LIGO – T1200210 – v1

This notebook is for analyzing the effect of ambient magnetic fields on shielded cables. Formula for common mode coupling taken from section 6.2 .2 of EMI control Methodology and Procedures, Donald R. White
For cable heights above the ground plane,
and total cable lengths << than the threat frequency' s wavelength,
the following general formula applies :

$$\frac{V_i}{B} = 2 \times L \times C \times \sin(\theta) * \sin\left(\frac{\pi * H}{\lambda} \cos(\alpha)\right)$$

Where :

V_i is the induced open loop voltage
B = Magnetic flux density in Tesla
H = Average loop height above ground
L = Loop length in meters
λ = Threat frequency in Hz
C = Velocity of propagation in m / sec
α = angle between plane of loop direction of propagation
θ = angle between direction of L and orthoganal plane to B

In the calculation below, an approximation derived from assumed diagonal polarizations is used. The typical threat frequencies in LIGO are low enough to ignore transmission line effects.

In[1]:=

```
Clear[B]
B = 0.001; (*Field strength in Gauss*)
H = 0.2; (*Average cable height above "ground" plane*)
L = 10; (*Cable length in meters*)
f = 60; (*Threat frequency in Hz*)
c = 3*^8; (*Speed of light in vacuum*)
λ = c / f; (*Wavelenght Calculation*)
v = 42 426 * L * Sin[2.22 * H / λ] * B; (*Open circuit voltage
 calculation with diagonal field polarization assumptions*)
v1 = ScientificForm[v, 3];
Labeled[Framed[v1],
 {"The induced open circuit voltage is ", "Volts rms"}, {Left, Right}]
```

Out[10]= The induced open circuit voltage is 3.77×10^{-5} Volts rms

Now, armed with the open circuit induced voltage,
we can make some assumptions to get the induced current in the loop

In[11]:=
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R = 12.7*^-3; (*aLIGO in-vacuum cable braid resistance in ohms per meter*)
P = 1; (*Miscellaneous parasitic resistance
 from connectors and mechanical interfaces in ground*)
Rt = R * L * 2 + P; (* Total loop resistance in ohms.  The
 factor of two is put in to account for the roundtrip
 nature of the current loop.  Big assumption here*)
Iloop = v / Rt; (* Total current in amperes flowing in the loop*)
Iloop1 = ScientificForm[Iloop, 3];
v1 = ScientificForm[v, 3];
Labeled[Framed[Iloop1],
 {"The induced current in the loop is ", "Amperes rms"}, {Left, Right}]
```

Out[17]= The induced current in the loop is $3. \times 10^{-5}$ Amperes rms

This induced current will manifest itself as an induced voltage for a single wire contained within the cable shield. The transfer impedance taken from LIGO – T0900158 is used to calculate the coupling

In[18]:=
```
T = 0.03; (*aLIGO in-vacuum cable braid tranfer impedance in ohms per meter*)
v2 = T * L * Iloop; (*The induced voltage for a single wire within the shield*)
vv2 = ScientificForm[v2, 3];
Labeled[Framed[vv2],
 {"The induced voltage in an inner wire within the shield is ", "Volts rms"},
 {Left, Right}]
```

Out[21]= The induced voltage in an inner wire within the shield is 9.01×10^{-6} Volts rms

The induced voltage will be actually applied to an instrumentation amplifier receiver,
so the common mode rejection ratio will reduce the observed signal

In[22]:=

```
Rtol = 0.01; (*Tolerance of components used
 in building our instrumentation amplifier receivers*)
gain = 0.5; (*Overall gain of the instrumentation amplifier*)
cmrr = (gain + 1) / (3 * Rtol);
(*Common mode rejection ratio based on the relationship between 3 resistors
 to a 4th as one might find in a typical discrete instrumentation amplifier*)
Labeled[Framed[cmrr],
 {"The common mode rejection ratio of the instrumentation amplifier will be ",
  "taking into account resistor matching and gain"}, {Left, Right}]
vout = v2 / cmrr; (*vout reduced by the cmrr*)
vout2 = ScientificForm[vout, 3];
Labeled[Framed[vout2],
 {"The voltage appearing at the output of the instrumentation
    amplifier will be ", "Volts rms"}, {Left, Right}]
```

Out[25]= The common mode rejection ratio of the instrumentation amplifier will be | 50. | taking into account resistor matching and gain

Out[28]= The voltage appearing at the output of the instrumentation amplifier will be | 1.8×10^{-7} | Volts rms