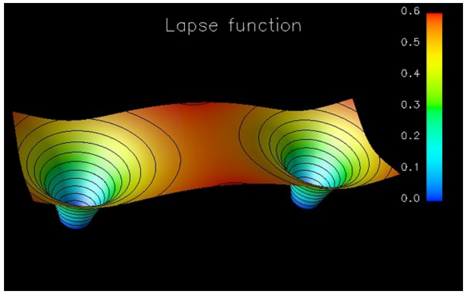
Testing the strong-field dynamics of general relativity with advanced gravitational wave detectors

T.G.F. Li, W. Del Pozzo, S. Vitale, C. Van Den Broeck, M. Agathos,
J. Veitch, K. Grover, T. Sidery, R. Sturani, A. Vecchio

GWADW 2012, Hawaii, 14/05/2012

NIKHEF



Statement of the problem

General Relativity has enjoyed important successes:

- Perihelium precession of Mercury
- Deflection of star light by the Sun
- Shapiro time delay
- Gravity Probe B
  - Geodetic effect
  - Frame dragging
- Expansion of the Universe
- Binary pulsars

Statement of the problem

General Relativity has enjoyed important successes:

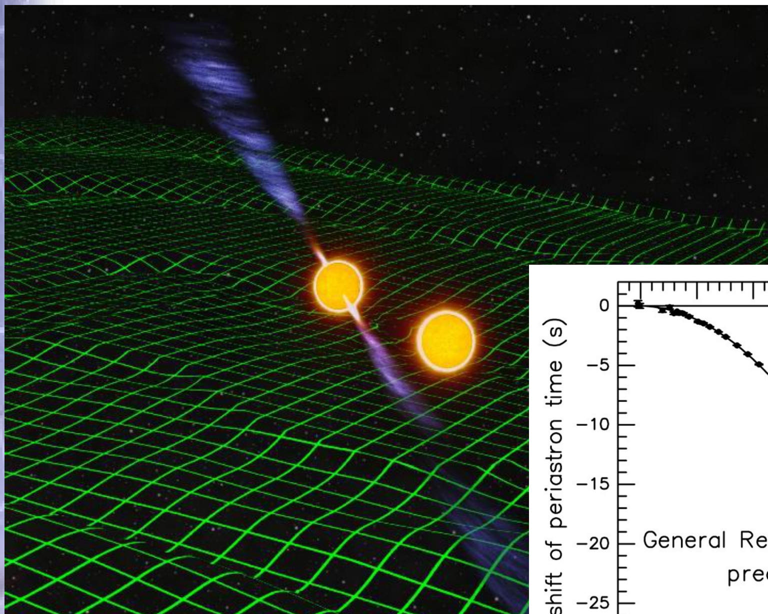
- Perihelium precession of Mercury [weak, static field]
- Deflection of star light by the Sun [weak, static field]
- Shapiro time delay [weak, static field]
- Gravity Probe B
  - Geodetic effect [weak, static field]
  - Frame dragging [weak, stationary field]
- Expansion of the Universe [dynamical but weak-field]
- Binary pulsars [dynamical but weak-field]

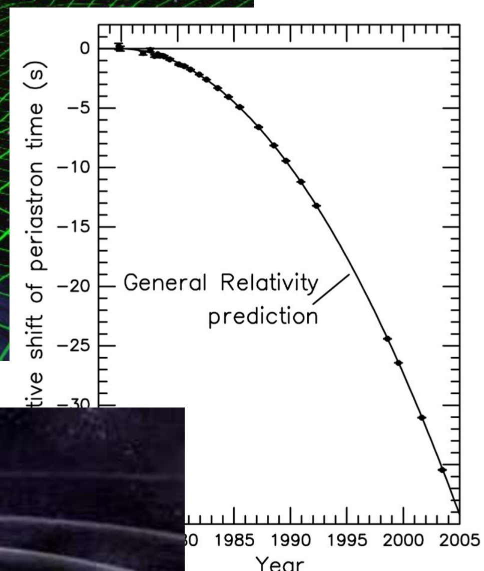
No tests of genuinely strong-field dynamics of the gravitational field

Ideal laboratories: coalescing binary neutron stars and black holes

→ *Need direct detection of gravitational waves*

Coalescence of binary neutron stars and black holes





- The observed binary pulsars do not suffice to study the dynamics of spacetime at strong gravitational fields:

  $GM/(c^2 R) \sim 10^{-6}$, $v/c \sim 10^{-3}$

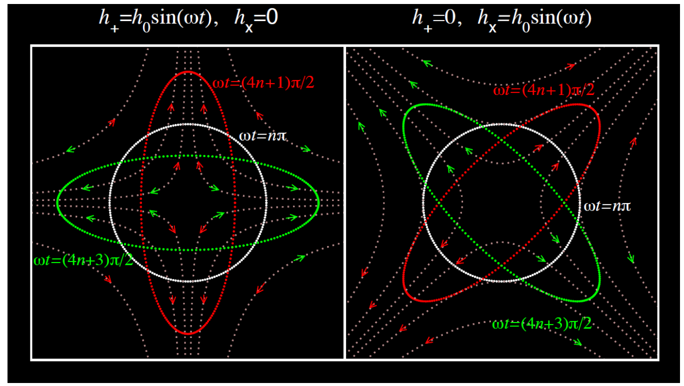
- Compare with binary neutron stars and/or black holes on the verge of merger:

  $GM/(c^2 R) \sim 0.2$, $v/c \sim 0.4$

Gravitational wave polarizations

- General relativity predicts that gravitational waves only have two polarizations

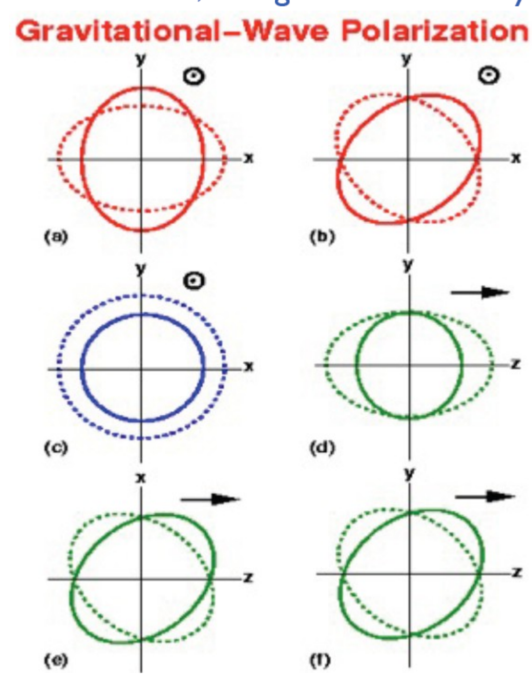
"Plus"



"Cross"

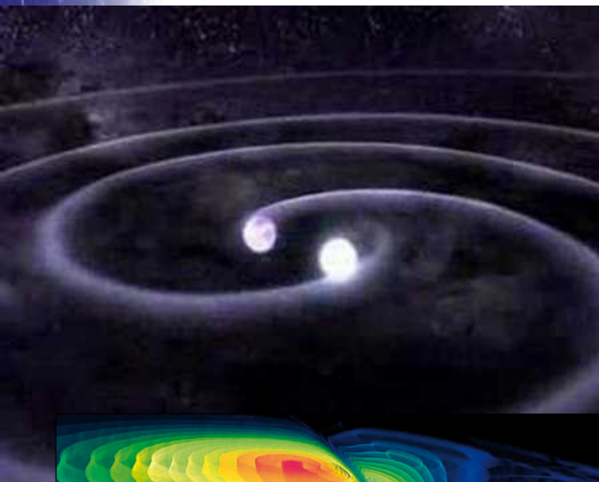
- In scalar-tensor theories: six polarizations

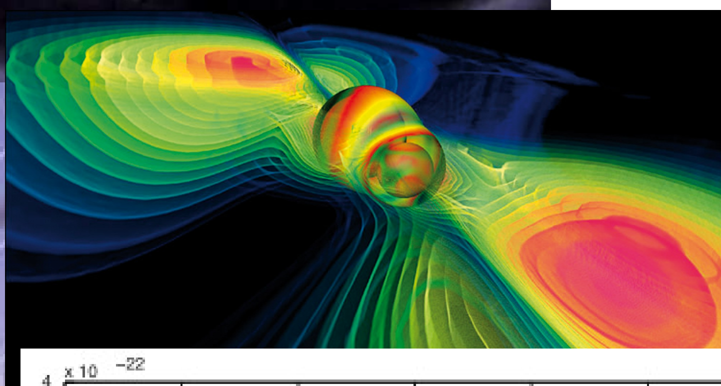
Cliff Will, Living Rev. in Relativity

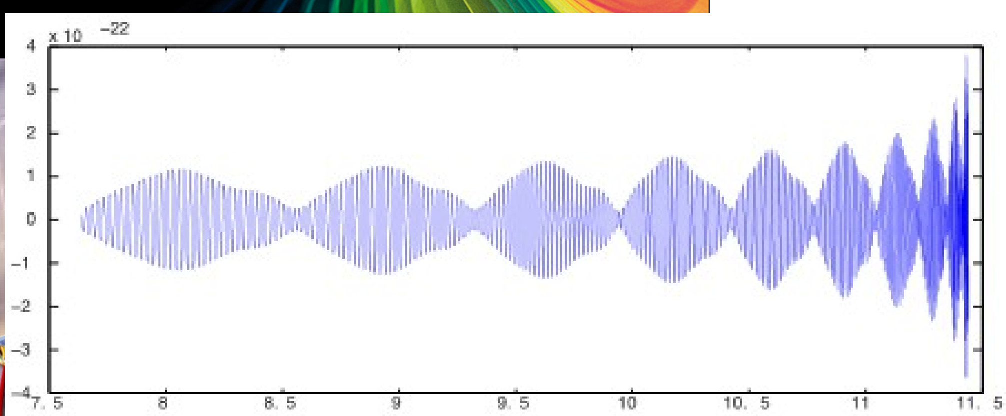


Single direct GW detection could rule out GR in a qualitative way

Testing GR with colliding binary objects







- Hulse-Taylor and similar binary pulsar: can study dissipative orbital dynamics only up to leading order in *(v/c)*
- Use inspiraling and merging binary black holes to study extreme strong-field dynamics of spacetime
- Most interesting effects occur starting at $(v/c)^3$ beyond leading-order:
  - Gravitational self-interaction: waves bouncing off the background spacetime ("tail effects")
  - Spin-orbit and spin-spin effects cause precession and even tumbling motion of orbital plane

  Extremely rich dynamics imprinted onto gravitational waveform

The inspiral of compact binaries

- Schematically: phase can be expressed in terms of *characteristic speed v(t)*

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

where the ψ_n and $\psi_n^{(l)}$ depend on masses and spins of component objects

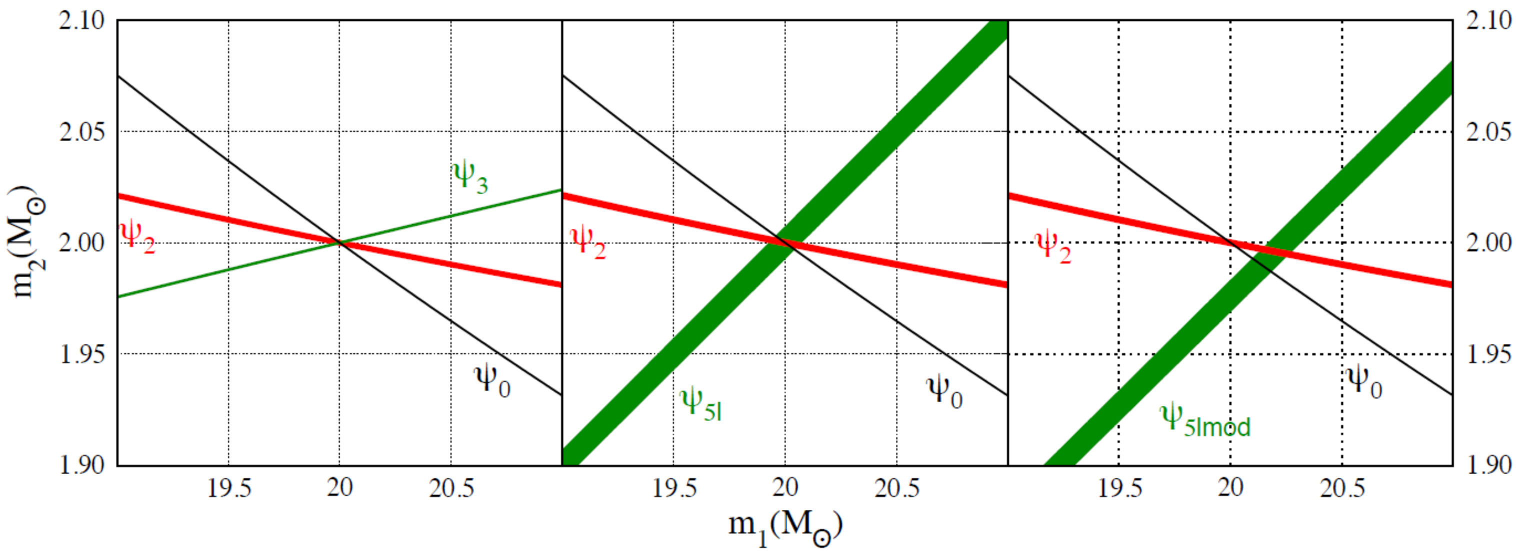
- Coefficients:
  - ψ_3 incorporates lowest-order "tail" effects (nonlinearity of GR), and spin-orbit interaction
  - ψ_4 has lowest-order spin-spin effects
  - $\psi_5^{(l)}$ is lowest-order non-zero "logarithmic" coefficient
- Modifications to GR:
  - "Massive graviton" would modify ψ_2
  - Scalar-tensor theories add $\psi_{\rm ST}(v/c)^{-7}$
  - Quadratic curvature terms in action add $\psi_{\rm QC}(v/c)^{-1}$
  - "Dynamical Chern-Simons theory" adds $\psi_{\rm CS}(v/c)^4$
  - Variable G adds $\psi_{G(t)}(v/c)^{-13}$

Cornish et al., PRD **84**, 062003 (2011)

The inspiral of compact binaries

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

- If no spins then coefficients only depend on the two component masses, hence only two independent coefficients
- Measure any two coefficients and see if third is consistent assuming GR correct:



Mishra et al., PRD **82**, 064010 (2010)

The TIGER analysis pipeline

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

In practice: Bayesian model selection between two hypotheses:

- GR hypothesis $\mathcal{H}_{\mathrm{GR}}$
- Hypothesis $\mathcal{H}_{\mathrm{modGR}}$ that there is a deviation of GR

- Ideally: $\mathcal{H}_{\mathrm{modGR}}$ negation of $\mathcal{H}_{\mathrm{GR}}$, but difficult in practice. Simple choice:

$\mathcal{H}_{\mathrm{modGR}} \leftrightarrow$ one or more of the ψ_n, $\psi_n^{(l)}$ are not as predicted by GR

- Construct an odds ratio:

$$O_{\mathrm{GR}}^{\mathrm{modGR}} \equiv \frac{P(\mathcal{H}_{\mathrm{modGR}}|d, I)}{P(\mathcal{H}_{\mathrm{GR}}|d, I)}$$

- Data d : could consist of data stretches associated with multiple detections of inspiraling binary neutron stars or black holes

$$d = \{d_1, d_2, \ldots, d_N\}$$

The TIGER analysis pipeline

$\mathcal{H}_{\rm modGR}$ $\leftrightarrow$ one or more of the ψ_n , $\psi_n^{(l)}$ are not as predicted by GR

- Odds ratio:

$$O_{\rm GR}^{\rm modGR} \equiv \frac{P(\mathcal{H}_{\rm modGR}|d, I)}{P(\mathcal{H}_{\rm GR}|d, I)}$$

- In practice, consider a set of testing coefficients $\{\psi_1, \psi_2, \ldots, \psi_N\}$
- Introduce auxiliary hypotheses:

$H_{i_1 i_2 \ldots i_k}$ $\leftrightarrow$ The coefficients $\{\psi_{i_1}, \psi_{i_2}, \ldots, \psi_{i_k}\} \subset \{\psi_1, \psi_2, \ldots, \psi_N\}$ differ from their GR values, but all other coefficients are as in GR

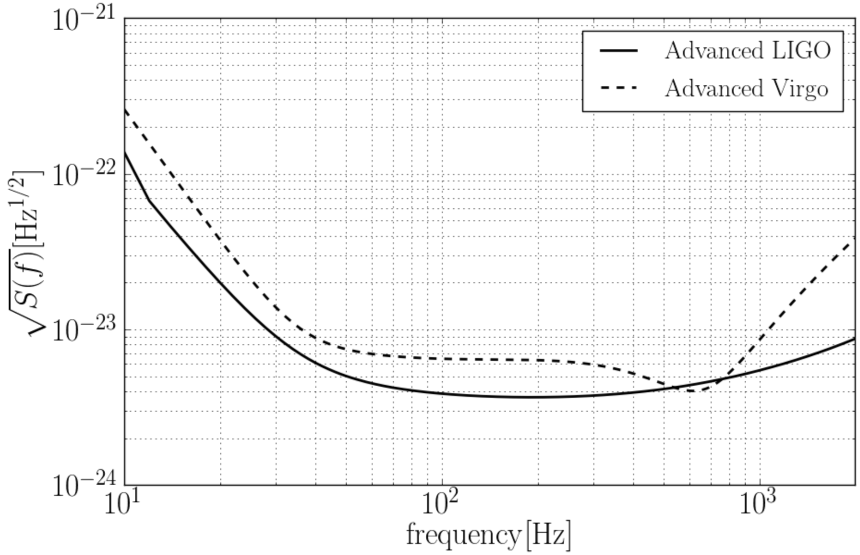
- These are logically exclusive, and their logical "or" is $\mathcal{H}_{\rm modGR}$:

$$\mathcal{H}_{\rm modGR} = \bigvee_{i_1 < i_2 < \ldots < i_k; k \leq N} H_{i_1 i_2 \ldots i_k}$$

- Computing the odds ratio involves comparing the evidences for the individual $H_{i_1 i_2 \ldots i_k}$ with the evidence for $\mathcal{H}_{\rm GR}$ and combining results

Li et al., PRD **85**, 082003 (2012)

How well will we do in practice?



Simulations

- Simulated stationary, Gaussian noise following predicted Advanced Virgo/LIGO noise curves
- Add simulated binary neutron star signals with or without deviation from GR
  - $m_1, m_2 \in [1,2]\ M_{sun}$
  - Uniform in volume
  - Distance $\in [100, 400]$ Mpc
  - Arbitrary orientations, sky positions
  - Signals: TaylorF2 (analytic frequency domain waveforms)
- Consider "catalogs" of 15 sources each ("plausible" detection rate: ~40 yr^{-1})
- Use three testing parameters $\{\psi_1, \psi_2, \psi_3\}$

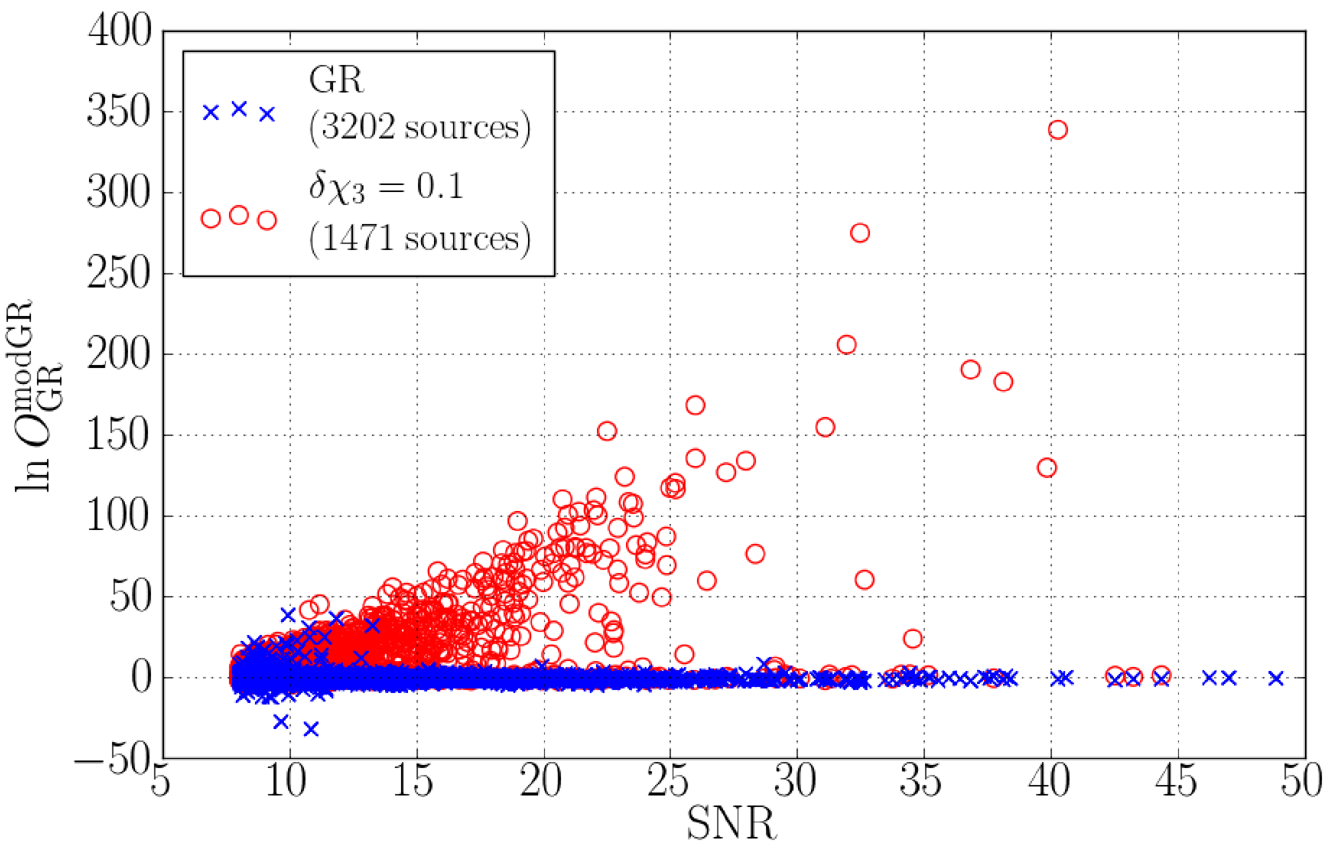
→ Odds ratio involves combining evidence for 2^3-1=7 auxiliary hypotheses

$$H_1, H_2, H_3, H_{12}, H_{13}, H_{23}, H_{123}$$

Example: Constant 10% shift at $(v/c)^3$

- Blue: Simulated signals which conform to GR
- Red: Simulated signals with 10% shift in phase coefficient ψ_3

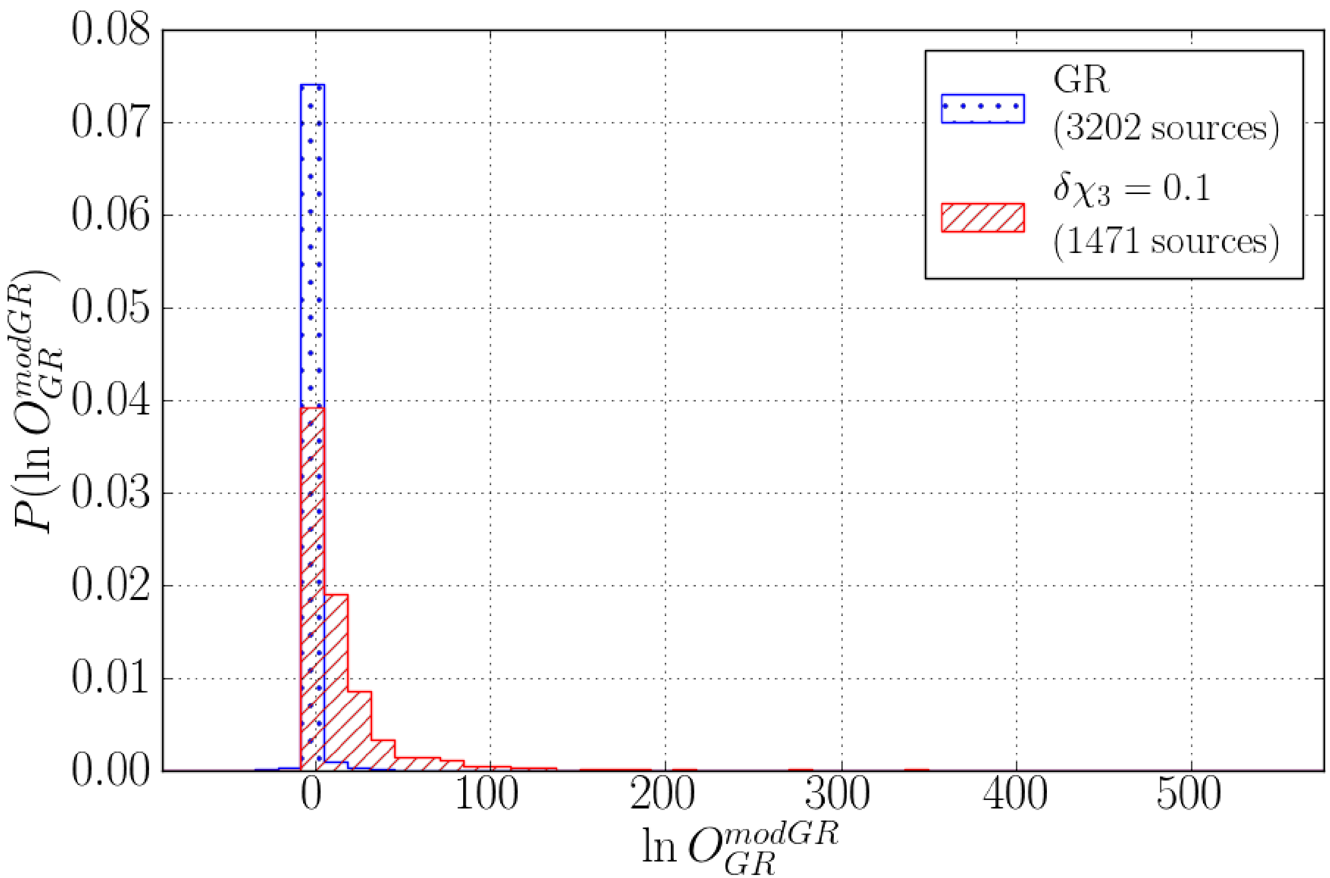
Log odds ratio against SNR for individual sources



Example: Constant 10% shift at $(v/c)^3$

- Blue: Simulated signals which conform to GR
- Red: Simulated signals with 10% shift in phase coefficient ψ_3

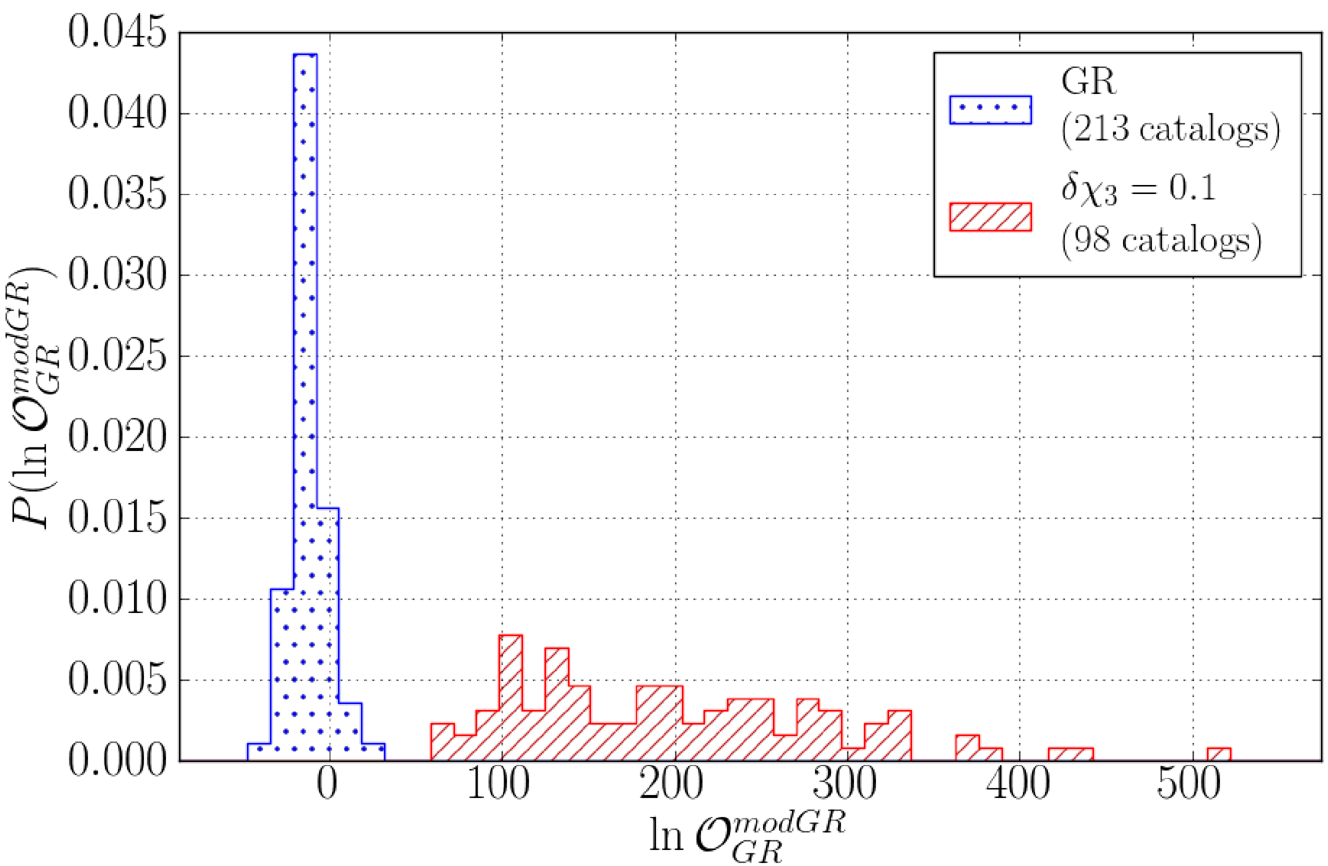
Histogram of log odds ratios for individual sources



Example: Constant 10% shift at $(v/c)^3$

- Blue: Simulated signals which conform to GR
- Red: Simulated signals with 10% shift in phase coefficient ψ_3

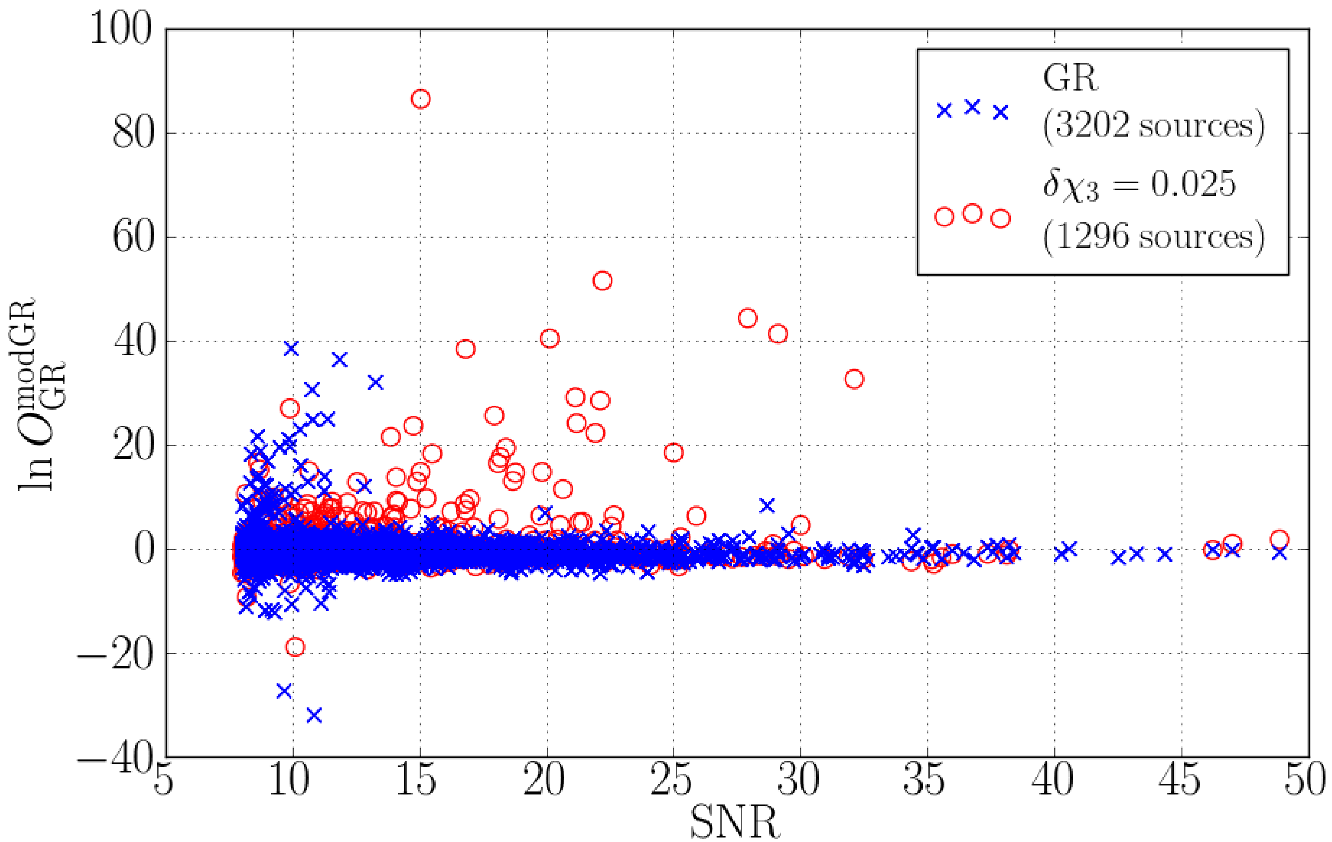
Histogram of log odds ratios for **catalogs of 15 sources each**



Example: Constant 2.5% shift at $(v/c)^3$

- Blue: Simulated signals which conform to GR
- Red: Simulated signals with 2.5% shift in phase coefficient ψ_3

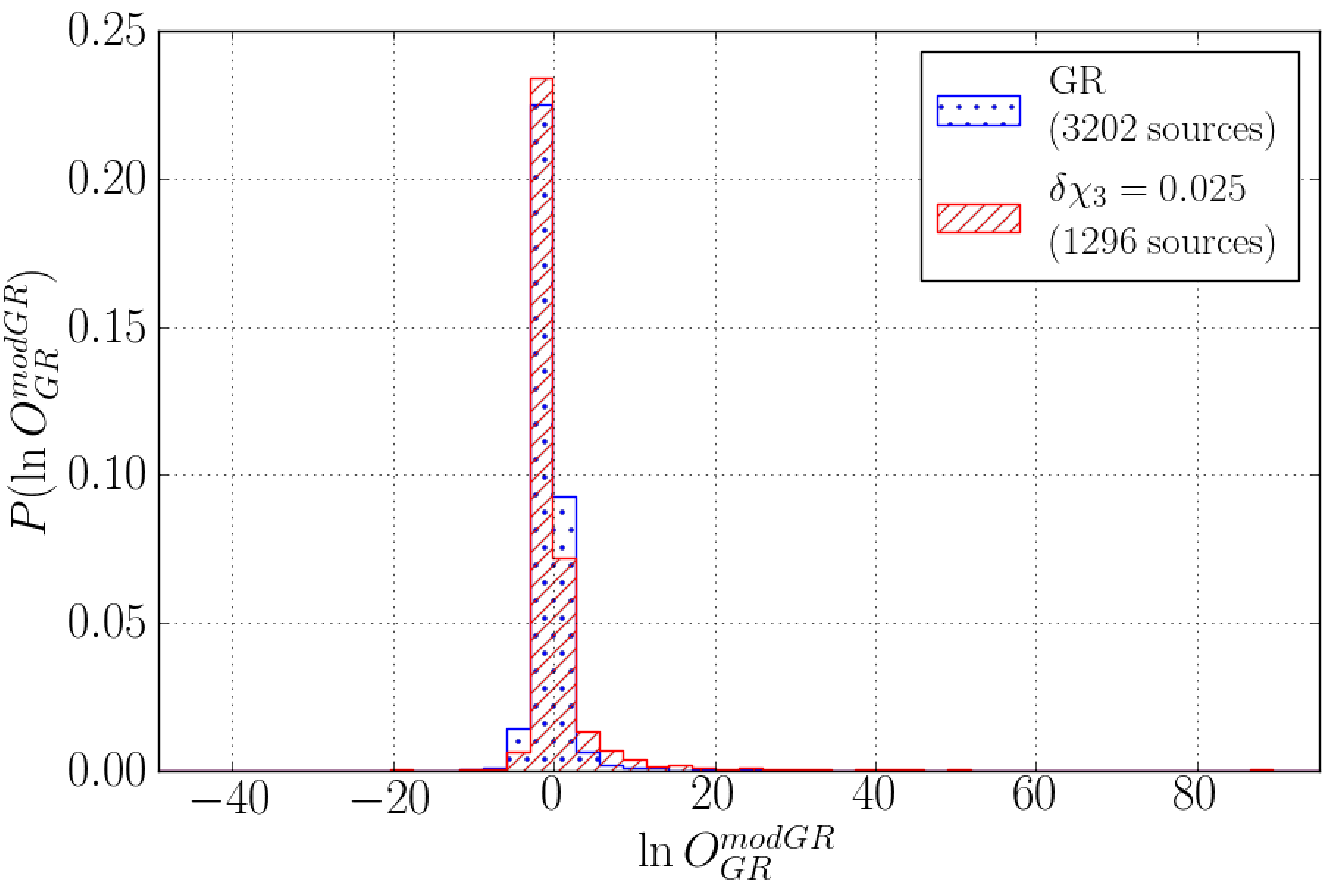
Log odds ratio against SNR for individual sources



Example: Constant 2.5% shift at $(v/c)^3$

- Blue: Simulated signals which conform to GR
- Red: Simulated signals with 2.5% shift in phase coefficient ψ_3

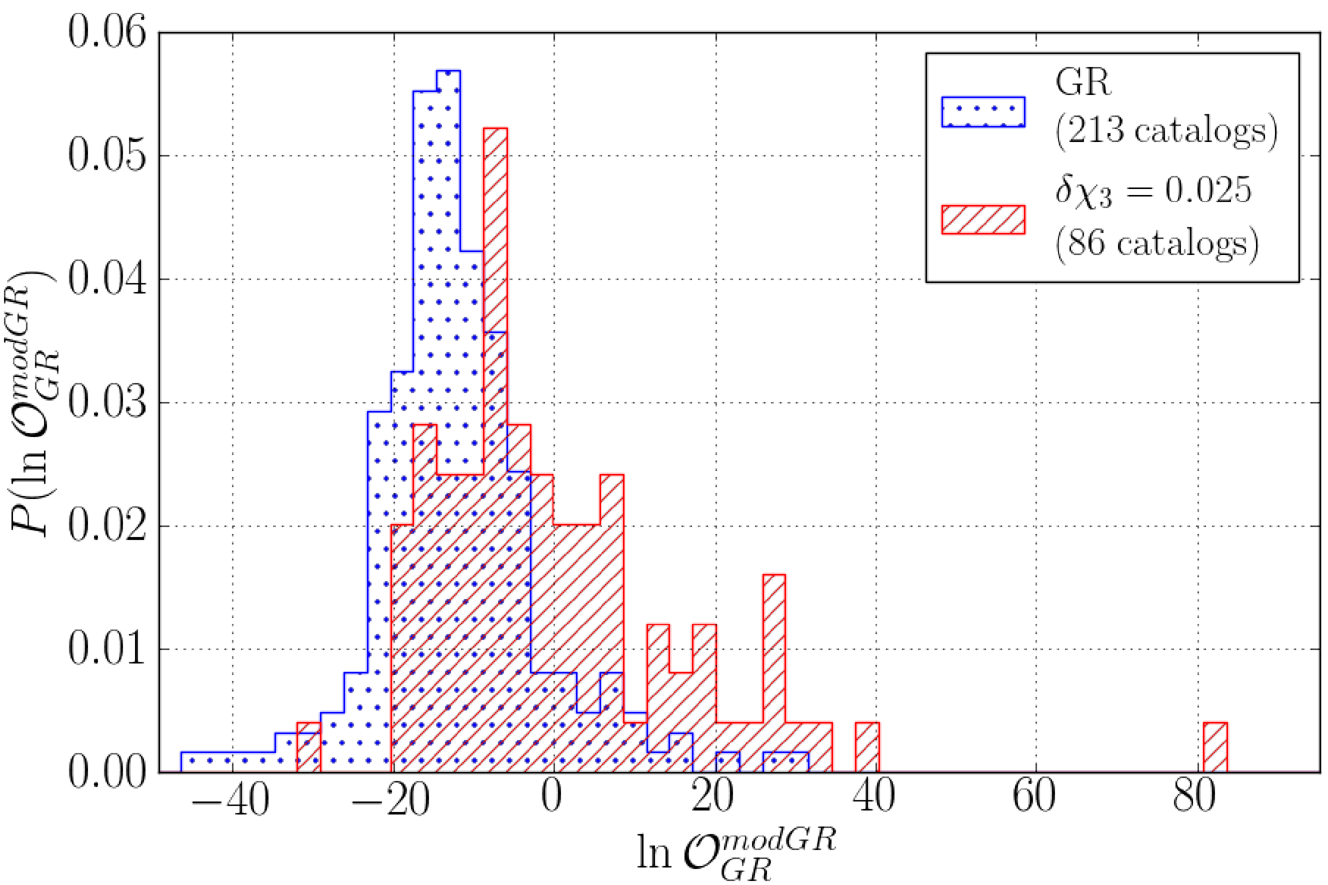
Histogram of log odds ratios for individual sources



Example: Constant 2.5% shift at $(v/c)^3$

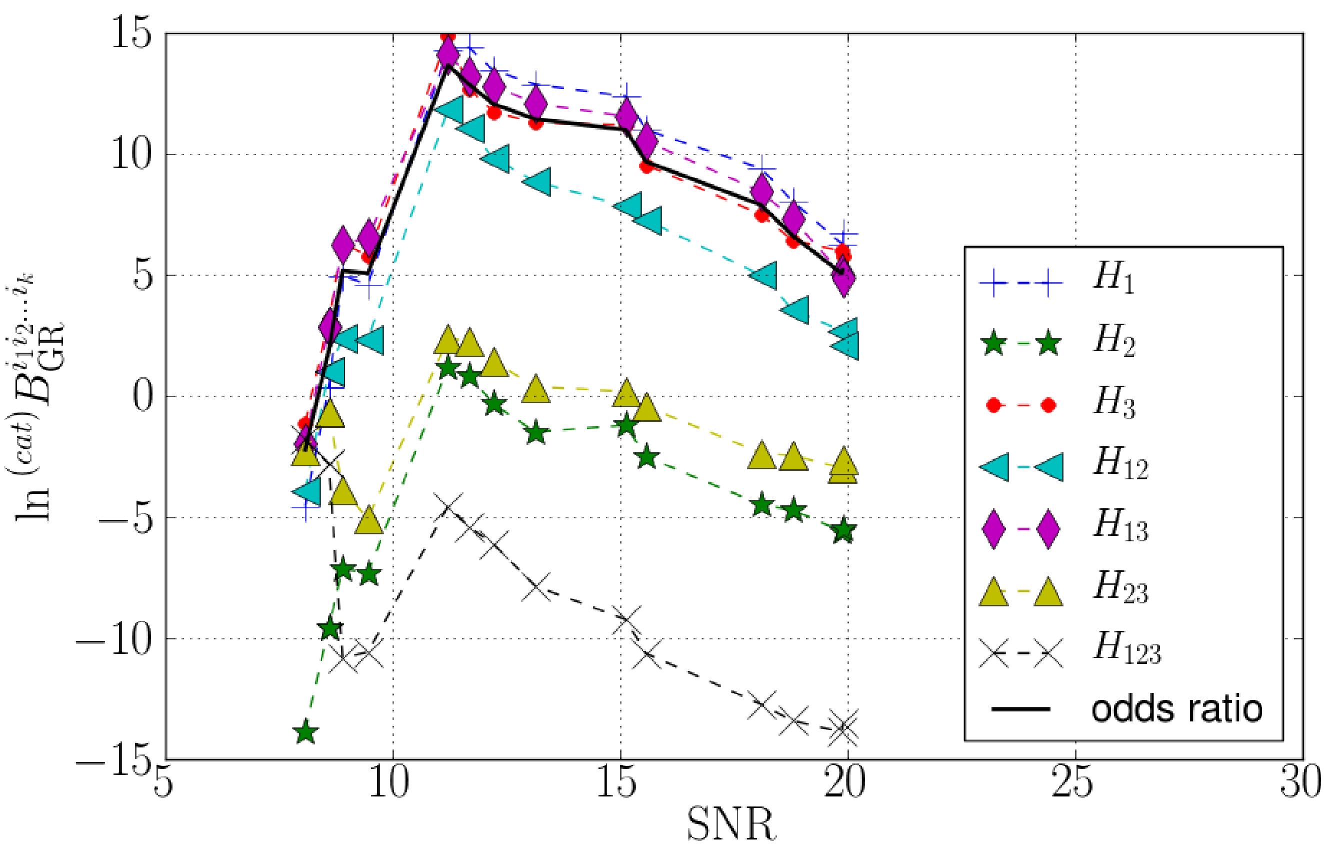
- Blue: Simulated signals which conform to GR
- Red: Simulated signals with 2.5% shift in phase coefficient ψ_3

Histogram of log odds ratios for **catalogs of 15 sources each**



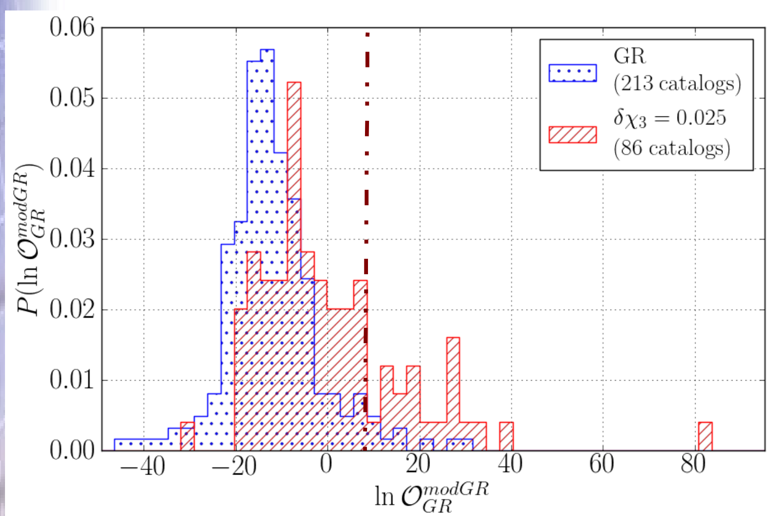
Example: Constant 2.5% shift at $(v/c)^3$

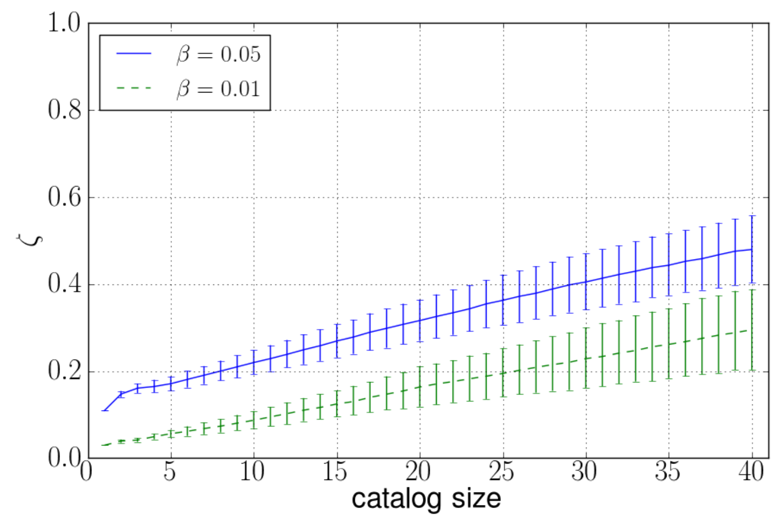
- Recall that odds ratio is computed from evidences for 7 auxiliary hypotheses $H_1, H_2, H_3, H_{12}, H_{13}, H_{23}, H_{123}$ against the GR hypothesis $\mathcal{H}_{\rm GR}$
- Also combine evidences from all sources in one's catalog
- Accumulation of evidences in an example catalog:



Example: Constant 2.5% shift at $(v/c)^3$

- Pick a maximum tolerable false alarm probability, β (e.g., 5%, or 1%, or ...)
- Use the log odds distribution for GR catalogs ("background") to set a threshold for the measured log odds ratio to overcome



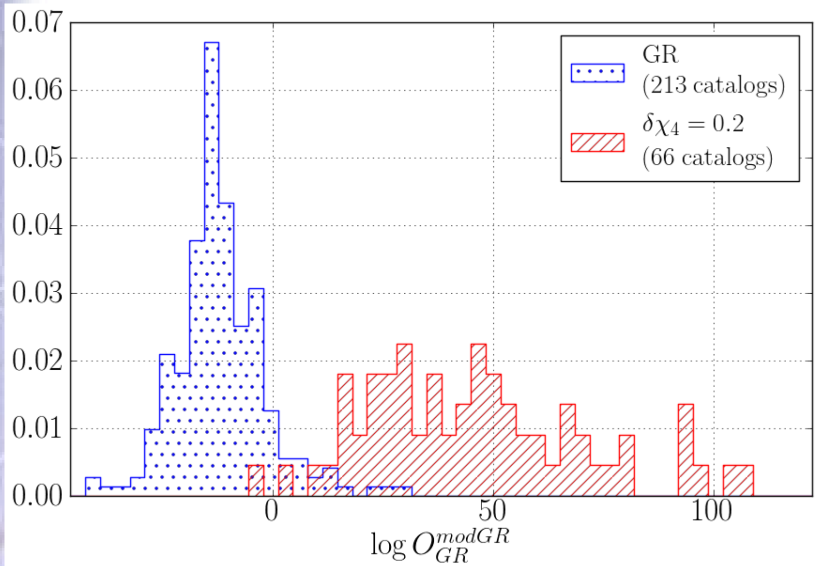


- In reality only one measured log odds ratio for the catalog of detected sources
- But, given a GR violation, can assess how likely it is that the measured log odds ratio is above threshold

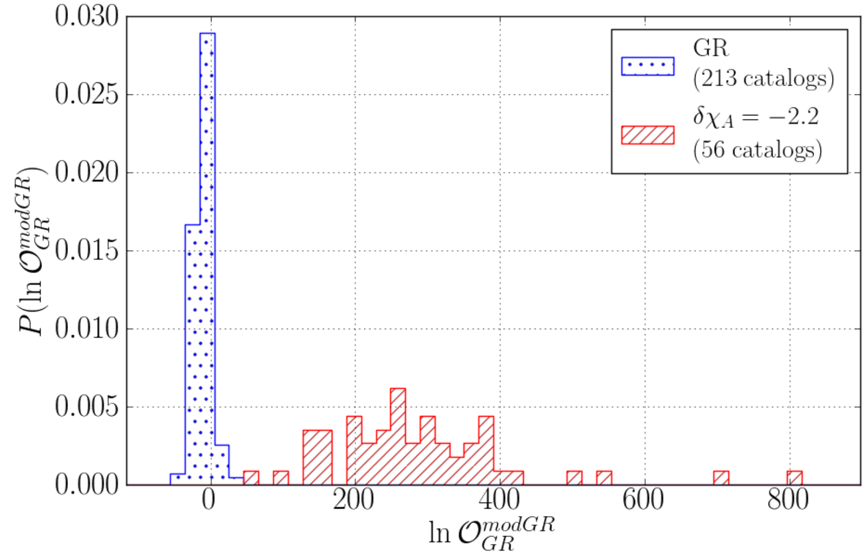
→ "Efficiency"

Other heuristic deviations

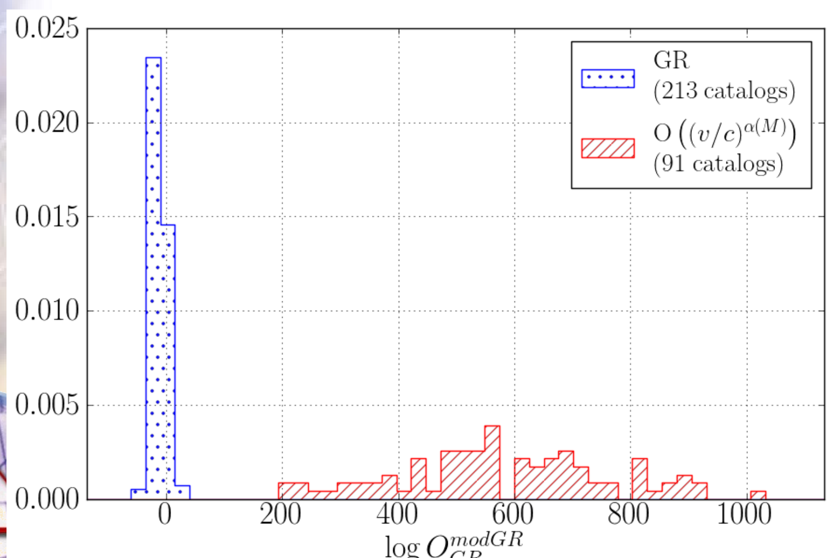
$(v/c)^4$



$(v/c)^{2.5}$



$(v/c)^{\alpha(M)}$



If a violation causes a change of more than half a cycle at f ~ 150 Hz then it will be found with essentially zero false alarm probability

→ Sensitive to *generic* violations of GR

Li et al., PRD **85**, 082003 (2012)
arXiv:1110.0530 [gr-qc]

Li et al., to appear in JPCS (2012)
arXiv:1111.5274 [gr-qc]

A note on parameter estimation

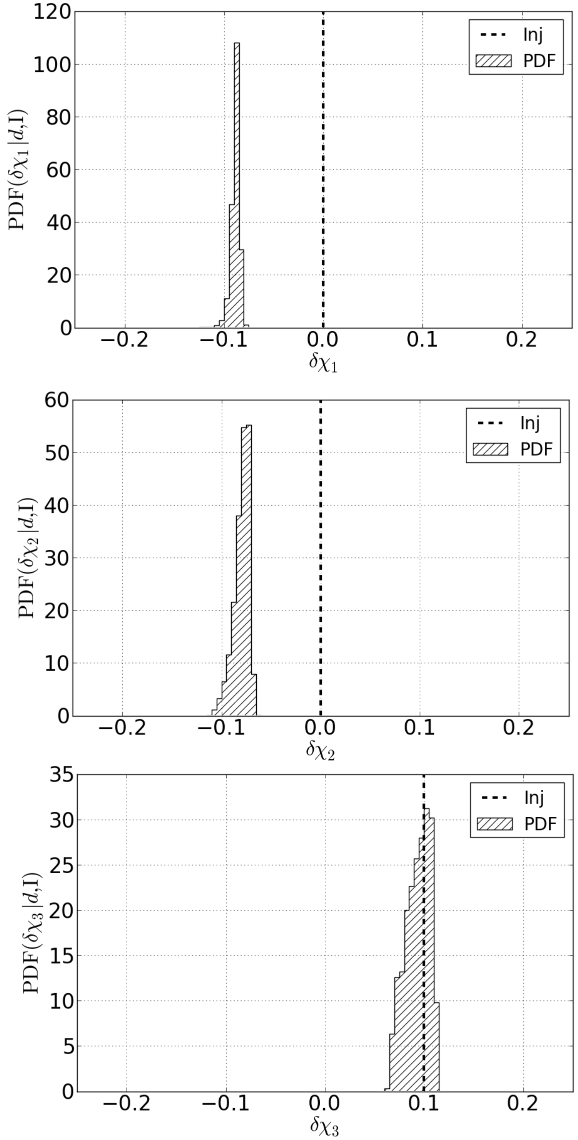
- Our test is based on model selection, which allows us to combine information from multiple sources
    - Asking a yes/no question whose correct answer is the same for every source!
- How GR violation manifests itself could be dependent on masses, charges... which vary from source to source!
    - Parameter estimation does not allow for combining information from multiple sources
- If a violation is found, we will nevertheless want to do parameter estimation as an aid to discover its precise nature
- Could fit signal against waveforms in which each phase coefficient in turn is free, and look at probability density function
- Or, use parameterized post-Einsteinian (ppE) formalism:

$$\Phi(f) \rightarrow \Phi(f) + \beta(\pi \mathcal{M} f)^b$$

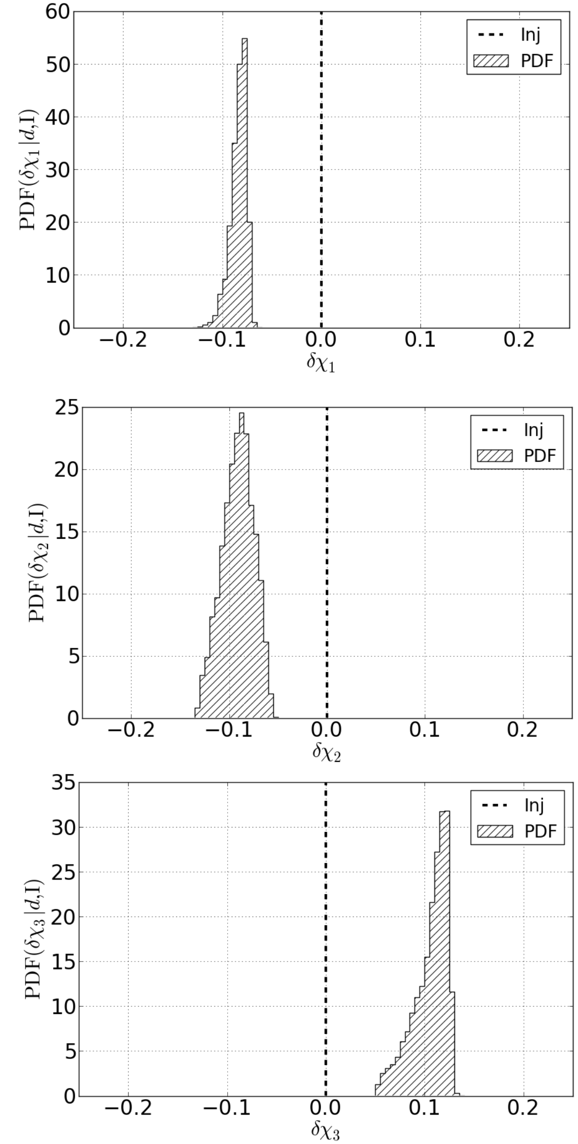
Yunes and Pretorius, PRD **80**,122003 (2009)

A note on parameter estimation

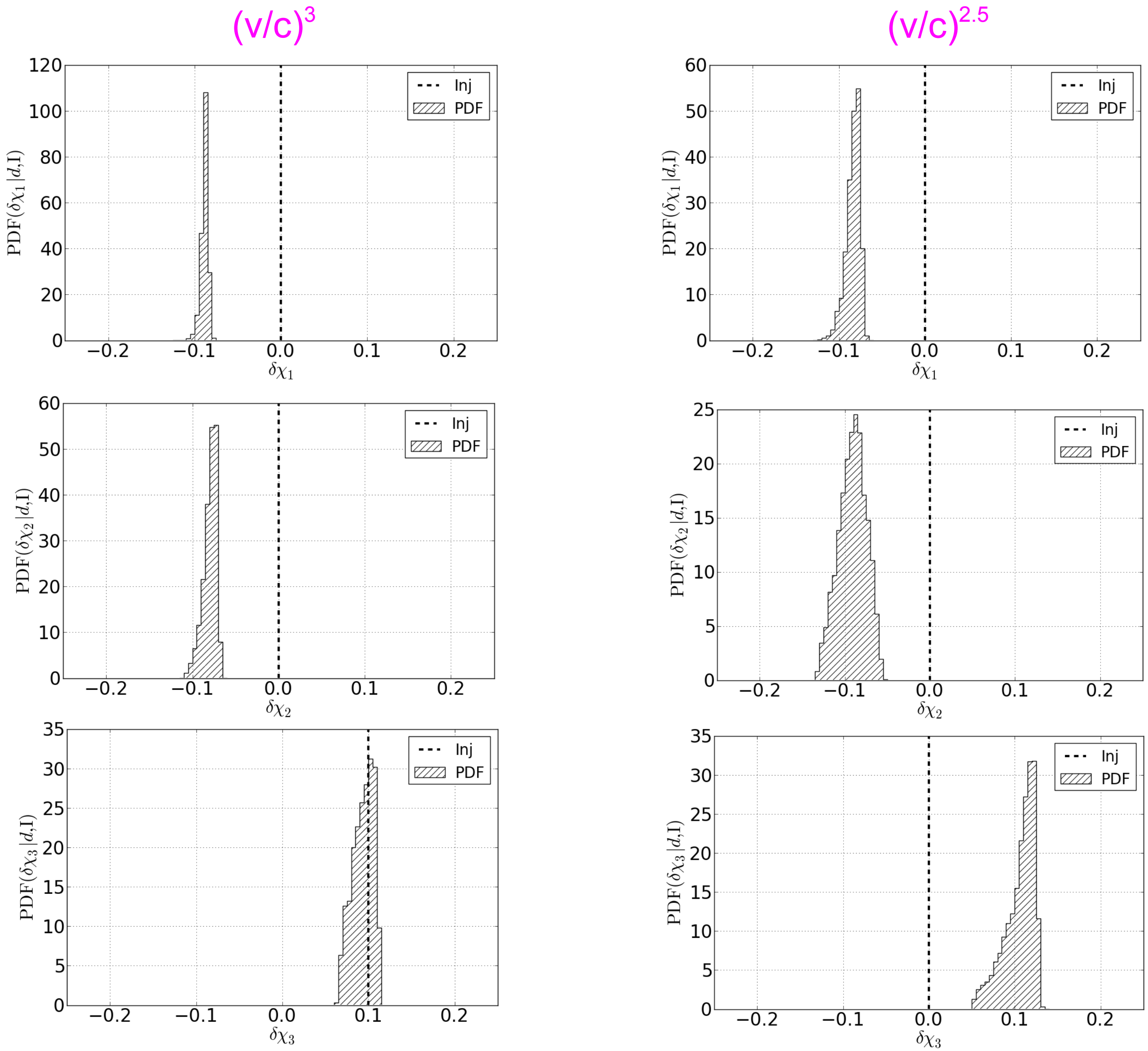
Violation A



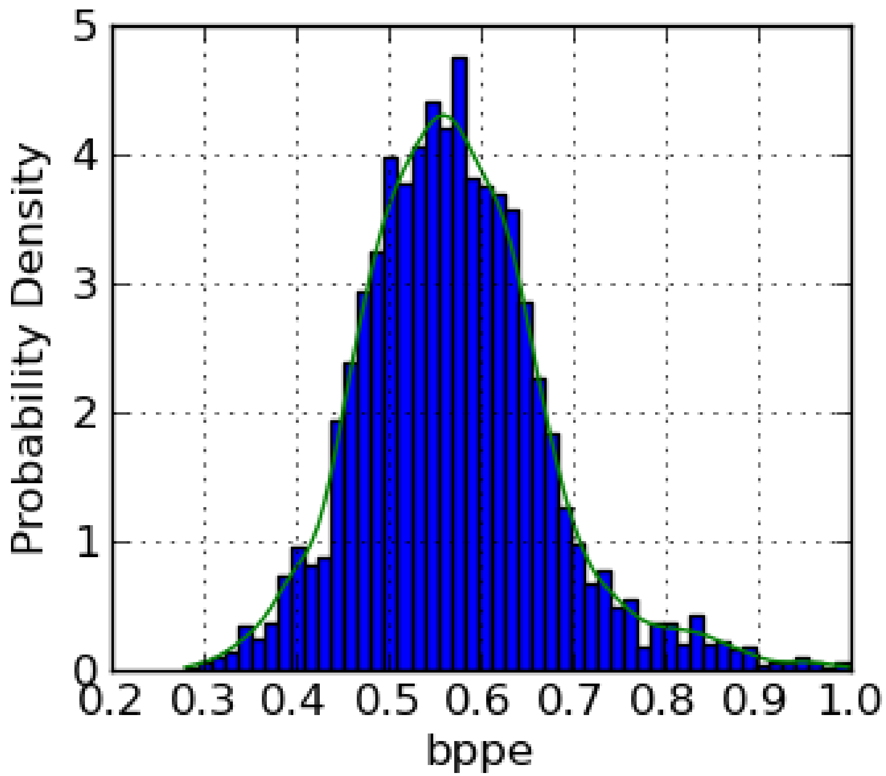
Violation B



A note on parameter estimation



A note on parameter estimation



- ppE formalism:

$$\Phi(f) \rightarrow \Phi(f) + \beta(\pi \mathcal{M} f)^{b}$$

- Source with large SNR: 37.5
- Deviation from GR in the signal:
  10% shift at $(v/c)^3$
- Mean of the distribution:
  b = 0.57

→ The formalism puts the deviation at $(v/c)^{6.71}$

With advanced detectors, can find GR violations of a few percent
But: could be difficult to identify the nature of the violation

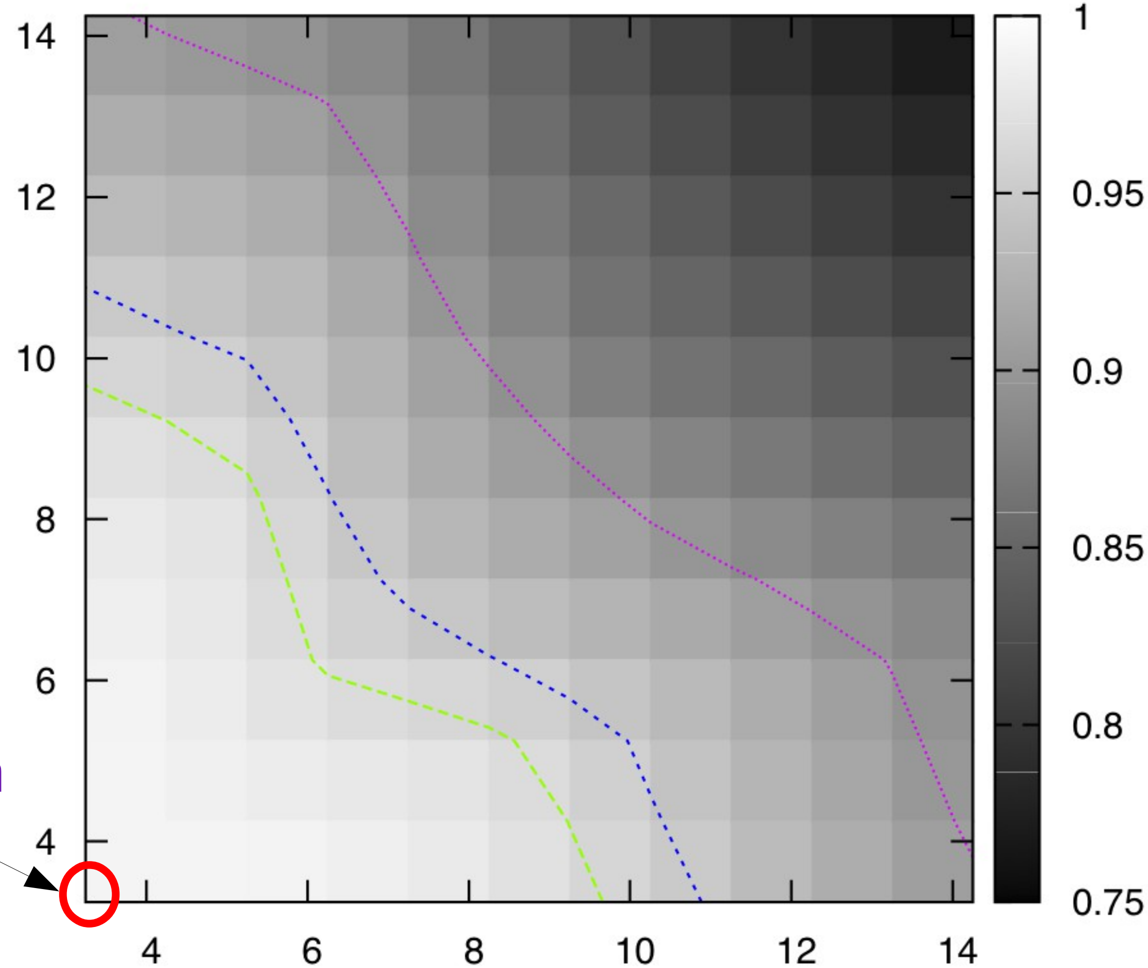
Possible concerns

- Are sufficiently accurate waveform approximants available?
- What about instrumental calibration errors?
- In the case of binary neutron stars, what about finite size and matter effects?
- Effect of spins?
- Can computational challenges be met?

Differences between waveform approximants?

- Match of TaylorF2 with non-spinning EOB inspiral-merger-ringdown optimized using numerical simulations

Buonanno et al.,
PRD **80**, 084043 (2009)



Most massive neutron star binaries

Effect of instrumental calibration errors?

Effect of calibration errors on Bayesian parameter estimation for gravitational wave signals from inspiral binary systems in the Advanced Detectors era

Salvatore Vitale,[1] Walter Del Pozzo,[1] Tjonnie G. F. Li,[1]
Chris Van Den Broeck,[1] Ilya Mandel,[2] Ben Aylott,[2] and John Veitch[3]

[1]*Nikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands*
[2]*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK*
[3]*School of Physics and Astronomy, Cardiff University, Cardiff CF24 3AA, UK*

By 2015 the advanced versions of the gravitational-wave detectors Virgo and LIGO will be online. They will collect data in coincidence with enough sensitivity to potentially deliver multiple detections of gravitation waves from inspirals of compact-object binaries. This work is focused on understanding the effects introduced by uncertainties in the calibration of the interferometers. We consider plausible calibration errors based on estimates obtained during LIGO's fifth and Virgo's third science runs, which include frequency-dependent amplitude errors of $\sim 10\%$ and frequency-dependent phase errors of ~ 3 degrees in each instrument. We quantify the consequences of such errors estimating the parameters of inspiraling binaries. We find that the systematics introduced by calibration errors on the inferred values of the chirp mass and mass ratio are smaller than 20% of the statistical measurement uncertainties in parameter estimation for 90% of signals in our mock catalog. Meanwhile, the calibration-induced systematics in the inferred sky location of the signal are smaller than $\sim 50\%$ of the statistical uncertainty. We thus conclude that calibration-induced errors at this level are not a significant detriment to accurate parameter estimation.

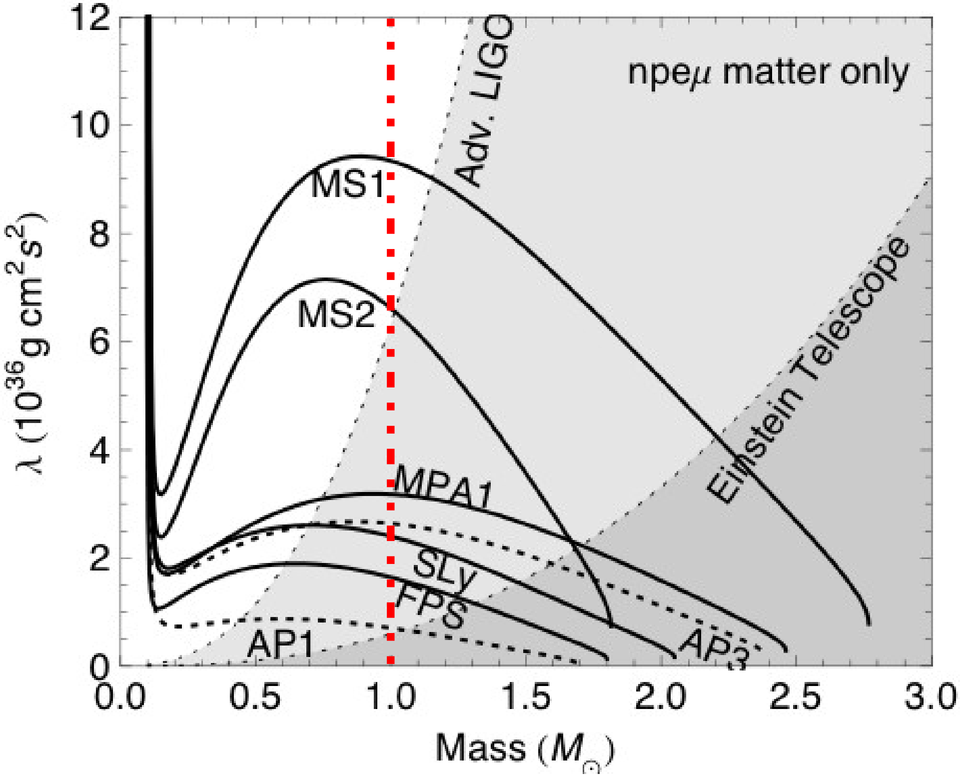
Vitale et al., PRD **85**, 064034 (2012)

Finite size and matter effects?

- A neutron star's finite size and deformability has effect on orbital motion
- Details depend on (currently unknown!) NS equation of state
- But, even for most extreme EOS, effect only becomes apparent at f > 450 Hz

→ Cut off template waveforms at 400 Hz

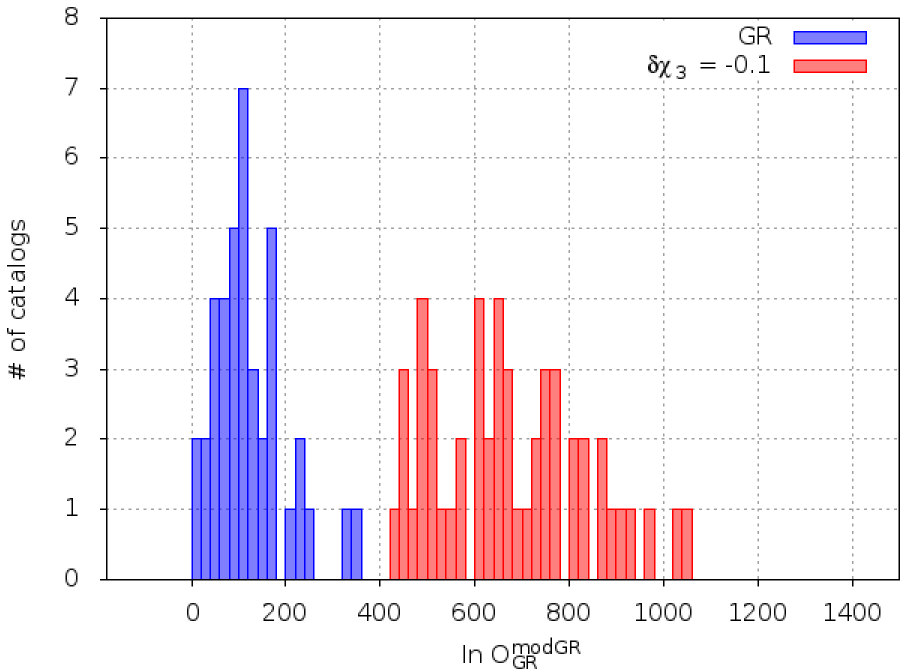
SNR loss < 1%



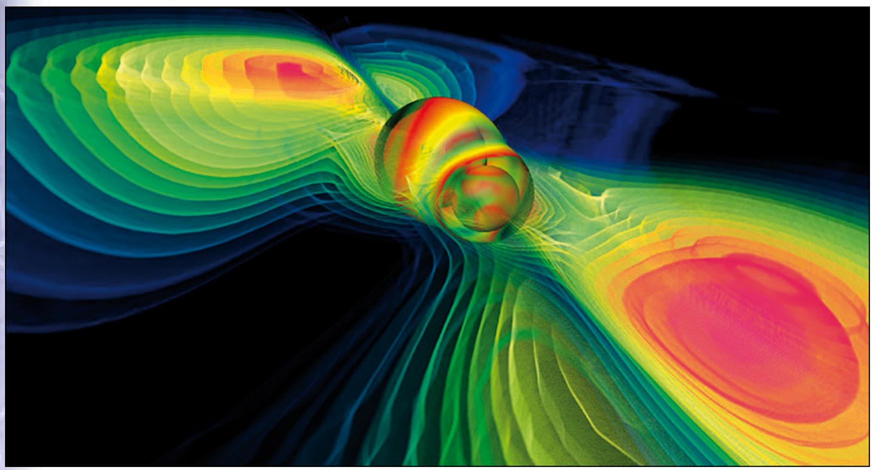
Hinderer et al., PRD **81**, 123016 (2010)

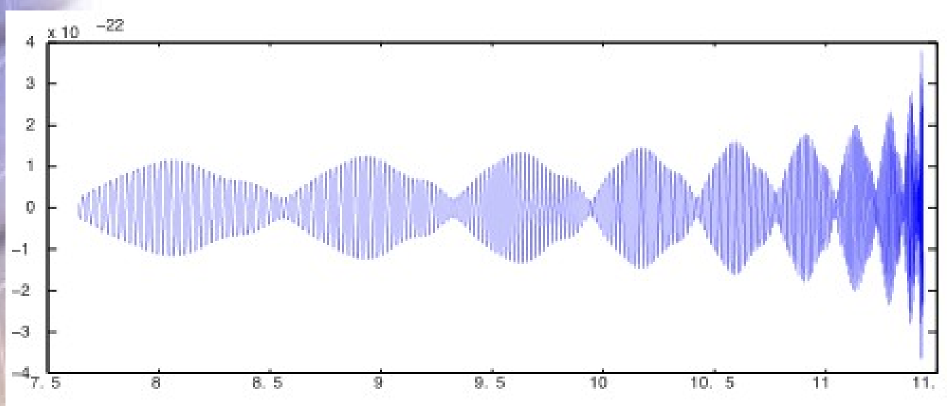
Effect of spins?

- Include spins in the simulated GR signals that are used to compute the background
    - Take dimensionless spins *S* to be Gaussian distributed around zero, width 0.05
    - Spins of all known neutron star binaries will be within ~0.5 σ
- Can do the same with foreground
- Template used for recovery: no spin



What about BH-BH and NS-BH?





- Rich dynamics due to spin-orbit, spin-spin interaction, but 6 extra parameters to deal with
- Partial degeneracies between spins and possible GR violations starting at $(v/c)^3$
- On the other hand, merger/ringdown in frequency band
- Waveform modeling is not a solved problem!
- Computational cost very high

Unclear what to expect, but is being explored

Summary and outlook

Gravitational waves will provide us with observational access to the genuinely strong-field dynamics of gravity

- Inspiral and merger of compact binaries:
  - Detections before end of decade virtually guaranteed
  - Will take us far beyond binary pulsar tests
  - Find deviations in phase coefficients of a few percent
  - Advanced detectors: hard to identify the nature of the deviation
- For binary neutron stars: reasonably mature pipeline
- NS-BH and BH-BH: richer dynamics, but...
  - Waveform modeling complicated problem
  - Computational cost very high
  - Under investigation...

Stay tuned!