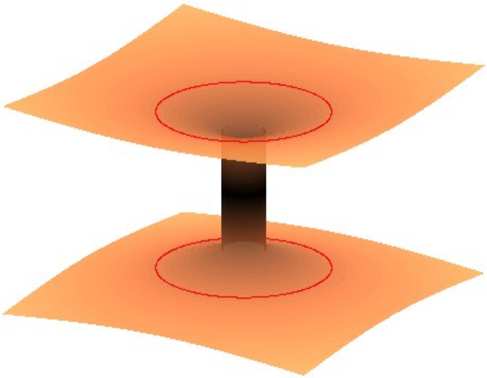
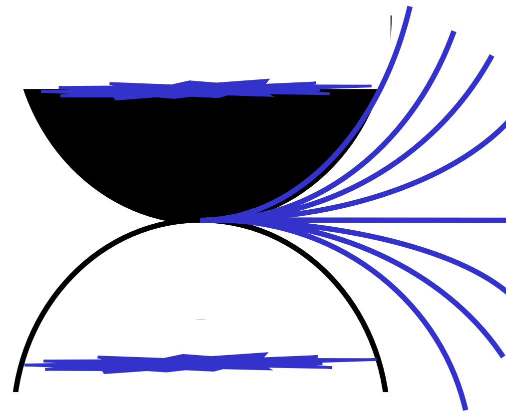
Black Holes And LIGO





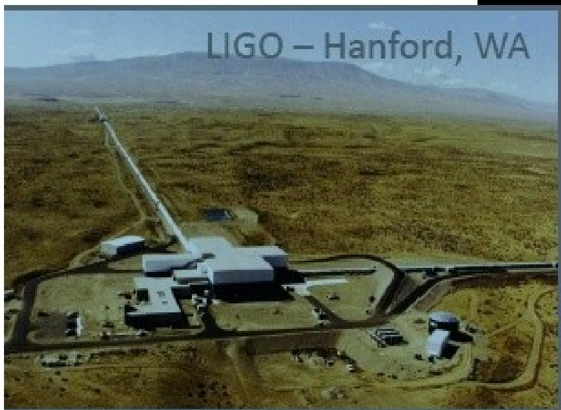
The Laser Interferometer Gravitational-wave Observatory: a Caltech/MIT collaboration supported by the National Science Foundation

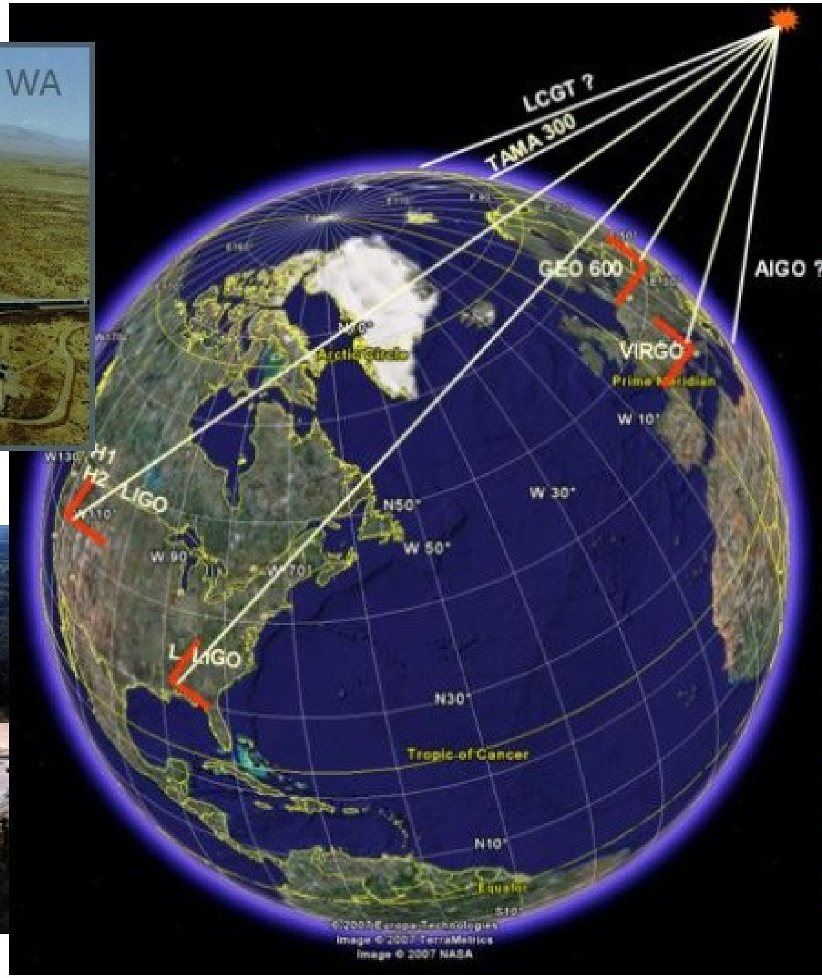
Gregory Mendell

LIGO Hanford Observatory

LIGO-G1200393

The LIGO-Virgo network

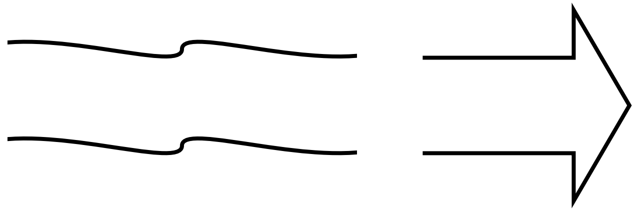




600+ Scientist and Engineers

Einstein Wondered:

Photo: *Albert Einstein at the first Solvay Conference, 1911; Public Domain*



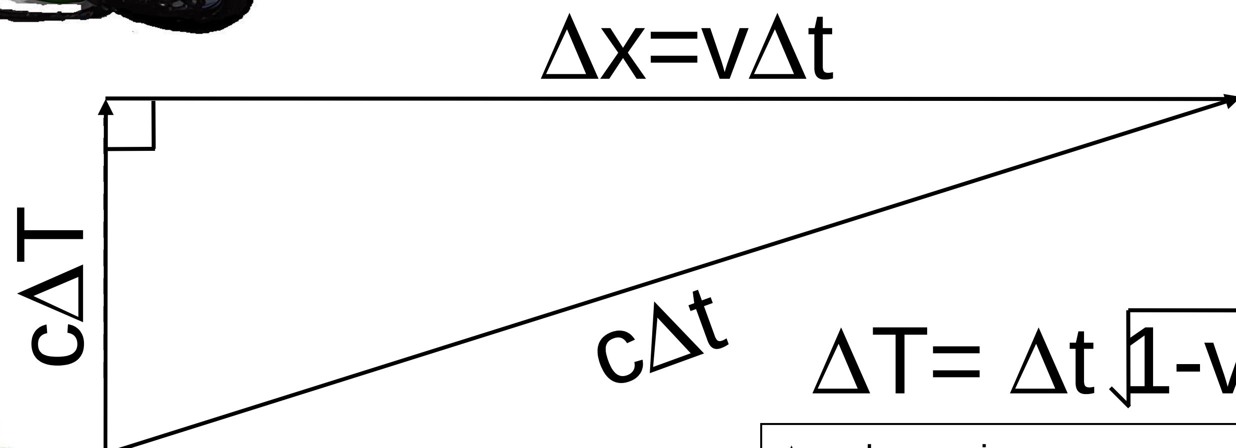
Can we catch light?

Mirror

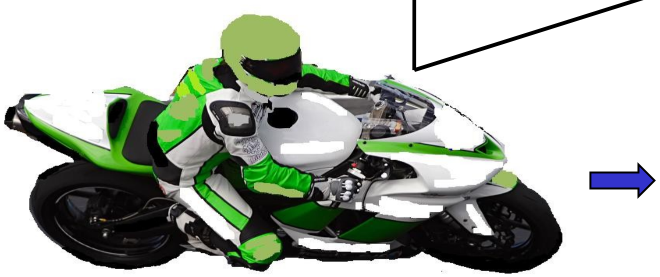
Time Dilation

Niels Bohr

Albert Einstein



$$\Delta T = \Delta t \sqrt{1 - v^2/c^2}$$



Δ = change in

T = time measured by motorcycle riders

t = time measured by observer at “rest”

v = speed of motercycles

c = speed of light

Start

Warning: thought experiment only; do not try this at home.

Motorcycle: http://en.wikipedia.org/wiki/Motorcycle_racing

The Pythagorean Theorem Of Spacetime

$$c^2\Delta T^2 + v^2\Delta t^2 = c^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - v^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - \Delta x^2$$

c = 1 light-year/year

$$\boxed{\Delta T^2 = \Delta t^2 - \Delta x^2}$$

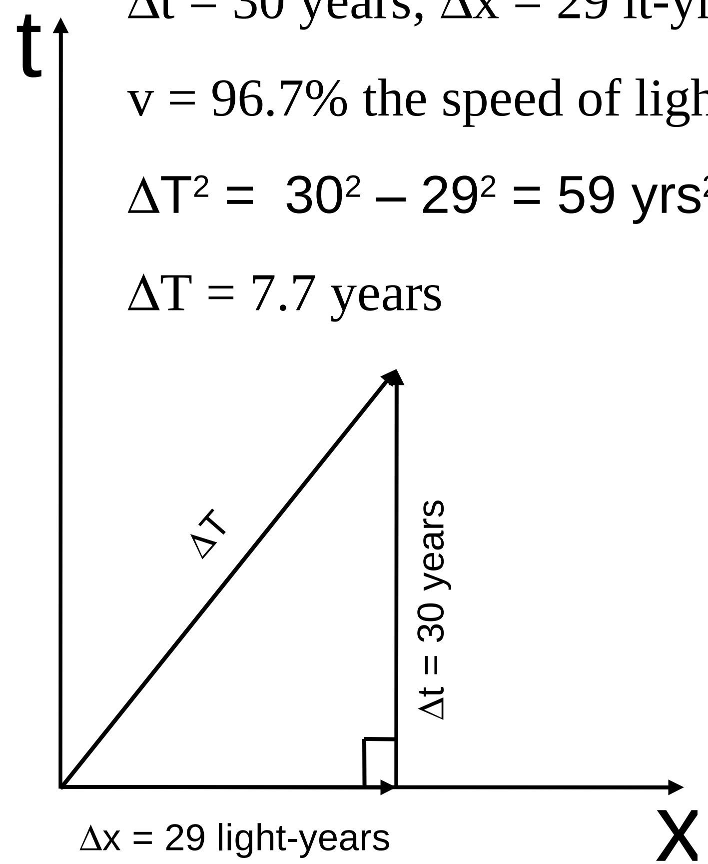
Pythagorean Thm. of Spacetime

Δt = 30 years; Δx = 29 lt-yrs.

v = 96.7% the speed of light

$\Delta T^2 = 30^2 - 29^2 = 59$ yrs^2

ΔT = 7.7 years



Spacetime

LIGO

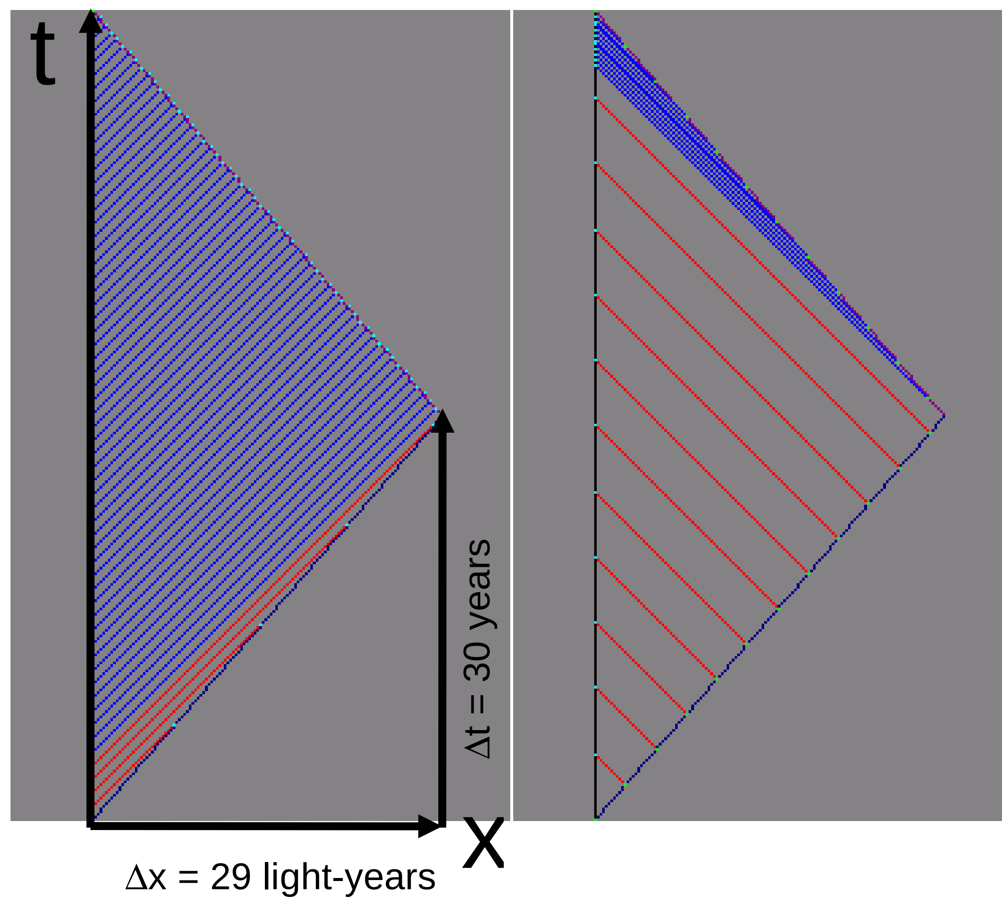
Spacetime Diagram

The Twin Paradox

- Imagine twins, Betty and Bob, separated 1 year after birth.

Baby Betty & Bob: ☺ ☺

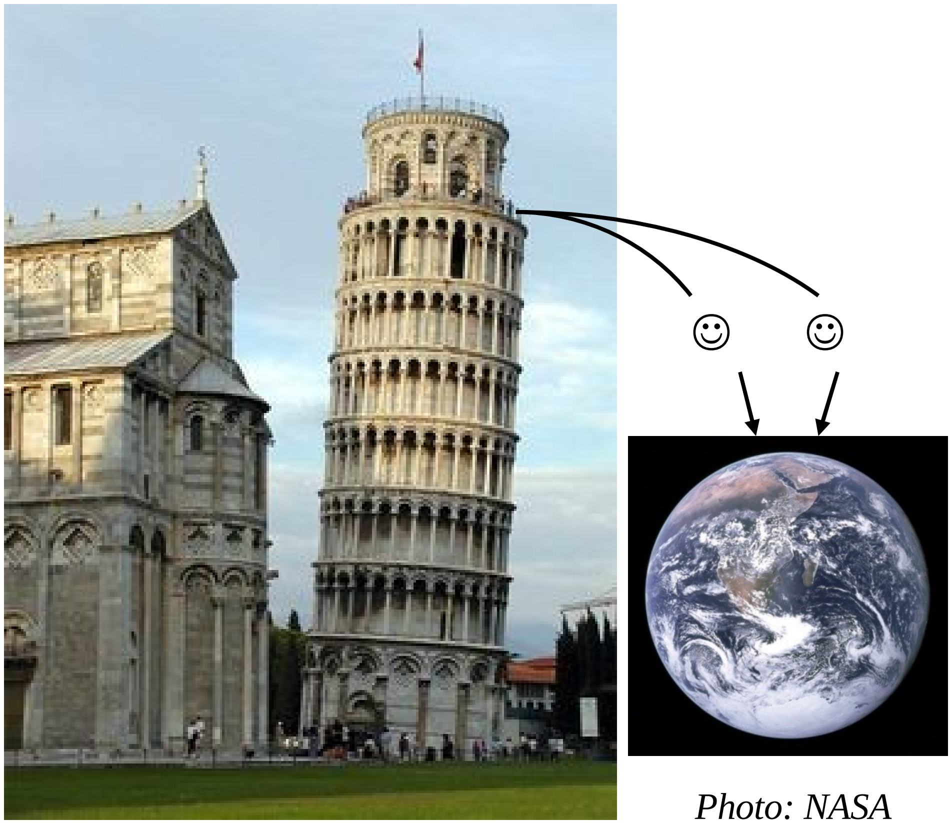
- Betty takes a rocket travelling at 96.67% the speed of light and travels 29 lt-yrs from Earth and back.
- When Betty returns she is sweet 16, and Bob is 61 years old!!!



$$\Delta T = 30 \text{ yrs} \sqrt{1-(.9667)^2} = 7.7 \text{ yrs}$$

Figure: http://en.wikipedia.org/wiki/Twin_paradox

Einstein's Happiest Thought: Gravity Disappears When You Free Fall



http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa

Photo: NASA

Einstein had this thought in 1907. This lead to the idea that gravity is the curvature of spacetime. Here I paraphrase a thought experiment I first heard from Kip Thorne. Suppose two friends jump parallel to each other off the Leaning Tower of Pisa. For the friends, gravity has disappeared, and they believe they are in empty space. Strangely though, they find their parallel paths converging at the center of the Earth. That can happen in empty space only if that space is not flat but curved. Einstein thought about the geometry of rotating objects, and other things, and after 8 more years produced General Relativity, which is a theory of gravity and spacetime. He had help from a mathematician, Marcel Grossmann.

Warning: thought experiment only; do not try this at home.

Pythagorean Theorem and Einstein's General Theory of Relativity

$\Delta \rightarrow d$ = infinitesimal change

$$dT^2 = g_{tt} dt^2 + g_{xx} dx^2$$

$$dT^2 = g_{\mu\nu} dx^\mu dx^\nu$$

In GR the components of a 4x4 symmetric matrix called the metric tensor define the curvature of spacetime.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}; R = g^{\mu\nu} R_{\mu\nu}$$

$$R^\alpha{}_{\mu\beta\nu} = \partial_\beta \Gamma^\alpha{}_{\mu\nu} - \partial_\nu \Gamma^\alpha{}_{\mu\beta} + \Gamma^\alpha{}_{\beta\gamma} \Gamma^\gamma{}_{\mu\nu} - \Gamma^\alpha{}_{\gamma\nu} \Gamma^\gamma{}_{\mu\beta}$$

$$\Gamma^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\partial_\nu g_{\mu\beta} + \partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu})$$

Einstein's Field Equations

$$\frac{dx^\alpha}{dT} = U^\alpha; \quad U_\alpha = g_{\alpha\beta} U^\beta$$

U = 4-Vel.; T = Proper Time

$$\frac{dU_\alpha}{dT} = \frac{1}{2} \partial_\alpha g_{\beta\gamma} U^\beta U^\gamma$$

Geodesic Equation

Schwarzschild Black Hole

$$c^2 dT^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$c^2 dT^2 = \left(1 - \frac{v_{esc}^2}{c^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{v_{esc}^2}{c^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Karl Schwarzschild

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

- Escape Velocity

$$R_s = \frac{2GM}{c^2}$$

- Schwarzschild Radius

Object	Schwarzschild Radius
You	1 thousand, million, million, millionth the thickness of a human hair
Earth	1 cm (size of marble)
Sun	3 km (2 miles)
Galaxy	~ trillion miles

Gravitational Time Dilation

$$\Delta T = \sqrt{1 - \frac{2GM}{rc^2}}\,\Delta t$$

Gravity slows time down!

Photo:http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa

Clock_Photos:http://en.wikipedia.org/wiki Cuckoo_clock

Gravity Slows Time

• Due to the orbital speed, clocks on the satellite lose 7 microseconds per day

•Due to the weaker gravitational field, clocks on the satellite gain 45 microseconds per day

•Satellite clocks gain a net of 38 microsecond per day

•Distance error = c*38 microseconds; c = 186,000 miles per second.

•Without calibrating clocks to account for Relativity, GPS distance would be off by 7 miles after one day!

See Scientific American, Sept. 1994

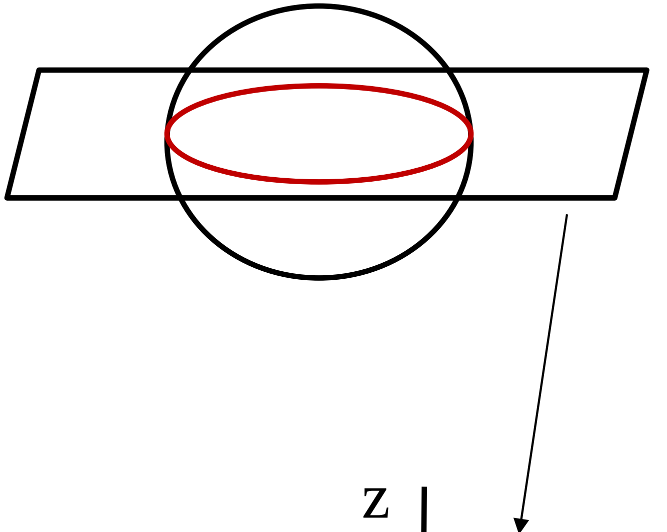
Illustration: NASA

Clock_Photos:http://en.wikipedia.org/wiki/Cuckoo_clock

Embedding Diagram

Schwarzschild for t = 0, θ = π /2:

$$ds^2 = \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 d\phi^2$$



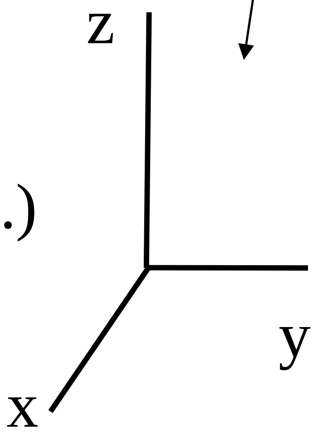
Flat space cylindrical coordinates:

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2$$

$z = f(r)$ (Surface of revolution about z-axis.)

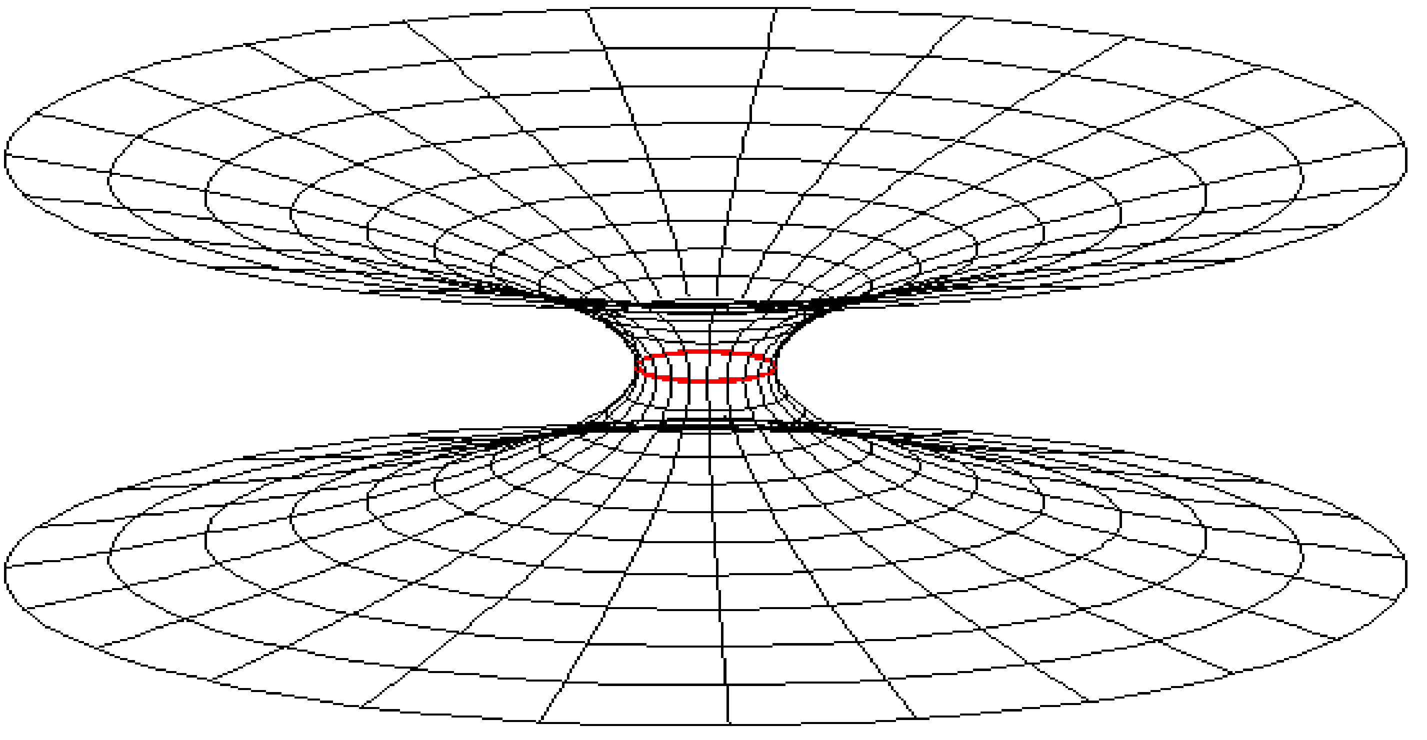
$$dz = f'(r) dr$$

$$ds^2 = [f'(r)^2 + 1] dr^2 + r^2 d\phi^2$$



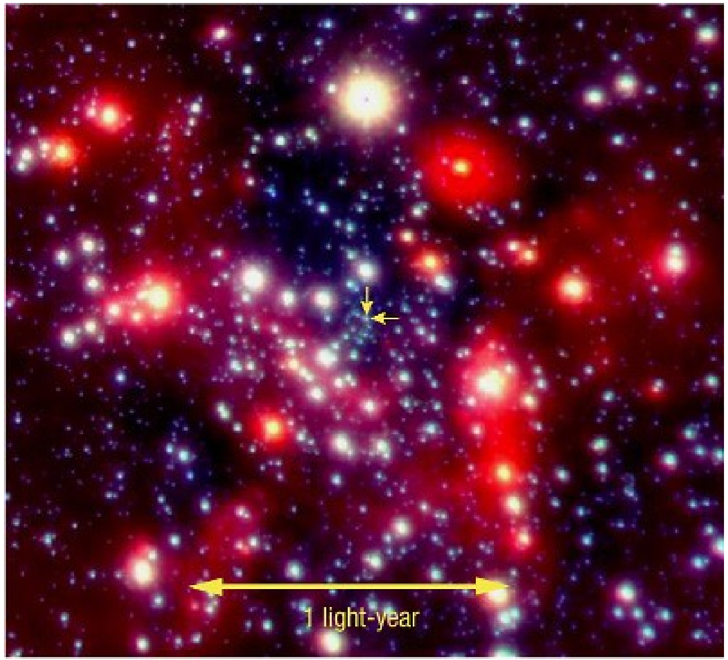
Einstein-Rosen Bridge

Our Universe

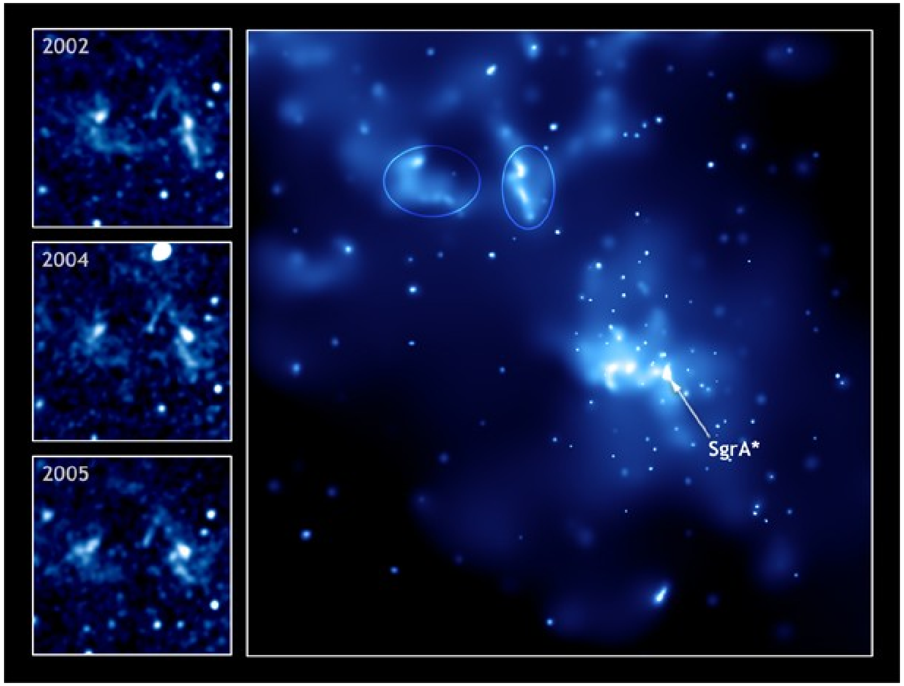


Another Universe?

The Center Of The Milky Way

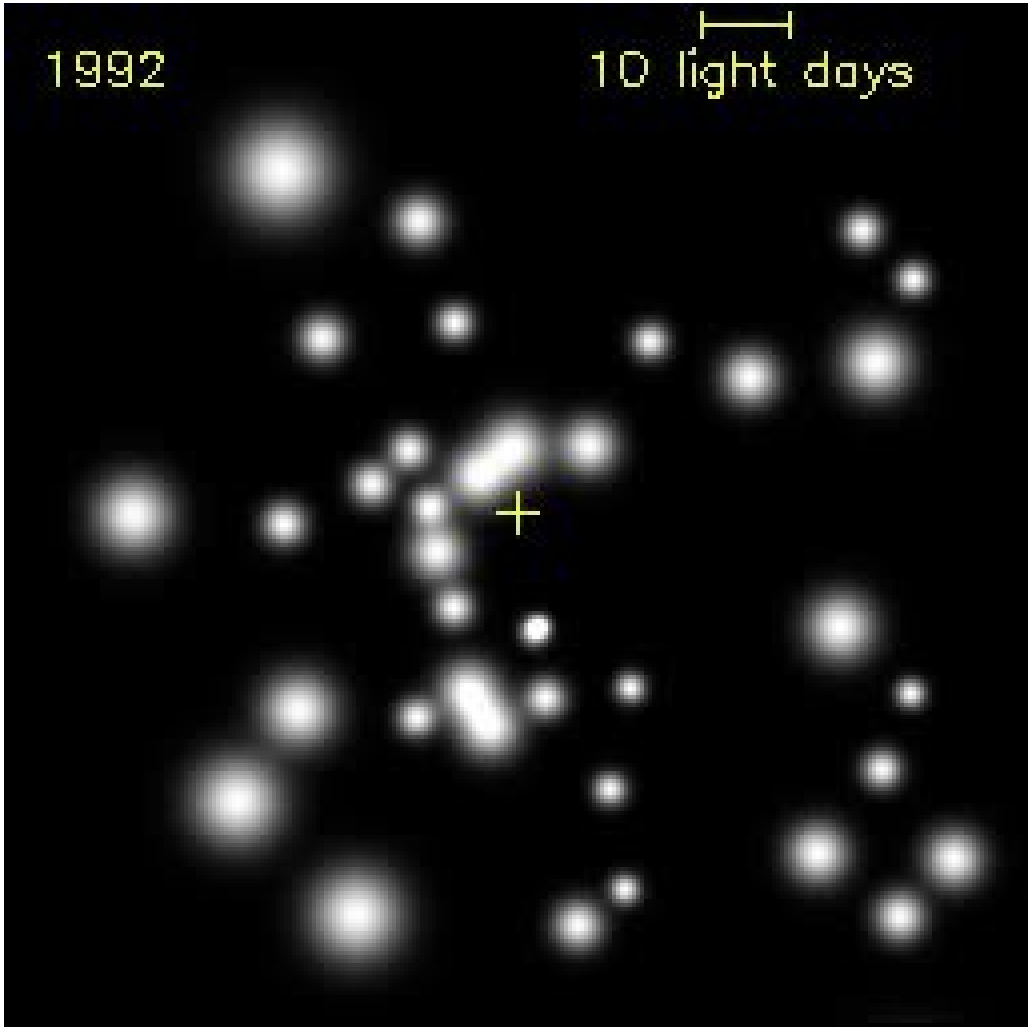


The Centre of the Milky Way
(VLT YEPUN + NACO)
ESO PR Photo 23a/02 (9 October 2002)



Credit: NASA/Chandra X-Ray Observatory

Zooming in on the galactic center…



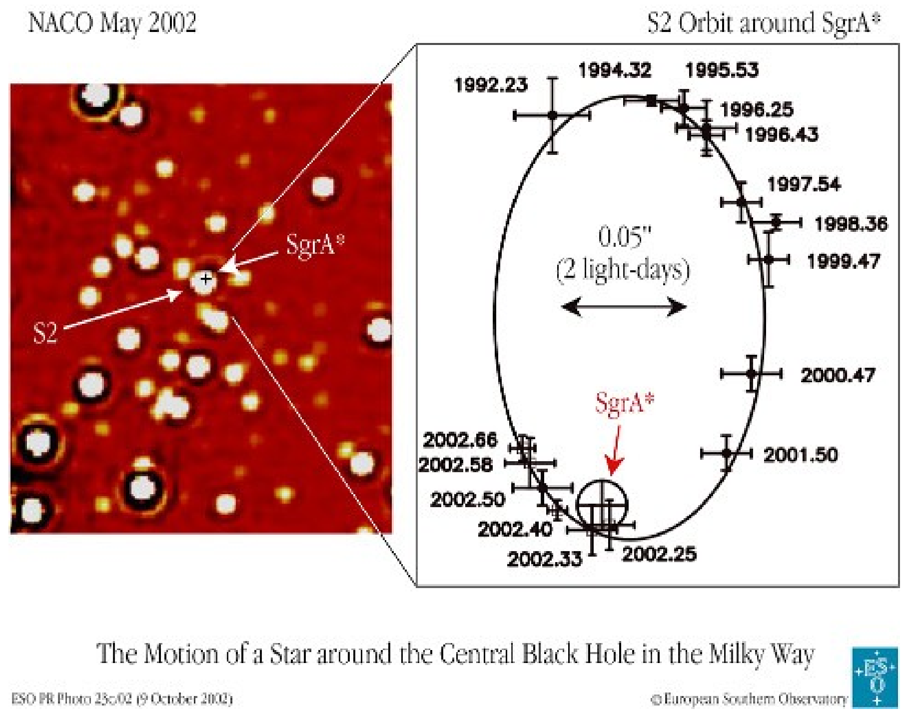
Credit: ESO PR Video Clip 02/02; ESO/European Organization for Astronomical Research in the Southern Hemisphere; Press Release 2002

Black Hole Detection

$$M = \frac{4\pi^2 a^3}{GT^2}$$

$$M = \frac{(900AU)^3}{(15yrs)^2}$$

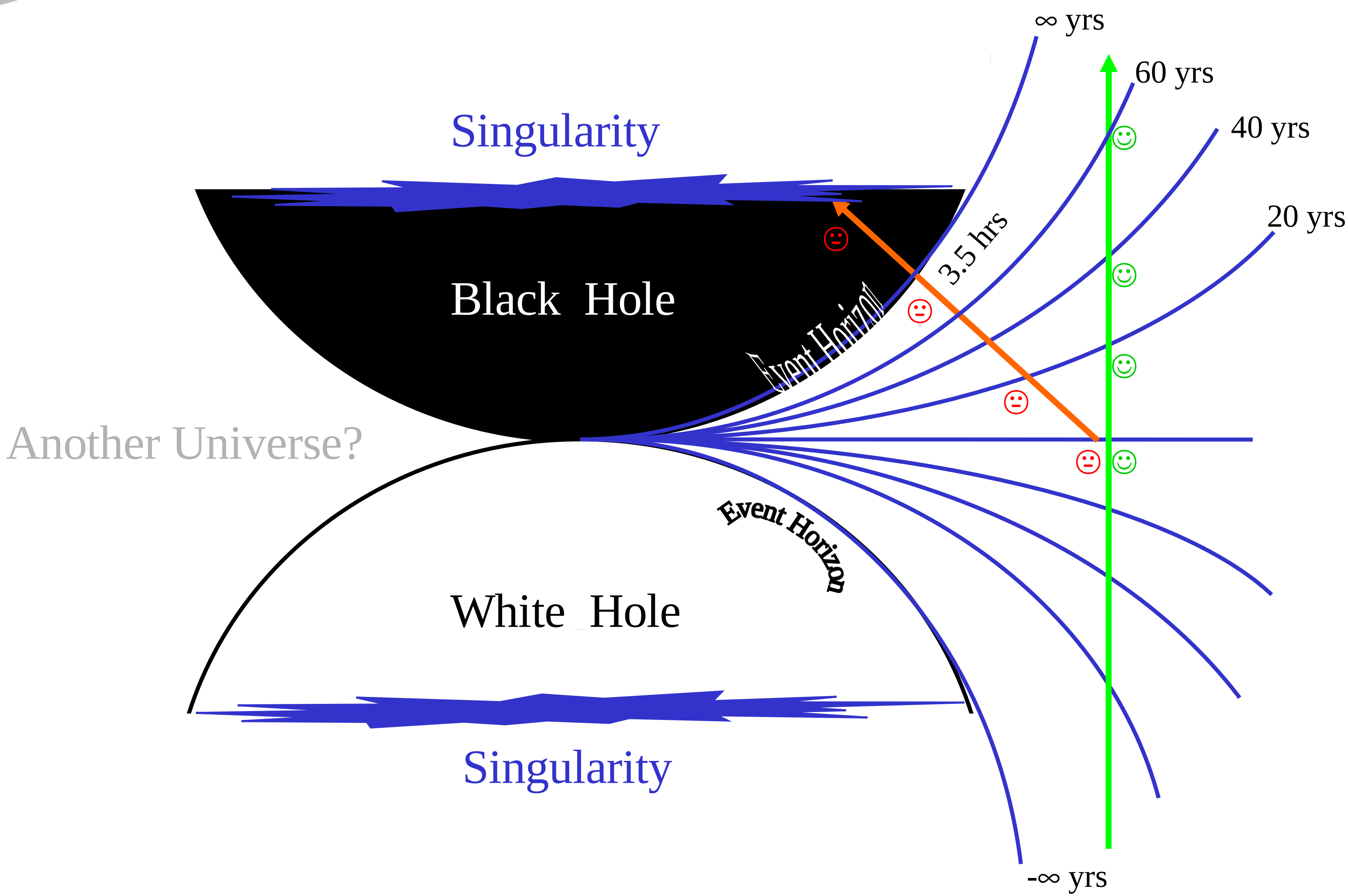
= 3 million Solar Masses



Conclusion: there is a Black Hole at the center of our Galaxy that has a mass 3 millions times (or more precisely 3.95 million times) that of the Sun. S2 orbits this Black Hole at a distance of 12000 Schwarzschild Radii.

LIGO

Falling Into A Black Hole



Embedding Diagram Inside The Black Hole

Schwarzschild for $r = R$, $\theta = \pi/2$:

$$ds^2 = c^2[2GM/(Rc^2)-1]dt^2 + R^2\phi^2.$$

Flat space cylindrical coordinates:

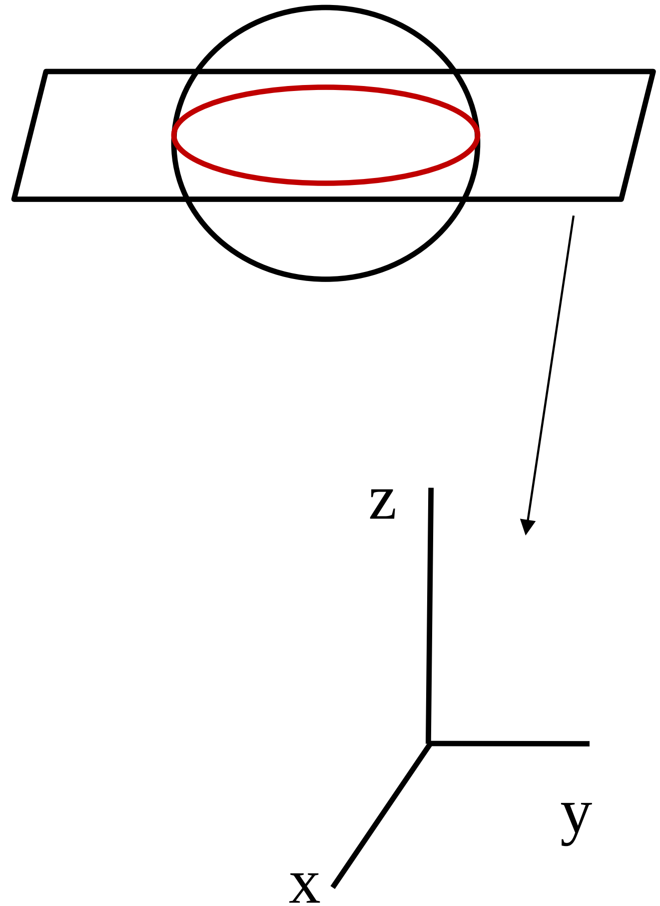
$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2.$$

Comparing, it looks like in the flat space $r = R =$ constant, so

$$ds^2 = dz^2 + R^2 d\phi^2.$$

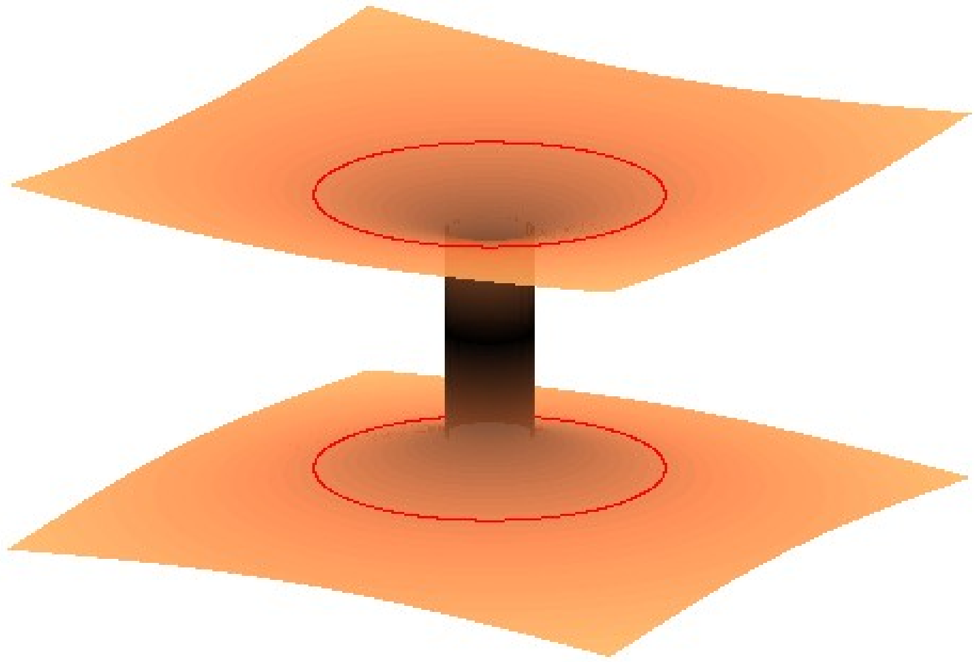
We need to match up:

$$dz^2 = c^2[2GM/(Rc^2)-1]dt^2.$$



Schwarzschild Worm Hole

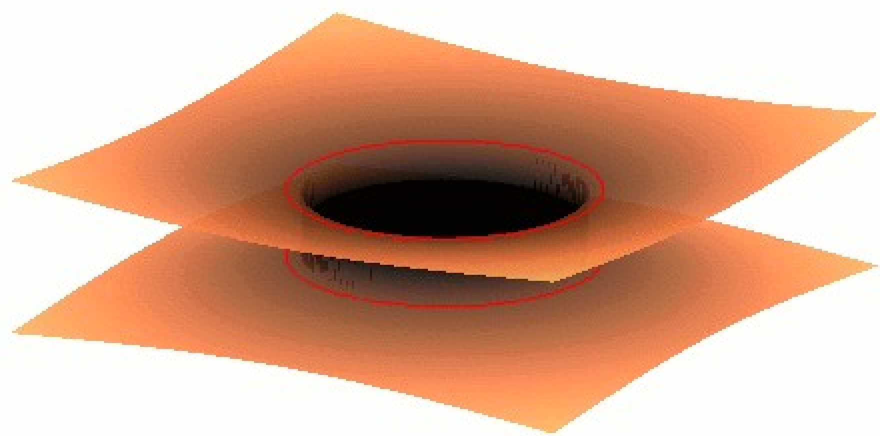
Our Universe



Another Universe

Embedding With Interior Dynamics

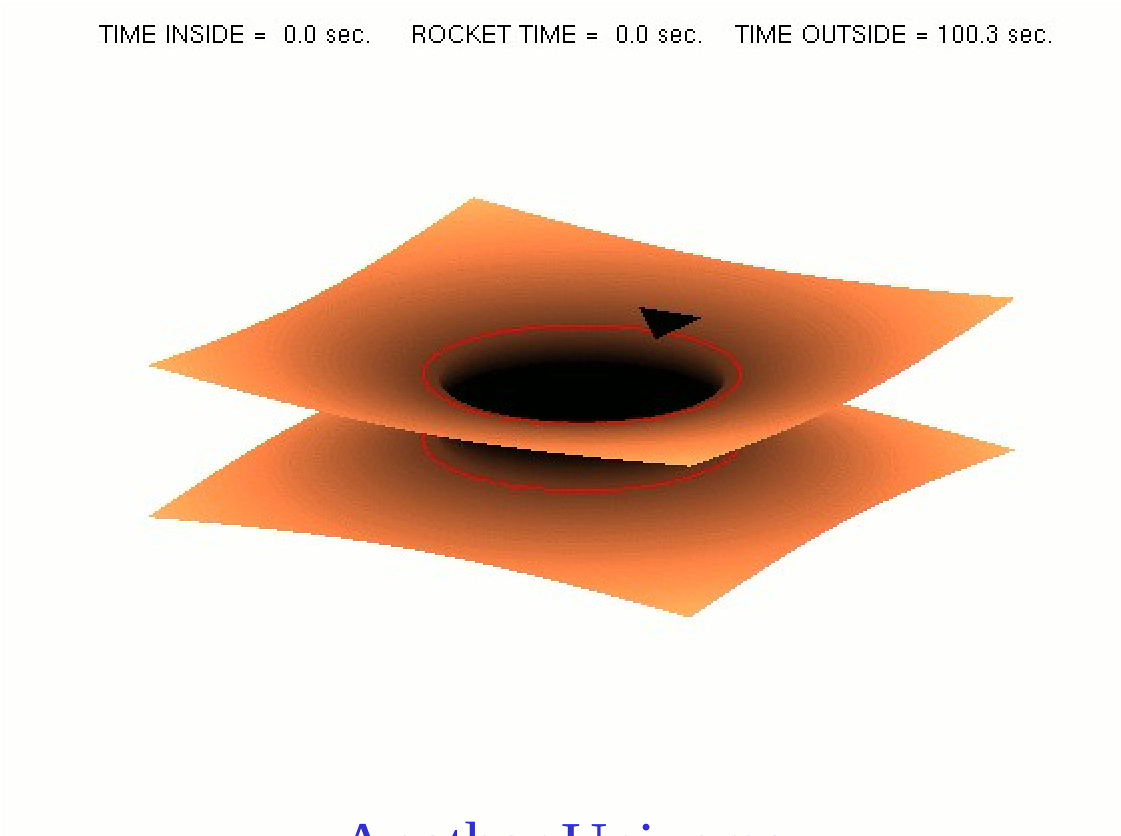
Our Universe



Another Universe

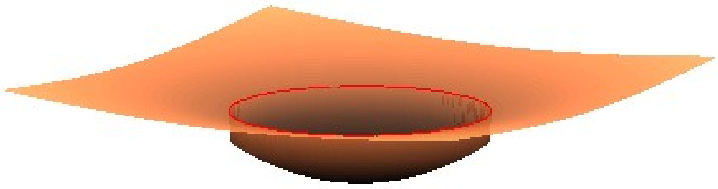
Nontraversable Wormhole

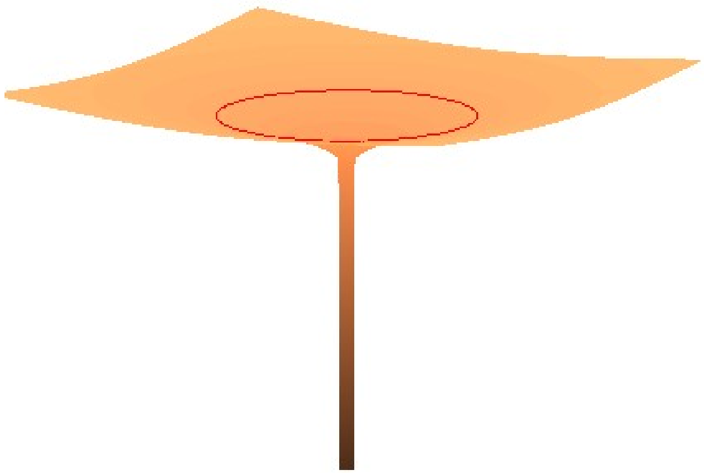
Our Universe



Another Universe

Stellar Collapse To Form A Black Hole

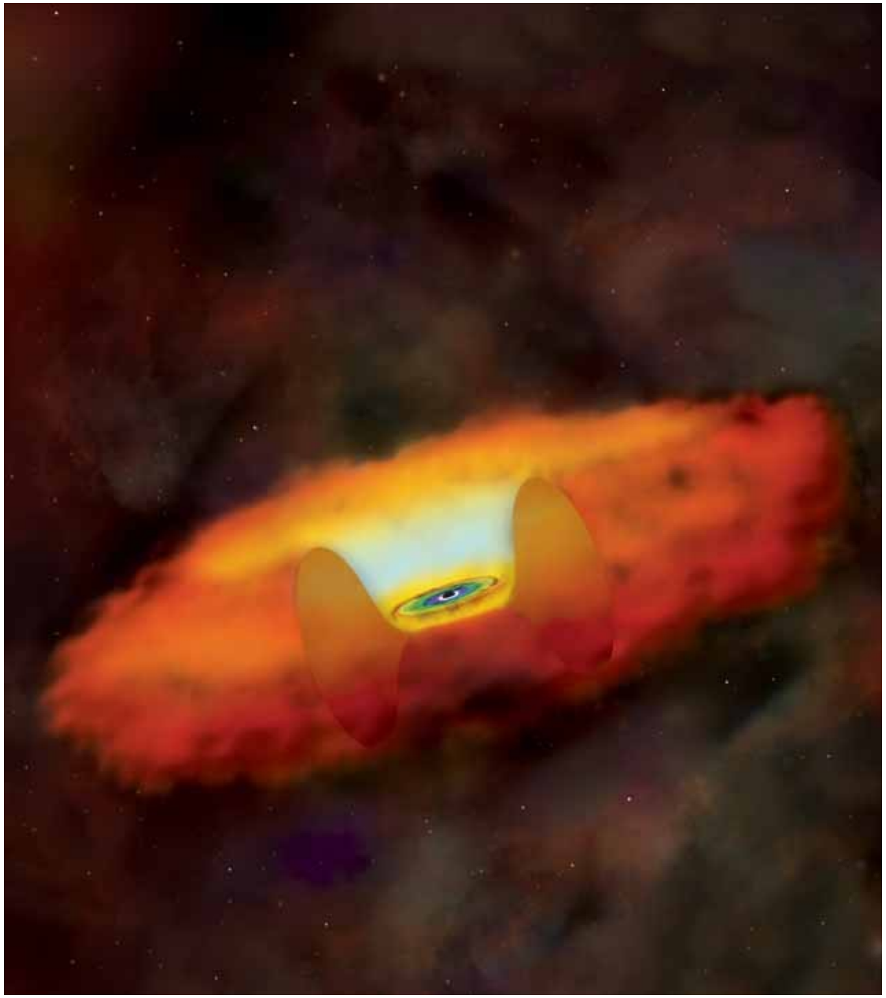




When pressure can no longer support a star's gravity its mass falls through its horizon.

And it collapses to a Singularity.

Black Holes & Accretion Disks



http://researchnews.osu.edu/archive/fuzzballpic.htm
(Illustration: CXC/M.Weiss)

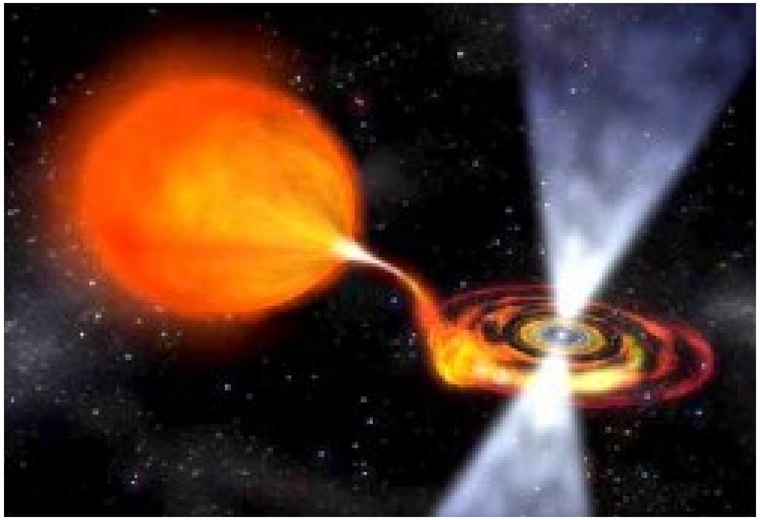
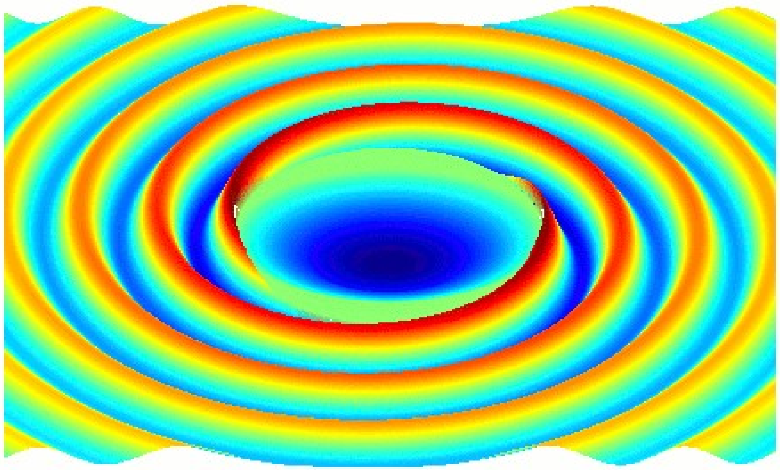


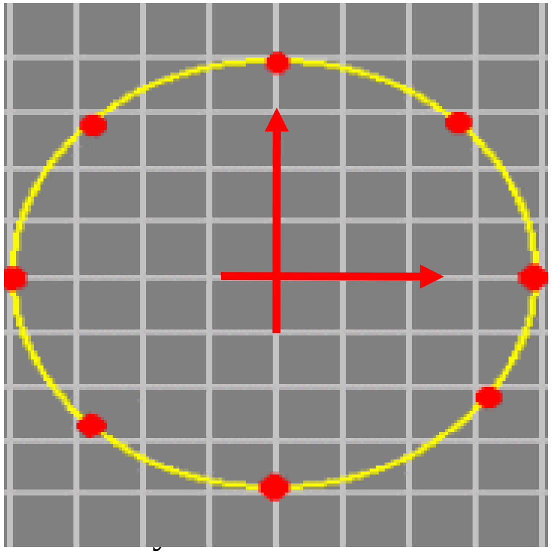
image by Dana Berry/NASA; NASA News Release posted July 2, 2003 on Spaceflight Now.

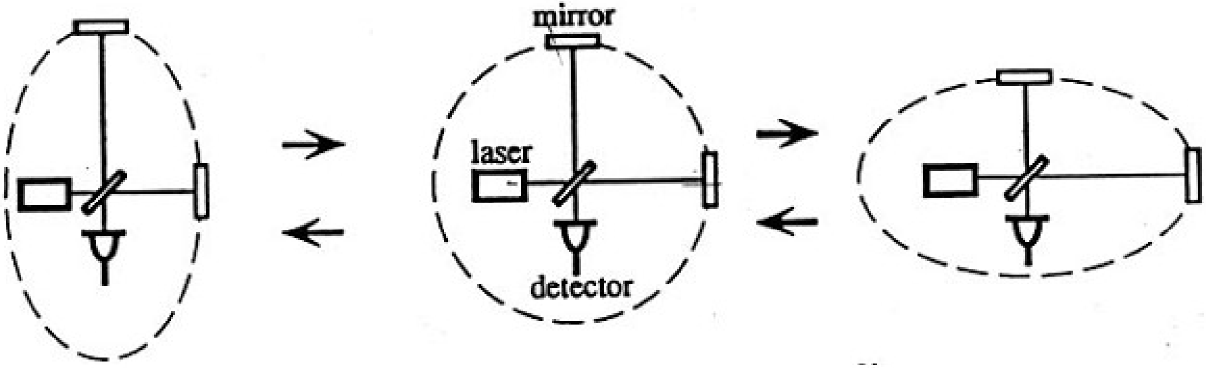
Gravitational Waves

Gravitational waves are ripples in spacetime when it is stirred up by rapidly changing motions of large concentrations of matter or energy. **The waves are extremely weak by the times they reach Earth.**

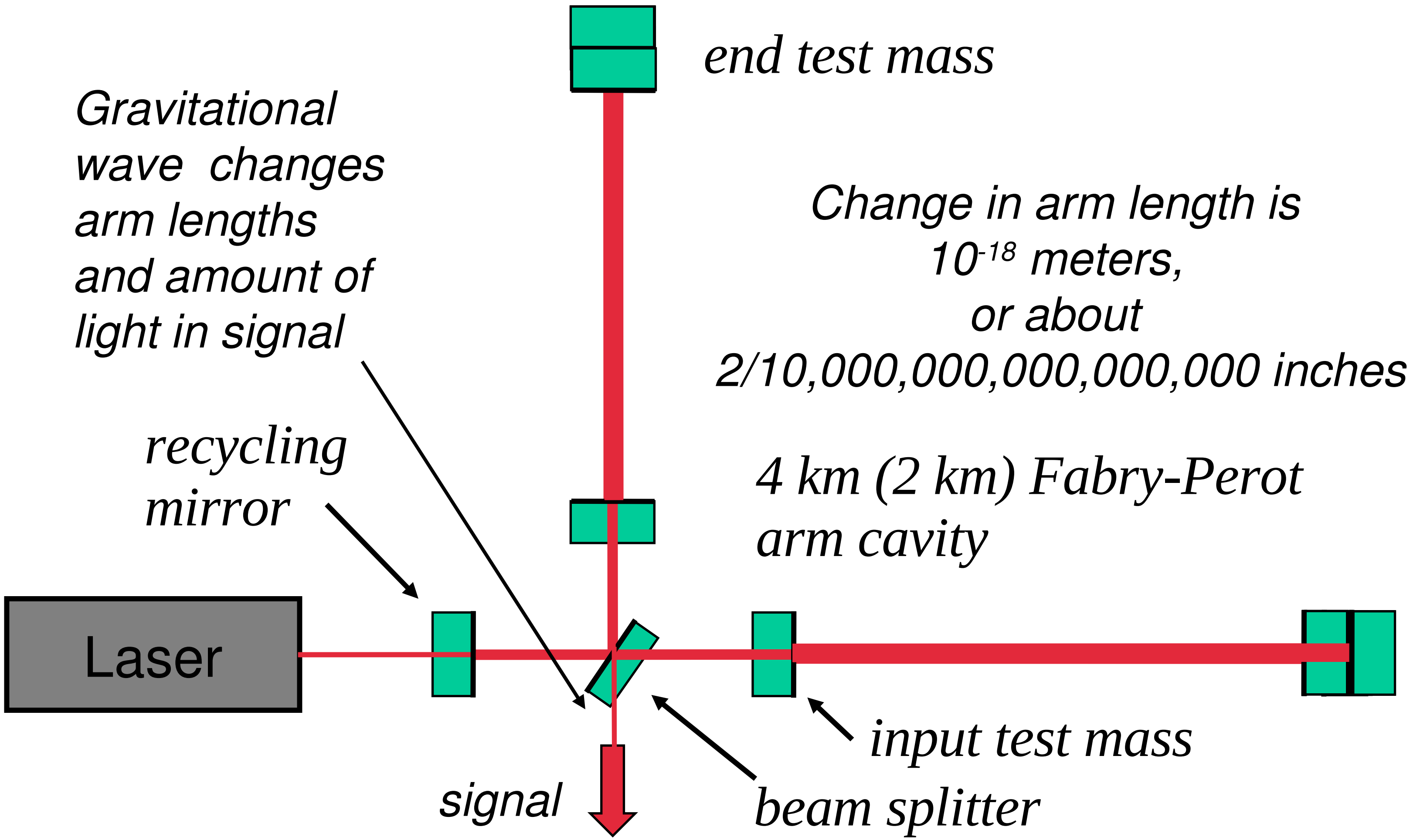
Illustration of Gravitational Waves:



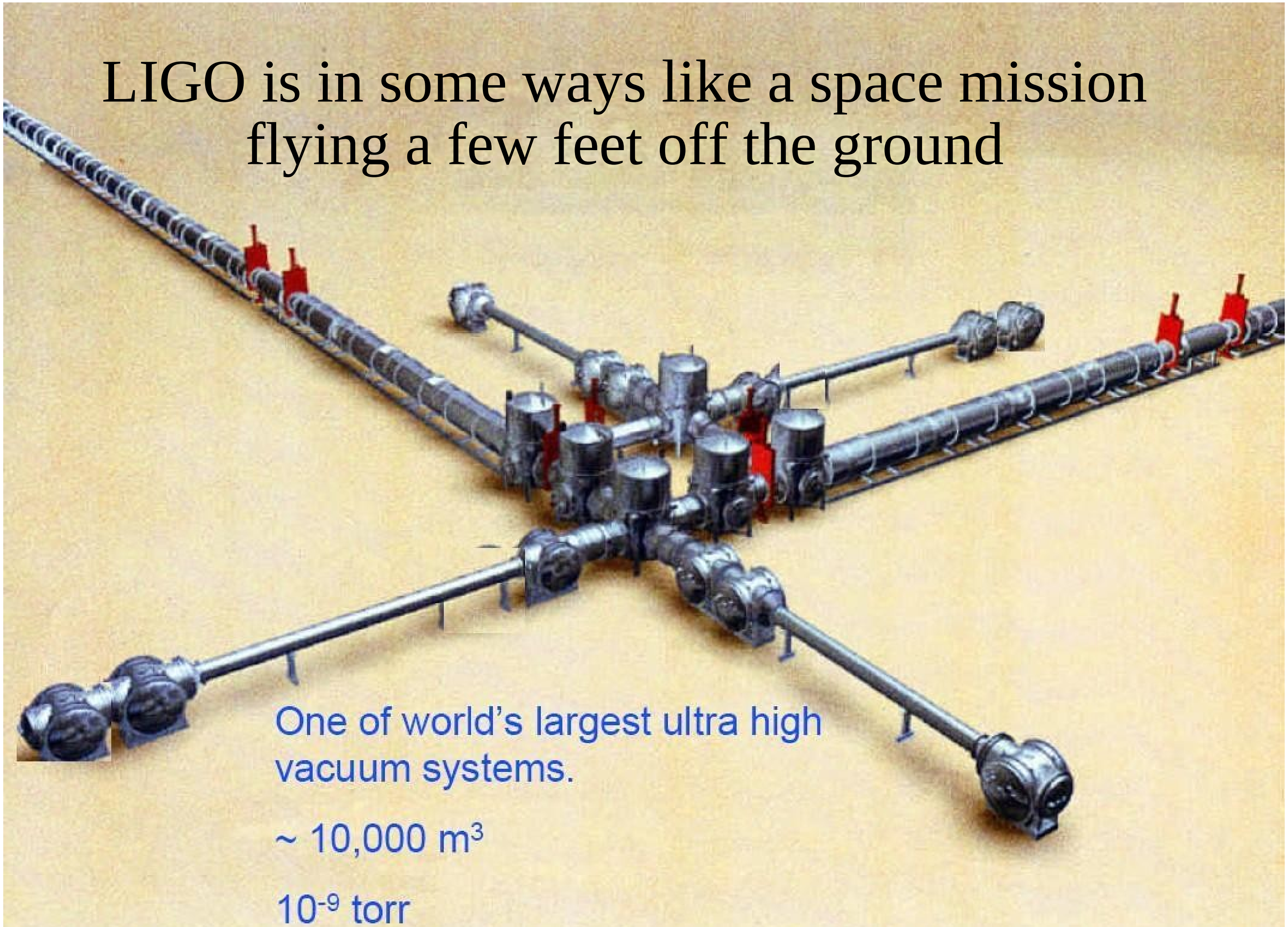




Sensing the Effect of a Gravitational Wave



LIGO is in some ways like a space mission flying a few feet off the ground



One of world's largest ultra high vacuum systems.

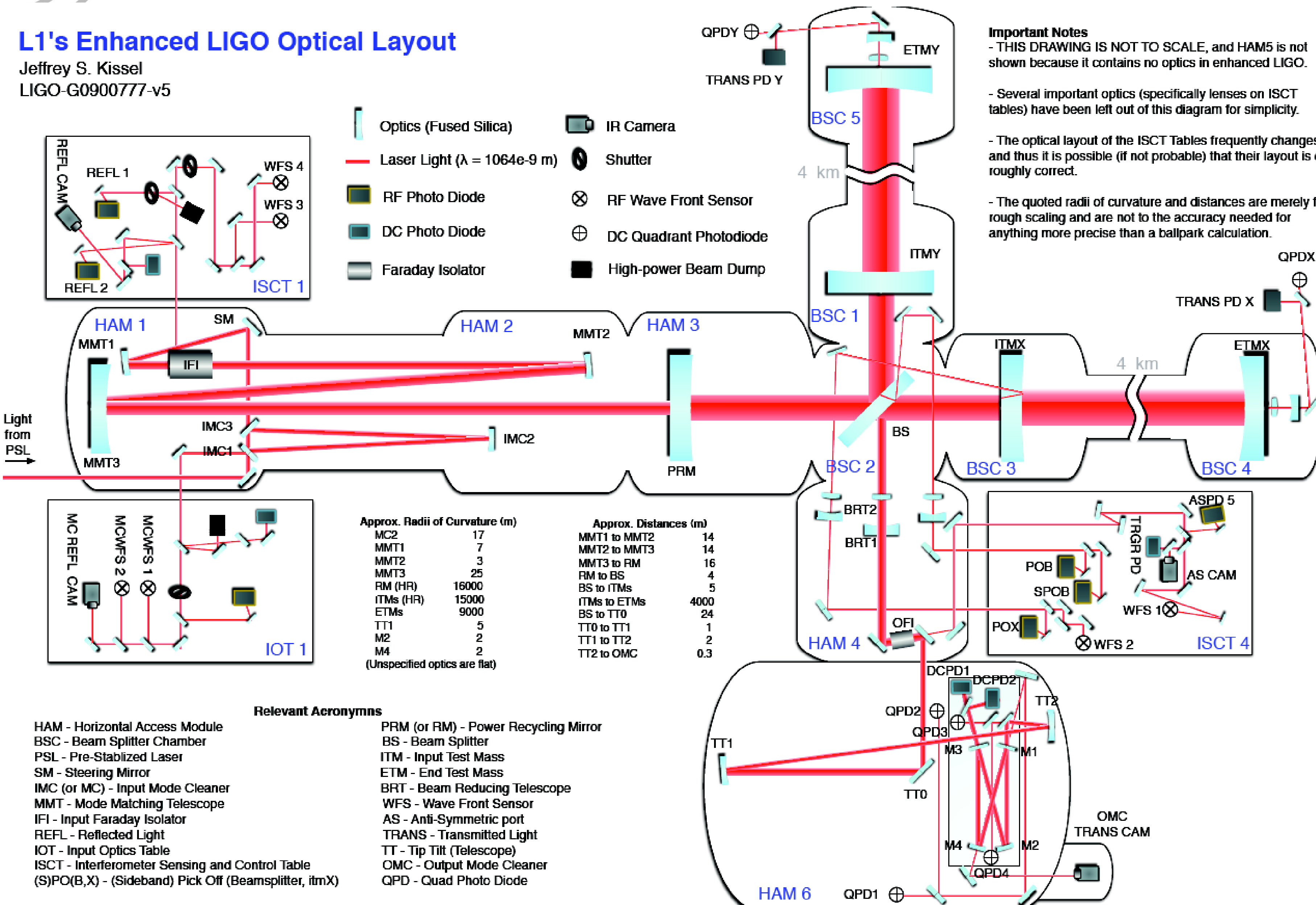
~ 10,000 m^3

10^{-9} torr

L1's Enhanced LIGO Optical Layout

Jeffrey S. Kissel

LIGO-G0900777-v5



Important Notes

- THIS DRAWING IS NOT TO SCALE, and HAM5 is not shown because it contains no optics in enhanced LIGO.
- Several important optics (specifically lenses on ISCT tables) have been left out of this diagram for simplicity.
- The optical layout of the ISCT Tables frequently changes and thus it is possible (if not probable) that their layout is only roughly correct.
- The quoted radii of curvature and distances are merely for rough scaling and are not to the accuracy needed for anything more precise than a ballpark calculation.

Approx. Radii of Curvature (m)

MC2	17
MMT1	7
MMT2	3
MMT3	25
RM (HR)	16000
ITMs (HR)	15000
ETMs	9000
TT1	5
M2	2
M4	2

(Unspecified optics are flat)

Approx. Distances (m)

MMT1 to MMT2	14
MMT2 to MMT3	14
MMT3 to RM	16
RM to BS	4
BS to ITMs	5
ITMs to ETMs	4000
BS to TT0	24
TT0 to TT1	1
TT1 to TT2	2
TT2 to OMC	0.3

Relevant Acronymns

HAM - Horizontal Access Module
BSC - Beam Splitter Chamber
PSL - Pre-Stablized Laser
SM - Steering Mirror
IMC (or MC) - Input Mode Cleaner
MMT - Mode Matching Telescope
IFI - Input Faraday Isolator
REFL - Reflected Light
IOT - Input Optics Table
ISCT - Interferometer Sensing and Control Table
(S)PO(B,X) - (Sideband) Pick Off (Beamsplitter, itmX)
PRM (or RM) - Power Recycling Mirror
BS - Beam Splitter
ITM - Input Test Mass
ETM - End Test Mass
BRT - Beam Reducing Telescope
WFS - Wave Front Sensor
AS - Anti-Symmetric port
TRANS - Transmitted Light
TT - Tip Tilt (Telescope)
OMC - Output Mode Cleaner
QPD - Quad Photo Diode

Binary Black Hole Coalescence

Show movies from: Simulating Extreme Spacetimes – SXS - Caltech – Cornell Project. http://www.black-holes.org/explore2.html

Credit: Introduction to LIGO & Gravitational Waves: http://www.ligo.org/science/GW-Inspiral.php

Credit: Scott Hughs, MIT group:

LIGO

LSC VIRGO

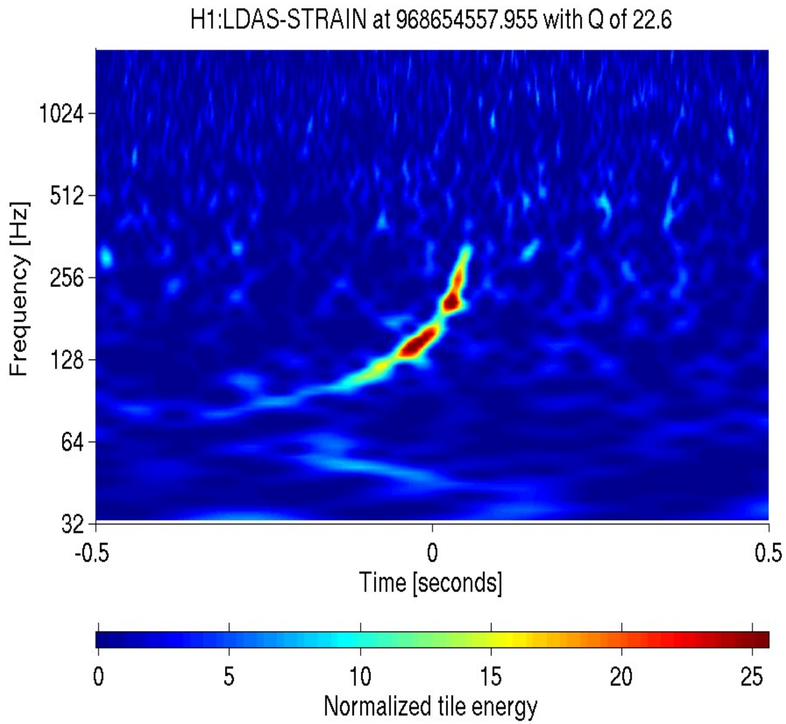
• During their 2009-2010 science runs, the LIGO Scientific Collaboration and the Virgo Collaboration did an end-to-end test with a **blind hardware injection of a fake signal into the detectors**.

• A signal was observed by several methods on Sept. 16, 2010. Subsequent analysis suggested it was a binary coalescence involving at least one black hole, with a 1/(7000 yr) false alarm rate.

•The Blind Injection Envelope was opened on March 14, 2011 revealing the Sept. 16, 2010 event was the fake injection of a neutron star – black hole coalescence signal.

• See: http://www.ligo.org/news/blind-injection.php; http://www.ligo.org/science/GW100916/

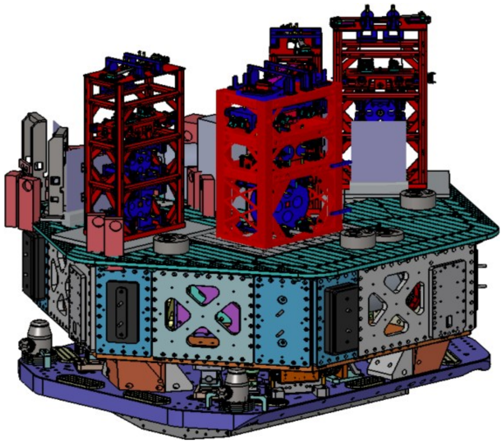
What will a detection look and sound like?

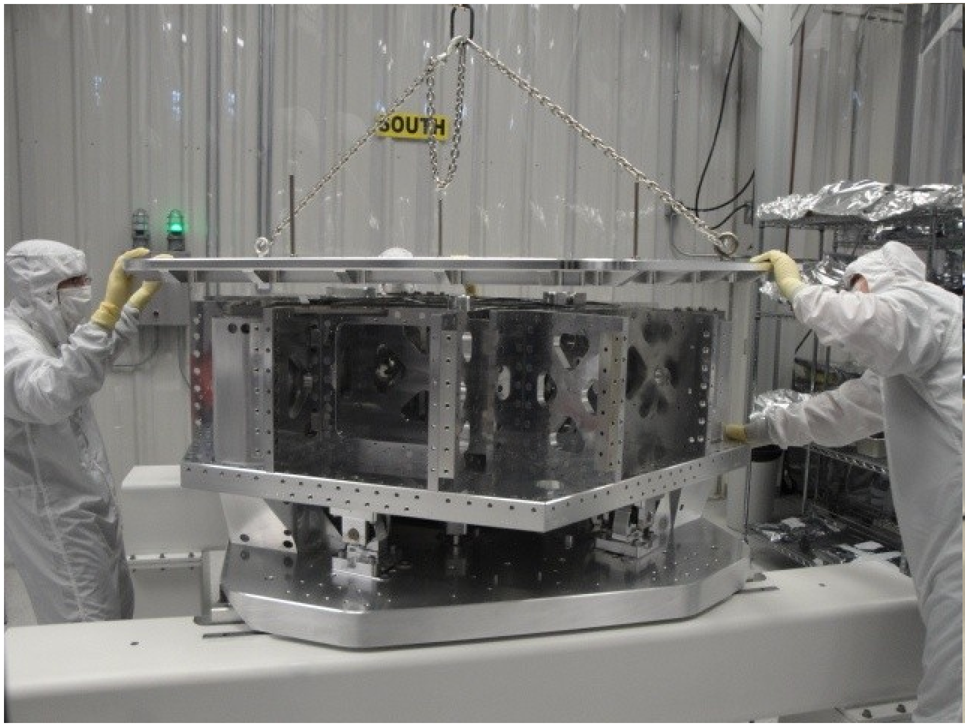


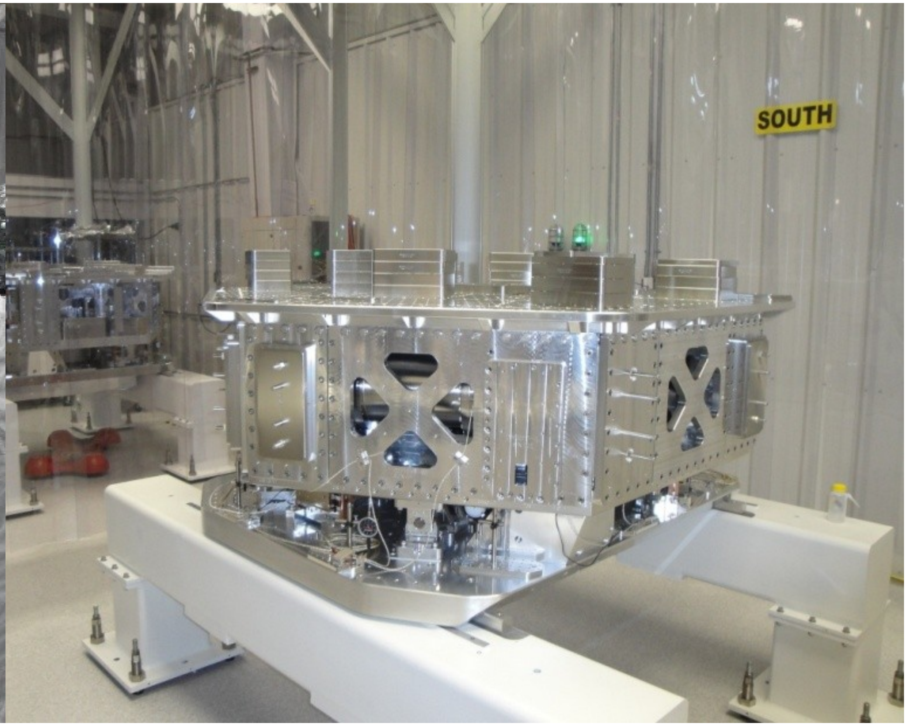
Data and Sound File from LIGO Hanford Observatory with **blind hardware injection of a fake signal.**

Advanced LIGO Seismic Isolation

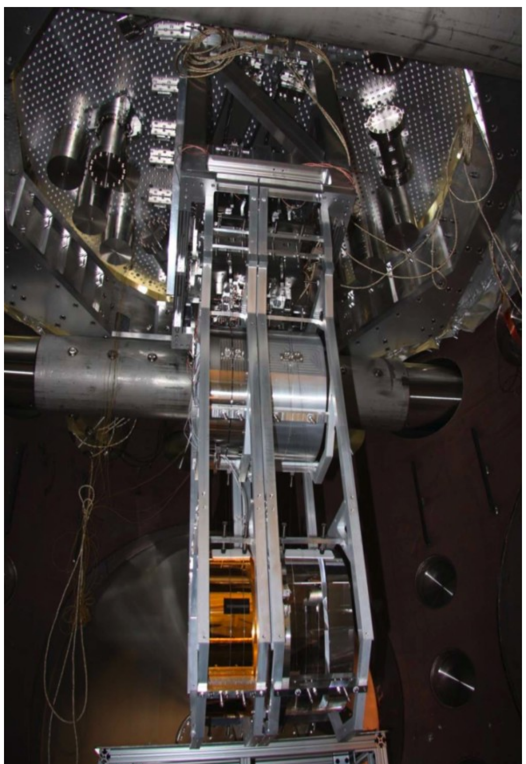
- Assembly of the Horizontal Access Module stacks is in full swing at both observatories.
- Active feedback control will be used.
- One assembly already was used in the Enhanced LIGO configuration.

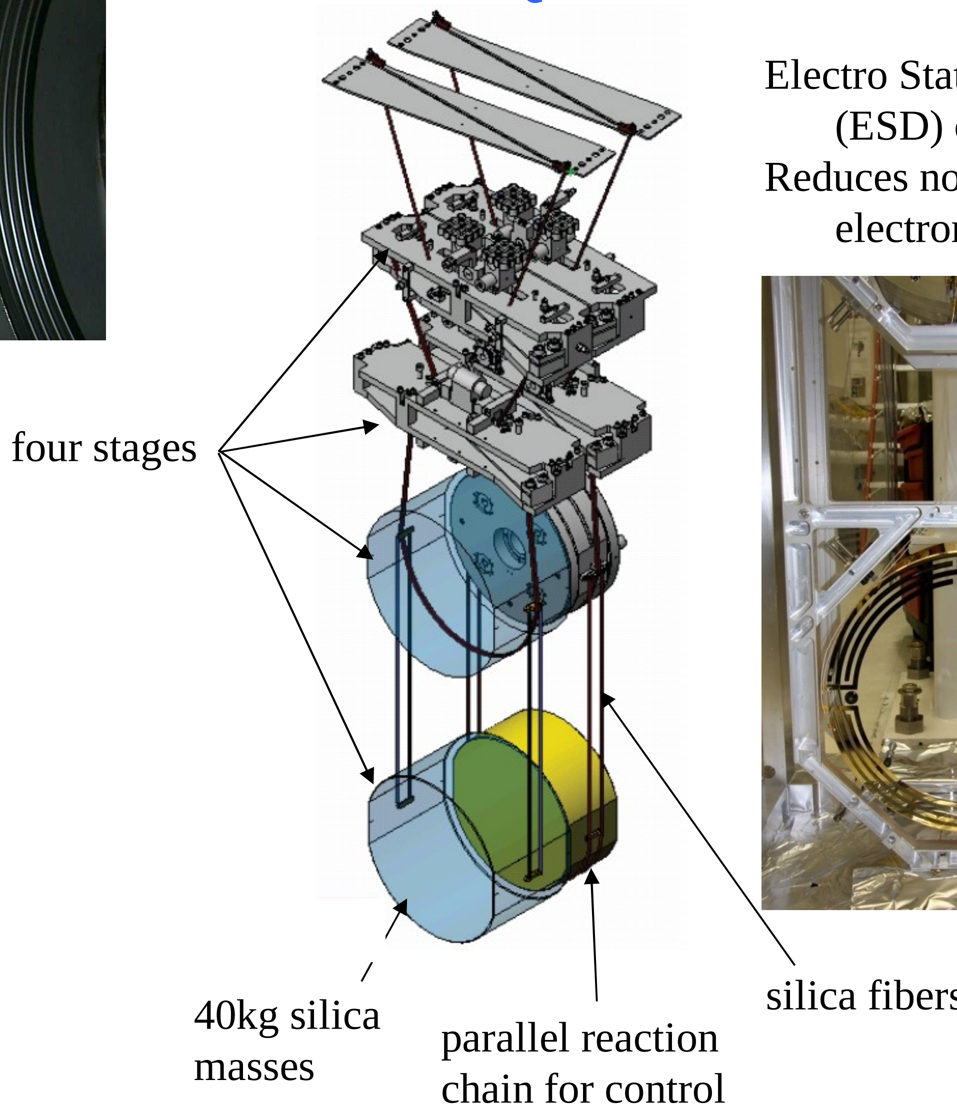




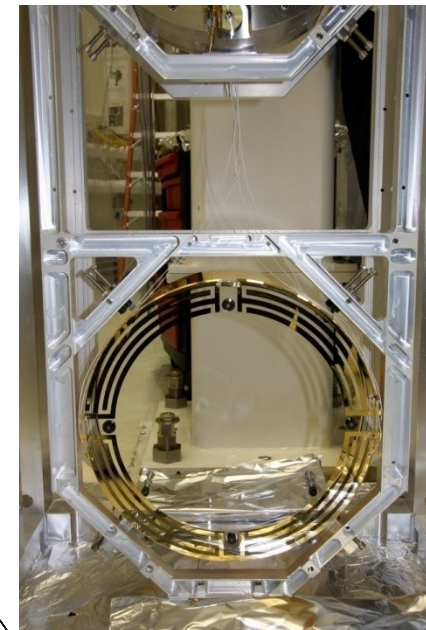


Suspension Systems
Initial Single vs. Advanced Quad Pendulum

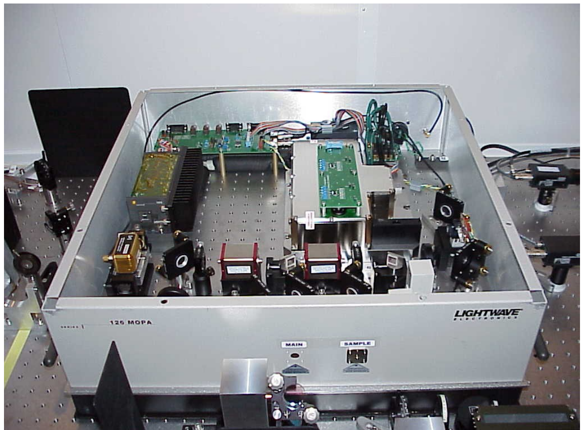


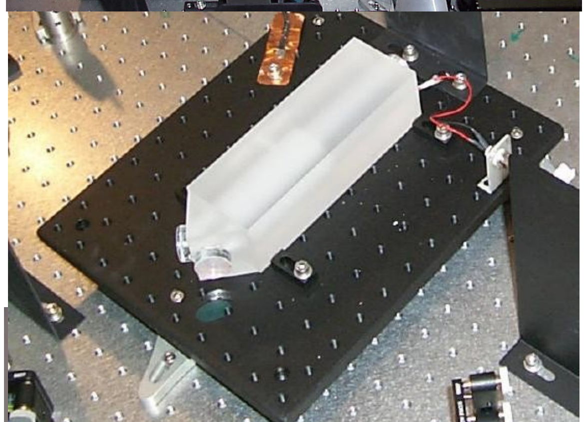


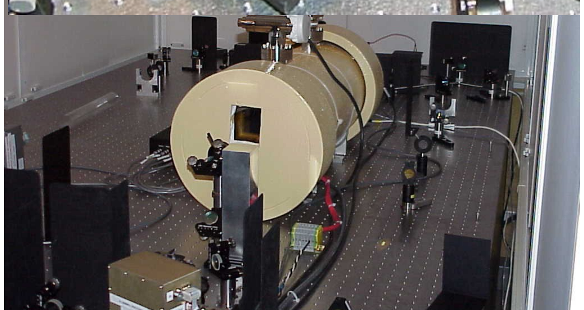
Electro Static Drive (ESD) on last stage:
Reduces noise from electromagnets



Nd:YAG Lasers: Initial LIGO 10W; Enhanced LIGO 35W; Advanced LIGO 150W



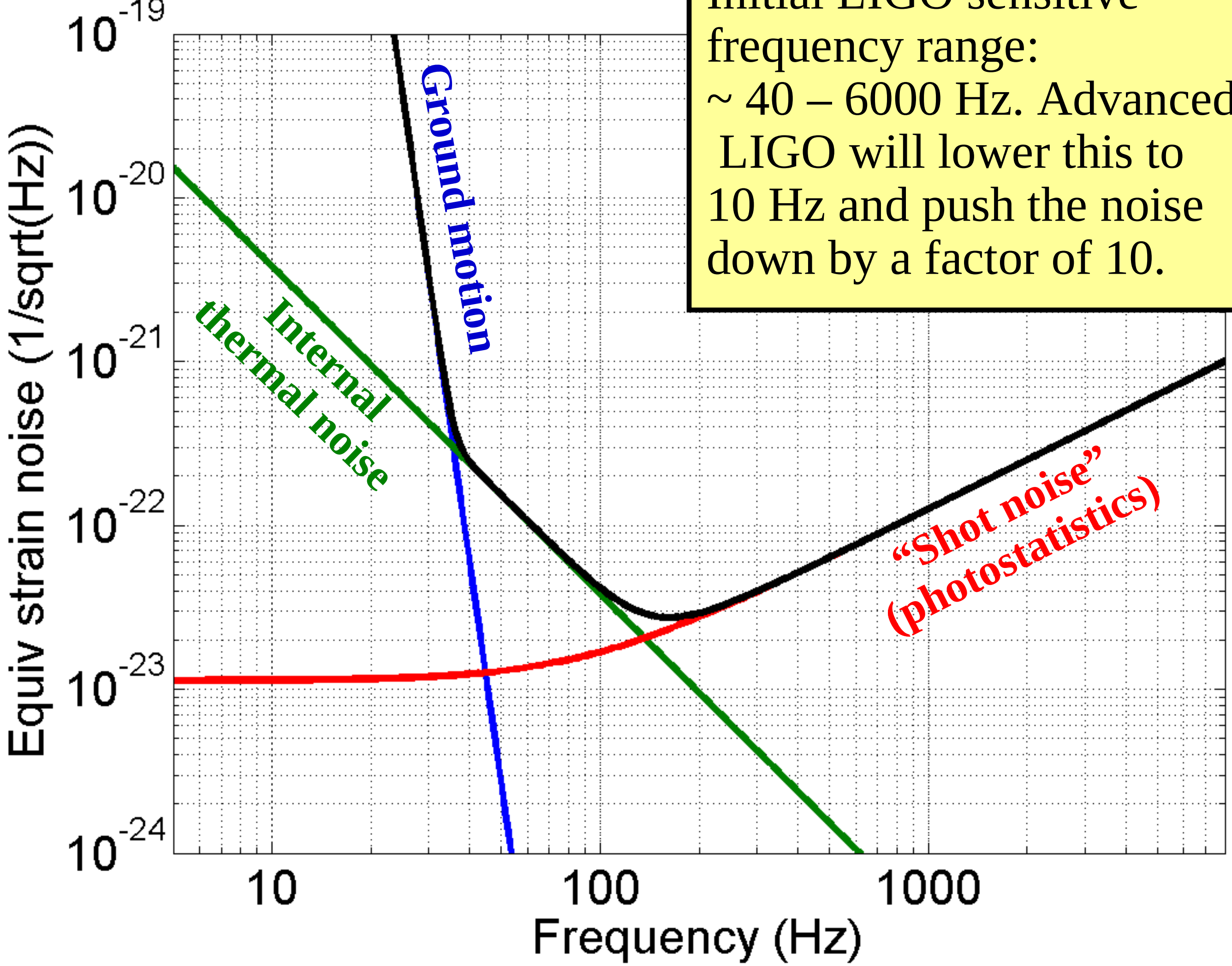




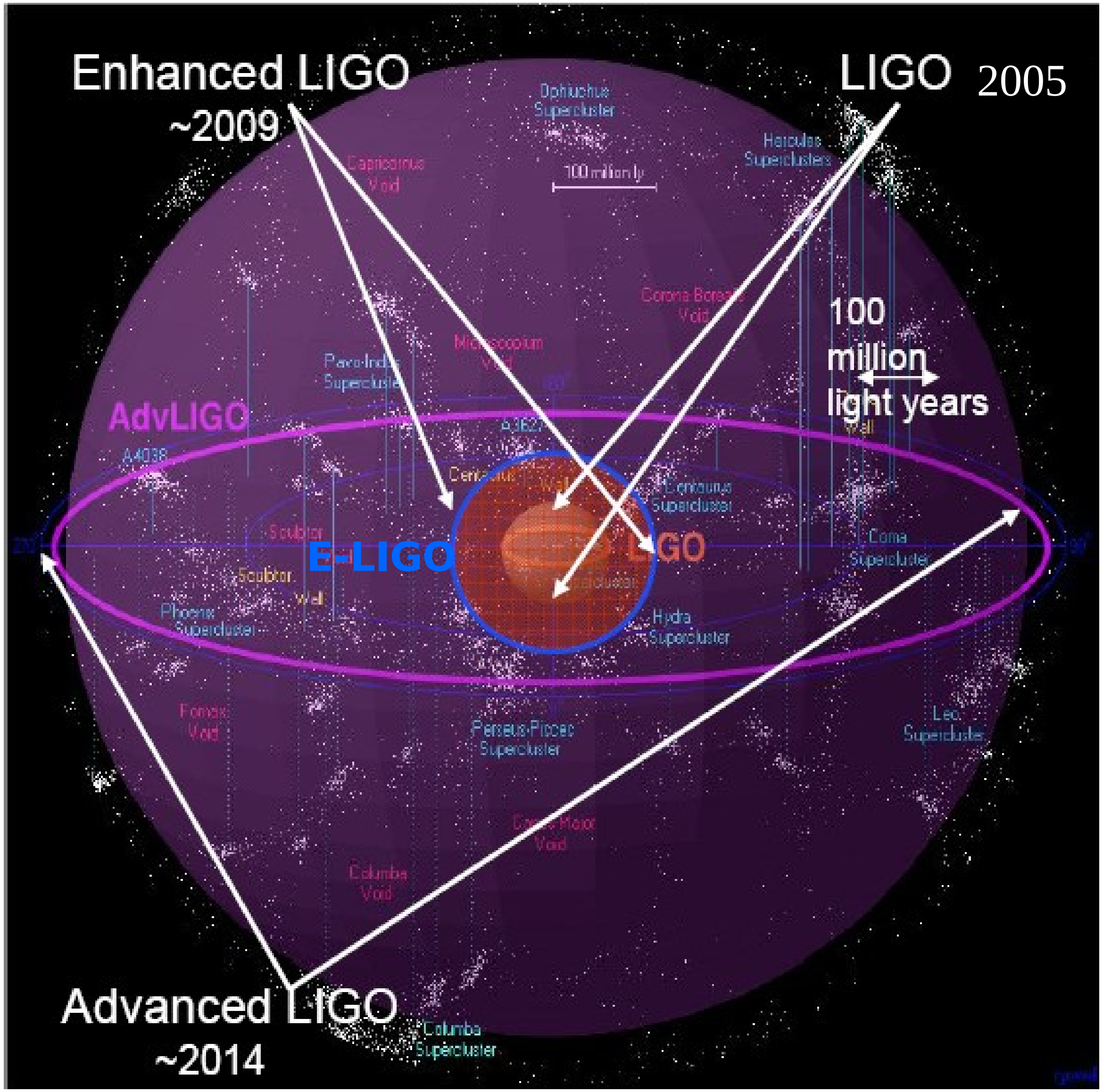
- Nd: YAG Neodymium-doped yttrium aluminum garnet.
- 1064 nanometers = infrared
- Stable to 1 part per million at 100 Hz.

Limiting Sources of Noise

Initial LIGO sensitive frequency range: ~ 40 – 6000 Hz. Advanced LIGO will lower this to 10 Hz and push the noise down by a factor of 10.



Reaching farther with Advance LIGO



Likely event rates per year:

- ~40 binary NS mergers
- ~10 NS-BH ??
- ~20 BH-BH ??

The LIGO Scientific Collaboration and Virgo Collaboration, Class. Quant. Grav. 27: 173001 (2010).

Other possible sources:

Intermediate-mass- ratio mergers to form a black hole.

Continuous signals from pulsars, low mass x-ray binaries, or unseen neutron stars.

Burst signals from supernovae (stellar core collapse) or cosmic strings.

Stochastic signal from the big bang or a population of sources.

The End

LIGO & Gravitational Waves

Gravitational waves carry information about the spacetime around black holes & other sources.

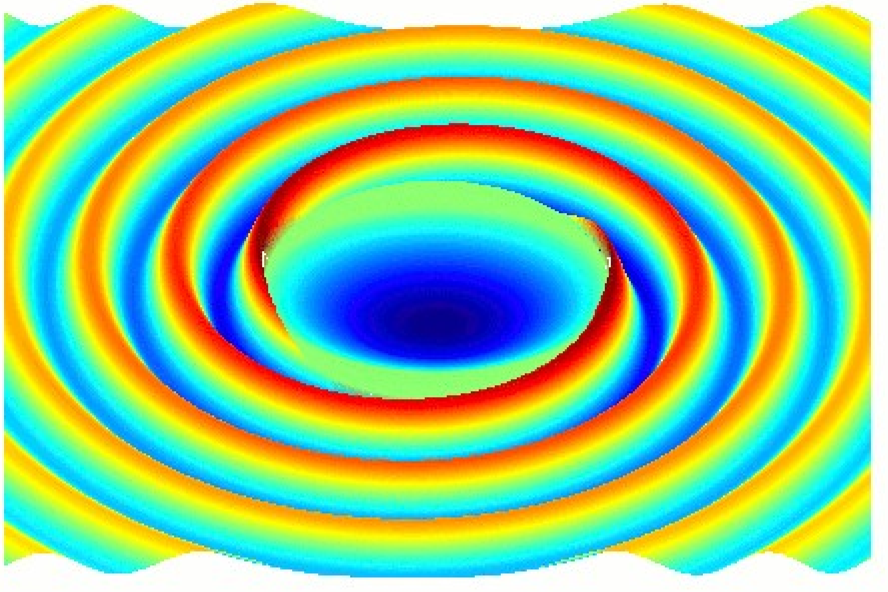
$$dT^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \bar{h}^{\mu\nu} = 0$$

$$h^{TT}_{\hat{\theta}\hat{\theta}} (\theta = \pi/2) \propto \frac{1}{r} \cos[2\pi f (t - r/c) + 2\phi]$$

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{2\pi i f (t - z/c)}$$



Detector Response

$$g_{\mu\upsilon}dx^{\mu}dx^{\upsilon}=0 \quad \text{(Light Travels On Null Geodesics)}$$

$$c^2dt^2-\begin{pmatrix}dx & 0 & 0\end{pmatrix}\begin{pmatrix}1+h_{xx} & h_{xy} & h_{xz}\\ h_{yx} & 1+h_{yy} & h_{yz}\\ h_{zx} & h_{zy} & 1+h_{zz}\end{pmatrix}\begin{pmatrix}dx\\ 0\\ 0\end{pmatrix}=0$$

$$c^2dt^2=(1+h_{xx})dx^2$$

$$c\int_0^{\Delta t}dt=\int_0^L\sqrt{1+h_{xx}}\,dx\cong\int_0^L\left(1+\frac{1}{2}h_{xx}\right)dx$$

$$c\Delta t=L_x=L+\frac{L}{2}h_{xx}$$

$$\frac{\Delta L}{L}=\frac{1}{2}(h_{xx}-h_{yy})=F_+(\theta,\phi,\psi)h_+(t)+F_\times(\theta,\phi,\psi)h_\times(t)$$

Black Holes After 1960

- Kruskal-Szekeres Coordinates, 1960
- Wormholes, Wheeler and Fuller, 1962
- Black Holes, popularized by Wheeler, 1968
- Penrose Process,1969
- Black Hole Evaporation, Hawking, 1974
- Time Machines, Morris and Thorne, 1988
- BH Information Theory?

Black Hole History

• Dark Stars, John Michell 1784 (Also Pierre-Simon Laplace, 1796)

• General Relativity, Einstein, 1915

• Spherically Symmetric Solution, Karl Schwarzschild, 1916

• Einstein-Rosen Bridge, 1935

LIGO

$E=mc^2$

$$c^2\Delta t^2 = c^2\Delta T^2 + v^2\Delta t^2$$

$$m^2c^4\Delta t^2 = m^2c^4\Delta T^2 + m^2c^2v^2\Delta t^2$$

$$m^2c^4\Delta t^2/\Delta T^2 = m^2c^4 + m^2c^2v^2\Delta t^2/\Delta T^2$$

$$[mc^2/(1-v^2/c^2)^{1/2}]^2 = [mc^2]^2 + [mv/(1-v^2/c^2)^{1/2}]^2c^2$$

$$[mc^2 + 1/2mv^2]^2 = E^2 = [mc^2]^2 + p^2c^2$$

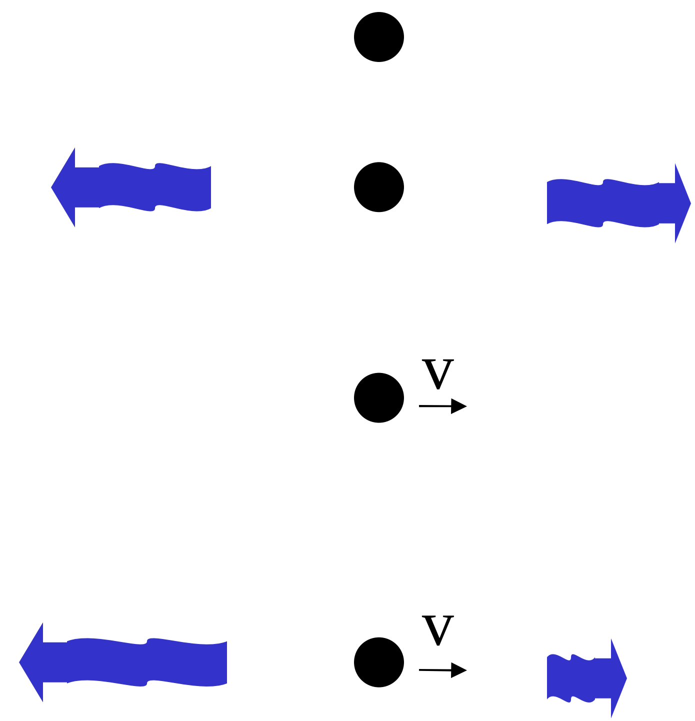
For v = 0: $E = mc^2$

Newtonian Momentum

Approximate to order $v^{2/}c^2$ == Newtonian Kinetic Energy

$E=mc^2$: What Einstein Said

"Does The Inertia Of A Body Depend Upon Its Energy-Content?" by A. Einstein, in "The Principle Of Relativity" translated by W. Perrett & G. B. Jeffery (Dover: 1952) from A. Einstein, Annalen der Physik, 17, 1905.



Consider a particle with energy U.

After it emits pulses of light with energy 0.5 E in opposite directions, the particles energy is H. Note that v does not change.

In a moving frame its energy is U + K1.

After it emits pulses of light with energy 0.5 γ E (1 +/- v/c) in opposite directions the particle's energy is H + K2. Note the relativistic blue/red shift factor is used.

By conservation of energy U = H + E and U + K1 = H + K2 + γ E. Thus, $(\gamma-1)E = K1 - K2 = \Delta K$ so $\frac{1}{2}(v^2/c^2)E = \frac{1}{2}(\Delta m)v^2$ to lowest order. The particle lost mass Δm. For Δm max. equal to m: $E = mc^2$.

LIGO

Einstein Wondered

- Einstein is famous for his thought experiments.
- In 1895, around age 16, he wondered, can we catch light?
- If yes, your image in a mirror would disappear. You would know your speed independent of any outside frame of reference. This would violate Galilean relativity.
- Einstein decides we cannot catch light; nothing can go faster than light.

The Speed of Light

c = 186,000 miles/s = 670,000,000 miles/hr

	v	Δt	ΔT
Car	60 mph	1 day	1 day - .35 nanoseconds
Plane	600 mph	1 day	1 day – 35 nanoseconds
Shuttle	17,000 mph	1 day	1 day – 28 microseconds
Voyager	38,000 mph	1 day	1 day – 140 microseconds
Andromeda	300,000 mph	1 day	1 day – 8.7 milliseconds
Electrons	99% c	1 day	3.4 hours

Photo: Stanford Linear Accelerator Center (SLAC); Public Domain

The faster you go the slower time goes!

Nothing can go faster than light!

Eddington Finkelstein Coordinates

If we introduce the following form of the Eddington Finkelstein time coordinate, t',

$t = t' - (2GM/c^2)\ln|rc^2/(2GM) - 1|$

outside the horizon, and

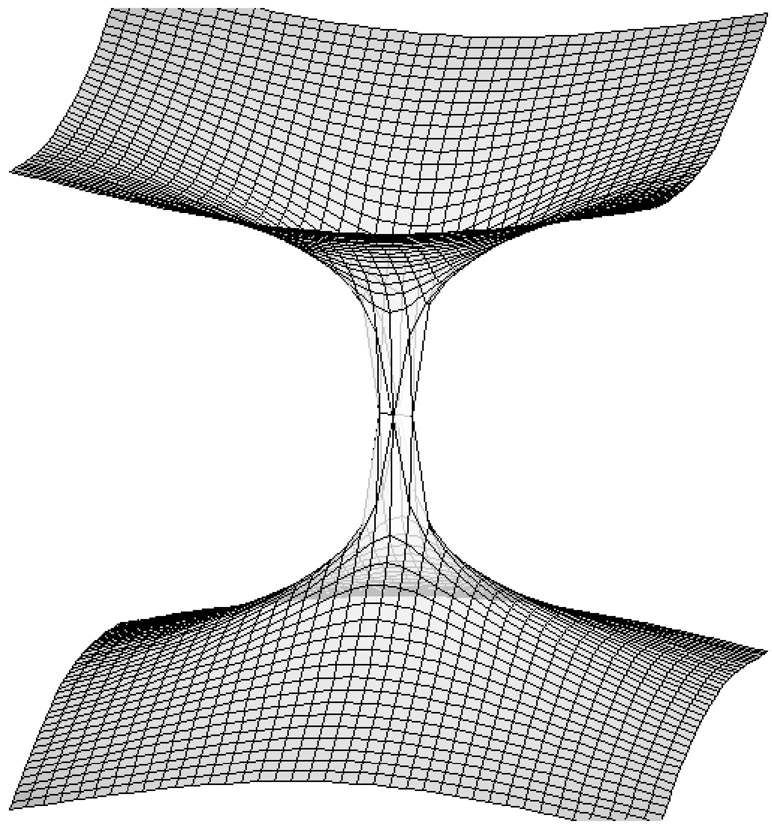
$t = t' - (2GM/c^2)\ln|1-rc^2/(2GM)|$

inside the horizon, then inside or outside, we get

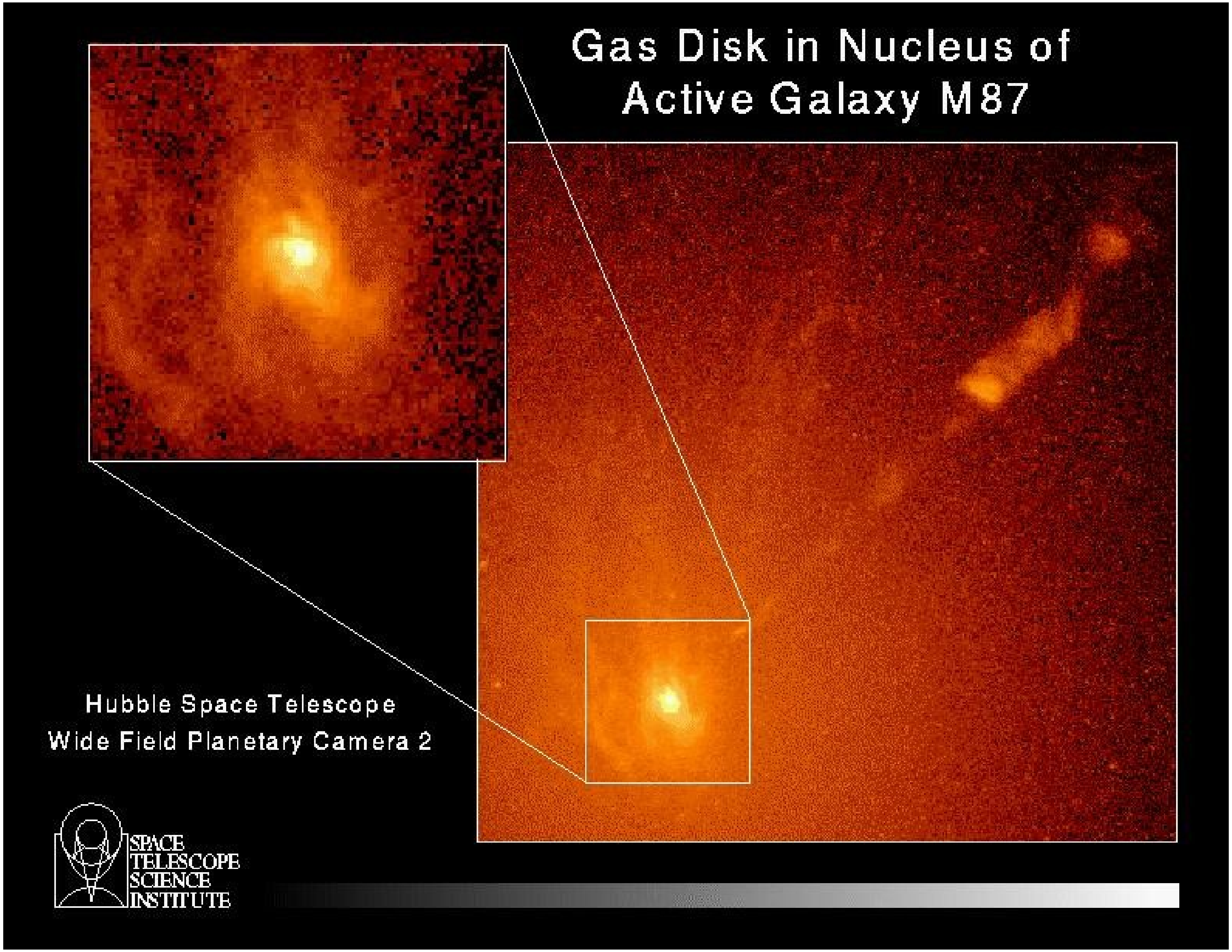
$ds^2 = -c^2[1-2GM/(rc^2)]dt'^2 + 4GM/(rc^2)dt'dr + [1+2GM/(rc^2)]dr^2 + r^2d\theta^2 + r^2\sin\theta^2 d\phi^2$.

Note that there is no coordinate singularity at the horizon.

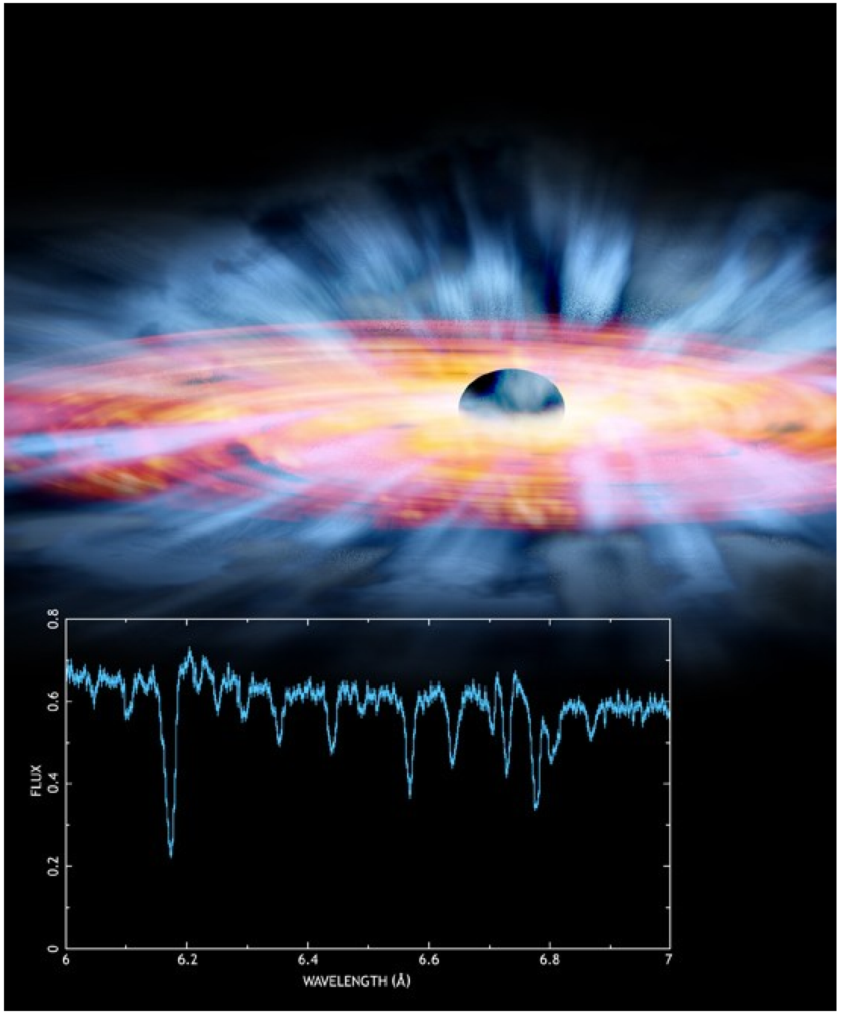
Schwarzschild Worm Hole



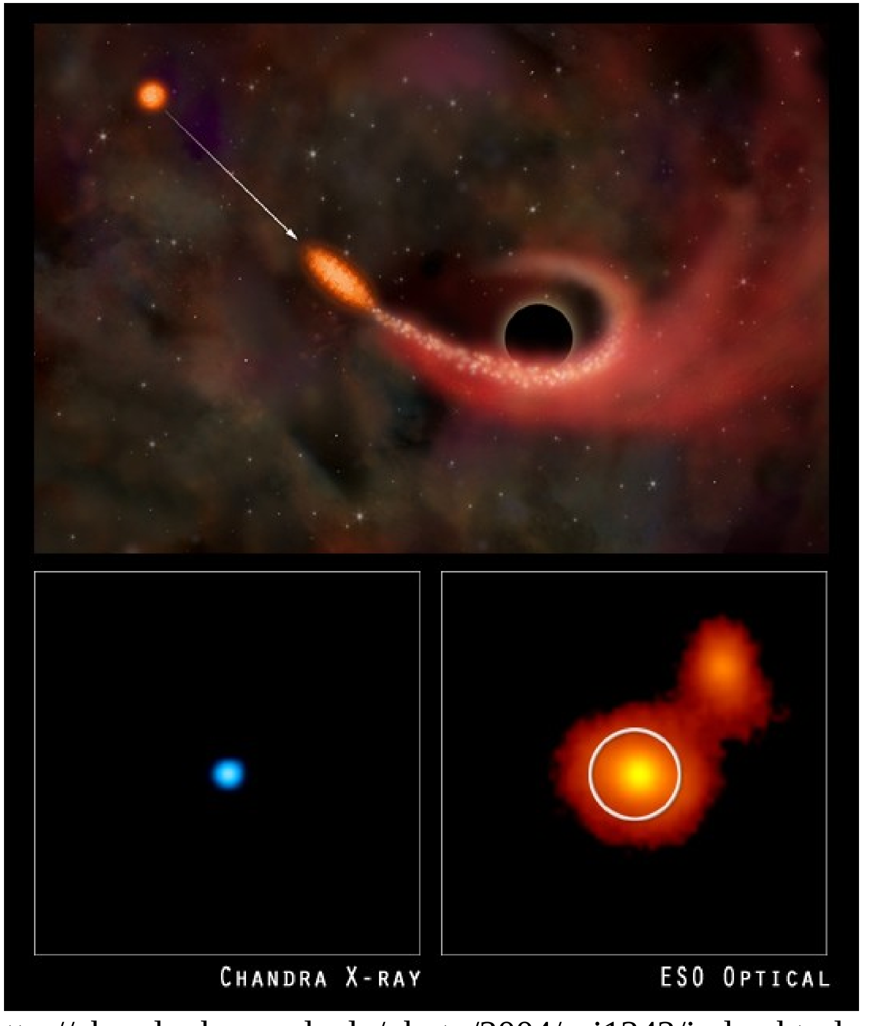
Black Hole Detection



Black Hole Detection

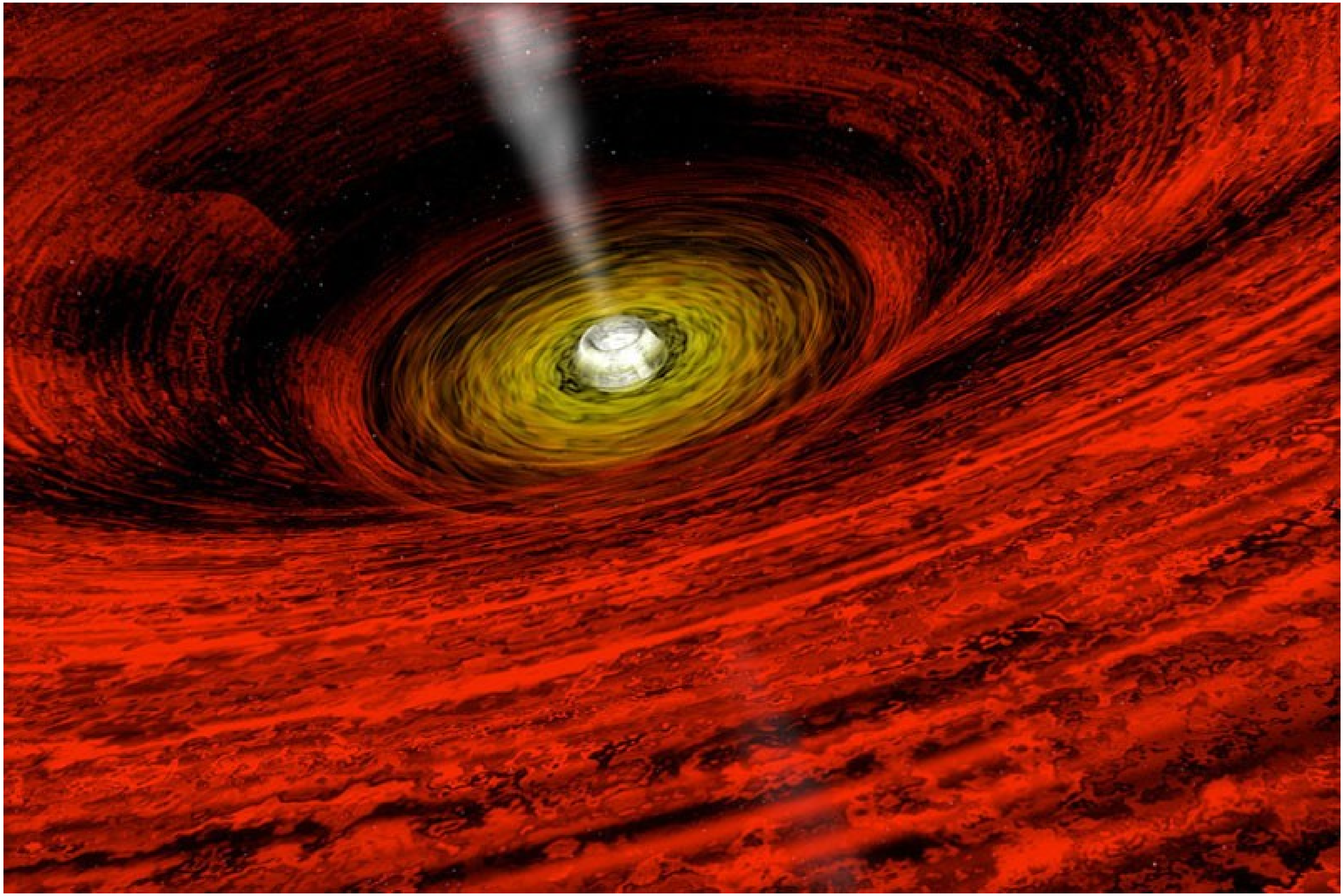


http://chandra.harvard.edu/photo/2006/j1655/; Credit: Illustration: NASA/CXC/M.Weiss; X-ray Spectrum: NASA/CXC/U.Michigan/J.Miller et al.



http://chandra.harvard.edu/photo/2004/rxj1242/index.html; Credit: Illustration: NASA/CXC/M.Weiss; X-ray: NASA/CXC/MPE/S.Komossa et al.; Optical: ESO/MPE/S.Komossa

Black Holes Detection



http://antwrp.gsfc.nasa.gov/apod/ap060528.html; GRO J1655-40: Evidence for a Spinning Black Hole; Drawing Credit: A. Hobart, CXC