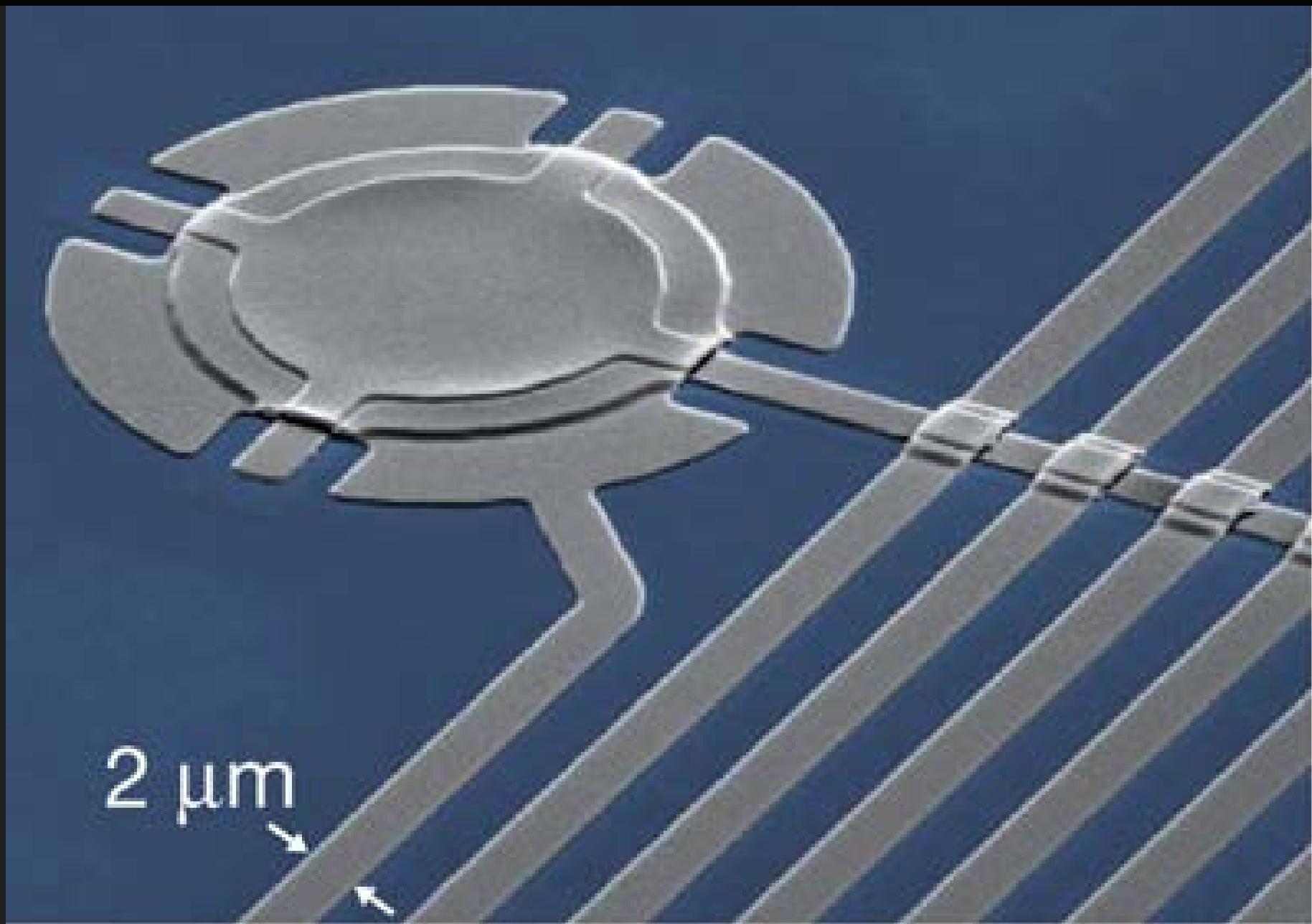
Aperture Size = 30.00 μ m
EHT = 10.00 kV
Pixel Size = 33.8 nm

Signal A = SE2
Signal B = SE2

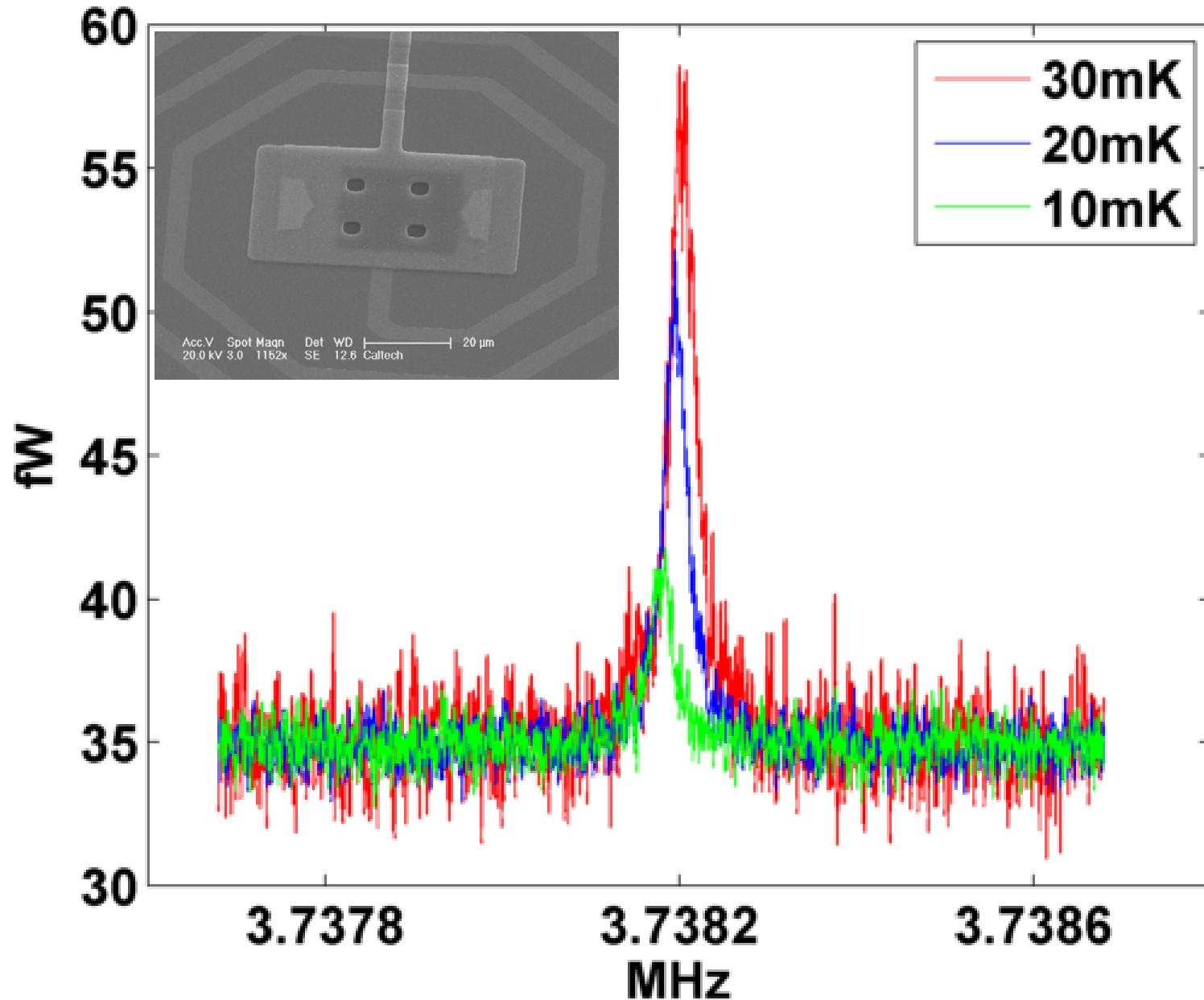
Date : 21 Dec 2007
Time : 13:51:59

CNF

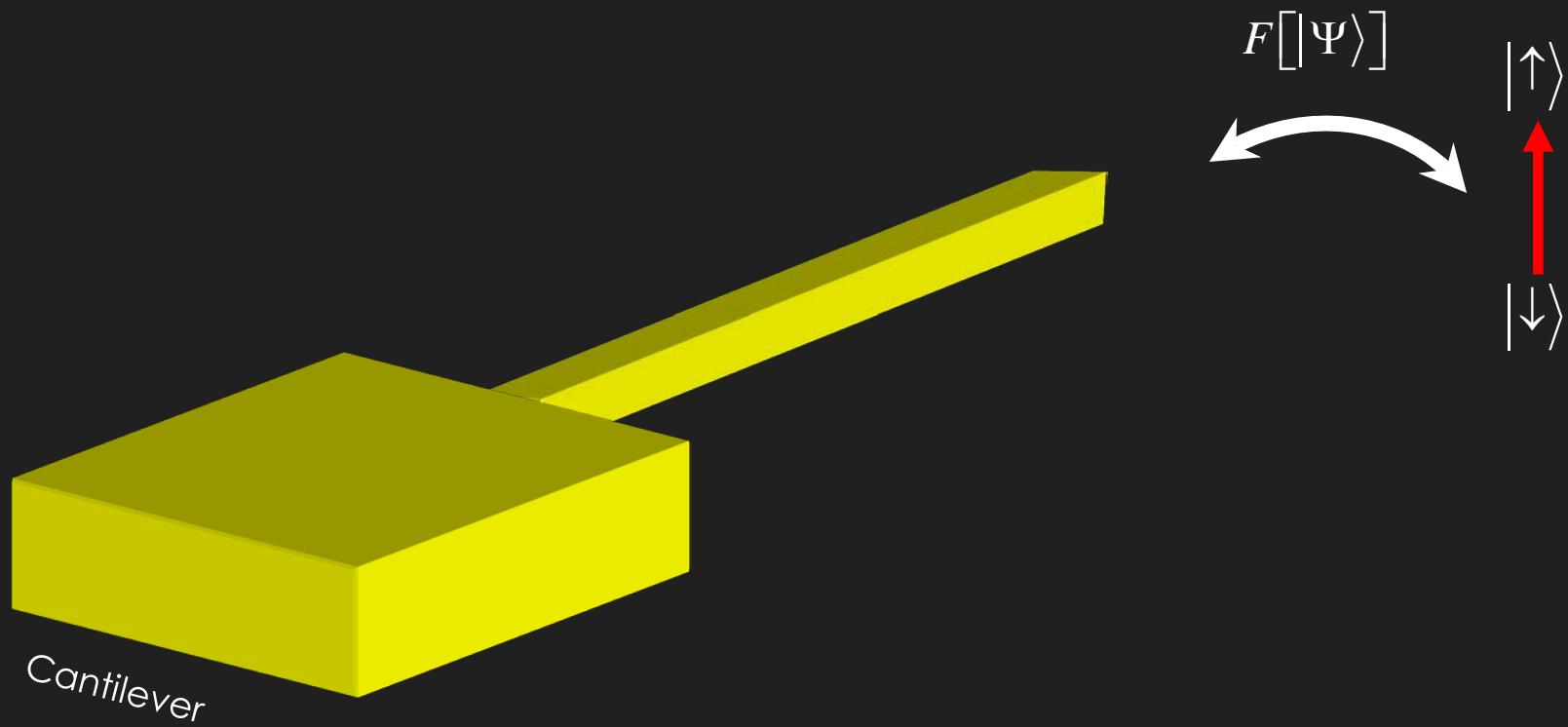
Electro-mechanical structures



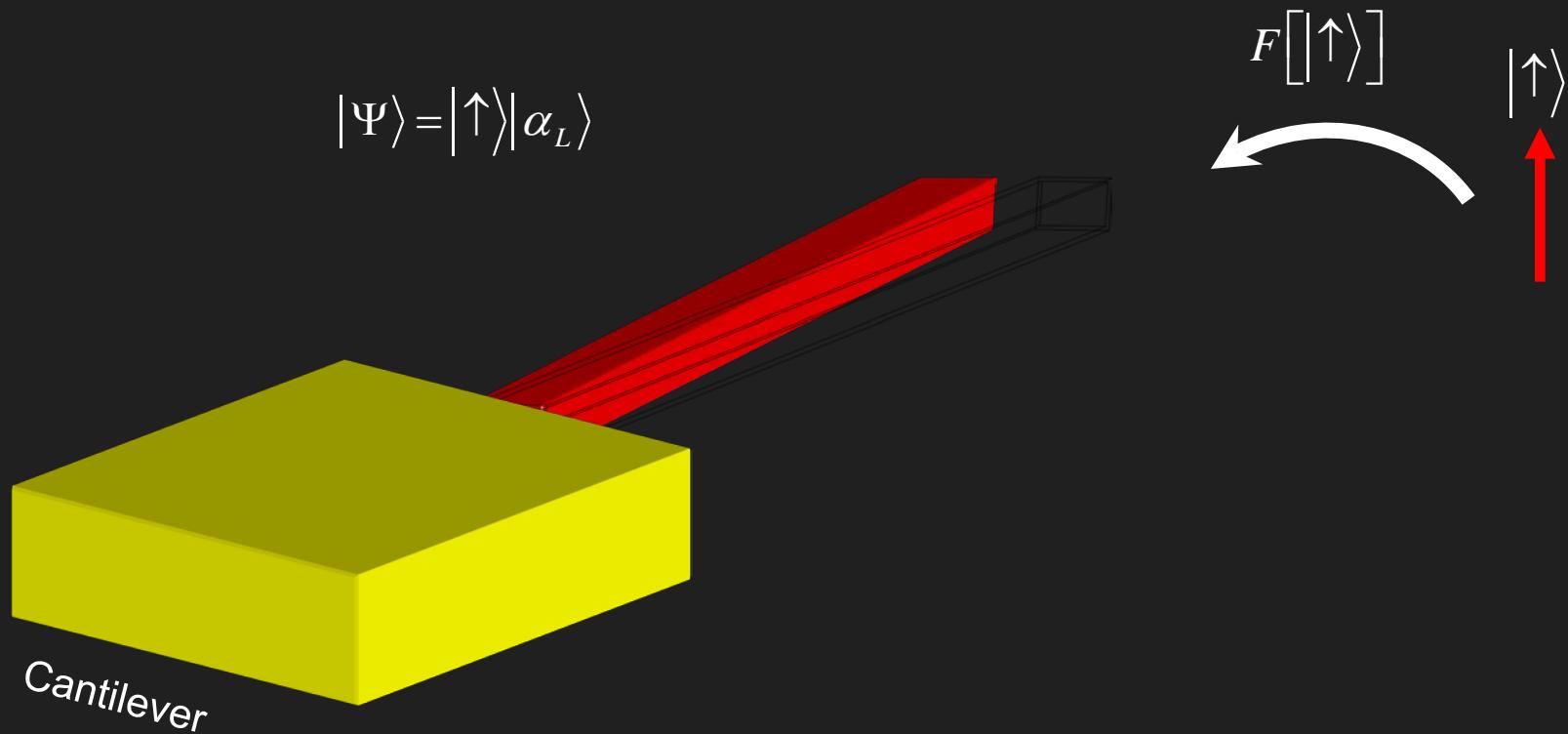
Membrane thermalizes down to 10mK



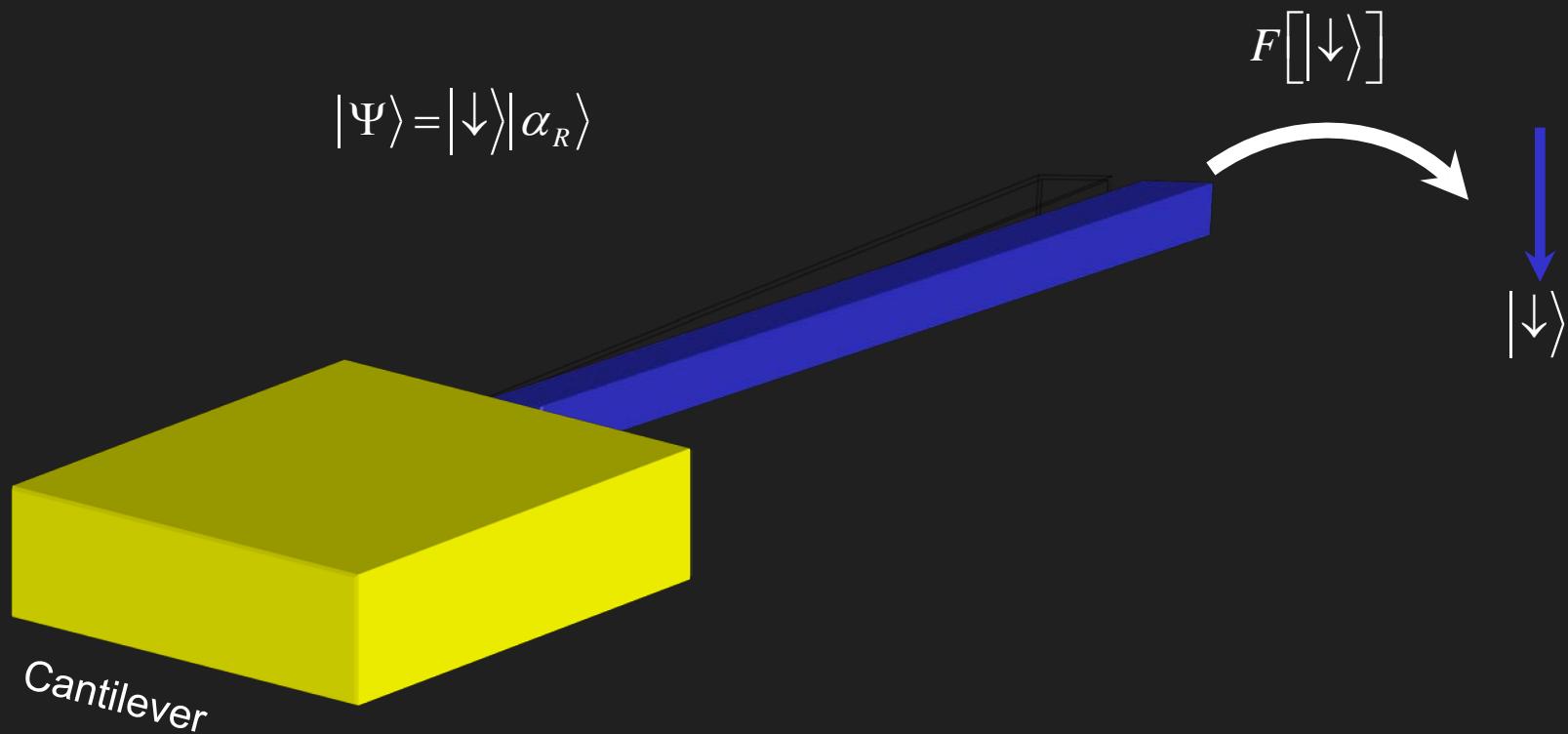
Schrödinger's Whisker



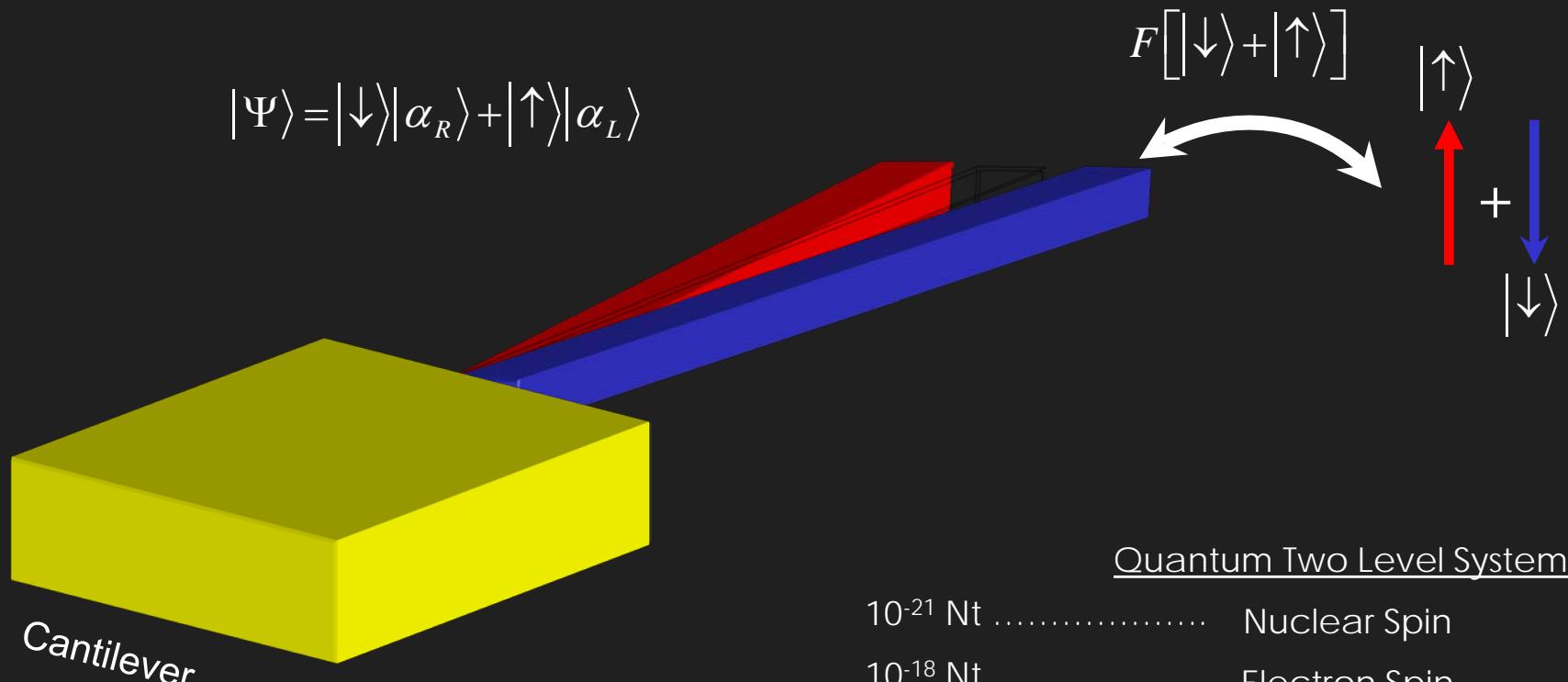
Schrödinger's Whisker



Schrödinger's Whisker



Schrödinger's Whisker



Quantum Two Level Systems

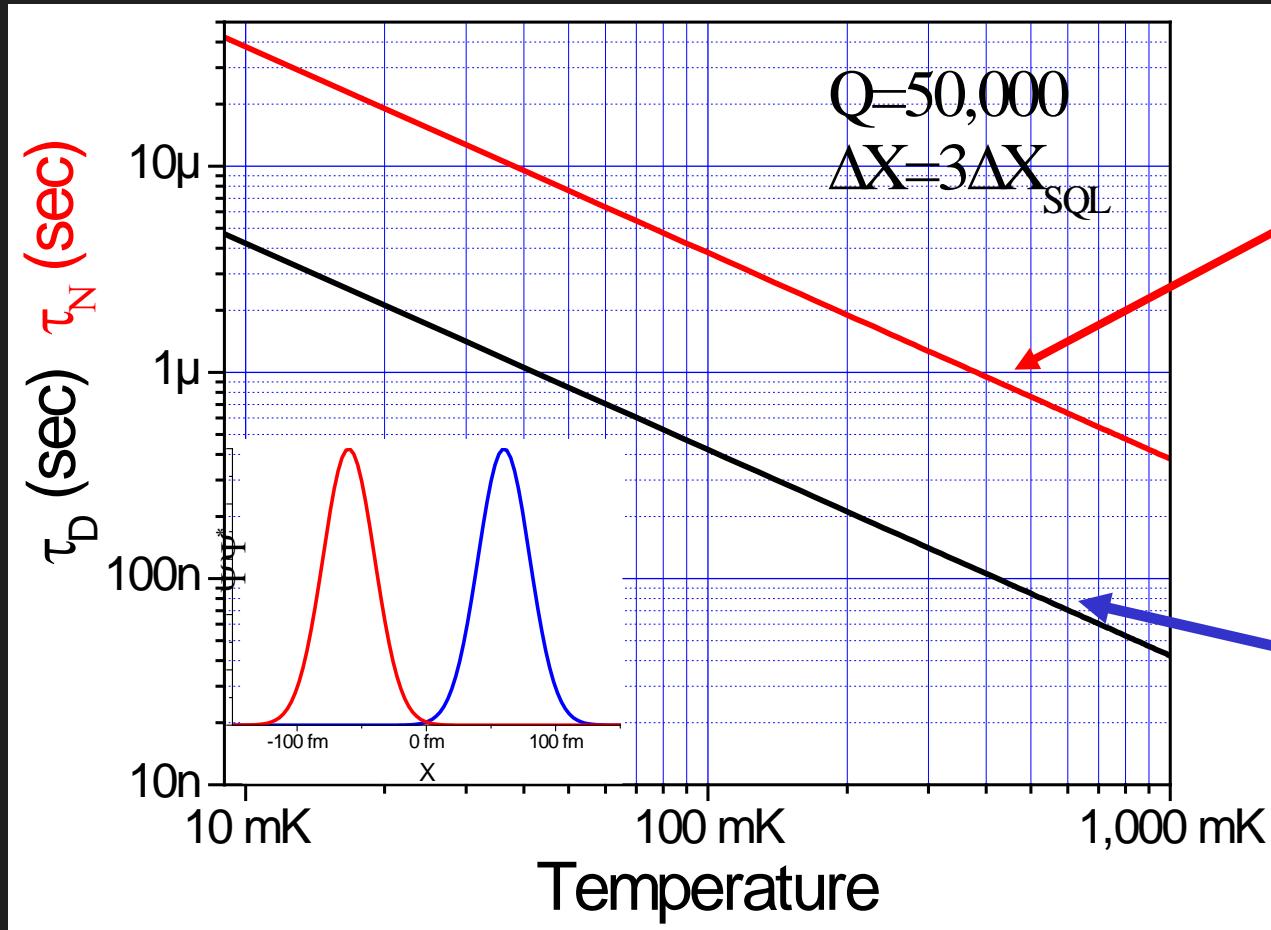
10^{-21} Nt Nuclear Spin

10^{-18} Nt..... Electron Spin

10^{-13} Nt..... Artificial Atoms - Qubits

Best qubits to date
 $t_1, t_2 \sim 10\text{-}100 \mu\text{sec}$

Coherence time and Fock state lifetime



Lifetime for number state:

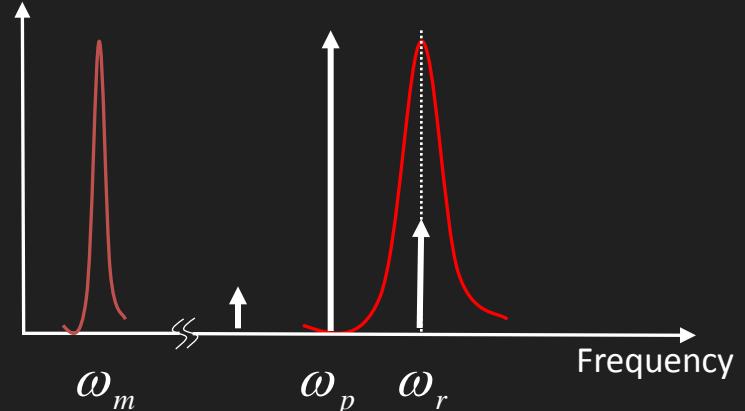
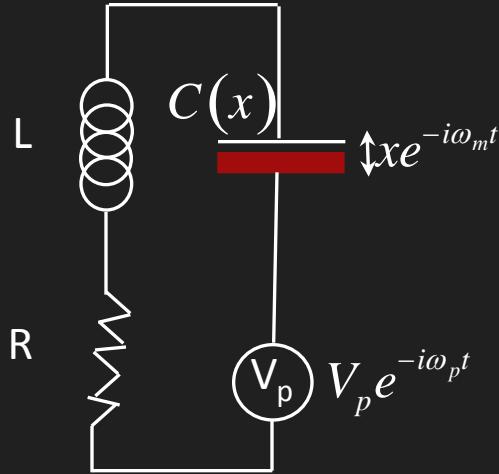
$$\tau_N = Q \frac{\hbar}{k_B T}$$

Decoherence time for superposition of coherent states:

$$\tau_D = \frac{\hbar^2}{2m\gamma_m k_B T (\Delta x)^2}$$

$$= Q \frac{\hbar}{k_B T} \left[\frac{\Delta x_{SQL}}{\Delta x} \right]^2$$

Parametric Transducer



$$\delta Q(t) = \delta C e^{-i\omega_m t} \cdot V_p e^{-i\omega_p t}$$

$$\omega_{sideband} = \omega_p \pm \omega_m$$

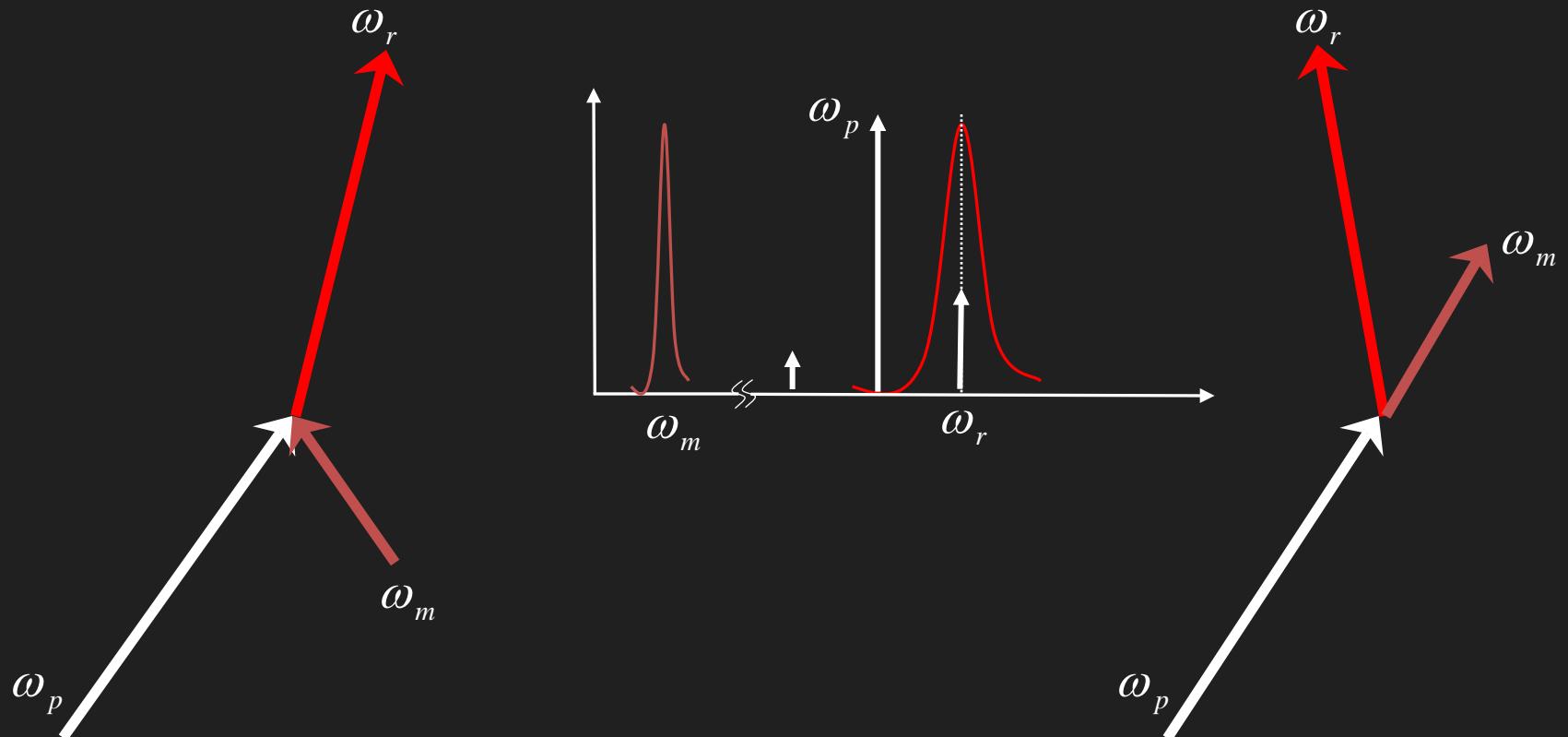
Johnson, Warren W., Bocko, Mark, Approaching the Quantum "Limit" for Force Detection. *Phys. Rev. Lett.* **47**, 1184-1187 (1981).

Braginsky, V. B., Khalili, F. Ya., *Quantum Measurement* (Cambridge Univ. Press, Cambridge, 1995).

Blair, Ivanov, Tobar, Turner, Kann, Heng, High Sensitivity Gravitational Wave Antenna with Parametric Transducer Readout. *Phys. Rev. Lett.* **74**, 1908 (1995).

Bocko, Mark F., Onofrio, Roberto, On the measurement of a weak classical force coupled to a harmonic oscillator: experimental progress. *Rev. Mod. Phys.* **68**, 755-799 (1996).

Up and down conversion -- Raman process



Up-conversion from pump
→ Cooling, positive damping of mechanics

$$\Gamma_- \propto n_m$$

Down-conversion from cavity
→ Heating of mechanics

$$\Gamma_+ \propto n_m + 1$$

Detailed Balance

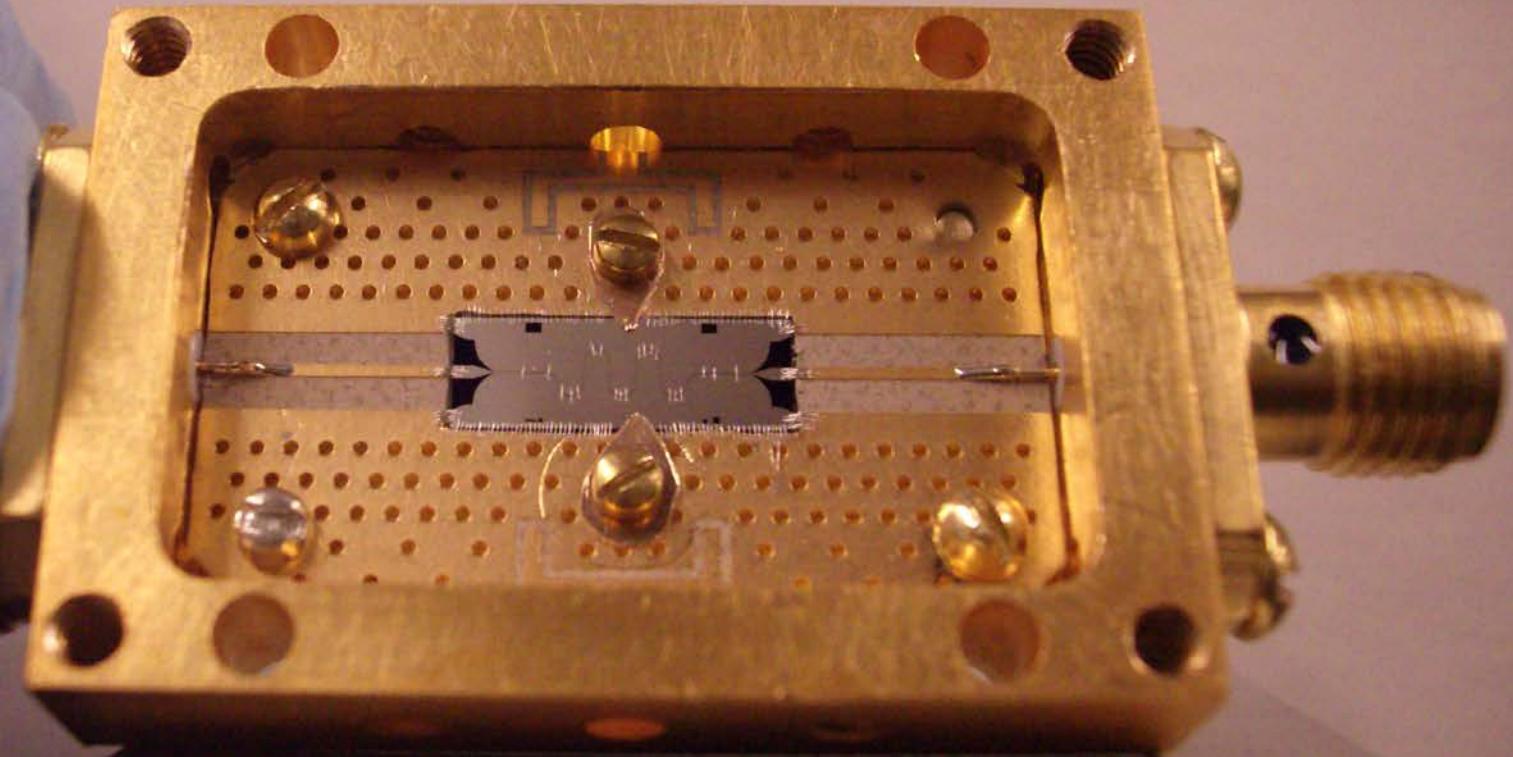


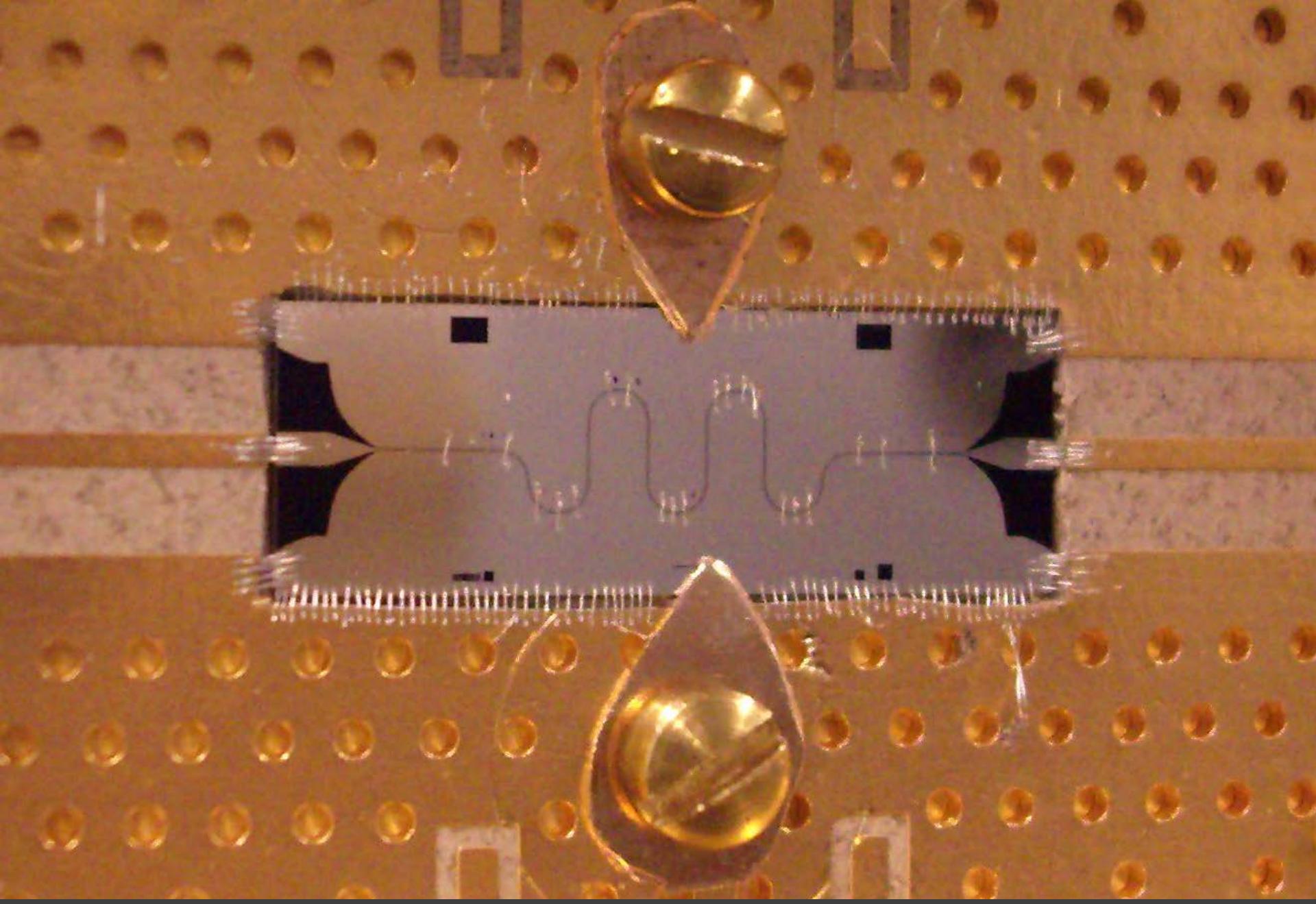
$$n_m = \frac{\Gamma_m^T n_m^T + \Gamma_{opt} n_{sr}}{\Gamma_m^T + \Gamma_{opt}}$$

$$\Gamma_{opt} = 4x_{zp}^2 \left(\frac{\partial \omega_{sr}}{\partial x} \right)^2 \frac{n_p}{\kappa}$$

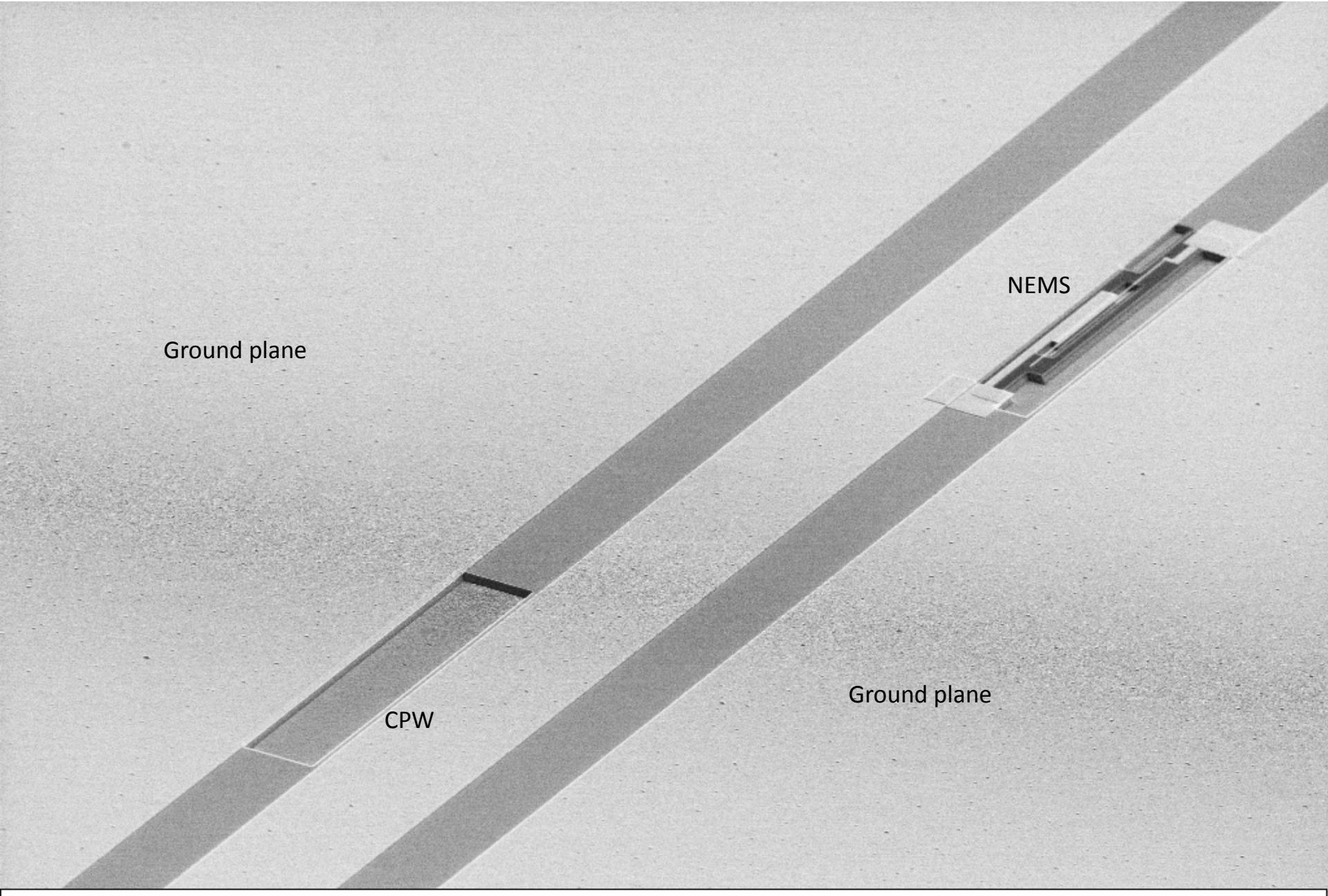
$$n_{sr} = \left(\frac{\kappa}{4\omega_m} \right)^2 + n_{sr}^T \left[1 + 2 \left(\frac{\kappa}{4\omega_m} \right)^2 \right]$$

Devices





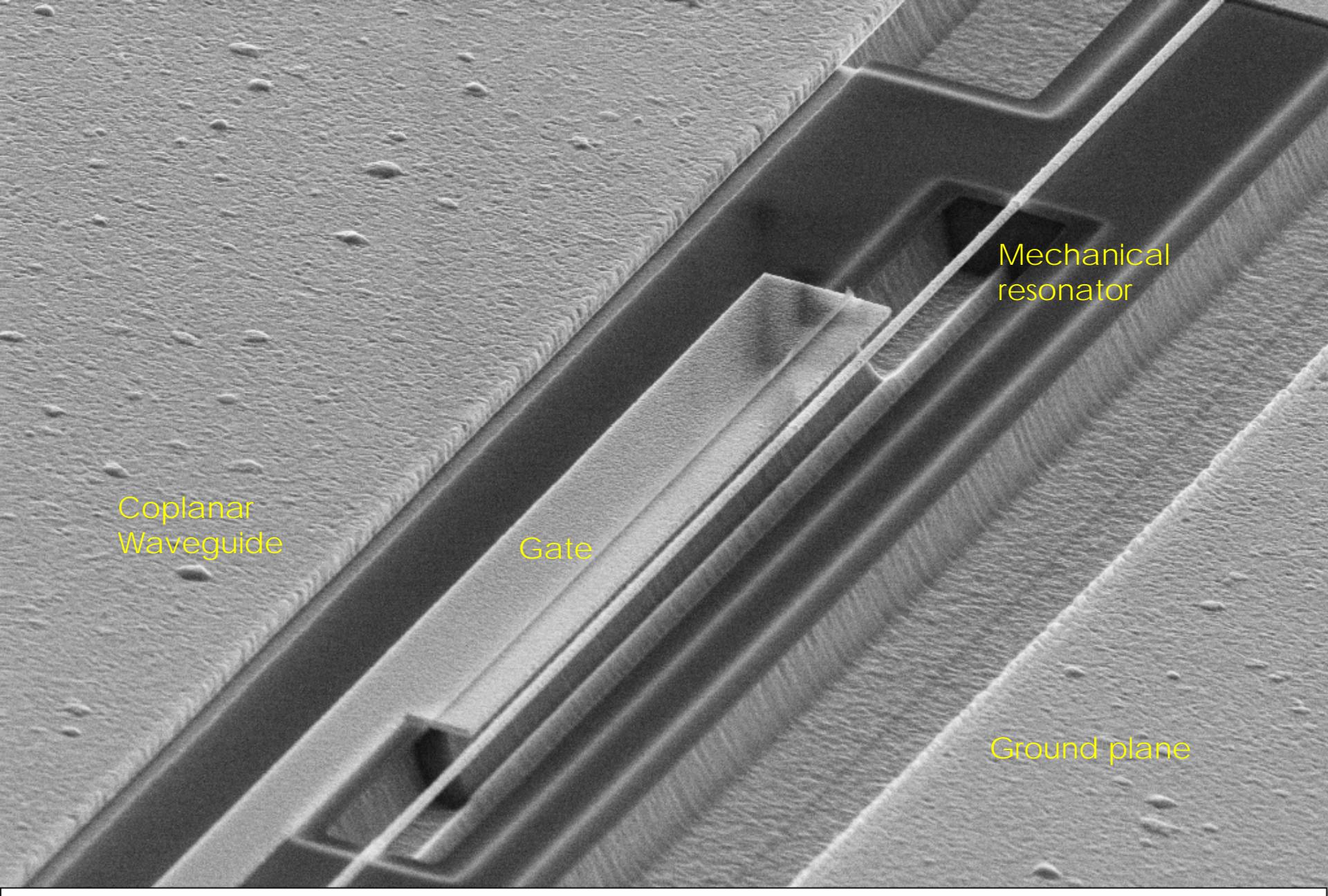
P. Day, R. LeDuc, B. Mazin, A. Vayonakis, J. Zmuidzinas, Nature **425**, 817 (2003).



10 μ m

WD = 7 mm Aperture Size = 20.00 μ m Signal A = InLens Date : 10 Apr 2008
Mag = 2.30 KX EHT = 5.00 kV Pixel Size = 153.8 nm Signal B = SE2 Time : 3:39:34

CNF



Coplanar
Waveguide

Gate

Mechanical
resonator

Ground plane

3 μ m

WD = 7 mm

Mag = 18.78 K X

Aperture Size = 20.00 μ m

EHT = 5.00 kV

Pixel Size = 18.8 nm

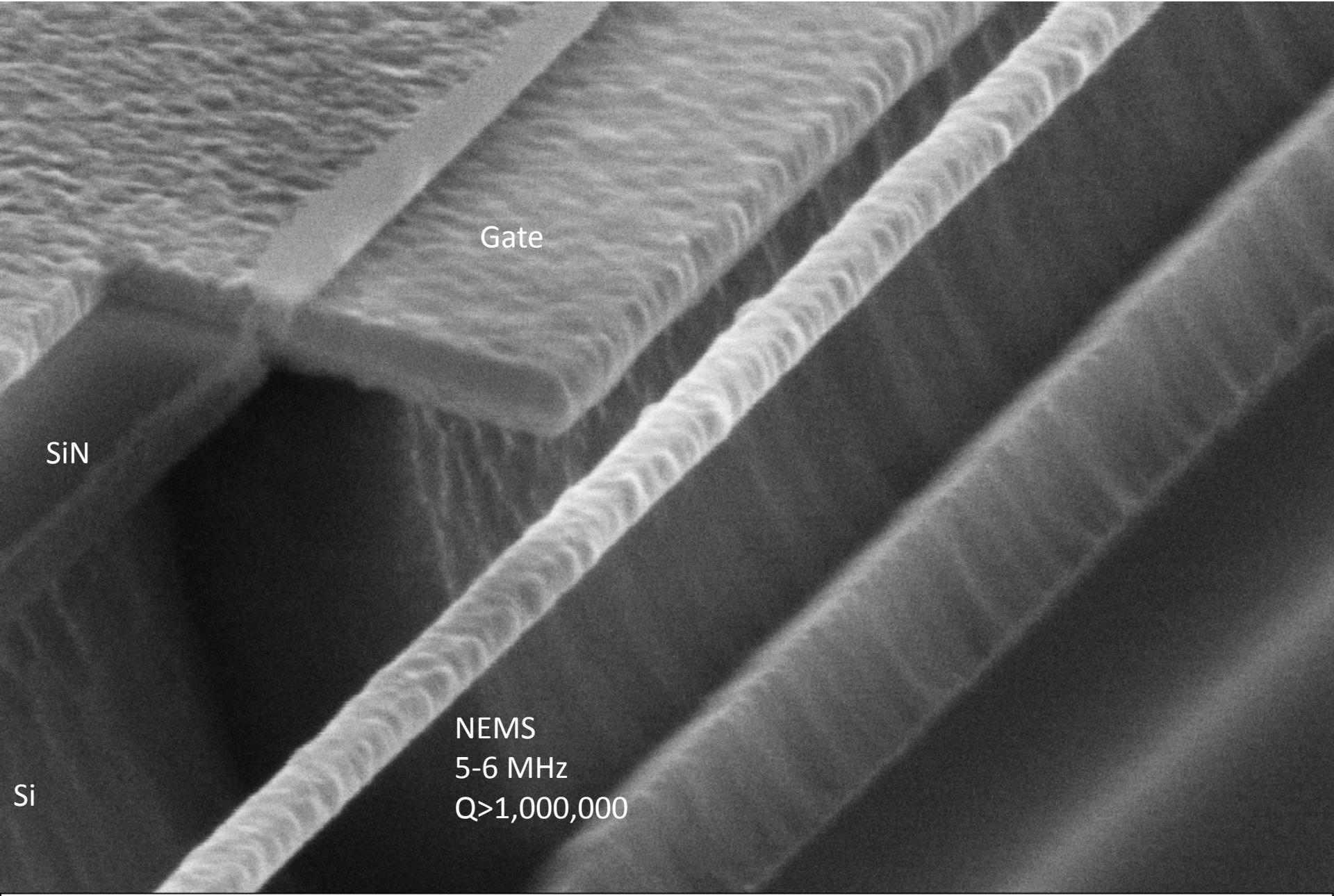
Signal A = InLens

Signal B = SE2

Date : 10 Apr 2008

Time : 3:32:46

CNF

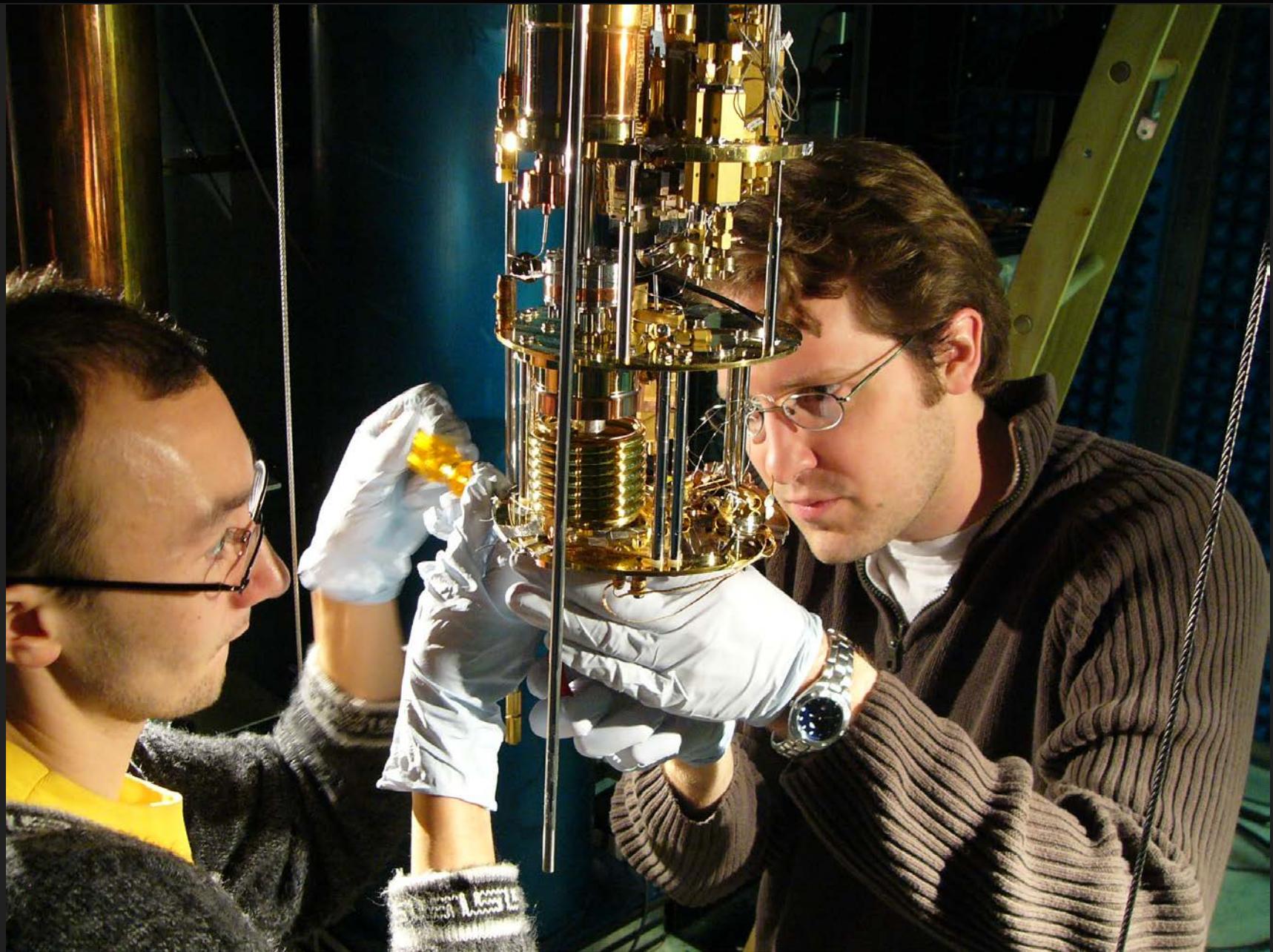


200nm

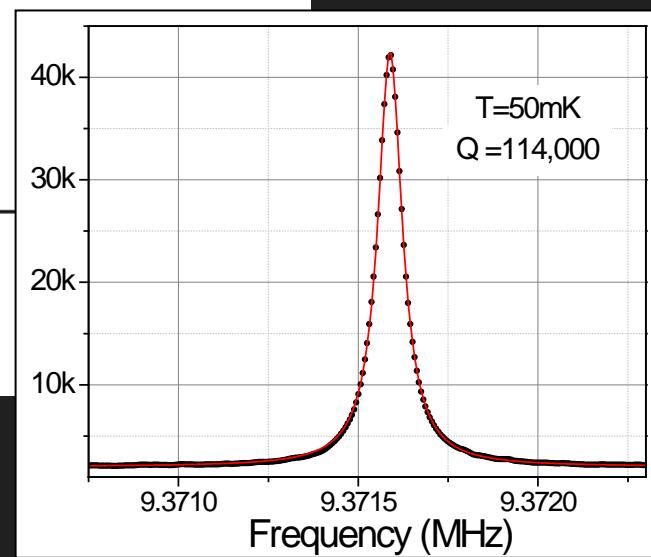
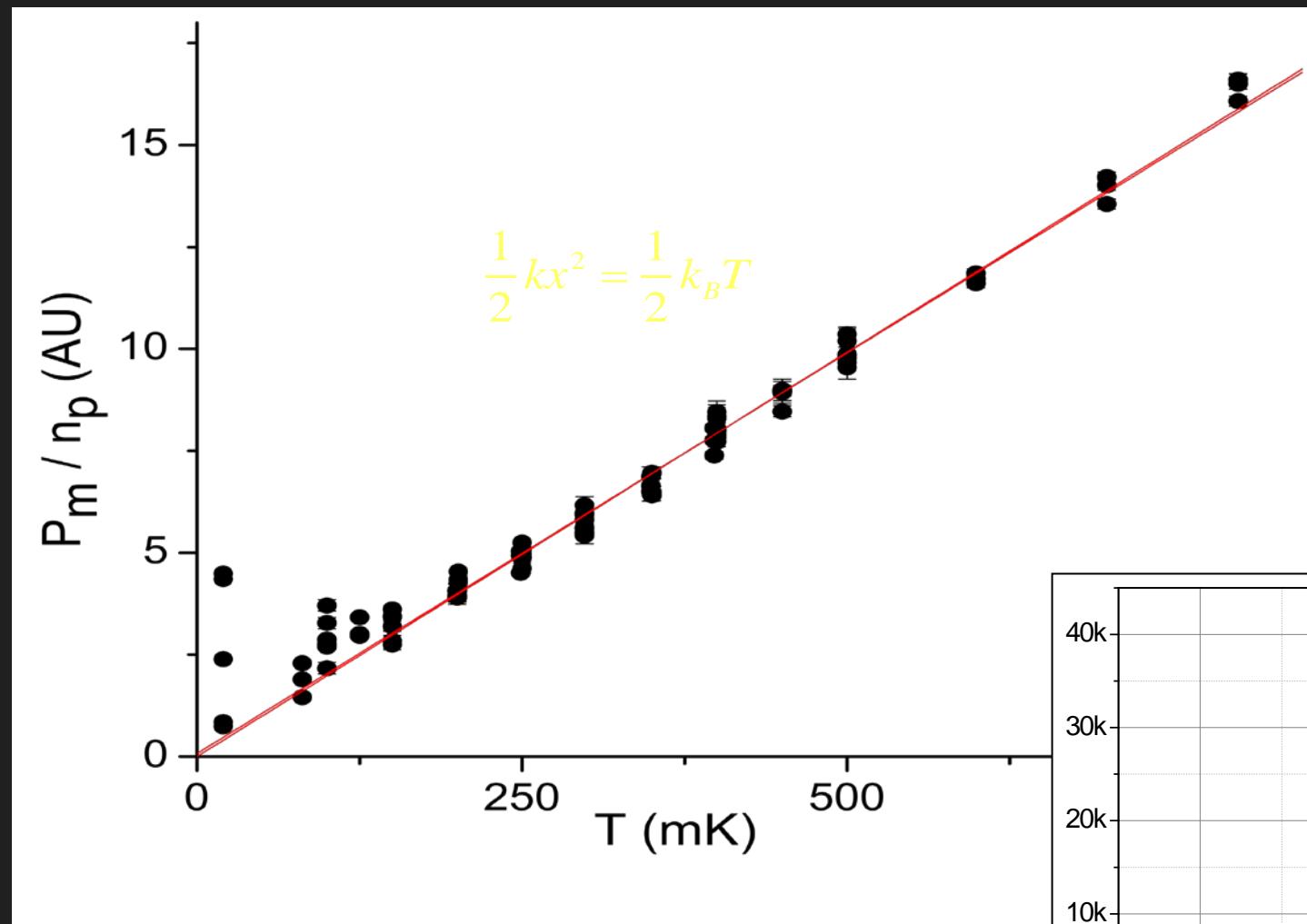
WD = 7 mm Aperture Size = 20.00 μm Signal A = InLens Date : 10 Apr 2008
Mag = 134.00 K X EHT = 5.00 kV Pixel Size = 2.6 nm Signal B = SE2 Time : 3:38:05

CNF

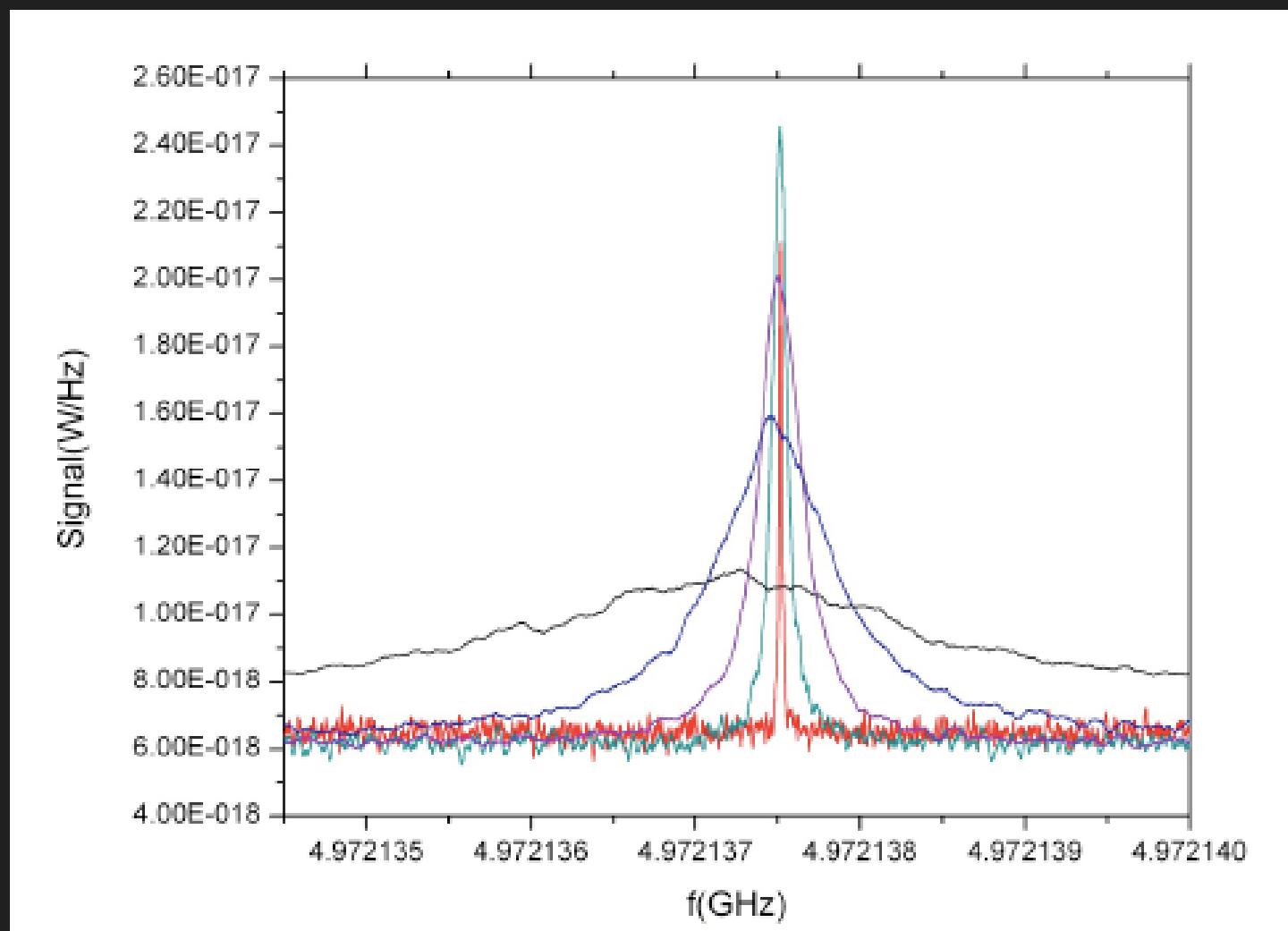
Installing samples onto the fridge



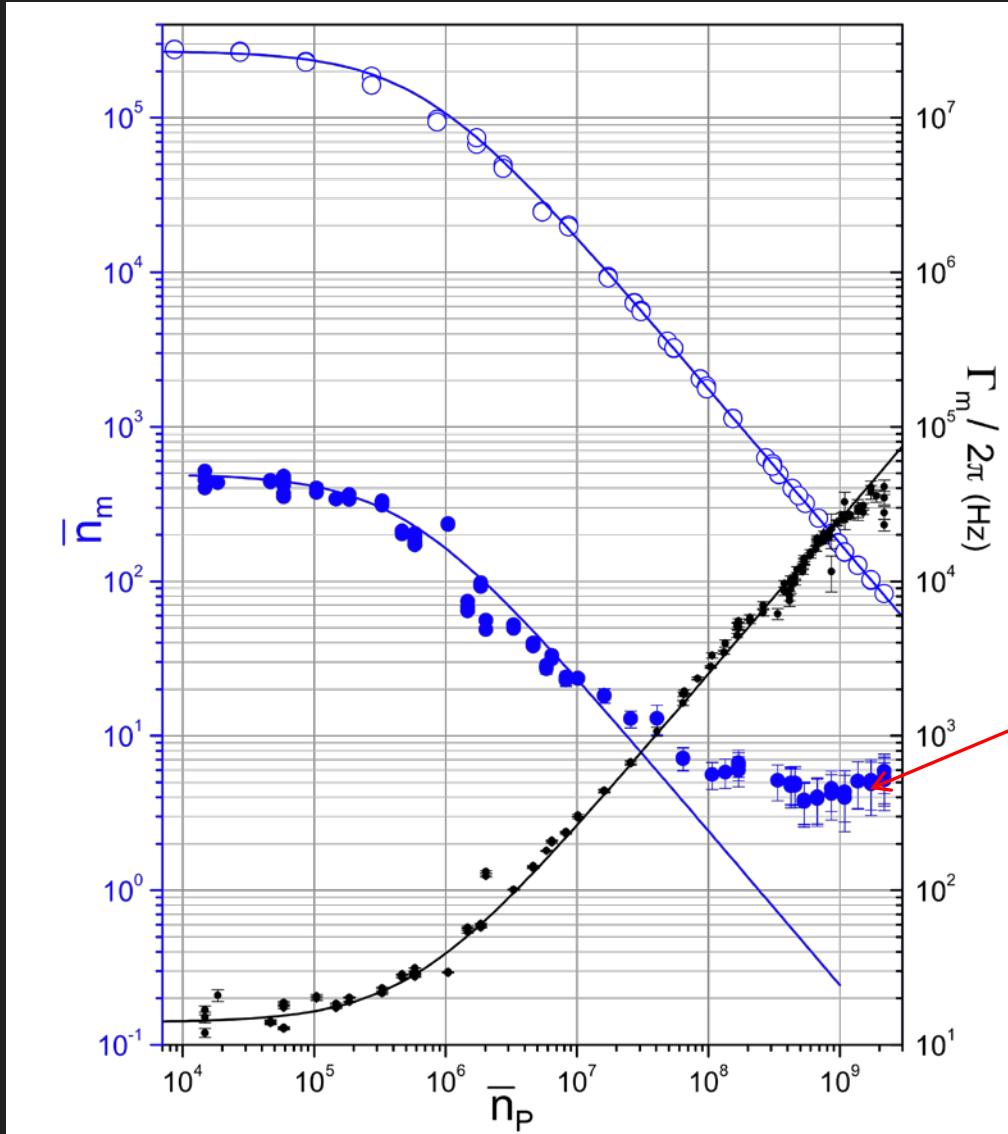
Brownian Motion – Equipartition



Damping and Cooling



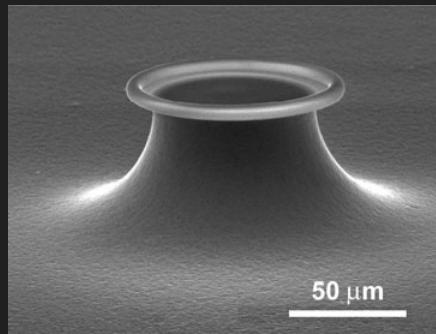
Damping and Cooling



$$n_m = 3.8 \pm 1.3$$

$$P_0 = \frac{1}{1 + n_m} = 0.2$$

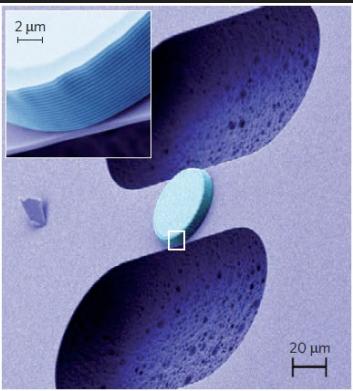
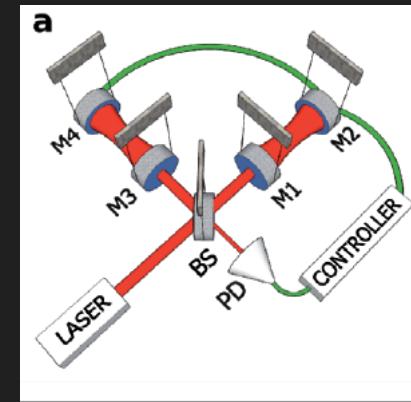
Progress in Opto-mechanics



Schliesser, Arcizet, Riviere, Anetsberger, Kippenberg, *Nature Phys.* **5**, (2009).

$N \sim \text{few}$
(58MHz)

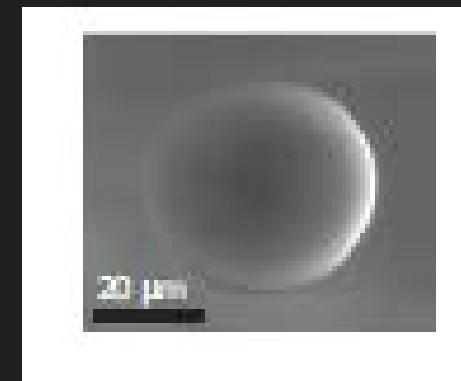
Abbott, et al., *New J. Phys.* **11**, (2009).



Groblacher, Hertzberg, Vanner, Cole, Gigan, Schwab, Aspelmeyer, *Nature Phys.* **5**, (2009).

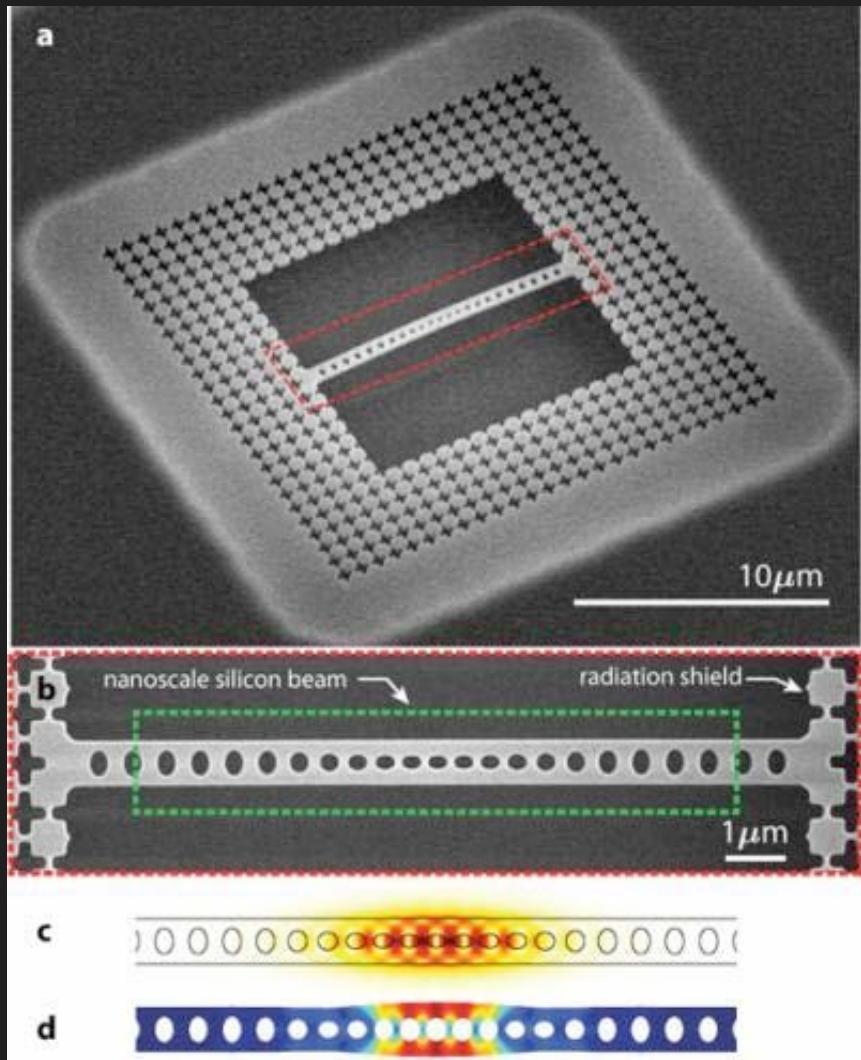
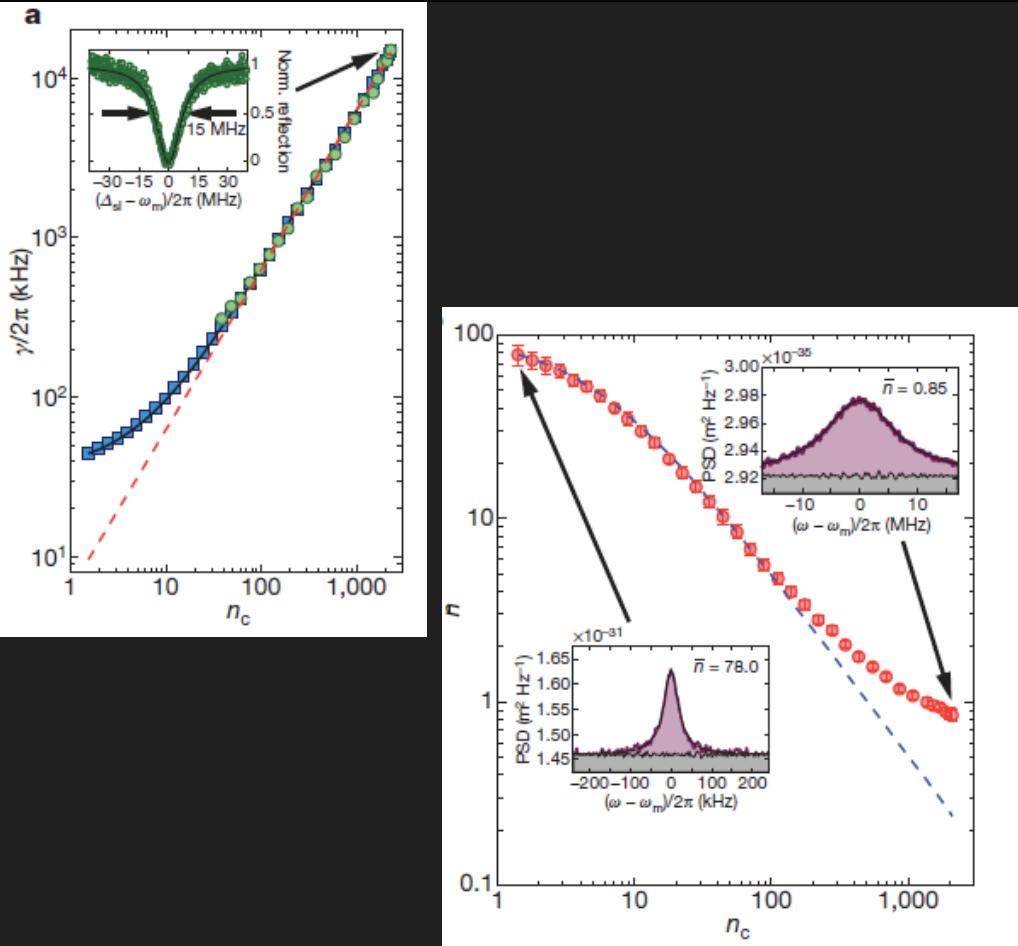
$N=30$
(1MHz)

Park and Wang, *Nature Phys.* **5**, (2009).



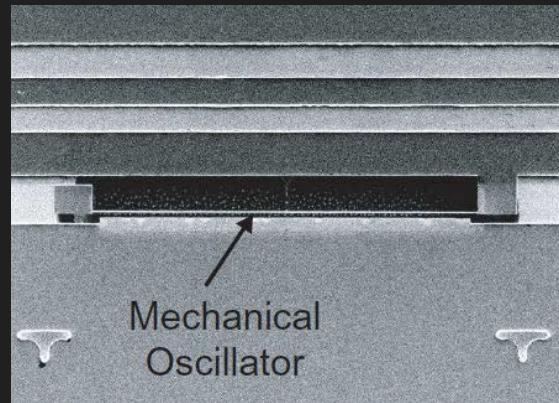
$N=37$
(100MHz)

Progress in Opto-mechanics -- ground state



J. Chan, T. Alegre, A. Safavi-Naeini, J. Hill, A. Krause, S. Groblacher, M. Aspelmeyer, O. Painter, *Nature* **478**, 389 (2011).
 A. Safavi-Naeini, J. Chan, J. Hill, T. Mayer Alegre, A. Krause, O. Painter, *PRL* (2012).

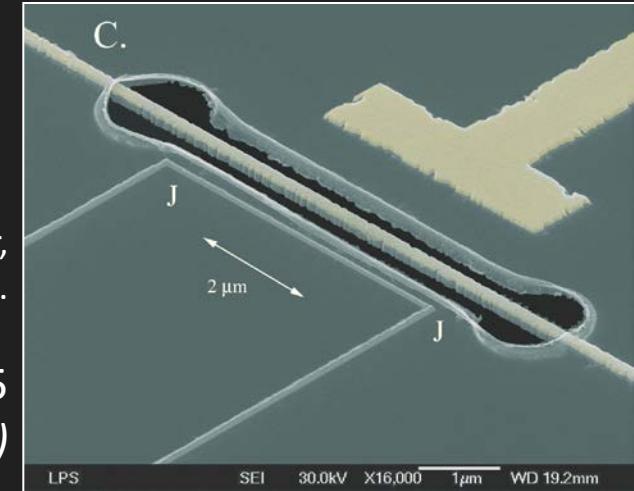
Progress in Electro-mechanics -- ground state



Teufel, Harlow, Regal, Lehnert, *Phys. Rev. Lett.*. **101**, (2008).

N=140
(1MHz)

Naik, LaHaye, Buu, Clerk, Blencowe, Armour,
Schwab, *Nature*. **443**, (2006).

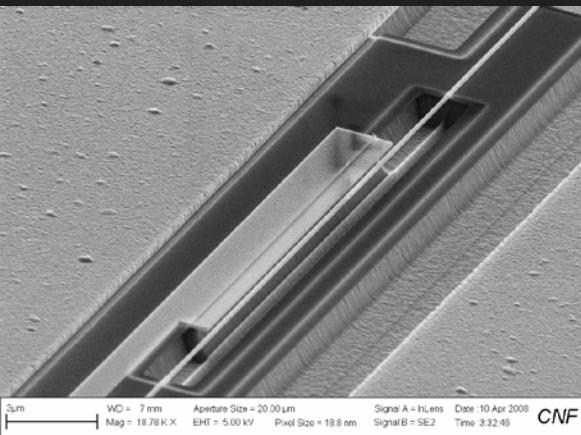


N=25
(20MHz)

This work....

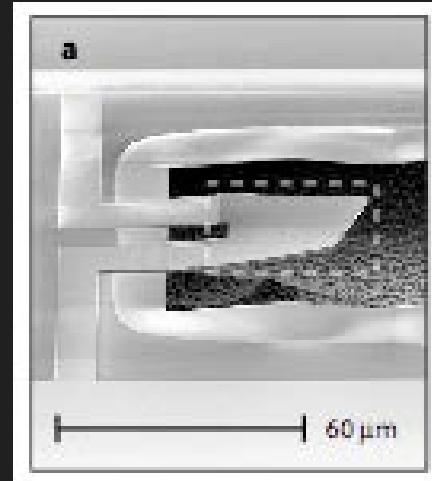
Rocheleau, Ndukum, Macklin, Hertzberg, Clerk,
Schwab, *Nature* (2010)

N=3.8 +/- 1
(5MHz)



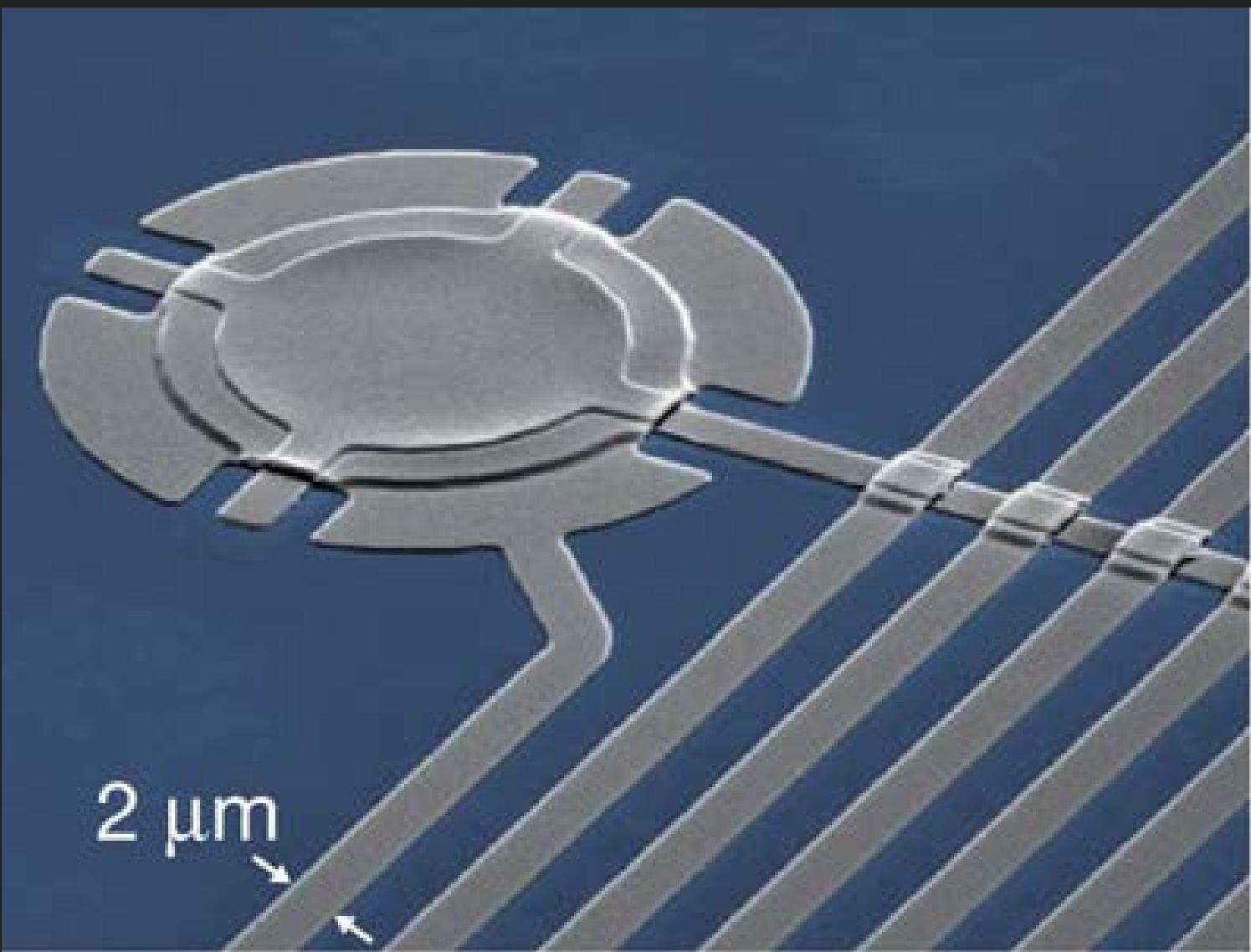
O'Connell, et al, *Nature* (2010).

N<0.1
(6GHz)

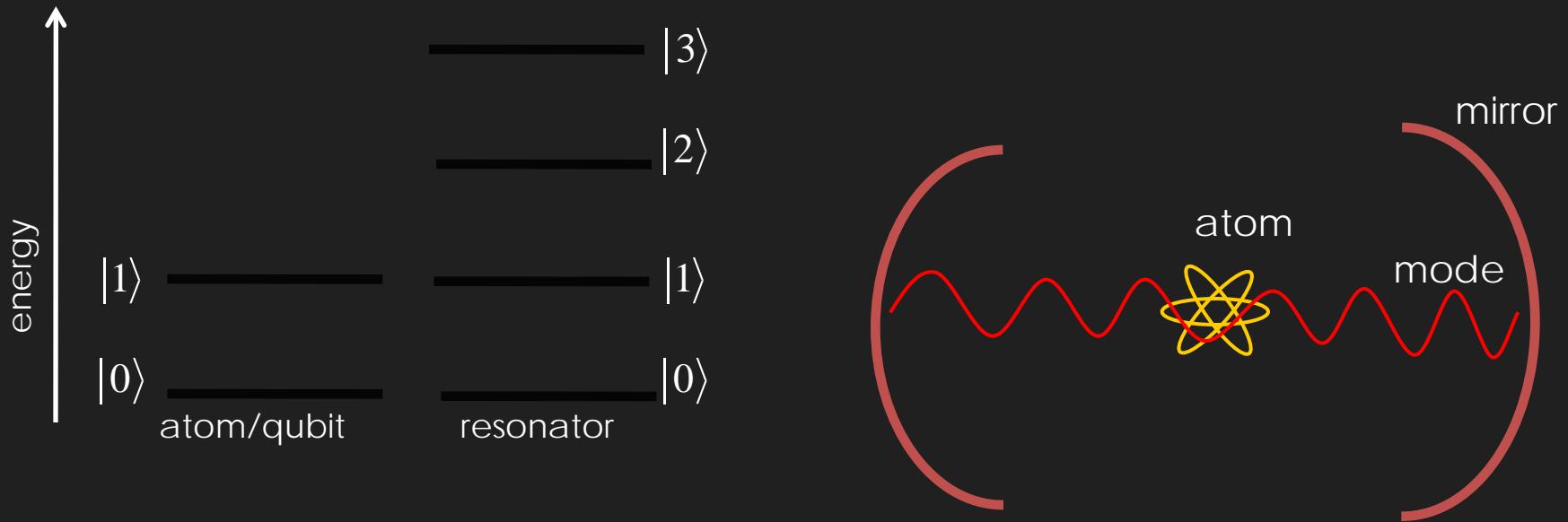


Progress in Electro-mechanics -- ground state

N=0.4 !



Control at the single quanta level



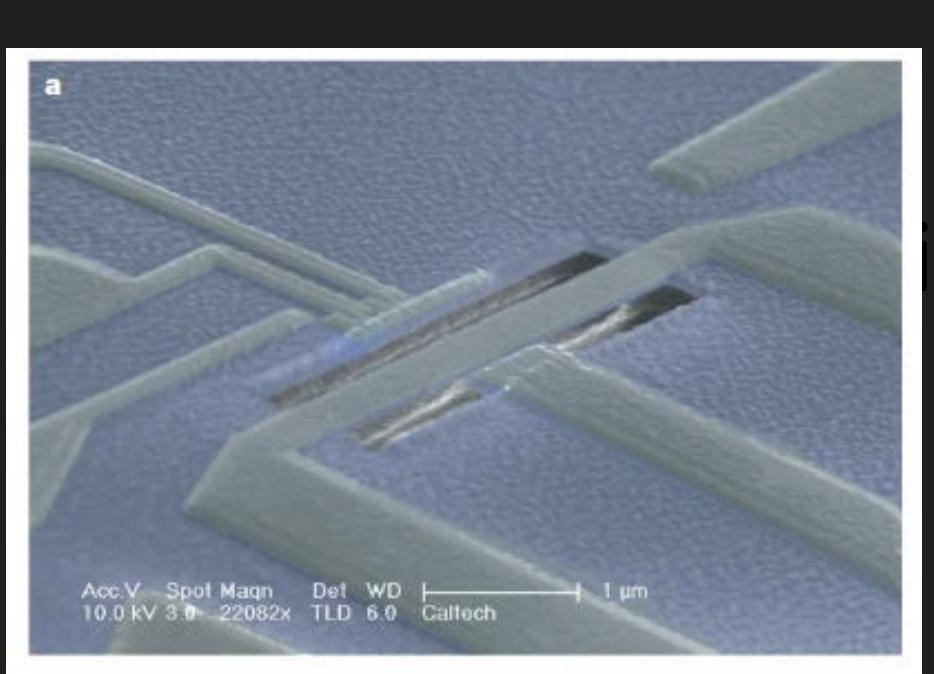
resonator

$$H_{total} = \hbar\omega_r a^\dagger a + \frac{1}{2} \hbar\omega_q \sigma_z + \hbar g (\sigma_+ + \sigma_-) (a + a^\dagger)$$

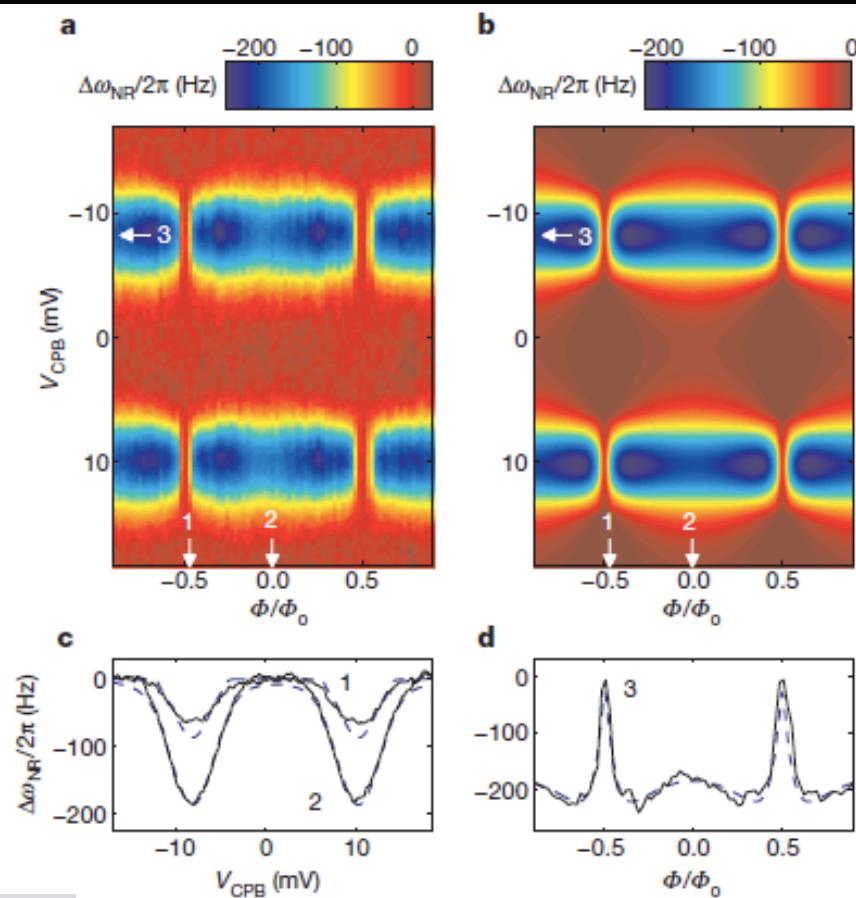
Two state system

Interaction with exchange of quanta

NEMS coupled to superconducting qubit



$$H = \hbar\omega_{NR}\hat{a}^\dagger\hat{a} + \frac{\Delta E}{2}\hat{\sigma}_z + \hbar\lambda(\hat{a} + \hat{a}^\dagger)\left(\frac{E_{el}}{\Delta E}\hat{\sigma}_z - \frac{E_j}{\Delta E}\hat{\sigma}_x\right)$$



nature

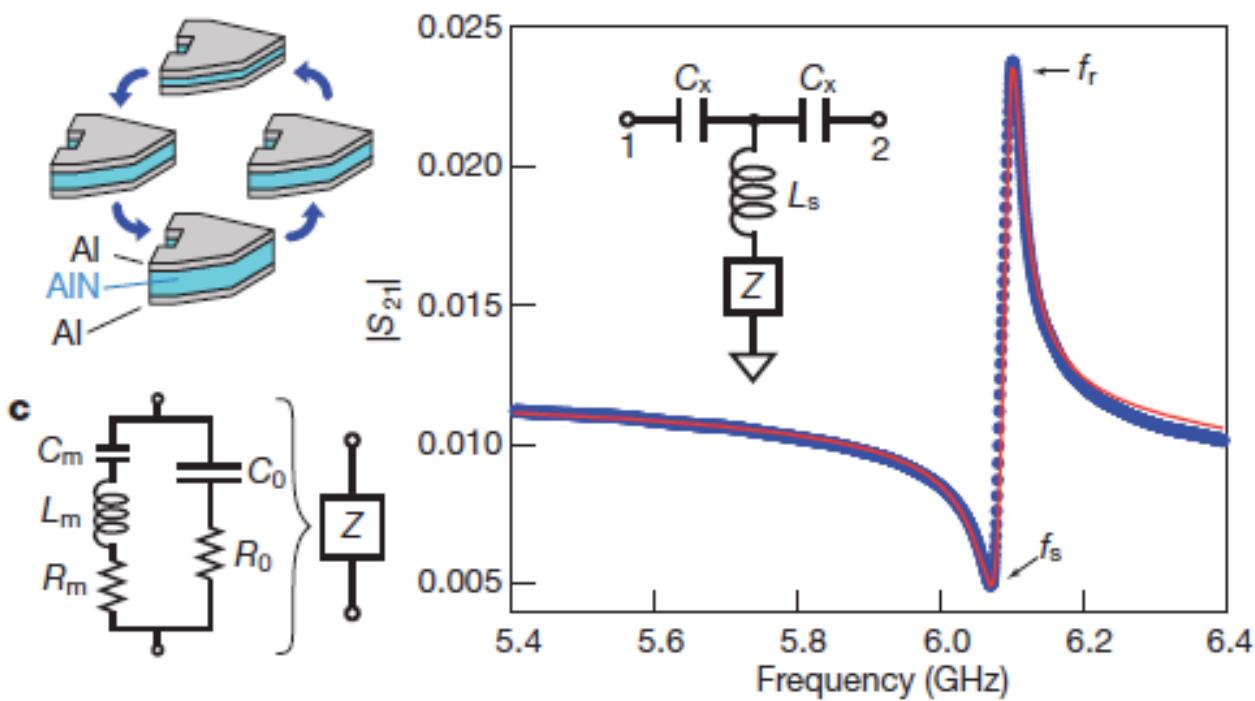
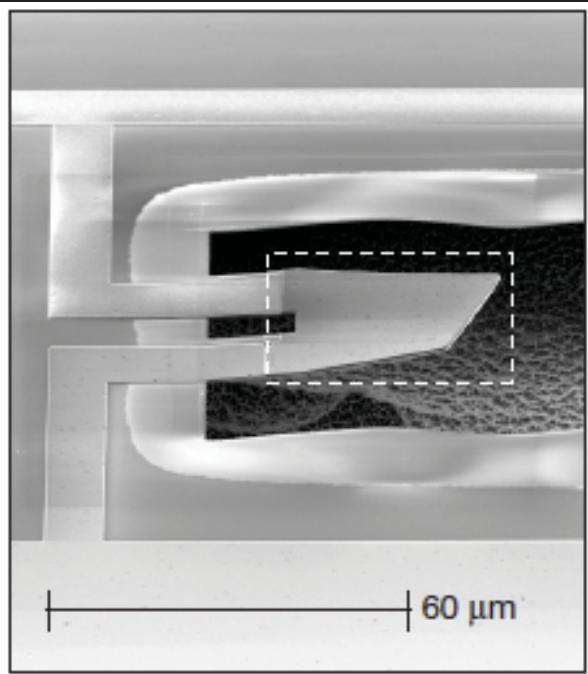
Vol 459 | 18 June 2009 | doi:10.1038/nature08093

LETTERS

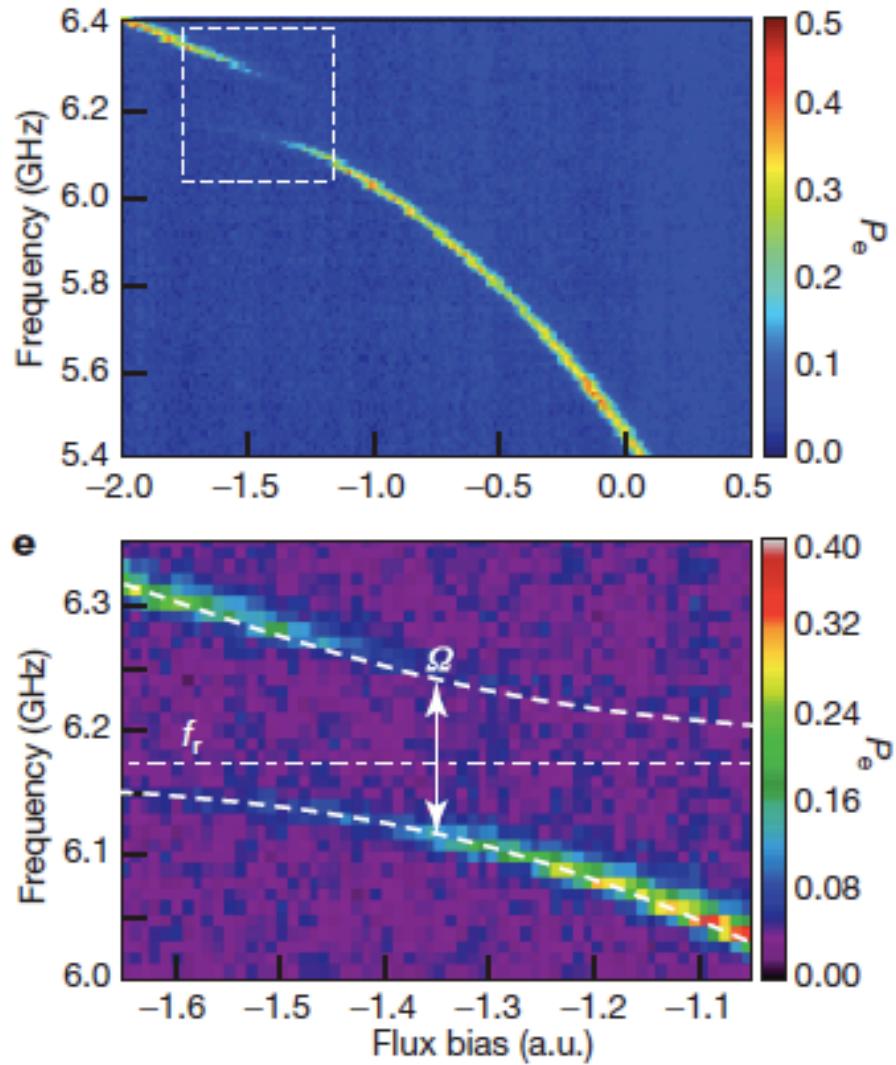
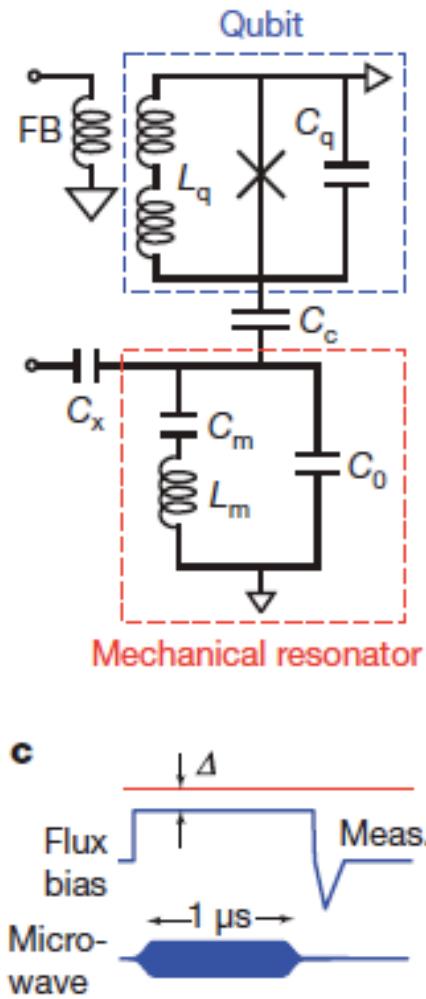
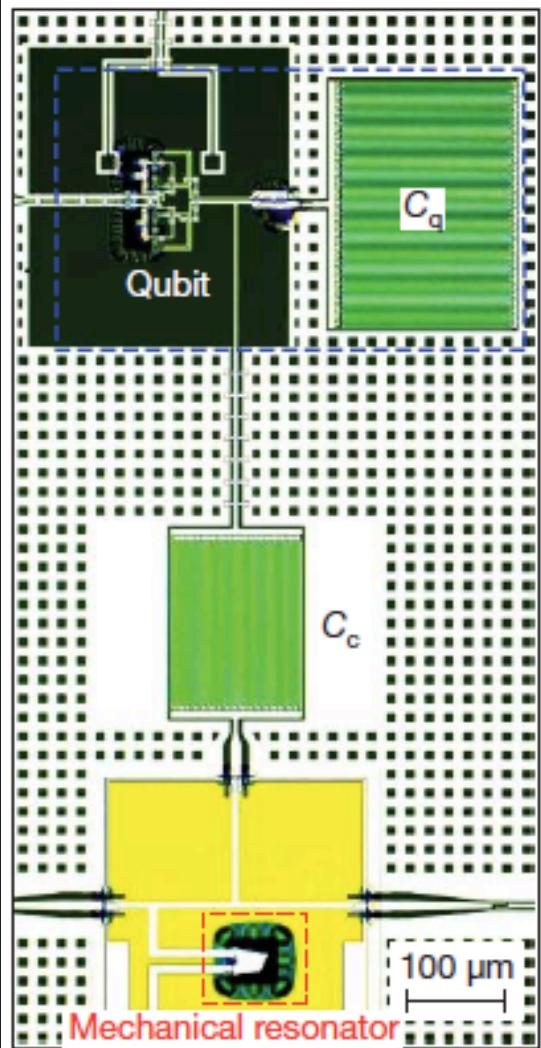
Nanomechanical measurements of a superconducting qubit

M. D. LaHaye¹, J. Suh¹, P. M. Echternach³, K. C. Schwab² & M. L. Roukes¹

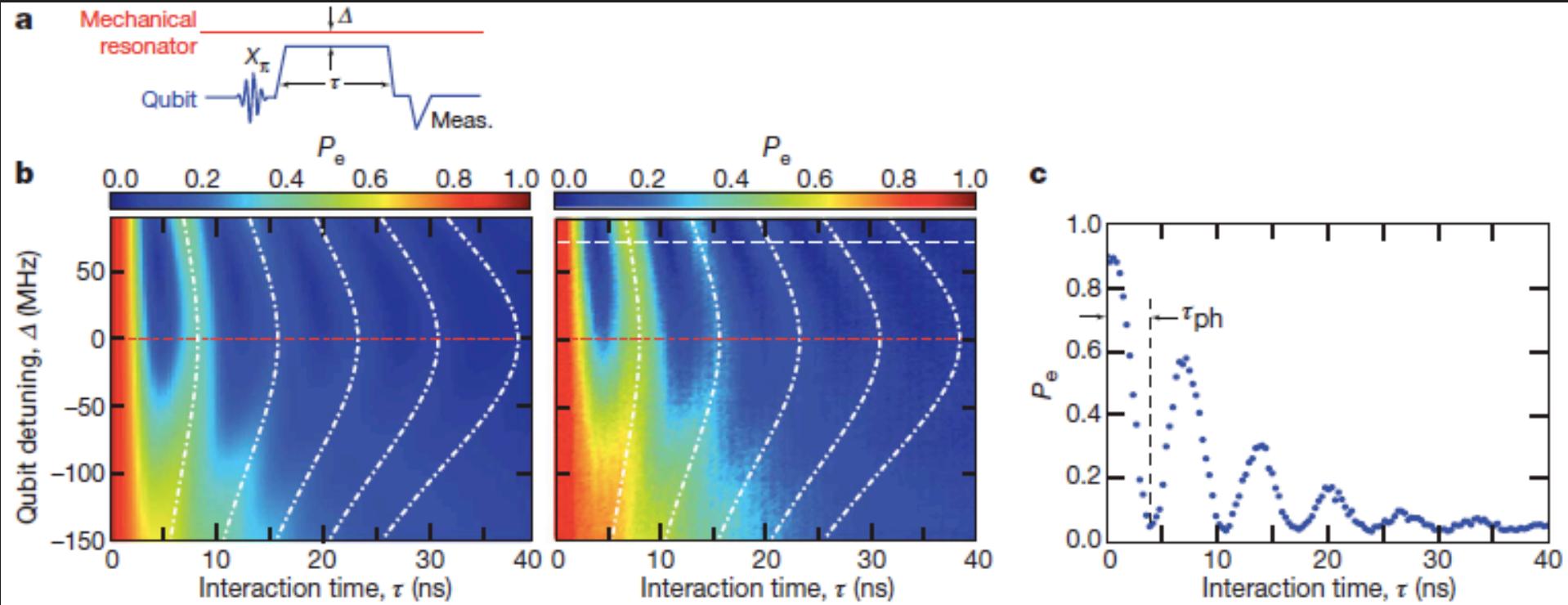
Control at the single quanta level



Coupling to a superconducting qubit



Exchanging a single quanta



The Standard Quantum Limit

$$\begin{aligned} & [\hat{x}, \hat{p}] = i\hbar \\ & \Delta x \cdot \Delta p \geq \frac{\hbar}{2} \\ & \hat{H} = \frac{1}{2}m\omega^2 \hat{x}^2 + \frac{1}{2m} \hat{p}^2 \\ & \frac{d\hat{x}}{dt} = -\frac{i}{\hbar} [\hat{x}, \hat{H}] \end{aligned}$$

The diagram illustrates the derivation of the standard quantum limit. It starts with the commutator relation $[\hat{x}, \hat{p}] = i\hbar$, followed by the uncertainty principle $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$. Below these, the Hamiltonian $\hat{H} = \frac{1}{2}m\omega^2 \hat{x}^2 + \frac{1}{2m} \hat{p}^2$ is given, and finally, the time evolution equation $\frac{d\hat{x}}{dt} = -\frac{i}{\hbar} [\hat{x}, \hat{H}]$. Red curved arrows connect the symbols \hat{x} and \hat{p} in the equations to their respective symbols in the text above, indicating the correspondence between the mathematical expressions and the physical variables.

QND Measurement of Quadratures

$$\hat{x} + i \frac{\hat{p}}{m\omega} = (\hat{X}_1 + i\hat{X}_2) e^{-i\omega t}$$

$$\hat{x}(t) = \hat{X}_1 \cos(\omega t) + \hat{X}_2 \sin(\omega t)$$

$$\hat{X}_1$$

$$\boxed{[\hat{X}_1, \hat{X}_2] = i \frac{\hbar}{m\omega}}$$

$$\boxed{\Delta X_1 \cdot \Delta X_2 \geq \frac{\hbar}{2m\omega}}$$

$$\hat{X}_2$$

$$\boxed{\frac{d\hat{X}_{1,2}}{dt} = \frac{\partial X_{1,2}}{\partial t} - \frac{i}{\hbar} [\hat{X}_{1,2}, \hat{H}] = 0}$$

Braginskii, V.B., Vorontsov, Yu.I., Quantum-mechanical limitations in macroscopic experiments and modern experimental design. *Sov. Phys. Usp.* **17**, 644 (1975).

Thorne, Kip S., Drever, Ronald W.P., Caves, Carlton M., Zimmerman, Mark, Sandberg, Vernon D., Quantum Nondemolition measurements of Harmonic Oscillators. *Phys. Rev. Lett.* **40**, 667-671 (1978).

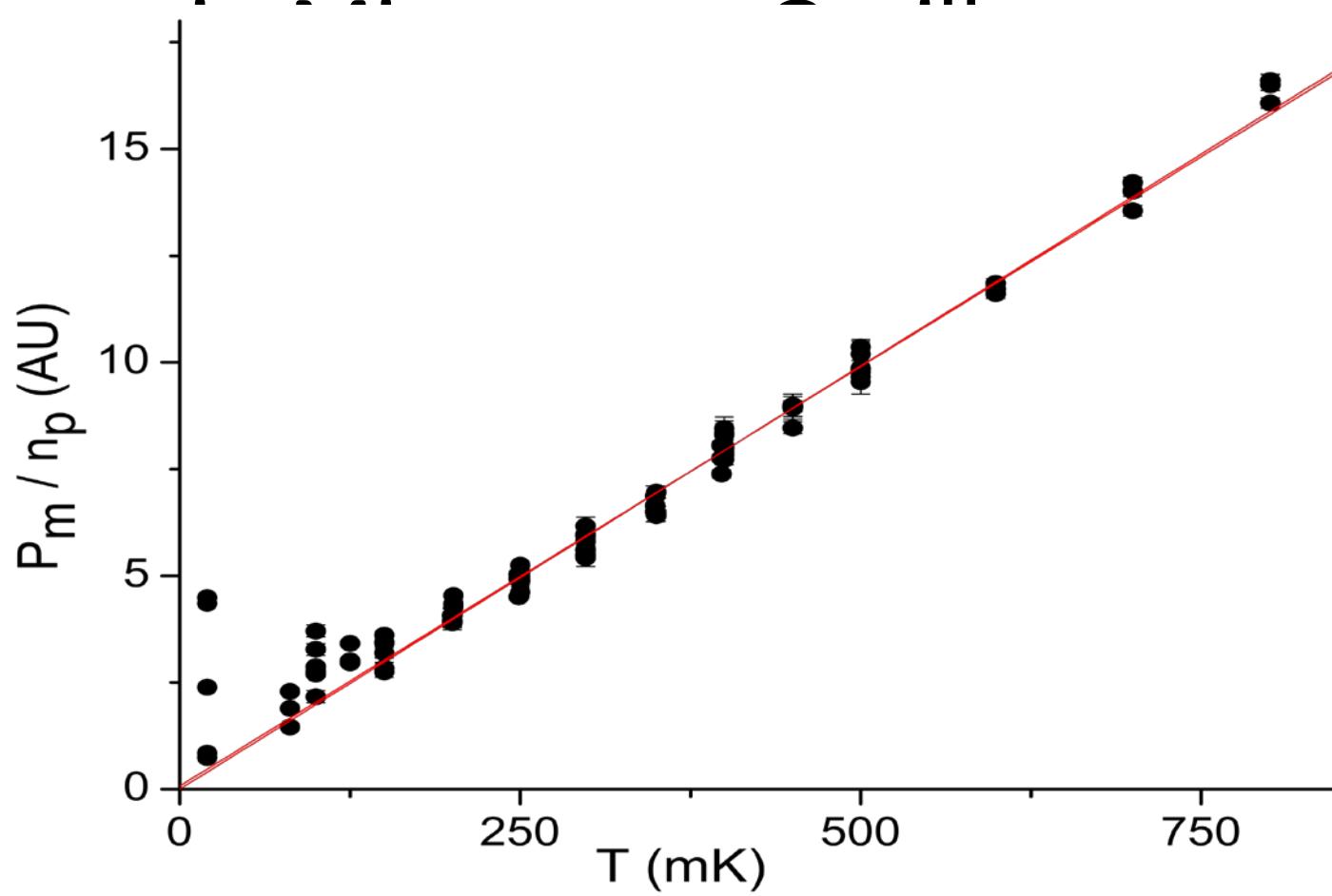
Summary, take home messages, and good things to know

- Small particles (photons, electrons, ect) don't behave in a way consistent with our classical/ancient notions of reality.
- Large molecules (Buckyballs and now larger) can act like waves, going both ways around an obstacle and interfering with itself.
- There is probably no limit to the size where quantum behavior stops; we are building small mechanical devices to see the wave behavior of larger objects.
- Our community has recently produced the quantum ground state of mechanical structures in both electro-mechanical and opto-mechanical devices.
- The Uncertainty Principle fluctuations of a mechanical mode have been recently measured.
- Manipulation of a single phonon, using a qubit, has recently been demonstrated.

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Mechanical Brownian Motion – Signal



Schrödinger's cat

• Wave function for a single atom: $|\Psi\rangle = |0\rangle_{atom} + |1\rangle_{atom}$

$$|\Psi\rangle = |1\rangle_{atom} + |0\rangle_{atom}$$

• After measurement:

$$|\Psi\rangle \rightarrow |1\rangle_{atom} \text{ or } |0\rangle_{atom}$$

• What if a microscopic degree of freedom is coupled to a macroscopic state?

$$|\Psi\rangle = |1\rangle_{atom} \otimes |\text{alive}\rangle_{cat} + |0\rangle_{atom} \otimes |\text{dead}\rangle_{cat}$$

• Can something as complex as a cat become entangled with microscopic entity?

• Before you look, is the cat in a superposition of alive and dead?

