Brownian Thermal Noise in Dielectric Coatings

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Dcc reference: LIGO-P1200012-v2

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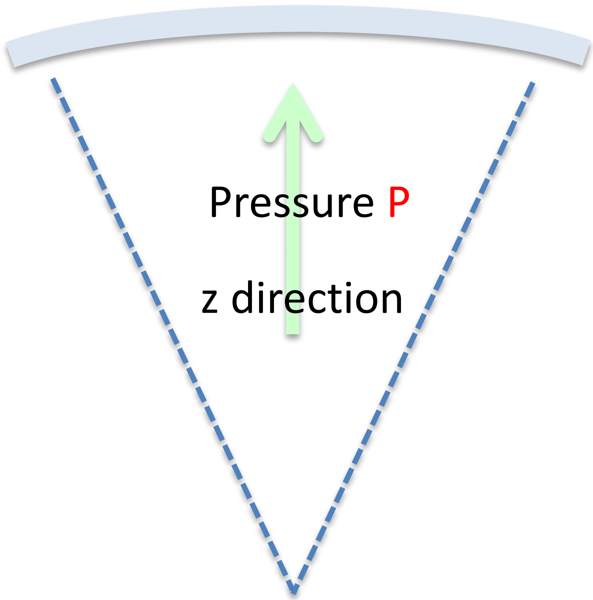
Two take home points

1. We were measuring the right (almost) loss angle! (there are two loss angles in each isotropic material)

2. Coherent backscattering effect in coating noise (negligible for Brownian noise)

Dcc link: LIGO-P1200012-v2

Two Dimensional flexural Rigidity for thin plates



Radius of curvature R

Thickness h, Young's Modulus Y, Poisson ratio σ

1. Free boundary and vertical stress is zero

$$T_{zz} = 0$$

2. Flexural rigidity is defined by

$$D = \frac{Yh^3}{12(1-\sigma^2)}$$

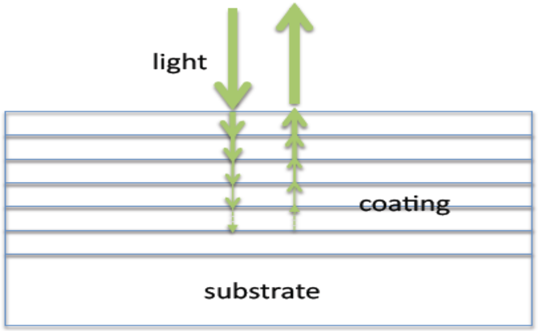
3. Hooke's law for 2-D thin plate

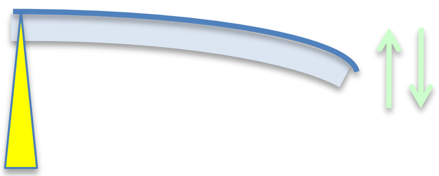
$$\frac{D}{R} = P$$

Coating flexural Rigidity and its loss angle

1. Light senses the coating thickness fluctuation + coating-substrate interface bending fluctuation

2. Bending is main noise source (70% of total noise) because substrate Young's modulus is relatively low

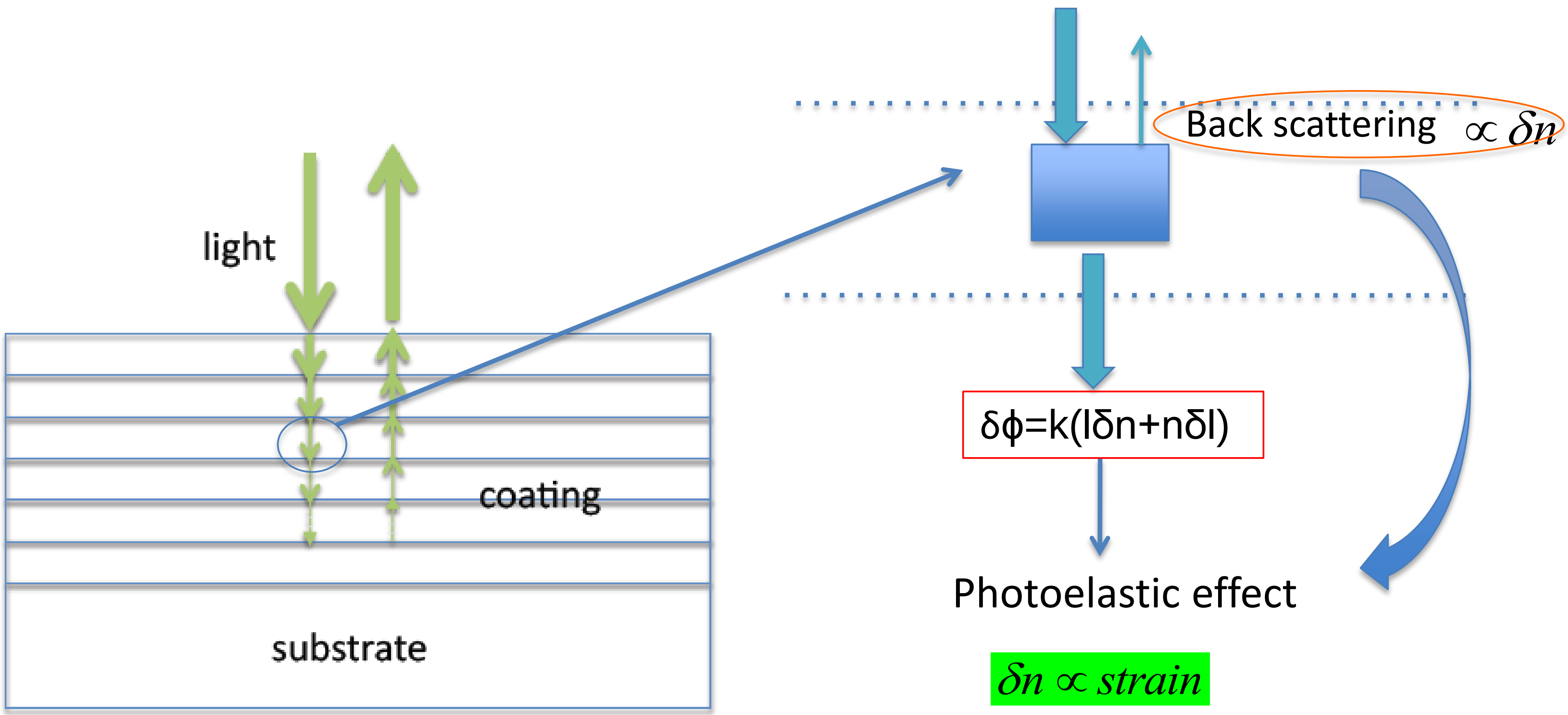




3. Bending loss angle = coating layer flexural rigidity loss angle
   = Drum mode loss angle in ringdown measurements

4. We were measuring the right loss angle !

Coherent light scattering induced Thermal noise



Thank You!

Bending is the main noise component

Position Noise = Sum of coating thickness fluctuation + interface fluctuation

$$S^{ij}_{u_{zz}u_{zz}}(\vec{x},z;\vec{x}',z') = \frac{4k_BT}{3\pi f}\frac{(1+\sigma_j)(1-2\sigma_j)}{Y_j(1-\sigma_j)^2}\left[\frac{1+\sigma_j}{2}\phi_{Bj}+(1-2\sigma_j)\phi_{Sj}\right]\delta_{ij}\delta^{(2)}(\vec{x}-\vec{x}')\delta(z-z')$$

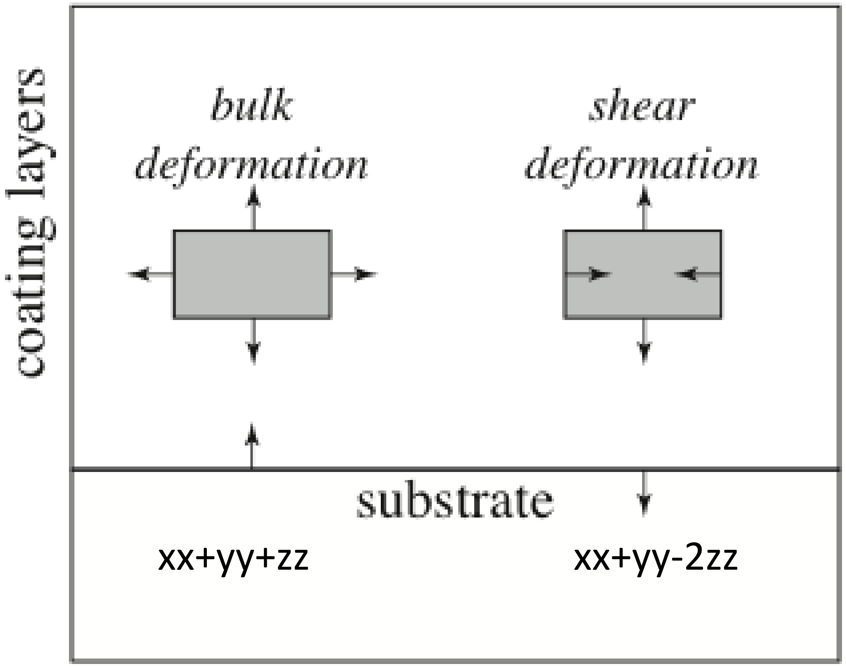
$$S_{ss}(\vec{x},\vec{x}') = \frac{4k_BT}{3\pi f}\frac{(1-\sigma_s-2\sigma_s^2)^2}{Y_s^2}\sum_j\frac{Y_j l_j}{(1-\sigma_j)^2}\left[\frac{1-2\sigma_j}{2}\phi_{Bj}+\frac{1-\sigma_j+\sigma_j^2}{1+\sigma_i}\phi_{Sj}\right]\delta^{(2)}(\vec{x}-\vec{x}')$$

$$S_{s\,u_{zz}}(\vec{x};\vec{x}',z') = \frac{2k_BT}{3\pi f}\frac{(1-\sigma_s-2\sigma_s^2)(1-\sigma_j-2\sigma_i^2)}{Y_s(1-\sigma_j)^2}[\phi_{Bj}-\phi_{Sj}]\delta^2(\vec{x}-\vec{x}')$$

Higher Substrate Young's Modulus -> Lower Thermal Noise

Substrate Material	Fused Silica	Silicon	Sapphire
Young's Modulus (10^{10} Pa)	7.2	16.5	34.5
Poisson Ratio	0.17	0.22	0.29
Brownian Noise (Spectra density)	1	0.4	0.32

Fluctuation induced by bulk and shear loss



ϕ_Θ $\qquad\qquad$ ϕ_Σ

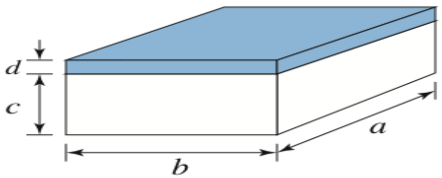
We assign a separate loss angle for bulk and shear energy.

$$U = \frac{1}{2} K\Theta^2 + \mu\Sigma^2$$

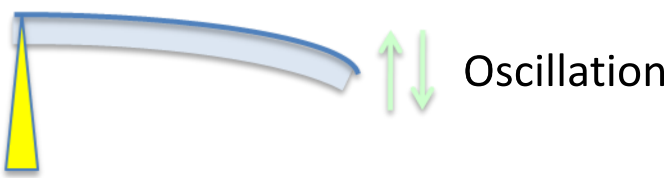
$$W_{diss}(per\_cycle) = \phi_\Theta \frac{1}{2} K\Theta^2 + \phi_\Sigma \mu\Sigma^2$$

- Bulk Noise:
  - xx+yy+zz — Coating thickness and interface
- Shear Noise:
  - xz+zx and yz+zy — No influence
  - xx-yy and xy+yx — Coating-Substrate Interface
  - xx+yy-2zz — Coating thickness and Interface

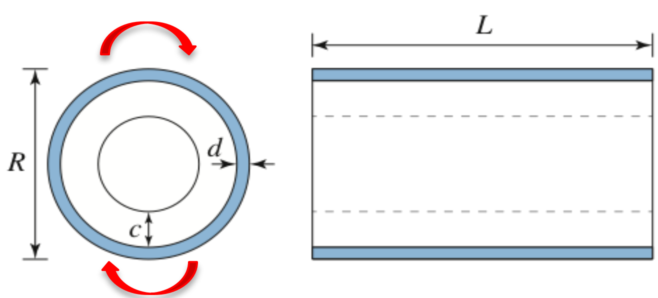
Ringdown measurements for loss angles



$$\phi = \frac{d}{c}\frac{Y_c}{Y_s}\frac{(1-\sigma_s^2)}{(1-\sigma_c)^2}[\phi_B(1-2\sigma_c)+2\phi_s\frac{1-\sigma_c+\sigma_c^2}{1+\sigma_c}]+\phi_{sub}$$

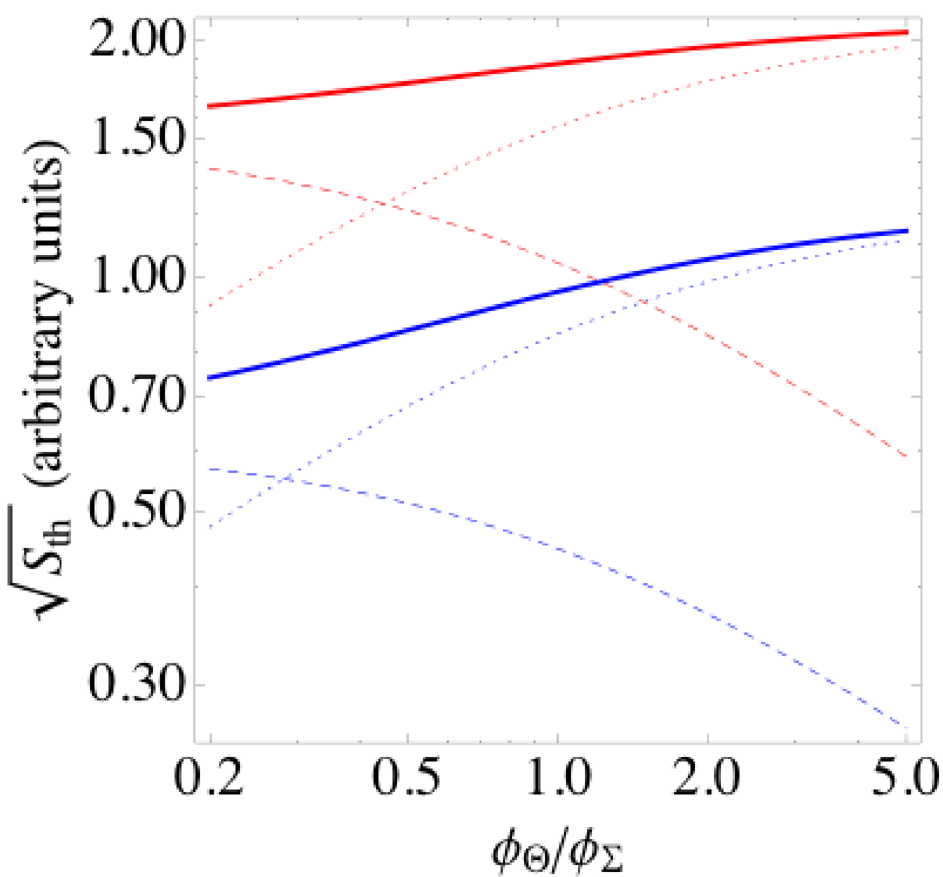


$$\phi = \frac{1}{Q}$$



$$\phi = \phi_{sub} + \frac{d}{c}\frac{Y_c}{Y_s}\frac{(1+\sigma_s)}{(1+\sigma_c)}\phi_S$$

Effect of uncertainties in loss angles



Baseline Parameters used for Coating Materials

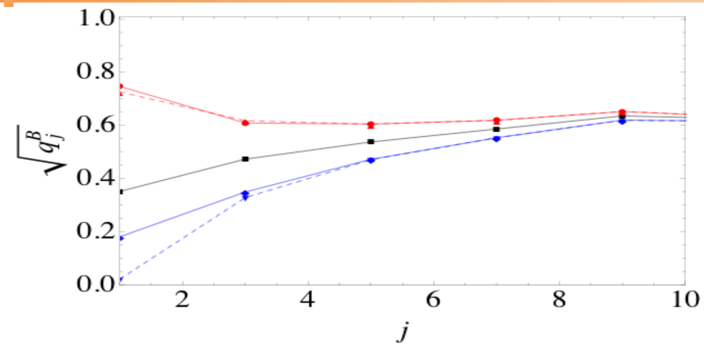
	Ti_2O_5	SiO_2
Refractive index	2.07	1.45
Poisson Ratio	0.23	0.17
Young's Modulus (Pa)	1.4×10^{11}	7×10^{10}
Loss Angle ($\phi_B=\phi_S$)	2×10^{-4}	4×10^{-5}

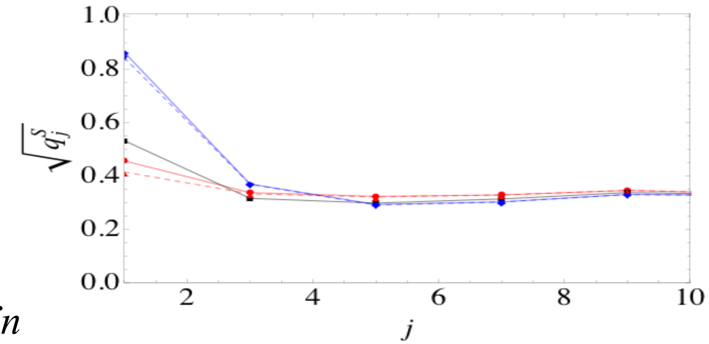
G. M.Harry et al., Class. Quantum Grav. 19, 897 (2002)

Red: Tantala. Blue:Silica. Bulk in dotted lines and shear in dashed lines

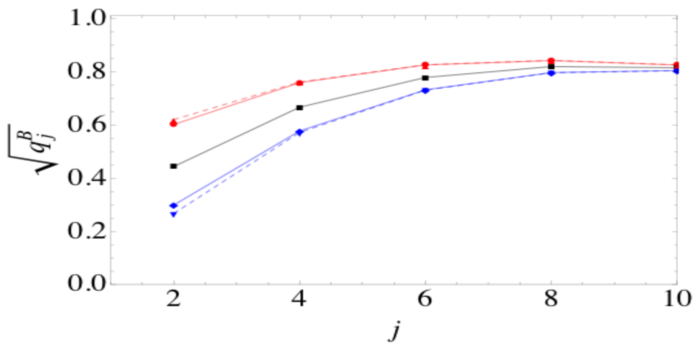
Coating Brownian noise: full calculation

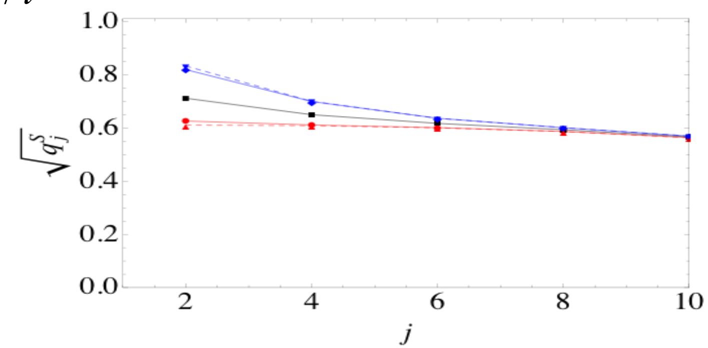
Silica





$$\beta = \frac{\delta n}{\delta l / l}$$





Bulk

Shear

Red for β=1, blue for β=−1, *dashed* for ignoring back-scattering terms

$$S = \sum_j (q_j^B \phi_B^j + q_j^S \phi_S^j) S_j$$

$$S_j \equiv \frac{4 k_B T \lambda_j (1 - \sigma_j - 2\sigma_j^2)}{3\pi f Y_j (1-\sigma_j)^2 A_{eff}}$$